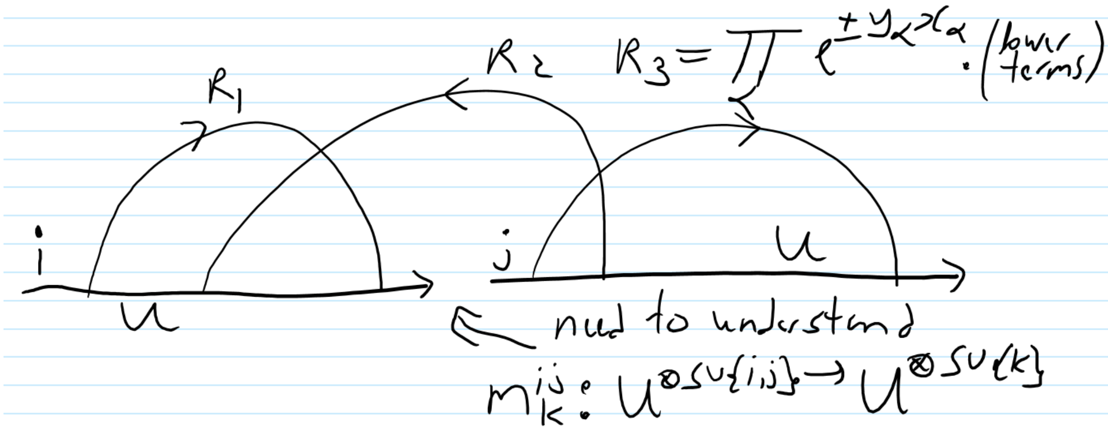
Dror Bar-Natan: Talks: Sydney-1708:

The Dogma is Wrong - Extra Details

Goal



Agenda

1. Quantizing and de-quantizing sl_2^ϵ.
2. Some understanding of sl_2^ϵ.
3. A full understanding of sl_2^ϵ at $\epsilon = 0$.
4. A full understanding of sl_2^ϵ at $\epsilon^2 = 0$.
5. Pushforwards of distributions, 0-dimensional QFT, Feynman diagrams and what had really happened here.

Some Shortcuts

```
ME[x_] := MatrixExp[x]; MB[x_, y_] := x.y - y.x; MF[x_] := MatrixForm[x];
```

Representing $\mathfrak{g}^\epsilon = \langle h, e, l, f\rangle / ([e, l] = -e,\ [f, l] = f,\ [e, f] = h - 2\epsilon l,\ [h, *] = 0)$

```
ρh = (-1  0
       0 -1); ρe = ( 0  0
                    -ϵ  0); ρl = (-(1 + 1/ϵ)/2        0
                                      0        (1 - 1/ϵ)/2); ρf = (0 1
                                                                   0 0);
Simplify@{MB[ρe, ρl] == -ρe, MB[ρf, ρl] == ρf, MB[ρe, ρf] == ρh - 2 ϵ ρl}
```

```
{True, True, True}
```

The Main $\mathfrak{g}_0 := \mathfrak{g}^\epsilon / (\epsilon = 0)$ Theorem.

The $\mathfrak{g}_0$ invariant of any S-component tangle T can be written in the form $Z(T) = \mathbb{O}\left(\omega e^{L+Q} \mid \prod_{i\in S} e_i l_i f_i\right)$, where ω is a scalar (meaning, a rational function in the variables h_i and their exponentials $t_i = e^{h_i}$), where $L = \sum a_{ij} h_i l_j$ is a balanced quadratic in the variables h_i and l_j with integer coefficients a_{ij} and where $Q = \sum b_{ij} e_i f_j$ is a balanced quadratic in the variables e_i and f_j with scalar coefficients b_{ij}. Furthermore, after setting $h_i = h$ and $t_i = t$ for all i, the invariant $Z(T)$ is poly-time computable.

Proof. Indeed, as we shall see, the following lemmas hold, and the rest is straight-forward.

Lemma 0. $R^s = e^{s(h\otimes l + e\otimes f)} = \mathbb{O}\left(\exp\left(s\,\mathrm{hl} + \frac{e^{sh}-1}{h}\,\mathrm{ef} \mid e\otimes \mathrm{lf}\right)\right)$.

Lemma 1. $\mathbb{O}\left(e^{\gamma \mathrm{l} + \beta \mathrm{e}} \mid \mathrm{le}\right) = \mathbb{O}\left(e^{\gamma \mathrm{l} + e^{\gamma} \beta \mathrm{e}} \mid \mathrm{el}\right)$.

Lemma 2. $\mathbb{O}\left(e^{\gamma \mathrm{l} + \beta \mathrm{f}} \mid \mathrm{fl}\right) = \mathbb{O}\left(e^{\gamma \mathrm{l} + e^{\gamma} \beta \mathrm{f}} \mid \mathrm{lf}\right)$.

Lemma 3. $\mathbb{O}\left(e^{\beta \mathrm{e} + \alpha \mathrm{f} + \delta \mathrm{ef}} \mid \mathrm{fe}\right) = \mathbb{O}\left(\nu\, e^{\nu(-\alpha\beta h + \beta e + \alpha f + \delta e f)} \mid e\, f\right)$, with $\nu = (1 + \mathrm{h}\delta)^{-1}$.

Some $\mathfrak{g}^\epsilon$ lemmas

Lemma 1. $\mathbb{O}\left(e^{\gamma l + \beta e} \mid l\, e\right) = \mathbb{O}\left(e^{\gamma l + e^{\gamma} \beta e} \mid e\, l\right)$.

Lemma 2. $\mathbb{O}\left(e^{\gamma l + \beta f} \mid f\, l\right) = \mathbb{O}\left(e^{\gamma l + e^{\gamma} \beta f} \mid l\, f\right)$.

Proofs.

```
MF /@ {ME[γ ρl].ME[β ρe], ME[e^γ β ρe].ME[γ ρl]}
```

$$\left\{\begin{pmatrix} e^{-\frac{\gamma}{2}-\frac{\gamma}{2\epsilon}} & 0 \\ -e^{\frac{\gamma}{2}-\frac{\gamma}{2\epsilon}}\beta\epsilon & e^{\frac{\gamma}{2}-\frac{\gamma}{2\epsilon}} \end{pmatrix}, \begin{pmatrix} e^{-\frac{\gamma}{2}-\frac{\gamma}{2\epsilon}} & 0 \\ -e^{\frac{\gamma}{2}-\frac{\gamma}{2\epsilon}}\beta\epsilon & e^{\frac{\gamma}{2}-\frac{\gamma}{2\epsilon}} \end{pmatrix}\right\}$$

```
MF /@ {ME[β ρf].ME[γ ρl], ME[γ ρl].ME[e^γ β ρf]}
```

$$\left\{\begin{pmatrix} e^{-\frac{\gamma}{2}-\frac{\gamma}{2\epsilon}} & e^{\frac{\gamma}{2}-\frac{\gamma}{2\epsilon}}\beta \\ 0 & e^{\frac{\gamma}{2}-\frac{\gamma}{2\epsilon}} \end{pmatrix}, \begin{pmatrix} e^{-\frac{\gamma}{2}-\frac{\gamma}{2\epsilon}} & e^{\frac{\gamma}{2}-\frac{\gamma}{2\epsilon}}\beta \\ 0 & e^{\frac{\gamma}{2}-\frac{\gamma}{2\epsilon}} \end{pmatrix}\right\}$$

Lemma 3 at $\delta = 0$. $\mathbb{O}\left(e^{\alpha f + \beta e} \mid f\, e\right) = \mathbb{O}\left(e^{c h + a e - 2\epsilon c l + b f} \mid e\, l\, f\right)$, with $\left\{a \to -\frac{\beta}{-1+\alpha\beta\epsilon},\ b \to -\frac{\alpha}{-1+\alpha\beta\epsilon},\ c \to \frac{\mathrm{Log}[1-\alpha\beta\epsilon]}{\epsilon}\right\}$.

Derivation.

```
ME[α ρf].ME[β ρe] // Simplify // MF
```

$$\begin{pmatrix} 1-\alpha\beta\epsilon & \alpha \\ -\beta\epsilon & 1 \end{pmatrix}$$

```
eqn = ME[α ρf].ME[β ρe] == ME[a ρe].ME[c (ρh - 2 ϵ ρl)].ME[b ρf]
```

$$\{\{1-\alpha\beta\epsilon, \alpha\}, \{-\beta\epsilon, 1\}\} == \left\{\left\{e^{c\epsilon}, b\, e^{c\epsilon}\right\}, \left\{-a\, e^{c\epsilon}\epsilon, e^{-c\epsilon} - a\, b\, e^{c\epsilon}\epsilon\right\}\right\}$$

```
sol = Solve[Thread[Flatten /@ eqn], {a, b, c}][[1]]
```

$$\left\{a \to -\frac{\beta}{-1+\alpha\beta\epsilon},\ b \to -\frac{\alpha}{-1+\alpha\beta\epsilon},\ c \to \text{ConditionalExpression}\left[\frac{2\,i\,\pi\,C[1] + \text{Log}[1-\alpha\beta\epsilon]}{\epsilon},\ C[1] \in \text{Integers}\right]\right\}$$

```
sol = sol /. C[1] → 0
```

$$\left\{a \to -\frac{\beta}{-1+\alpha\beta\epsilon},\ b \to -\frac{\alpha}{-1+\alpha\beta\epsilon},\ c \to \frac{\text{Log}[1-\alpha\beta\epsilon]}{\epsilon}\right\}$$

Lemma 3 for $\mathfrak{g}_0$.

```
Limit[{a, b, c} /. sol, ϵ → 0]
```

$\{\beta, \alpha, -\alpha\beta\}$

And so in $\mathfrak{g}_0$, $\mathbb{O}\left(e^{\alpha f+\beta e} \mid f e\right) = \mathbb{O}\left(e^{\alpha f+\beta e-\alpha\beta h} \mid e\,l\,f\right)$. Hence $\mathbb{O}\left(e^{\alpha f+\beta e+\delta e f} \mid f e\right) = e^{\delta\partial_\alpha\partial_\beta}\,\mathbb{O}\left(e^{\alpha f+\beta e} \mid f e\right) = e^{\delta\partial_\alpha\partial_\beta}\,\mathbb{O}\left(e^{\alpha f+\beta e-\alpha\beta h} \mid e\,l\,f\right) = \mathbb{O}(\psi \mid e\,l\,f)$, where $\psi = e^{\delta\partial_\alpha\partial_\beta}\,e^{\alpha f+\beta e-\alpha\beta h}$ satisfies $\psi_{\delta=0} = e^{\alpha f+\beta e-\alpha\beta h}$ and $\partial_\delta\psi = \partial_{\alpha,\beta}\psi$.

```
With[{ψ = ν e^(ν (δ e f - α β h + α f + β e)) /. ν → (1 + δ h)^-1}, Simplify@{∂_δ ψ - ∂_{α,β} ψ, ψ /. δ → 0}]
```

$\{0,\ e^{f\alpha+e\beta-h\alpha\beta}\}$

A Lemma 3 for $\mathfrak{g}_k := \mathfrak{g}^\epsilon/(\epsilon^{k+1}=0)$.

Lemma 3$_k$. $\mathbb{O}\left(e^{\beta e+\alpha f+\delta e f} \mid f e\right) = \mathbb{O}\left(\nu\,e^{\nu(-\alpha\beta h+\beta e+\alpha f+\delta e f)}\,\Lambda_k(\epsilon, e, l, f, \alpha, \beta, \delta) \mid e\,l\,f\right)$, with $\nu = (1+h\,\delta)^{-1}$ and where for any fixed k, $\Lambda_k(\epsilon, e, l, f, \alpha, \beta, \delta)$ is a fixed polynomial of degree at most $4\,k$ in e, $\sqrt{l}$, f, α, β, with scalar coefficients.

Comment. Even better, $\log(\Lambda_k)$ is of degree at most $2\,k+2$ in said variables.

Comment. And hence the $\mathfrak{g}_k$ invariant is computable in polynomial time.

Proof of Lemma 3$_k$. We know that $\mathbb{O}\left(e^{\alpha f+\beta e} \mid f e\right) = \mathbb{O}\left(e^{c h+a e-2\epsilon c l+b f} \mid e\,l\,f\right)$, with $\left\{a \to -\frac{\beta}{-1+\alpha\beta\epsilon},\ b \to -\frac{\alpha}{-1+\alpha\beta\epsilon},\ c \to \frac{\text{Log}[1-\alpha\beta\epsilon]}{\epsilon}\right\}$. Expanding in ϵ we get $\mathbb{O}\left(e^{\alpha f+\beta e} \mid f e\right) = \mathbb{O}\left(\lambda_\epsilon(\alpha,\beta)\,e^{\alpha f+\beta e-\alpha\beta h} \mid e\,l\,f\right) = \mathbb{O}\left(\lambda_\epsilon(\partial_f,\partial_e)\,e^{\alpha f+\beta e-\alpha\beta h} \mid e\,l\,f\right)$ and so $\mathbb{O}\left(e^{\alpha f+\beta e+\delta e f} \mid f e\right) = \mathbb{O}\left(\lambda_\epsilon(\partial_f,\partial_e)\,e^{\delta\partial_\alpha\partial_\beta}\,e^{\alpha f+\beta e-\alpha\beta h} \mid e\,l\,f\right) = \mathbb{O}\left(\lambda_\epsilon(\partial_f,\partial_e)\,\nu\,e^{\nu(-\alpha\beta h+\beta e+\alpha f+\delta e f)} \mid e\,l\,f\right)$.

```
DP_{α_→D_f_, β_→D_e_}[P_][λ_] :=
 Total[CoefficientRules[P, {α, β}] /. ({m_, n_} → c_) :→ c D[λ, {f, m}, {e, n}]]
```

```
(* "D" for Detailed *)
DΛ_k_[h_, e_, l_, f_, α_, β_, δ_] := Module[
  {ρh, ρe, ρl, ρf, eqn, a, b, c, sol, λ, q, ν},
  ρh = ( -1  0 ; 0  -1 ); ρe = ( 0  0 ; -ϵ  0 ); ρl = ( -(1 + 1/ϵ)/2  0 ; 0  (1 - 1/ϵ)/2 ); ρf = ( 0 1 ; 0 0 );
  eqn = ME[α ρf].ME[β ρe] == ME[a ρe].ME[c (ρh - 2 ϵ ρl)].ME[b ρf];
  sol = Solve[Thread[Flatten /@ eqn], {a, b, c}][[1]] /. C[1] → 0;
  λ = Simplify[e^(-f α-e β+h α β) Normal@Series[e^(c h + a e - 2 ϵ c l + b f) /. sol, {ϵ, 0, k}]];
  q = e^(ν (f α+e β-h α β+e f δ));
  Collect[q^-1 DP_(α→D_f, β→D_e)[λ][q] /. ν → (1 + h δ)^-1, ϵ, Simplify]
 ];
```

```
DΛ₁[h, e, l, f, α, β, δ]
```

$$1+\frac{1}{2\,(1+h\,\delta)^4}\Big(2\,e\,\alpha\,\beta^2-h\,\alpha^2\,\beta^2+4\,e\,\beta\,\delta-4\,h\,\alpha\,\beta\,\delta+2\,e^2\,\beta^2\,\delta-2\,h\,\delta^2+4\,e\,h\,\beta\,\delta^2-4\,h^2\,\alpha\,\beta\,\delta^2+e^2\,h\,\beta^2\,\delta^2-4\,h^2\,\delta^3-2\,h^3\,\delta^4+4\,l\,(1+h\,\delta)^2\,\big(\alpha\,(\beta+f\,\delta)+\delta\,(1+e\,\beta+e\,f\,\delta+h\,\delta)\big)+f^2\,\delta\,(\alpha+e\,\delta)\,\big(\alpha\,(2+h\,\delta)+e\,\delta\,(4+3\,h\,\delta)\big)+2\,f\,\big(\alpha^2\,\beta+2\,\alpha\,\delta\,(1+h\,\delta+e\,\beta\,(2+h\,\delta))+e\,\delta^2\,(4+6\,h\,\delta+2\,h^2\,\delta^2+e\,\beta\,(3+2\,h\,\delta))\big)\Big)\,\epsilon$$

```
DΛ2[h, e, l, f, α, β, δ]
```

$$
\begin{aligned}
&1 + \frac{1}{2\,(1+h\,\delta)^4}\Big(2\,e\,\alpha\,\beta^2 - h\,\alpha^2\,\beta^2 + 4\,e\,\beta\,\delta - 4\,h\,\alpha\,\beta\,\delta + 2\,e^2\,\beta^2\,\delta - 2\,h\,\delta^2 + 4\,e\,h\,\beta\,\delta^2 - 4\,h^2\,\alpha\,\beta\,\delta^2 + e^2\,h\,\beta^2\,\delta^2 - \\
&\quad 4\,h^2\,\delta^3 - 2\,h^3\,\delta^4 + 4\,l\,(1+h\,\delta)^2\,\big(\alpha\,(\beta+f\,\delta) + \delta\,(1+e\,\beta+e\,f\,\delta+h\,\delta)\big) + \\
&\quad f^2\,\delta\,(\alpha+e\,\delta)\,\big(\alpha\,(2+h\,\delta)+e\,\delta\,(4+3\,h\,\delta)\big) + 2\,f\,\big(\alpha^2\,\beta + 2\,\alpha\,\delta\,(1+h\,\delta+e\,\beta\,(2+h\,\delta)) + \\
&\qquad e\,\delta^2\,(4+6\,h\,\delta+2\,h^2\,\delta^2+e\,\beta\,(3+2\,h\,\delta))\big)\Big)\,\epsilon + \frac{1}{24\,(1+h\,\delta)^8} \\
&\Big(24\,l\,(1+h\,\delta)^4\,\big((\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2 + 4\,\delta\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta) + 2\,\delta^2\,(1+h\,\delta)^2\big) + \\
&\quad 48\,l^2\,(1+h\,\delta)^4\,\big((\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2 + 4\,\delta\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta) + 2\,\delta^2\,(1+h\,\delta)^2\big) + \\
&\quad 24\,f\,(\alpha+e\,\delta)\,(1+h\,\delta)^3 \\
&\quad\ \big((\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2 + 6\,\delta\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta) + 6\,\delta^2\,(1+h\,\delta)^2\big) + 48\,f\,l\,(\alpha+e\,\delta) \\
&\quad\ (1+h\,\delta)^3\,\big((\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2 + 6\,\delta\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta) + 6\,\delta^2\,(1+h\,\delta)^2\big) + 24\,e \\
&\quad\ (\beta+f\,\delta)\,(1+h\,\delta)^3\,\big((\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2 + 6\,\delta\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta) + 6\,\delta^2\,(1+h\,\delta)^2\big) + \\
&\quad 48\,e\,l\,(\beta+f\,\delta)\,(1+h\,\delta)^3\,\big((\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2 + 6\,\delta\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta) + \\
&\qquad 6\,\delta^2\,(1+h\,\delta)^2\big) + 12\,(\beta+f\,\delta)^2\,(e+e\,h\,\delta)^2 \\
&\quad\ \big((\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2 + 8\,\delta\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta) + 12\,\delta^2\,(1+h\,\delta)^2\big) + 12\,(\alpha+e\,\delta)^2 \\
&\quad\ (f+f\,h\,\delta)^2\,\big((\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2 + 8\,\delta\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta) + 12\,\delta^2\,(1+h\,\delta)^2\big) + \\
&\quad 24\,e\,f\,(1+h\,\delta)^2\,\big((\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3 + 9\,\delta\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta) + \\
&\qquad 18\,\delta^2\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^2 + 6\,\delta^3\,(1+h\,\delta)^3\big) - 8\,h\,(1+h\,\delta)^2\,\big((\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3 + \\
&\qquad 9\,\delta\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta) + 18\,\delta^2\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^2 + 6\,\delta^3\,(1+h\,\delta)^3\big) - \\
&\quad 24\,h\,l\,(1+h\,\delta)^2\,\big((\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3 + 9\,\delta\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta) + \\
&\qquad 18\,\delta^2\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^2 + 6\,\delta^3\,(1+h\,\delta)^3\big) - \\
&\quad 12\,f\,h\,(\alpha+e\,\delta)\,(1+h\,\delta)\,\big((\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3 + 12\,\delta\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta) + \\
&\qquad 36\,\delta^2\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^2 + 24\,\delta^3\,(1+h\,\delta)^3\big) - \\
&\quad 12\,e\,h\,(\beta+f\,\delta)\,(1+h\,\delta)\,\big((\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3 + 12\,\delta\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta) + \\
&\qquad 36\,\delta^2\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^2 + 24\,\delta^3\,(1+h\,\delta)^3\big) + \\
&\quad 3\,h^2\,\big((\alpha+e\,\delta)^4\,(\beta+f\,\delta)^4 + 16\,\delta\,(\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3\,(1+h\,\delta) + 72\,\delta^2\,(\alpha+e\,\delta)^2 \\
&\qquad (\beta+f\,\delta)^2\,(1+h\,\delta)^2 + 96\,\delta^3\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^3 + 24\,\delta^4\,(1+h\,\delta)^4\big)\Big)\,\epsilon^2
\end{aligned}
$$

```
Λk_[h_, e_, l_, f_, α_, β_, δ_] := Λk[h, e, l, f, α, β, δ] = Module[{λ},
   λ = Normal@Series[ⅇ^((f α+e β)/(1-α β ϵ)) (1 - α β ϵ)^(-2 l + h/ϵ), {ϵ, 0, k}] /. ⅇ- → 1;
   Collect[DP_{α→D_f, β→D_e}[λ][ⅇ^((f α+e β+e f δ)/(1+h δ))] /. ⅇ- → 1, ϵ, Simplify]];
```

```
Simplify[DΛ2[h, e, l, f, α, β, δ] == Λ2[h, e, l, f, α, β, δ]]
```

```
True
```

The Main $\mathfrak{g}_k$ Theorem

The $\mathfrak{g}_k$ invariant of any S-component tangle T can be written in the form $Z(T) = \mathbb{O}\left(\omega\, e^{L+Q+P} \;\middle|\; \prod_{i\in S} e_i\, l_i\, f_i\right)$, where ω is a scalar (meaning, a rational function in the variables h_i and their exponentials $t_i = e^{h_i}$), where $L = \sum a_{ij} h_i l_j$ is a balanced quadratic in the variables h_i and l_j with integer coefficients a_{ij}, where $Q = \sum b_{ij} e_i f_j$ is a balanced quadratic in the variables e_i and f_j with scalar coefficients b_{ij}, and where P is a polynomial in $\{\epsilon, e_i, l_i, f_i\}$ (with scalar coefficients) whose ϵ^d-term is of degree at most $2d+2$ in $\left\{e_i, \sqrt{l_i}, f_i\right\}$. Furthermore, after setting $h_i = h$ and $t_i = t$ for all i, the invariant $Z(T)$ is poly-time computable.

Partial Proof. Indeed,

0. $R^{\pm} = ?$, $n^{\pm} = ?$.
1. $\mathbb{O}\left(\mathcal{P}(l, e)\, e^{\gamma l + \beta e} \mid l\, e\right) = \mathbb{O}\left(\mathcal{P}(\partial_\gamma, \partial_\beta)\, e^{\gamma l + e^{\gamma} \beta e} \mid e\, l\right)$,
2. $\mathbb{O}\left(\mathcal{P}(l, f)\, e^{\gamma l + \beta f} \mid f\, l\right) = \mathbb{O}\left(\mathcal{P}(\partial_\gamma, \partial_\beta)\, e^{\gamma l + e^{\gamma} \beta f} \mid l\, f\right)$,
3. $\mathbb{O}\left(\mathcal{P}(e, f)\, e^{\beta e + \alpha f + \delta e f} \mid f\, e\right) = \mathbb{O}\left(\nu\, \mathcal{P}(\partial_\beta, \partial_\alpha)\, e^{\nu(-\alpha \beta h + \beta e + \alpha f + \delta e f)}\, \Lambda_k(\epsilon, e, l, f, \alpha, \beta, \delta) \mid e\, f\right)$, with $\nu = (1 + \mathrm{h}\delta)^{-1}$, and $\Lambda_k(\epsilon, e, l, f, \alpha, \beta, \delta)$ as above.

Pushforwards of distributions, 0-dimensional QFT, Feynman diagrams and what had really happened here.