

$$\Delta = -\hbar \eta_i \xi_j c_k + (\xi_i + \xi_j) x_k + (\eta_i + \eta_j) y_k;$$

Simplify@With[{ \mathbb{E} = MatrixExp},

$$\mathbb{E}[\xi_i \rho_X] \cdot \mathbb{E}[\eta_i \rho_Y] \cdot \mathbb{E}[\xi_j \rho_X] \cdot \mathbb{E}[\eta_j \rho_Y] ==$$

$$\mathbb{E}[\partial_{x_k} \Delta \rho_X] \cdot \mathbb{E}[\partial_{y_k} \Delta \rho_Y] \cdot \mathbb{E}[\partial_{c_k} \Delta \rho_C]$$