

Pensieve header: Twisting the LT crossings.

```
In[*]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\USC-240205"];
```

```
In[*]:= Once[<< KnotTheory`];
```

Loading KnotTheory` version of October 29, 2024, 10:29:52.1301.
Read more at <http://katlas.org/wiki/KnotTheory>.

```
In[*]:=  $\theta[x_] /;$  NumericQ[x] := UnitStep[x]
```

```
In[*]:=  $\omega 2[v_][p_] :=$  Module[{q = Expand[p], n, c},  
  If[q ===  $\theta$ ,  $\theta$ , c = Coefficient[q,  $\omega$ , n = Exponent[q,  $\omega$ ]];  
   $c v^n + \omega 2[v][q - c (\omega + \omega^{-1})^n]$ ];
```

```
In[*]:= sign[ $\mathcal{E}_]$  := Module[{n, d, v, p, rs, e, k},  
  {n, d} = NumeratorDenominator[ $\mathcal{E}$ ];  
  {n, d} /=  $\omega^{\text{Exponent}[n, \omega] / 2 + \text{Exponent}[n, \omega, \text{Min}] / 2}$ ;  
  p = Factor[ $\omega 2[v] @ n * \omega 2[v] @ d /. v \rightarrow 4 u^2 - 2$ ];  
  rs = Solve[p ==  $\theta$ , u, Reals];  
  If[rs === {}, Sign[p /. u  $\rightarrow \theta$ ],  
  rs = Union@ (u /. rs);  
  Sign[ $(-1)^{e = \text{Exponent}[p, u]}$  Coefficient[p, u, e] + Sum[  
    k =  $\theta$ ; While[(d = RootReduce[ $\partial_{\{u, ++k\}} p /. u \rightarrow r$ ]) ==  $\theta$ ];  
    If[EvenQ[k],  $\theta$ , 2 Sign[d]] *  $\theta[u - r]$ ,  
    {r, rs}]  
  ]  
]
```

```
In[*]:= SetAttributes[B, Orderless];  
CF[b_B] := RotateLeft[#, First@Ordering[#] - 1] & /@ DeleteCases[b, {}]
```

```
In[*]:= CF[ $\mathcal{E}_]$  := Module[{ $\gamma$ s = Union@Cases[ $\mathcal{E}$ ,  $\gamma_ | \bar{\gamma}_$ ,  $\infty$ ]},  
  Total[CoefficientRules[ $\mathcal{E}$ ,  $\gamma$ s] /. (ps_  $\rightarrow$  c_)  $\Rightarrow$  Factor[c] Times @@  $\gamma$ sps ]
```

```
In[*]:= CF[{}] = {};  
CF[C_List] := Module[{ $\gamma$ s = Union@Cases[C,  $\gamma_$ ,  $\infty$ ],  $\gamma$ },  
  CF /@ DeleteCases[ $\theta$ ] [  
    RowReduce[Table[ $\partial_{\gamma} r$ , {r, C}, { $\gamma$ ,  $\gamma$ s}]] .  $\gamma$ s ]
```

```
In[*]:= ( $\mathcal{E}_$ )* :=  $\mathcal{E} /. \{\bar{\gamma} \rightarrow \gamma, \gamma \rightarrow \bar{\gamma}, \omega \rightarrow \omega^{-1}, c\_Complex \rightarrow c^*\}$ ;  
r_Rule* := {r, r*}
```

```
In[*]:= RulesOf[ $\gamma_i$  + rest_.] := ( $\gamma_i \rightarrow -rest$ )+;
CF[PQ[C_, q_]] := Module[{nC = CF[C]},
  PQ[nC, CF[q /. Union@@RulesOf /@nC]] ]
```

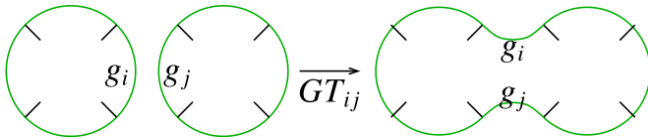
```
In[*]:= CF[ $\Sigma_b$ [ $\sigma$ _, pq_]] :=  $\Sigma_{CF[b]}$ [ $\sigma$ , CF[pq]]
```

Pretty-Printing.

```
In[*]:= Format[ $\Sigma_{b_B}$ [ $\sigma$ _, PQ[C_, q_]]] := Module[{ $\gamma_S$ },
   $\gamma_S$  =  $\gamma_{\#}$  & /@ Join@@b;
  Column[{TraditionalForm@ $\sigma$ ,
    TableForm[Join[
      Prepend[""] /@ Table[TraditionalForm[ $\partial_c r$ ], {r, C}, {c,  $\gamma_S$ }],
      {Prepend[""] [
        Join@@(b /. {l_, m___, r_} => {DisplayForm@RowBox[{"(", l}],
          m, DisplayForm@RowBox[{r, ")"}]}) /. i_Integer =>  $\gamma_i$  ]},
      MapThread[Prepend, {Table[TraditionalForm[ $\partial_{r,c} q$ ], {r,  $\gamma_S^*$ }, {c,  $\gamma_S$ }],  $\gamma_S^*$ }
    ], TableAlignments -> Center
  ], Center] ];
```

The Face-Centric Core.

```
In[*]:=  $\Sigma_{b1}$ [ $\sigma1$ _, PQ[C1_, q1_]]  $\oplus$   $\Sigma_{b2}$ [ $\sigma2$ _, PQ[C2_, q2_]] ^=
  CF@ $\Sigma_{Join[b1,b2]}$ [ $\sigma1 + \sigma2$ , PQ[C1  $\cup$  C2, q1 + q2]];
```



GT for Gap Touch:

```
In[*]:= GT_{i_,j_}@ $\Sigma_B$ [{li___,i_,ri___},{lj___,j_,rj___},bs___][ $\sigma$ _, PQ[C_, q_]] :=
  CF@ $\Sigma_B$ [{ri,li,j,rj,lj,i},bs][ $\sigma$ _, PQ[C  $\cup$  { $\gamma_i - \gamma_j$ }, q]]
```

Cordon:

```
In[*]:= Cordon_{i_}@ $\Sigma_B$ [{li___,i_,ri___},bs___][ $\sigma$ _, PQ[C_, q_]] :=
  Module[{ $\phi = \partial_{\gamma_i} C$ ,  $\lambda = \partial_{\gamma_i, \gamma_i} q$ , n $\sigma = \sigma$ , nC, nq, p},
    {p} = FirstPosition[ (# != 0) & /@  $\phi$ , True, {0}];
    {nC, nq} = Which[
      p > 0, {C, q} /. ( $\gamma_i \rightarrow -C[[p]] / \phi[[p]]$ )+ /. ( $\gamma_i \rightarrow \theta$ )+,
       $\lambda \neq 0$ , (n $\sigma$  += sign[ $\lambda$ ]; {C, q} /. ( $\gamma_i \rightarrow -(\partial_{\gamma_i} q) / \lambda$ )+ /. ( $\gamma_i \rightarrow \theta$ )+),
       $\lambda == 0$ , {C  $\cup$  { $\partial_{\gamma_i} q$ }, q} /. ( $\gamma_i \rightarrow \theta$ )+];
    CF@ $\Sigma_B$ [Most@{ri,li},bs][n $\sigma$ , PQ[nC, nq] /. ( $\gamma_{Last@{ri,li}} \rightarrow \gamma_{First@{ri,li}}$ )+] ]
```

Strand Operations. c for contract, mc for magnetic contract:

In[*]:= **LT@X**_{-1,2,3,-4}

Out[*]=

$$\begin{matrix}
 & & & & \theta & & & & \\
 & & & & & & & & \\
 & & & & & & & & \\
 \overline{\gamma}_{-4} & (\overline{\gamma}_{-4} & \overline{\gamma}_{-1} & \overline{\gamma}_2 & & \overline{\gamma}_3) & & & \\
 & \theta & 1 - \omega & \theta & & \omega - 1 & & & \\
 \overline{\gamma}_{-1} & \frac{\omega-1}{\omega} & \frac{a1 \omega + a3 \omega + \omega^2 - 2 \omega + 1}{\omega} & \omega - 1 & & -2 (\omega - 1) & & & \\
 \overline{\gamma}_2 & \theta & -\frac{\omega-1}{\omega} & \theta & & \frac{\omega-1}{\omega} & & & \\
 \overline{\gamma}_3 & -\frac{\omega-1}{\omega} & \frac{2 (\omega-1)}{\omega} & 1 - \omega & & -\frac{a1 \omega + a3 \omega - \omega^2 + 2 \omega - 1}{\omega} & & &
 \end{matrix}$$

Solve $\left[\frac{\omega \overline{a2} + \omega - 1}{\omega} == \theta, \overline{a2} \right]$

Out[*]=

$$\left\{ \left\{ \overline{a2} \rightarrow \frac{1 - \omega}{\omega} \right\} \right\}$$

In[*]:= **Simplify** $\left[\left(\frac{1 - \omega}{\omega} \right)^* \right]$

Out[*]=

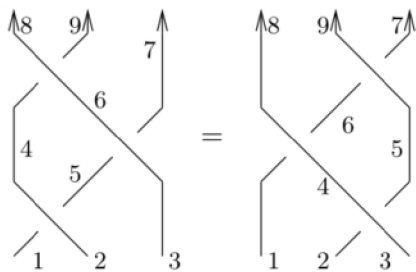
-1 + ω

Evaluation on Tangles and Knots.

```

In[*]:= LT[K_] := Fold[mc[#1@#2] &,  $\Sigma_B$ [{0, PQ[{ }, 0]}], List@@ (LT /@ PD@K) ] /.
   $\theta$ [c_ + u] /; Abs[c] ≥ 1 =>  $\theta$ [c];
LTSig[K_] := LT[K][[1]]
    
```

Reidemeister 3



In[*]:= **R3L** = **PD**[**X**_{-2,5,4,-1}, **X**_{-3,7,6,-5}, **X**_{-6,9,8,-4}];
R3R = **PD**[**X**_{-3,5,4,-2}, **X**_{-4,6,8,-1}, **X**_{-5,7,9,-6}];
LT@**R3L** == **LT**@**R3R**

Out[*]=

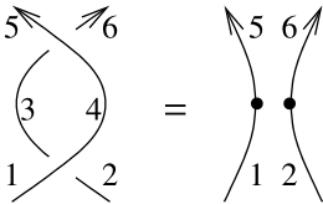
True

In[*]:= **LT@R3L**

Out[*]=

	$(\gamma_{-3}$	γ_7	γ_9	γ_8	γ_{-1}	γ_{-2}
$\bar{\gamma}_{-3}$	$\frac{a1 \omega + a3 \omega + \omega^2 + 1}{\omega}$	$2 (\omega - 1)$	-2ω	2	0	-2
$\bar{\gamma}_7$	$-\frac{2 (\omega - 1)}{\omega}$	0	$\frac{2 (\omega - 1)}{\omega}$	0	0	0
$\bar{\gamma}_9$	$-\frac{2}{\omega}$	$-2 (\omega - 1)$	$-\frac{a1 \omega + a3 \omega - \omega^2 - 1}{\omega}$	$-\frac{2}{\omega}$	0	$\frac{2}{\omega}$
$\bar{\gamma}_8$	2	0	-2ω	$-\frac{a1 \omega + a3 \omega - \omega^2 - 1}{\omega}$	0	$-\frac{2}{\omega}$
$\bar{\gamma}_{-1}$	0	0	0	0	0	0
$\bar{\gamma}_{-2}$	-2	0	2ω	-2ω	0	$\frac{a1 \omega + a3 \omega + \omega^2 + 1}{\omega}$

Reidemeister 2b



In[*]:= **LT@PD**[**X**_{-2,4,3,-1}, **X̄**_{-4,6,5,-3}]

Out[*]=

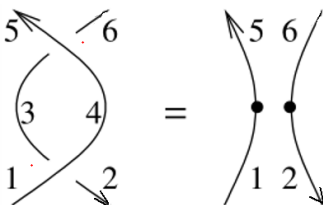
		0		
	1	0	-1	0
	$(\gamma_{-2}$	γ_6	γ_5	γ_{-1})
$\bar{\gamma}_{-2}$	0	0	0	0
$\bar{\gamma}_6$	0	0	0	0
$\bar{\gamma}_5$	0	0	0	0
$\bar{\gamma}_{-1}$	0	0	0	0

In[*]:= **LT@PD**[**X**_{-2,4,3,-1}, **X̄**_{-4,6,5,-3}] == **GT**_{5,-2}@**LT@PD**[**P**_{-1,5}, **P**_{-2,6}]

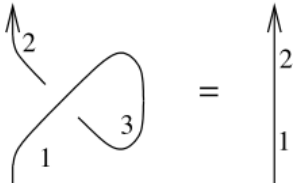
Out[*]=

True

Reidemeister 2c



Reidemeister 1

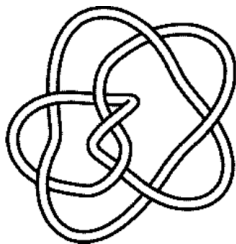


In[*]:= `LT@PD[X-3,3,2,-1] == LT@P-1,2`

Out[*]=

True

A Knot



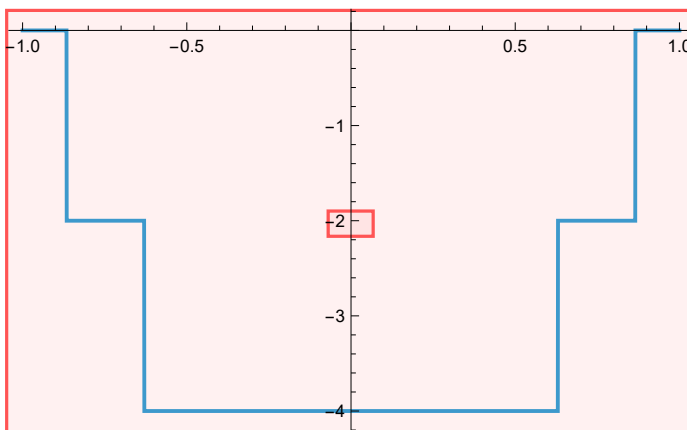
In[*]:= `f = LTSig[Knot[8, 5]]`

Out[*]=

$$2\theta\left[-\frac{\sqrt{3}}{2} + u\right] - 2\theta\left[\frac{\sqrt{3}}{2} + u\right] - 2\theta\left[u - \sqrt[4]{-0.630\dots}\right] + 2\theta\left[u - \sqrt[4]{0.630\dots}\right]$$

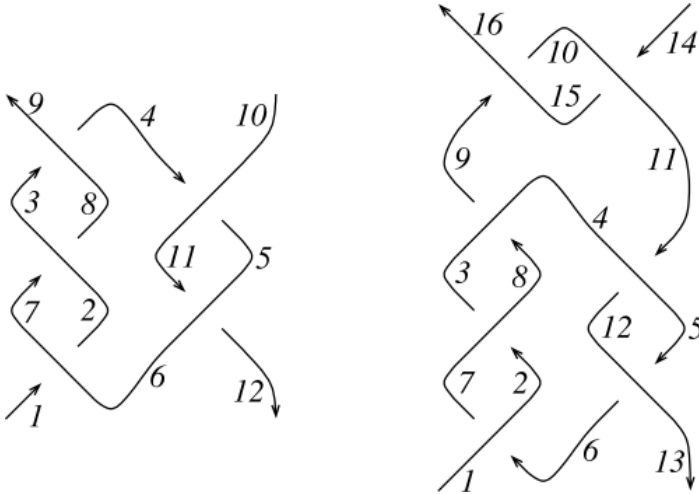
In[*]:= `Plot[f, {u, -1, 1}]`

Out[*]=



Some Tangles

The Conway-Kinoshita-Terasaka Tangles.



$$In[*]:= T1 = PD[\bar{X}_{-6,2,7,-1}, \bar{X}_{-2,8,3,-7}, \bar{X}_{-8,4,9,-3}, X_{-11,6,12,-5}, X_{-4,11,5,-10}];$$

$$T2 = PD[X_{-6,2,7,-1}, X_{-2,8,3,-7}, X_{-8,4,9,-3}, \bar{X}_{-12,6,13,-5}, \bar{X}_{-4,12,5,-11}, \bar{X}_{-10,15,11,-14}, \bar{X}_{-15,10,16,-9}];$$

$$In[*]:= LT[T1]$$

Out[*]=

$$-2 \theta \left(u - \frac{\sqrt{3}}{2} \right) + 2 \theta \left(u + \frac{\sqrt{3}}{2} \right) - 1$$

	Υ_{-10}	Υ_9	Υ_{-1}	Υ_{12}
$\bar{\Upsilon}_{-10}$	θ	$-2(\omega - 1)$	θ	$2(\omega - 1)$
$\bar{\Upsilon}_9$	$\frac{2(\omega - 1)}{\omega}$	$\frac{2\omega}{\omega^2 - \omega + 1}$	$-\frac{2(\omega - 1)}{\omega}$	$-\frac{2\omega}{\omega^2 - \omega + 1}$
$\bar{\Upsilon}_{-1}$	θ	$2(\omega - 1)$	θ	$-2(\omega - 1)$
$\bar{\Upsilon}_{12}$	$-\frac{2(\omega - 1)}{\omega}$	$-\frac{2\omega}{\omega^2 - \omega + 1}$	$\frac{2(\omega - 1)}{\omega}$	$\frac{2\omega}{\omega^2 - \omega + 1}$

$$In[*]:= LT[T2]$$

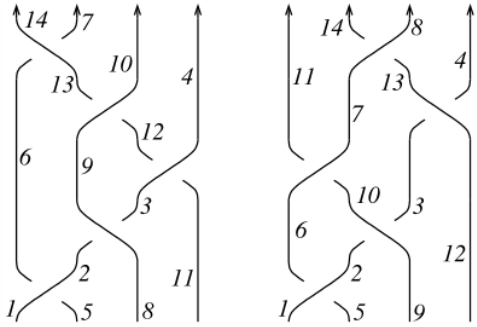
Out[*]=

$$\theta$$

	Υ_{-14}	Υ_{16}	Υ_{-1}	Υ_{13}
$\bar{\Upsilon}_{-14}$	θ	$-2(\omega - 1)$	θ	$2(\omega - 1)$
$\bar{\Upsilon}_{16}$	$\frac{2(\omega - 1)}{\omega}$	$-\frac{2(\omega - 1)^2 \omega}{\omega^4 - 3\omega^3 + 5\omega^2 - 3\omega + 1}$	$-\frac{2(\omega - 1)}{\omega}$	$\frac{2(\omega - 1)^2 \omega}{\omega^4 - 3\omega^3 + 5\omega^2 - 3\omega + 1}$
$\bar{\Upsilon}_{-1}$	θ	$2(\omega - 1)$	θ	$-2(\omega - 1)$
$\bar{\Upsilon}_{13}$	$-\frac{2(\omega - 1)}{\omega}$	$\frac{2(\omega - 1)^2 \omega}{\omega^4 - 3\omega^3 + 5\omega^2 - 3\omega + 1}$	$\frac{2(\omega - 1)}{\omega}$	$-\frac{2(\omega - 1)^2 \omega}{\omega^4 - 3\omega^3 + 5\omega^2 - 3\omega + 1}$

Some Braids

Examples with non-trivial codimension.



```
In[*]:= PD[X[5, 2, 6, 1], X[2, 9, 3, 10], X[10, 7, 11, 6], X[3, 12, 4, 13], X[13, 8, 14, 7]] /.
x : X[i_, j_, k_, l_] => If[PositiveQ@x, X[-i, j, k, -l], X[-j, k, l, -i]]
```

```
Out[*]= PD[X-5,2,6,-1, X̄-9,3,10,-2, X-10,7,11,-6, X̄-12,4,13,-3, X-13,8,14,-7]
```

```
In[*]:= B1 = PD[X-5,2,6,-1, X̄-8,3,9,-2, X-11,4,12,-3, X-12,10,13,-9, X̄-13,7,14,-6];
B2 = PD[X-5,2,6,-1, X̄-9,3,10,-2, X-10,7,11,-6, X̄-12,4,13,-3, X-13,8,14,-7];
```

```
In[*]:= LT[B1]
```

```
Out[*]=
```

					0		
	1	0	-1	0	0	$\frac{1}{\omega}$	0
	0	0	0	-1	0	$\frac{1}{\omega}$	0
	(γ_{-11}	γ_4	γ_{10}	γ_7	γ_{14}	γ_{-1}	
$\bar{\gamma}_{-11}$	0	0	0	0	0	0	0
$\bar{\gamma}_4$	0	0	0	0	0	0	0
$\bar{\gamma}_{10}$	0	0	0	0	0	$-\frac{a1 \omega + a3 \omega + \omega^2 - 1}{\omega^2}$	0
$\bar{\gamma}_7$	0	0	0	0	0	$-\frac{a1 \omega - a3 \omega + \omega^2 - 2 \omega + 1}{\omega^2}$	0
$\bar{\gamma}_{14}$	0	0	$-a1 \omega - a3 \omega + \omega^2 - 1$	$-a1 \omega - a3 \omega + \omega^2 - 2 \omega + 1$	$\frac{a1 \omega + a3 \omega - \omega^2 + 2 \omega - 1}{\omega}$	$-\frac{2 (\omega - 1)}{\omega}$	
$\bar{\gamma}_{-1}$	0	0	0	0	$2 (\omega - 1)$	0	0
$\bar{\gamma}_{-5}$	0	0	$a1 \omega + a3 \omega - \omega^2 + 1$	$a1 \omega + a3 \omega - \omega^2 + 2 \omega - 1$	$-2 (a1 + a3)$	$\frac{2 (\omega - 1)}{\omega}$	
$\bar{\gamma}_{-8}$	0	0	0	0	0	0	0

```
In[*]:= LT[B2]
```

```
Out[*]=
```

					0		
	(γ_{-12}	γ_4	γ_8	γ_{14}	γ_{11}	γ_{-1}	
$\bar{\gamma}_{-12}$	$\frac{a1 \omega + a3 \omega + \omega^2 - 2 \omega + 1}{\omega}$	0	$-2 (\omega - 1)$	$\frac{2 (\omega - 1)^2}{\omega}$	$\frac{2 (\omega - 1)}{\omega^2}$	0	0
$\bar{\gamma}_4$	0	0	0	0	0	0	0
$\bar{\gamma}_8$	$\frac{2 (\omega - 1)}{\omega}$	0	$-\frac{a1 \omega + a3 \omega - \omega^2 + 2 \omega - 1}{\omega}$	$-\frac{2 (\omega - 1)^2}{\omega}$	$-\frac{2 (\omega - 1)}{\omega^2}$	0	0
$\bar{\gamma}_{14}$	$\frac{2 (\omega - 1)^2}{\omega}$	0	$-\frac{2 (\omega - 1)^2}{\omega}$	$-\frac{a1 \omega + a3 \omega - 3 \omega^2 + 6 \omega - 3}{\omega}$	$-\frac{2 (\omega - 1)^2}{\omega^2}$	0	0
$\bar{\gamma}_{11}$	$-2 (\omega - 1) \omega$	0	$2 (\omega - 1) \omega$	$-2 (\omega - 1)^2$	$-\frac{a1 \omega + a3 \omega - \omega^2 + 2 \omega - 1}{\omega}$	$-\frac{2 (\omega - 1)}{\omega}$	
$\bar{\gamma}_{-1}$	0	0	0	0	$2 (\omega - 1)$	0	0
$\bar{\gamma}_{-5}$	$2 (\omega - 1) \omega$	0	$-2 (\omega - 1) \omega$	$2 (\omega - 1) \omega$	$-2 (\omega - 1)$	$\frac{2 (\omega - 1)}{\omega}$	
$\bar{\gamma}_{-9}$	$-\frac{2 (\omega - 1) (2 \omega - 1)}{\omega}$	0	$\frac{2 (\omega - 1) (2 \omega - 1)}{\omega}$	$-\frac{2 (\omega - 1) (2 \omega - 1)}{\omega}$	$\frac{2 (\omega - 1)^2}{\omega^2}$	0	0