

Warning. Conventions on this page change randomly from line to line.

The Algebra. $\mathcal{U}_{\hbar;\alpha\beta}$ conventions: $q = e^{\hbar\alpha\beta}$, $H = \langle a, x \rangle / ([a, x] = \alpha x)$ with

$$A = e^{-\hbar\beta a}, \quad xA = qAx, \quad S(a, A, x) = (-a, A^{-1}, -A^{-1}x),$$

$$\Delta(a, A, x) = (a_1 + a_2, A_1A_2, x_1 + A_1x_2)$$

and dual $H^* = \langle b, y \rangle / ([b, y] = -\beta y)$ with

$$B = e^{-\hbar ab}, \quad By = qyB, \quad S(b, B, y) = (-b, B^{-1}, -yB^{-1}),$$

$$\Delta(b, B, y) = (b_1 + b_2, B_1B_2, y_1B_2 + y_2).$$

Pairing by $(a, x)^* = \hbar \langle b, y \rangle$ making $\langle y^l b^j, a^j x^k \rangle = \delta_{ij} \delta_{kl} i! [k]_q!$. Then $\mathcal{U} = H^{*cop} \otimes H$ with $(\phi f)(\psi g) = \langle \psi_1 S^{-1} f_3 \rangle \langle \psi_3, f_1 \rangle \langle \phi \psi_2 \rangle (f_2 g)$. With the central $t := \beta a - \alpha b$, $T := e^{\hbar t} = A^{-1}B$ get

$$[a, y] = -\alpha y, \quad xy - qyx = (1 - TA^2)/\hbar.$$

Benkart-Witherspoon, [2017-06/BW.nb](#): At $\alpha\beta\hbar = \sigma - \rho$, represented by $y \rightarrow \begin{pmatrix} 0 & 0 \\ -e^\rho & 0 \end{pmatrix}$, $a \rightarrow \frac{\alpha}{\rho - \sigma} \begin{pmatrix} \rho & 0 \\ 0 & \sigma \end{pmatrix}$, $A \rightarrow \begin{pmatrix} e^\rho & 0 \\ 0 & e^\sigma \end{pmatrix}$,

$$x \rightarrow \frac{e^\rho - e^\sigma}{\hbar e^{\rho + \sigma}} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad t \rightarrow \frac{\rho + \sigma}{\hbar} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad T \rightarrow \frac{1}{e^{\rho + \sigma}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad b \rightarrow$$

$$\frac{\beta}{\sigma - \rho} \begin{pmatrix} \sigma & 0 \\ 0 & \rho \end{pmatrix}, \quad B \rightarrow \begin{pmatrix} e^{-\sigma} & 0 \\ 0 & e^{-\rho} \end{pmatrix}.$$

The R-Matrix. With $[n]_q := (q^n - 1)/(q - 1)$, $[n]_q! := [1]_q \cdots [n]_q$ and $e_q^x := \sum_{n \geq 0} \frac{x^n}{[n]_q!}$, we have the mysterious Quesne formula

of [arXiv:math-ph/0305003](#): $e_q^x = e^x \exp\left(\sum_{k \geq 2} \frac{(1-q)^k x^k}{k(1-q^k)}\right)$. Then $R_{ij} := \mathbb{O}(yb \otimes ax: e^{\hbar ba} e_q^{\hbar yx})$.

xa Swaps.

ay Swaps.

xy Swaps.

The Drinfel'd Element.

Putting Everything Together.
