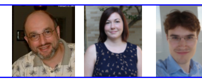
The Dogma is Wrong

Follows Rozansky [Ro1, Ro2, Ro3] and Overbay [Ov], joint with van der Veen. More at ωεβ/talks.



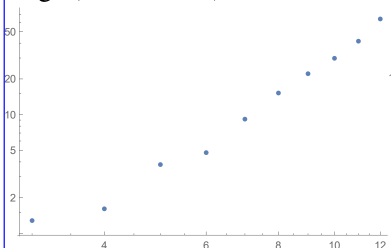
Thanks for the invitation!

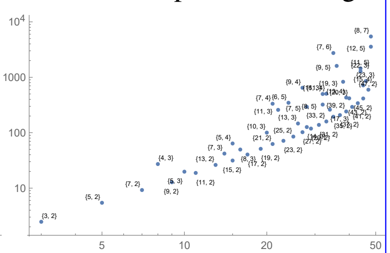
ωεβ:=http://drorbn.net/Sydney-1708/

Abstract. It has long been known that there are knot invariants associated to semi-simple Lie algebras, and there has long been a dogma as for how to extract them: "quantize and use representation theory". We present an alternative and better procedure: "centrally extend, approximate by solvable, and learn how to re-order exponentials in a universal enveloping algebra". While equivalent to the old invariants via a complicated process, our invariants are in practice stronger, faster to compute (poly-time vs. exp-time), and clearly carry topological information.

KiW 43 Abstract (ωεβ/kiw). Whether or not you like the formulas on this page, they describe the strongest truly computable knot invariant we know.

Experimental Analysis (ωεβ/Exp). Log-log plots of computation time (sec) vs. crossing number, for all knots with up to 12 crossings (mean times) and for all torus knots with up to 48 crossings:

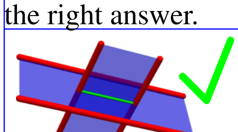




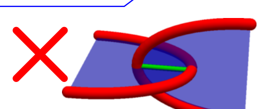
Power. On the 250 knots with at most 10 crossings, the pair (ω, ρ_1) attains 250 distinct values, while (Khovanov, HOMFLY-PT) attains only 249 distinct values. To 11 crossings the numbers are (802, 788, 772) and to 12 they are (2978, 2883, 2786).

Genus. Up to 12 xings, always ρ_1 is symmetric under $t \leftrightarrow t^{-1}$. With ρ_1^+ denoting the positive-degree part of ρ_1, always $\deg \rho_1^+ \leq 2g-1$, where g is the 3-genus of K (equality for 2530 knots). This gives a lower bound on g in terms of ρ_1 (conjectural, but undoubtedly true). This bound is often weaker than the Alexander bound, yet for 10 of the 12-xing Alexander failures it does give the right answer.

Ribbon Knots.

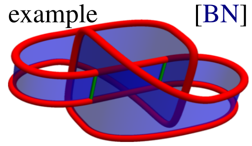


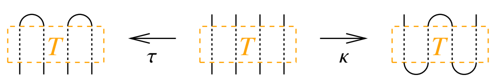
a ribbon singularity

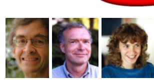


a clasp singularity

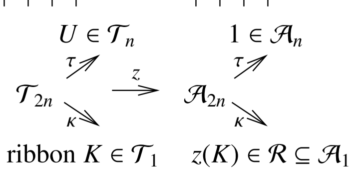
example [BN]

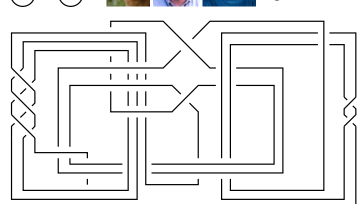






Gompf, Scharlemann, Thompson [GST]





[Vo]: Works for Alexander!

with $\mathcal{R} := \kappa(\tau^{-1}(1))$

$A^+ = -t^8 + 2t^7 - t^6 - 2t^4 + 5t^3 - 2t^2 - 7t + 13$

$\rho_1^+ = 5t^{15} - 18t^{14} + 33t^{13} - 32t^{12} + 2t^{11} + 42t^{10} - 62t^9 - 8t^8 + 166t^7 - 242t^6 + 108t^5 + 132t^4 - 226t^3 + 148t^2 - 11t - 36$

Faster is better, leaner is meaner!

dog·ma (dôg′mə, dŏg′-) The Free Dictionary, ωεβ/TFD

n. pl. **dog·mas** or **dog·ma·ta** (-mə-tə)

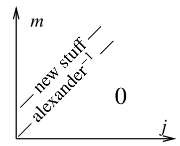
1. A doctrine or a corpus of doctrines relating to matters such as morality and faith, set forth in an authoritative manner by a religion.
2. A principle or statement of ideas, or a group of such principles or statements, especially when considered to be authoritative or accepted uncritically: *"Much education consists in the instilling of unfounded dogmas in place of a spirit of inquiry" (Bertrand Russell).*

Theorem ([BNG], conjectured [MM], elucidated [Ro1]). Let $J_d(K)$ be the coloured Jones polynomial of K, in the d-dimensional representation of sl_2. Writing

Melvin, Morton, Garoufalidis

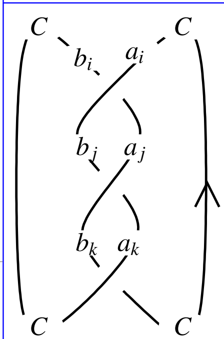
$$\left.\frac{(q^{1/2}-q^{-1/2})J_d(K)}{q^{d/2}-q^{-d/2}}\right|_{q=e^\hbar} = \sum_{j,m\geq 0} a_{jm}(K) d^j \hbar^m,$$

"below diagonal" coefficients vanish, $a_{jm}(K) = 0$ if $j > m$, and "on diagonal" coefficients give the inverse of the Alexander polynomial: $\left(\sum_{m=0}^\infty a_{mm}(K)\hbar^m\right) \cdot \omega(K)(e^\hbar) = 1$.



"Above diagonal" we have Rozansky's Theorem [Ro3, (1.2)]:

$$J_d(K)(q) = \frac{q^d - q^{-d}}{(q-q^{-1})\omega(K)(q^d)}\left(1 + \sum_{k=1}^\infty \frac{(q-1)^k \rho_k(K)(q^d)}{\omega^{2k}(K)(q^d)}\right).$$



The Yang-Baxter Technique. Given an algebra A (typically $\hat{\mathcal{U}}(\mathfrak{g})$ or $\hat{\mathcal{U}}_q(\mathfrak{g})$) and elements

$$R = \sum a_i \otimes b_i \in A \otimes A \quad \text{and} \quad C \in A,$$

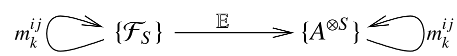
form

$$Z = \sum_{i,j,k} C a_i b_j a_k C^2 b_i a_j b_k C.$$

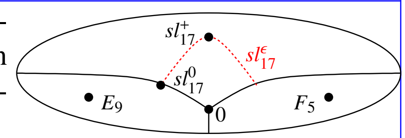
Problem. Extract information from Z.

The Dogma. Use representation theory. In principle finite, but *slow*.

The Loyal Opposition. For certain algebras, work in a homomorphic poly-dimensional "space of formulas".



The (fake) moduli of Lie algebras on V, a quadratic variety in $(V^*)^{\otimes 2} \otimes V$ is on the right. We care about $sl_{17}^k := sl_{17}^\epsilon / (\epsilon^{k+1} = 0)$.



Why are "solvable algebras" any good? Contrary to common beliefs, computations in semi-simple Lie algebras are just awful:

```
In[1]:= MatrixExp[( a b
                    c d )] // FullSimplify // MatrixForm    Enter
```

Yet in solvable algebras, exponentiation is fine and even BCH, $z = \log(e^x e^y)$, is bearable:

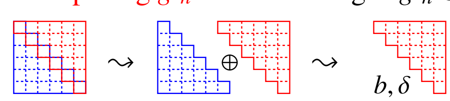
```
In[2]:= MatrixExp[( a b
                    0 c )] // MatrixForm
```

Out[2]//MatrixForm=

$$\begin{pmatrix} e^a & \frac{b(e^a - e^c)}{a-c} \\ 0 & e^c \end{pmatrix}$$

```
In[3]:= MatrixExp[( a1 b1
                    0  c1 )].MatrixExp[( a2 b2
                                         0  c2 )] //
          MatrixLog // PowerExpand // Simplify //
        MatrixForm    Enter
```

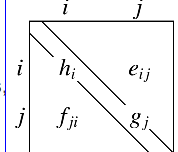
Recomposing gl_n. Half is enough! $gl_n \oplus \mathfrak{a}_n = \mathcal{D}(\triangledown, b, \delta)$:



$b(\triangledown) = b : \triangledown \otimes \triangledown \to \triangledown$

$b(\triangle) \rightsquigarrow \delta : \triangledown \to \triangledown \otimes \triangledown$

Now define $gl_n^\epsilon := \mathcal{D}(\triangledown, b, \epsilon\delta)$. Schematically, this is $[\triangledown, \triangledown] = \triangledown$, $[\triangle, \triangle] = \epsilon\triangle$, and $[\triangledown, \triangle] = \triangle + \epsilon\triangledown$. In detail, it is



$$[e_{ij}, e_{kl}] = \delta_{jk}e_{il} - \delta_{li}e_{kj} \qquad [f_{ij}, f_{kl}] = \epsilon\delta_{jk}f_{il} - \epsilon\delta_{li}f_{kj}$$

$$[e_{ij}, f_{kl}] = \delta_{jk}(\epsilon\delta_{j<k}e_{il} + \delta_{il}(h_i + \epsilon g_i)/2 + \delta_{i>l}f_{il}) - \delta_{li}(\epsilon\delta_{k<j}e_{kj} + \delta_{kj}(h_j + \epsilon g_j)/2 + \delta_{k>j}f_{kj})$$

$$[g_i, e_{jk}] = (\delta_{ij} - \delta_{ik})e_{jk} \qquad [h_i, e_{jk}] = \epsilon(\delta_{ij} - \delta_{ik})e_{jk}$$

$$[g_i, f_{jk}] = (\delta_{ij} - \delta_{ik})f_{jk} \qquad [h_i, f_{jk}] = \epsilon(\delta_{ij} - \delta_{ik})f_{jk}$$

The sl_2 Example. Let $\mathfrak{g}^\epsilon = \langle h,e,l,f\rangle/([h,\cdot]=0,\ [e,l]=-e,\ [f,l]=f,\ [e,f]=h-2\epsilon l)$ and let $\mathfrak{g}_k=\mathfrak{g}^\epsilon/(\epsilon^{k+1}=0)$.

The Main $\mathfrak{g}_k$ Theorem. The $\mathfrak{g}_k$-invariant of any S-component tangle T can be written in the form

$$Z(T)=\mathbb{O}\left(\omega\mathbb{e}^{L+Q+P}\colon \bigotimes_{i\in S} e_il_if_i\right),$$

where ω is a scalar (meaning, a rational function in the variables h_i and their exponentials $t_i:=\mathbb{e}^{h_i}$), where $L=\sum a_{ij}h_il_j$ is a balanced quadratic in the variables h_i and l_j with integer coefficients, where $Q=\sum b_{ij}e_if_j$ is a balanced quadratic in the variables e_i and f_j with scalar coefficients b_{ij}, and where P is a polynomial in $\{\epsilon,e_i,l_i,f_i\}$ (with scalar coefficients) whose ϵ^d-term is of degree at most $2d+2$ in $\{e_i,\sqrt{l_i},f_i\}$. Furthermore, after setting $h_i=h$ and $t_i=t$ for all i, the invariant $Z(T)$ is poly-time computable.

The Main $\mathfrak{g}_k$ Lemma. The following "re-ordering relations" hold:

$$\mathbb{O}\left(\mathbb{e}^{\gamma l+\beta e}\colon le\right)=\mathbb{O}\left(\mathbb{e}^{\gamma l+\mathbb{e}^\gamma\beta e}\colon el\right)\quad\text{(and similarly for } fl\to lf),$$

$$\mathbb{O}\left(\mathbb{e}^{\beta e+\alpha f+\delta ef}\colon fe\right)=\mathbb{O}\left(\nu\mathbb{e}^{\nu(-\alpha\beta h+\beta e+\alpha f+\delta ef)+\lambda_k(\epsilon,e,l,f,\alpha,\beta,\delta)}\colon elf\right),$$

with $\nu=(1+h\delta)^{-1}$ and where $\lambda_k(\epsilon,e,l,f,\alpha,\beta,\delta)$ is some fixed polynomial of degree at most $2k+2$ in $\epsilon,e,\sqrt{l},f,\alpha,\beta,\delta$, with scalar coefficients.

Demo Programs.

ωεβ/Demo

Formatting

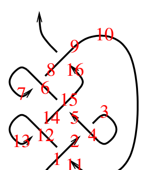
```
CF[ℰ_] := Module[{vars = Union@Cases[ℰ, e_ | l_ | f_, ∞]},
  If[vars === {}, Factor[ℰ],
   Total[CoefficientRules[ℰ, vars] /.
     (p_ → c_) :> Factor[c] Times @@ (vars^p)] ]];
CF[ℰ_𝔼] := CF /@ ℰ;
```

Preparation

```
𝔼[i_, j_, s_] := 𝔼[1, (-1)^s l_j, (-t)^s e_i f_j,
   t^s e_i l_(1+s) i-s j f_j + (-1)^s l_i l_j + (-t^2)^s e_i^2 f_j^2 / 4];
𝔼[i_, s_] := 𝔼[1, 0, 0, s l_i];
𝔼 /: 𝔼[1, L1_, Q1_, P1_] 𝔼[1, L2_, Q2_, P2_] :=
  𝔼[1, L1 + L2, Q1 + Q2, P1 + P2];
```

Preparing the Trefoil

```
z1 = (𝔼[1, 11, 0] 𝔼[4, 2, -1] 𝔼[15, 5, 0] ×
   𝔼[6, 8, -1] 𝔼[9, 16, 0] 𝔼[12, 14, -1] ×
   𝔼[3, -1] 𝔼[7, +1] 𝔼[10, -1] 𝔼[13, +1])
```



$$\mathbb{E}\Big[1,\ -l_2+l_5-l_8+l_{11}-l_{14}+l_{16},\ -\frac{e_4f_2}{t}+e_{15}f_5-\frac{e_6f_8}{t}+e_1f_{11}-\frac{e_{12}f_{14}}{t}+e_9f_{16},\ -\frac{e_4^2f_2^2}{4t^2}+\frac14 e_{15}^2f_5^2-\frac{e_6^2f_8^2}{4t^2}+\frac14 e_1^2f_{11}^2-\frac{e_{12}^2f_{14}^2}{4t^2}+\frac14 e_9^2f_{16}^2+e_1f_{11}l_1+\frac{e_4f_2l_2}{t}-l_3-l_2l_4+l_7+\frac{e_6f_8l_8}{t}-l_6l_8+e_9f_{16}l_9-l_{10}+l_1l_{11}+l_{13}+\frac{e_{12}f_{14}l_{14}}{t}-l_{12}l_{14}+e_{15}f_5l_{15}+l_5l_{15}+l_9l_{16}\Big]$$

Differential Polynomials

(Implementing $P(\partial_\alpha,\partial_\beta)(f)$)

```
DP_{x_→D_α_, y_→D_β_}[P_][f_] :=
 Total[CoefficientRules[P, {x, y}] /.
   ({m_, n_} → c_) :> c D[f, {α, m}, {β, n}]]
```

le and *fl* Sorts

```
S_{l_j_ (x:e|f)_i_ → k_}[𝔼[ω_, L_, Q_, P_]] :=
 With[{λ = ∂_{l_j} L, α = ∂_{x_i} Q, q = e^γ β x_k + γ l_k}, CF[
   𝔼[ω, L /. l_j → l_k, t^λ α x_k + (Q /. x_i → 0),
    e^-q DP_{l_j→D_γ, x_i→D_β}[P][e^q] /. {β → α / ω, γ → λ Log[t]}] ]];
```

The Λόγος

```
Λ[k_] := ((t - 1) (2 (α β + δ μ)^2 - α^2 β^2) - 4 e_k l_k f_k δ^2 μ^2 -
   δ (1 + μ) (f_k^2 α^2 + e_k^2 β^2) - e_k^2 f_k^2 δ^3 (1 + 3 μ) -
   2 (α β + 2 δ μ + e_k f_k δ^2 (1 + 2 μ) + 2 l_k δ μ^2) (f_k α + e_k β) -
   4 (l_k μ^2 + e_k f_k δ (1 + μ)) (α β + δ μ)) (1 + t) / 4;
```

fe Sorts

```
S_{f_i_ e_j_ → k_}[𝔼[ω_, L_, Q_, P_]] :=
 With[{q = ((1 - t) α β + β e_k + α f_k + δ e_k f_k) / μ}, CF[
   𝔼[μ ω, L, μ ω q + μ (Q /. f_i | e_j → 0),
     μ^4 e^-q DP_{f_i→D_α, e_j→D_β}[P][e^q] + ω^4 Λ[k]] /. μ → 1 + (t - 1) δ /.
    {α → ω^-1 (∂_{f_i} Q /. e_j → 0), β → ω^-1 (∂_{e_j} Q /. f_i → 0),
     δ → ω^-1 ∂_{f_i,e_j} Q}]];
```

Elf Merges

```
m_{i_,j_→k_}[Z_𝔼] := Module[{x, z},
  CF[(Z // S_{f_i e_j→x} // S_{l_i e_x→x} // S_{f_x l_j→x}) /. z_{-i|j|x} → z_k]]
```

Rewriting the Trefoil

(by merging 16 elves)

```
(Do[z1 = z1 // m_{1,k→1}, {k, 2, 16}]; z1)
```

$$\mathbb{E}\left[\frac{1-t+t^2}{t},\ 0,\ 0,\ \frac{(-1+t)\,(1-t+t^2)^2\,(1-t+2t^2)}{t^3}-\frac{2\,(1+t)\,(1-t+t^2)^3\,e_1f_1}{t^4}-\frac{2\,(-1+t)\,(1+t)\,(1-t+t^2)^3\,l_1}{t^4}\right]$$

Readout

```
ρ1[𝔼[ω_, _, _, P_]] := CF[ t ((P /. e_ | l_ | f_ → 0) - t ω^3 (∂_t ω)) / ((t - 1)^2 ω^2) ]
```

$\rho_1(3_1)$

```
ρ1[z1] // Expand
```

$\frac{1}{t}+t$

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[BNG] D. Bar-Natan and S. Garoufalidis, *On the Melvin-Morton-Rozansky conjecture,* Invent. Math. **125** (1996) 103–133.

[GST] R. E. Gompf, M. Scharlemann, and A. Thompson, *Fibered Knots and Potential Counterexamples to the Property 2R and Slice-Ribbon Conjectures,* Geom. and Top. **14** (2010) 2305–2347, arXiv:1103.1601.

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[Ov] A. Overbay, *Perturbative Expansion of the Colored Jones Polynomial,* University of North Carolina PhD thesis, ωεβ/Ov.

[Ro1] L. Rozansky, *A contribution of the trivial flat connection to the Jones polynomial and Witten's invariant of 3d manifolds, I,* Comm. Math. Phys. **175-2** (1996) 275–296, arXiv:hep-th/9401061.

[Ro2] L. Rozansky, *The Universal R-Matrix, Burau Representation and the Melvin-Morton Expansion of the Colored Jones Polynomial,* Adv. Math. **134-1** (1998) 1–31, arXiv:q-alg/9604005.

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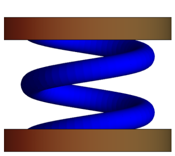
[Vo] H. Vo, University of Toronto Ph.D. thesis, in preparation.



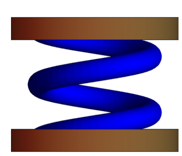


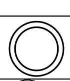
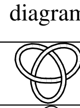
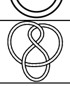
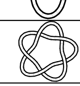
diagram	n_k^t Alexander's ω^+ / Today's / Rozansky's ρ_1^+		genus / ribbon / unknotting number / amphicheiral	diagram	n_k^t Alexander's ω^+ / Today's / Rozansky's ρ_1^+		genus / ribbon / unknotting number / amphicheiral
	0_1^a	1	0 / ✔		3_1^a	$t-1$	1 / ✘
	0		0 / ✔		t		1 / ✘
	4_1^a	$3-t$	1 / ✘		5_1^a	t^2-t+1	2 / ✘
	0		1 / ✔		$2t^3+3t$		2 / ✘

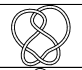
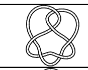
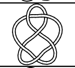
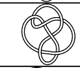
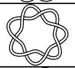
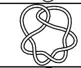
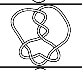
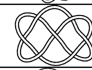
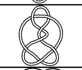
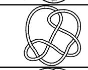
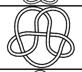
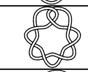
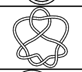
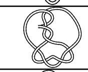
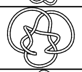
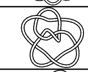
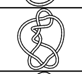
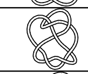
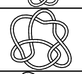
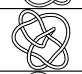
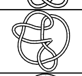
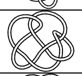
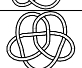
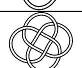
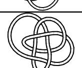
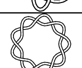
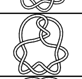
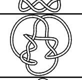
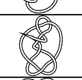
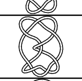
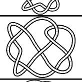
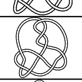
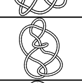
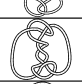
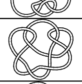
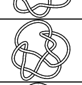
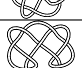
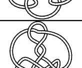
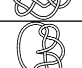
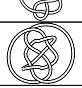
diagram	n_k^t Alexander's ω^+ Today's / Rozansky's ρ_1^+	genus / ribbon unknotting number / amphicheiral	diagram	n_k^t Alexander's ω^+ Today's / Rozansky's ρ_1^+	genus / ribbon unknotting number / amphicheiral
	5_2^a $2t-3$ $5t-4$	1 / ✗ 1 / ✗		6_1^a $5-2t$ $t-4$	1 / ✓ 1 / ✗
	6_2^a $-t^2+3t-3$ t^3-4t^2+4t-4	2 / ✗ 1 / ✗		6_3^a t^2-3t+5 0	2 / ✗ 1 / ✓
	7_1^a t^3-t^2+t-1 $3t^5+5t^3+6t$	3 / ✗ 3 / ✗		7_2^a $3t-5$ $14t-16$	1 / ✗ 1 / ✗
	7_3^a $2t^2-3t+3$ $-9t^3+8t^2-16t+12$	2 / ✗ 2 / ✗		7_4^a $4t-7$ $32-24t$	1 / ✗ 2 / ✗
	7_5^a $2t^2-4t+5$ $9t^3-16t^2+29t-28$	2 / ✗ 2 / ✗		7_6^a $-t^2+5t-7$ $t^3-8t^2+19t-20$	2 / ✗ 1 / ✗
	7_7^a t^2-5t+9 $8-3t$	2 / ✗ 1 / ✗		8_1^a $7-3t$ $5t-16$	1 / ✗ 1 / ✗
	8_2^a $-t^3+3t^2-3t+3$ $2t^5-8t^4+10t^3-12t^2+13t-12$	3 / ✗ 2 / ✗		8_3^a $9-4t$ 0	1 / ✗ 2 / ✓
	8_4^a $-2t^2+5t-5$ $3t^3-8t^2+6t-4$	2 / ✗ 2 / ✗		8_5^a $-t^3+3t^2-4t+5$ $-2t^5+8t^4-13t^3+20t^2-22t+24$	3 / ✗ 2 / ✗
	8_6^a $-2t^2+6t-7$ $5t^3-20t^2+28t-32$	2 / ✗ 2 / ✗		8_7^a t^3-3t^2+5t-5 $-t^5+4t^4-10t^3+12t^2-13t+12$	3 / ✗ 1 / ✗
	8_8^a $2t^2-6t+9$ $-t^3+4t^2-12t+16$	2 / ✓ 2 / ✗		8_9^a $-t^3+3t^2-5t+7$ 0	3 / ✓ 1 / ✓
	8_{10}^a t^3-3t^2+6t-7 $-t^5+4t^4-11t^3+16t^2-21t+20$	3 / ✗ 2 / ✗		8_{11}^a $-2t^2+7t-9$ $5t^3-24t^2+39t-44$	2 / ✗ 1 / ✗
	8_{12}^a $t^2-7t+13$ 0	2 / ✗ 2 / ✓		8_{13}^a $2t^2-7t+11$ $-t^3+4t^2-14t+20$	2 / ✗ 1 / ✗
	8_{14}^a $-2t^2+8t-11$ $5t^3-28t^2+57t-68$	2 / ✗ 1 / ✗		8_{15}^a $3t^2-8t+11$ $21t^3-64t^2+120t-140$	2 / ✗ 2 / ✗
	8_{16}^a t^3-4t^2+8t-9 $t^5-6t^4+17t^3-28t^2+35t-36$	3 / ✗ 2 / ✗		8_{17}^a $-t^3+4t^2-8t+11$ 0	3 / ✗ 1 / ✓
	8_{18}^a $-t^3+5t^2-10t+13$ 0	3 / ✗ 2 / ✓		8_{19}^n t^3-t^2+1 $-3t^5-4t^2-3t$	3 / ✗ 3 / ✗
	8_{20}^n t^2-2t+3 $4t-4$	2 / ✓ 1 / ✗		8_{21}^n $-t^2+4t-5$ $t^3-8t^2+16t-20$	2 / ✗ 1 / ✗
	9_1^a $t^4-t^3+t^2-t+1$ $4t^7+7t^5+9t^3+10t$	4 / ✗ 4 / ✗		9_2^a $4t-7$ $30t-40$	1 / ✗ 1 / ✗
	9_3^a $2t^3-3t^2+3t-3$ $-13t^5+12t^4-25t^3+20t^2-32t+24$	3 / ✗ 3 / ✗		9_4^a $3t^2-5t+5$ $23t^3-28t^2+46t-44$	2 / ✗ 2 / ✗
	9_5^a $6t-11$ $100-65t$	1 / ✗ 2 / ✗		9_6^a $2t^3-4t^2+5t-5$ $13t^5-24t^4+45t^3-52t^2+68t-64$	3 / ✗ 3 / ✗
	9_7^a $3t^2-7t+9$ $23t^3-56t^2+99t-108$	2 / ✗ 2 / ✗		9_8^a $-2t^2+8t-11$ $3t^3-16t^2+29t-28$	2 / ✗ 2 / ✗
	9_9^a $2t^3-4t^2+6t-7$ $13t^5-24t^4+55t^3-72t^2+98t-96$	3 / ✗ 3 / ✗		9_{10}^a $4t^2-8t+9$ $-40t^3+72t^2-114t+120$	2 / ✗ 2, 3 / ✗
	9_{11}^a $-t^3+5t^2-7t+7$ $-2t^5+16t^4-41t^3+52t^2-66t+64$	3 / ✗ 2 / ✗		9_{12}^a $-2t^2+9t-13$ $5t^3-36t^2+84t-100$	2 / ✗ 1 / ✗
	9_{13}^a $4t^2-9t+11$ $-40t^3+92t^2-154t+168$	2 / ✗ 2, 3 / ✗		9_{14}^a $2t^2-9t+15$ $-t^3+8t^2-35t+60$	2 / ✗ 1 / ✗
	9_{15}^a $-2t^2+10t-15$ $-5t^3+40t^2-108t+136$	2 / ✗ 2 / ✗		9_{16}^a $2t^3-5t^2+8t-9$ $-13t^5+36t^4-80t^3+120t^2-161t+168$	3 / ✗ 3 / ✗
	9_{17}^a t^3-5t^2+9t-9 $t^5-8t^4+23t^3-32t^2+28t-24$	3 / ✗ 2 / ✗		9_{18}^a $4t^2-10t+13$ $40t^3-108t^2+193t-220$	2 / ✗ 2 / ✗
	9_{19}^a $2t^2-10t+17$ $t^3-8t^2+20t-24$	2 / ✗ 1 / ✗		9_{20}^a $-t^3+5t^2-9t+11$ $2t^5-16t^4+47t^3-84t^2+117t-124$	3 / ✗ 2 / ✗
	9_{21}^a $-2t^2+11t-17$ $-5t^3+44t^2-127t+164$	2 / ✗ 1 / ✗		9_{22}^a $t^3-5t^2+10t-11$ $-t^5+8t^4-24t^3+38t^2-40t+36$	3 / ✗ 1 / ✗
	9_{23}^a $4t^2-11t+15$ $40t^3-128t^2+243t-288$	2 / ✗ 2 / ✗		9_{24}^a $-t^3+5t^2-10t+13$ $-4t^2+16t-20$	3 / ✗ 1 / ✗
	9_{25}^a $-3t^2+12t-17$ $12t^3-70t^2+153t-188$	2 / ✗ 2 / ✗		9_{26}^a $t^3-5t^2+11t-13$ $-t^5+8t^4-31t^3+64t^2-85t+92$	3 / ✗ 1 / ✗
	9_{27}^a $-t^3+5t^2-11t+15$ $t^3-8t^2+24t-32$	3 / ✓ 1 / ✗		9_{28}^a $t^3-5t^2+12t-15$ $t^5-8t^4+30t^3-68t^2+105t-120$	3 / ✗ 1 / ✗
	9_{29}^a $t^3-5t^2+12t-15$ $t^5-8t^4+26t^3-48t^2+59t-56$	3 / ✗ 2 / ✗		9_{30}^a $-t^3+5t^2-12t+17$ $2t^3-10t^2+25t-32$	3 / ✗ 1 / ✗

diagram	n_k^t	Alexander's ω^+ / Today's / Rozansky's ρ_1^+	genus / ribbon	unknotting number / amphicheiral	diagram	n_k^t	Alexander's ω^+ / Today's / Rozansky's ρ_1^+	genus / ribbon	unknotting number / amphicheiral
	9^a_{31}	$t^3-5t^2+13t-17$ $t^5-8t^4+33t^3-80t^2+132t-152$	3 / ✘	2 / ✘		9^a_{32}	$t^3-6t^2+14t-17$ $-t^5+10t^4-42t^3+94t^2-133t+148$	3 / ✘	2 / ✘
	9^a_{33}	$-t^3+6t^2-14t+19$ $t^3-10t^2+30t-40$	3 / ✘	1 / ✘		9^a_{34}	$-t^3+6t^2-16t+23$ $3t^3-18t^2+43t-56$	3 / ✘	1 / ✘
	9^a_{35}	$7t-13$ $90t-144$	1 / ✘	2, 3 / ✘		9^a_{36}	$-t^3+5t^2-8t+9$ $-2t^5+16t^4-44t^3+66t^2-87t+88$	3 / ✘	2 / ✘
	9^a_{37}	$2t^2-11t+19$ $t^3-8t^2+22t-28$	2 / ✘	2 / ✘		9^a_{38}	$5t^2-14t+19$ $62t^3-204t^2+382t-452$	2 / ✘	2, 3 / ✘
	9^a_{39}	$-3t^2+14t-21$ $-12t^3+84t^2-210t+268$	2 / ✘	1 / ✘		9^a_{40}	$t^3-7t^2+18t-23$ $t^5-12t^4+57t^3-144t^2+229t-264$	3 / ✘	2 / ✘
	9^a_{41}	$3t^2-12t+19$ $3t^3-20t^2+70t-108$	2 / ✔	2 / ✘		9^n_{42}	$-t^2+2t-1$ $-t^3+2t^2+t-4$	2 / ✘	1 / ✘
	9^n_{43}	$-t^3+3t^2-2t+1$ $-2t^5+8t^4-7t^3+2t^2-5t+4$	3 / ✘	2 / ✘		9^n_{44}	t^2-4t+7 $-2t^2+9t-12$	2 / ✘	1 / ✘
	9^n_{45}	$-t^2+6t-9$ $t^3-14t^2+47t-60$	2 / ✘	1 / ✘		9^n_{46}	$5-2t$ $3t-12$	1 / ✔	2 / ✘
	9^n_{47}	t^3-4t^2+6t-5 $-t^5+6t^4-15t^3+16t^2-10t+12$	3 / ✘	2 / ✘		9^n_{48}	$-t^2+7t-11$ $-t^3+12t^2-42t+52$	2 / ✘	2 / ✘
	9^n_{49}	$3t^2-6t+7$ $-21t^3+38t^2-61t+60$	2 / ✘	3 / ✘		10^a_{1}	$9-4t$ $14t-40$	1 / ✘	1 / ✘
	10^a_{2}	$-t^4+3t^3-3t^2+3t-3$ $3t^7-12t^6+16t^5-20t^4+24t^3-24t^2+27t-24$	4 / ✘	3 / ✘		10^a_{3}	$13-6t$ $11t-28$	1 / ✔	2 / ✘
	10^a_{4}	$-3t^2+7t-7$ $4t^3-8t^2+t+8$	2 / ✘	2 / ✘		10^a_{5}	$t^4-3t^3+5t^2-5t+5$ $-2t^7+8t^6-20t^5+28t^4-36t^3+36t^2-39t+36$	4 / ✘	2 / ✘
	10^a_{6}	$-2t^3+6t^2-7t+7$ $9t^5-36t^4+56t^3-72t^2+81t-84$	3 / ✘	3 / ✘		10^a_{7}	$-3t^2+11t-15$ $14t^3-72t^2+135t-160$	2 / ✘	1 / ✘
	10^a_{8}	$-2t^3+5t^2-5t+5$ $7t^5-20t^4+23t^3-28t^2+26t-24$	3 / ✘	2 / ✘		10^a_{9}	$-t^4+3t^3-5t^2+7t-7$ $-t^7+4t^6-10t^5+20t^4-25t^3+28t^2-28t+28$	4 / ✘	1 / ✘
	10^a_{10}	$3t^2-11t+17$ $-5t^3+24t^2-71t+100$	2 / ✘	1 / ✘		10^a_{11}	$-4t^2+11t-13$ $16t^3-52t^2+68t-72$	2 / ✘	2, 3 / ✘
	10^a_{12}	$2t^3-6t^2+10t-11$ $-5t^5+20t^4-50t^3+72t^2-89t+92$	3 / ✘	2 / ✘		10^a_{13}	$2t^2-13t+23$ $t^3-12t^2+51t-84$	2 / ✘	2 / ✘
	10^a_{14}	$-2t^3+8t^2-12t+13$ $9t^5-52t^4+119t^3-180t^2+225t-236$	3 / ✘	2 / ✘		10^a_{15}	$2t^3-6t^2+9t-9$ $-3t^5+12t^4-24t^3+24t^2-17t+12$	3 / ✘	2 / ✘
	10^a_{16}	$-4t^2+12t-15$ $-16t^3+56t^2-76t+80$	2 / ✘	2 / ✘		10^a_{17}	$t^4-3t^3+5t^2-7t+9$ 0	4 / ✘	1 / ✔
	10^a_{18}	$-4t^2+14t-19$ $16t^3-68t^2+121t-140$	2 / ✘	1 / ✘		10^a_{19}	$2t^3-7t^2+11t-11$ $3t^5-16t^4+35t^3-40t^2+30t-24$	3 / ✘	2 / ✘
	10^a_{20}	$-3t^2+9t-11$ $14t^3-56t^2+88t-104$	2 / ✘	2 / ✘		10^a_{21}	$-2t^3+7t^2-9t+9$ $9t^5-44t^4+80t^3-104t^2+121t-124$	3 / ✘	2 / ✘
	10^a_{22}	$-2t^3+6t^2-10t+13$ $-t^5+4t^4-10t^3+24t^2-37t+44$	3 / ✔	2 / ✘		10^a_{23}	$2t^3-7t^2+13t-15$ $-5t^5+24t^4-67t^3+108t^2-137t+144$	3 / ✘	1 / ✘
	10^a_{24}	$-4t^2+14t-19$ $24t^3-116t^2+221t-268$	2 / ✘	2 / ✘		10^a_{25}	$-2t^3+8t^2-14t+17$ $9t^5-52t^4+131t^3-232t^2+314t-344$	3 / ✘	2 / ✘
	10^a_{26}	$-2t^3+7t^2-13t+17$ $-t^5+4t^4-10t^3+28t^2-49t+60$	3 / ✘	1 / ✘		10^a_{27}	$2t^3-8t^2+16t-19$ $5t^5-28t^4+87t^3-164t^2+229t-252$	3 / ✘	1 / ✘
	10^a_{28}	$4t^2-13t+19$ $-8t^3+36t^2-100t+136$	2 / ✘	2 / ✘		10^a_{29}	$t^3-7t^2+15t-17$ $t^5-12t^4+52t^3-104t^2+124t-128$	3 / ✘	2 / ✘
	10^a_{30}	$-4t^2+17t-25$ $24t^3-148t^2+345t-440$	2 / ✘	1 / ✘		10^a_{31}	$4t^2-14t+21$ $-4t^2+9t-12$	2 / ✘	1 / ✘
	10^a_{32}	$-2t^3+8t^2-15t+19$ $t^5-4t^4+13t^3-40t^2+78t-96$	3 / ✘	1 / ✘		10^a_{33}	$4t^2-16t+25$ 0	2 / ✘	1 / ✔
	10^a_{34}	$3t^2-9t+13$ $-5t^3+20t^2-52t+68$	2 / ✘	2 / ✘		10^a_{35}	$2t^2-12t+21$ $-t^3+12t^2-47t+76$	2 / ✔	2 / ✘
	10^a_{36}	$-3t^2+13t-19$ $14t^3-88t^2+208t-264$	2 / ✘	2 / ✘		10^a_{37}	$4t^2-13t+19$ 0	2 / ✘	2 / ✔
	10^a_{38}	$-4t^2+15t-21$ $24t^3-128t^2+270t-336$	2 / ✘	2 / ✘		10^a_{39}	$-2t^3+8t^2-13t+15$ $9t^5-52t^4+125t^3-204t^2+263t-280$	3 / ✘	2 / ✘
	10^a_{40}	$2t^3-8t^2+17t-21$ $-5t^5+28t^4-89t^3+176t^2-258t+288$	3 / ✘	2 / ✘		10^a_{41}	$t^3-7t^2+17t-21$ $t^5-12t^4+54t^3-120t^2+157t-164$	3 / ✘	2 / ✘
	10^a_{42}	$-t^3+7t^2-19t+27$ $2t^3-8t^2+11t-12$	3 / ✔	1 / ✘		10^a_{43}	$-t^3+7t^2-17t+23$ 0	3 / ✘	2 / ✔

diagram	n_k^t	Alexander's ω^+	genus / ribbon	Today's / Rozansky's ρ_1^+	unknotting number / amphicheiral
	10^a_{44}	$t^3 - 7t^2 + 19t - 25$	3 / ✘	$t^5 - 12t^4 + 56t^3 - 140t^2 + 220t - 248$	1 / ✘
	10^a_{45}	$-t^3 + 7t^2 - 21t + 31$	3 / ✘	0	2 / ✔
	10^a_{46}	$-t^4 + 3t^3 - 4t^2 + 5t - 5$	4 / ✘	$-3t^7 + 12t^6 - 21t^5 + 34t^4 - 43t^3 + 52t^2 - 55t + 56$	3 / ✘
	10^a_{47}	$t^4 - 3t^3 + 6t^2 - 7t + 7$	4 / ✘	$-2t^7 + 8t^6 - 23t^5 + 38t^4 - 56t^3 + 60t^2 - 68t + 64$	2, 3 / ✘
	10^a_{48}	$t^4 - 3t^3 + 6t^2 - 9t + 11$	4 / ✔	$t^5 - 2t^4 + 2t^3 - 3t + 4$	2 / ✘
	10^a_{49}	$3t^3 - 8t^2 + 12t - 13$	3 / ✘	$30t^5 - 94t^4 + 196t^3 - 292t^2 + 372t - 392$	3 / ✘
	10^a_{50}	$-2t^3 + 7t^2 - 11t + 13$	3 / ✘	$-9t^5 + 44t^4 - 94t^3 + 150t^2 - 186t + 200$	2 / ✘
	10^a_{51}	$2t^3 - 7t^2 + 15t - 19$	3 / ✘	$-5t^5 + 24t^4 - 73t^3 + 134t^2 - 194t + 212$	2, 3 / ✘
	10^a_{52}	$2t^3 - 7t^2 + 13t - 15$	3 / ✘	$-3t^5 + 16t^4 - 37t^3 + 50t^2 - 49t + 44$	2 / ✘
	10^a_{53}	$6t^2 - 18t + 25$	2 / ✘	$93t^3 - 346t^2 + 680t - 828$	2, 3 / ✘
	10^a_{54}	$2t^3 - 6t^2 + 10t - 11$	3 / ✘	$-3t^5 + 12t^4 - 24t^3 + 26t^2 - 21t + 16$	2, 3 / ✘
	10^a_{55}	$5t^2 - 15t + 21$	2 / ✘	$66t^3 - 246t^2 + 488t - 596$	2 / ✘
	10^a_{56}	$-2t^3 + 8t^2 - 14t + 17$	3 / ✘	$-9t^5 + 52t^4 - 133t^3 + 234t^2 - 312t + 340$	2 / ✘
	10^a_{57}	$2t^3 - 8t^2 + 18t - 23$	3 / ✘	$-5t^5 + 28t^4 - 93t^3 + 194t^2 - 300t + 340$	2 / ✘
	10^a_{58}	$3t^2 - 16t + 27$	2 / ✘	$3t^3 - 28t^2 + 94t - 140$	2 / ✘
	10^a_{59}	$t^3 - 7t^2 + 18t - 23$	3 / ✘	$-t^5 + 12t^4 - 55t^3 + 128t^2 - 181t + 196$	1 / ✘
	10^a_{60}	$-t^3 + 7t^2 - 20t + 29$	3 / ✘	$5t^3 - 40t^2 + 122t - 176$	1 / ✘
	10^a_{61}	$-2t^3 + 5t^2 - 6t + 7$	3 / ✘	$-7t^5 + 20t^4 - 27t^3 + 36t^2 - 35t + 36$	2, 3 / ✘
	10^a_{62}	$t^4 - 3t^3 + 6t^2 - 8t + 9$	4 / ✘	$-2t^7 + 8t^6 - 23t^5 + 40t^4 - 63t^3 + 76t^2 - 89t + 88$	2 / ✘
	10^a_{63}	$5t^2 - 14t + 19$	2 / ✘	$66t^3 - 220t^2 + 416t - 496$	2 / ✘
	10^a_{64}	$-t^4 + 3t^3 - 6t^2 + 10t - 11$	4 / ✘	$-t^7 + 4t^6 - 11t^5 + 24t^4 - 37t^3 + 52t^2 - 60t + 64$	2 / ✘
	10^a_{65}	$2t^3 - 7t^2 + 14t - 17$	3 / ✘	$-5t^5 + 24t^4 - 71t^3 + 124t^2 - 169t + 180$	2 / ✘
	10^a_{66}	$3t^3 - 9t^2 + 16t - 19$	3 / ✘	$30t^5 - 112t^4 + 279t^3 - 480t^2 + 662t - 724$	3 / ✘
	10^a_{67}	$-4t^2 + 16t - 23$	2 / ✘	$24t^3 - 140t^2 + 312t - 392$	2 / ✘
	10^a_{68}	$4t^2 - 14t + 21$	2 / ✘	$8t^3 - 40t^2 + 117t - 164$	2 / ✘
	10^a_{69}	$t^3 - 7t^2 + 21t - 29$	3 / ✘	$-t^5 + 12t^4 - 68t^3 + 212t^2 - 397t + 476$	2 / ✘
	10^a_{70}	$t^3 - 7t^2 + 16t - 19$	3 / ✘	$-t^5 + 12t^4 - 53t^3 + 114t^2 - 146t + 152$	2 / ✘
	10^a_{71}	$-t^3 + 7t^2 - 18t + 25$	3 / ✘	$t^3 - 2t^2 - t + 4$	1 / ✘
	10^a_{72}	$-2t^3 + 9t^2 - 16t + 19$	3 / ✘	$-9t^5 + 60t^4 - 167t^3 + 298t^2 - 410t + 448$	2 / ✘
	10^a_{73}	$t^3 - 7t^2 + 20t - 27$	3 / ✘	$t^5 - 12t^4 + 65t^3 - 194t^2 + 350t - 416$	1 / ✘
	10^a_{74}	$-4t^2 + 16t - 23$	2 / ✘	$24t^3 - 136t^2 + 290t - 360$	2 / ✘
	10^a_{75}	$-t^3 + 7t^2 - 19t + 27$	3 / ✔	$-4t^3 + 36t^2 - 117t + 172$	2 / ✘
	10^a_{76}	$-2t^3 + 7t^2 - 12t + 15$	3 / ✘	$-9t^5 + 44t^4 - 104t^3 + 184t^2 - 245t + 272$	2, 3 / ✘
	10^a_{77}	$2t^3 - 7t^2 + 14t - 17$	3 / ✘	$-5t^5 + 24t^4 - 71t^3 + 132t^2 - 189t + 208$	2, 3 / ✘
	10^a_{78}	$-t^3 + 7t^2 - 16t + 21$	3 / ✘	$2t^5 - 24t^4 + 105t^3 - 244t^2 + 390t - 448$	2 / ✘
	10^a_{79}	$t^4 - 3t^3 + 7t^2 - 12t + 15$	4 / ✘	0	2, 3 / ✔
	10^a_{80}	$3t^3 - 9t^2 + 15t - 17$	3 / ✘	$30t^5 - 112t^4 + 260t^3 - 426t^2 + 568t - 616$	3 / ✘
	10^a_{81}	$-t^3 + 8t^2 - 20t + 27$	3 / ✘	0	2 / ✔
	10^a_{82}	$-t^4 + 4t^3 - 8t^2 + 12t - 13$	4 / ✘	$t^7 - 6t^6 + 19t^5 - 42t^4 + 64t^3 - 78t^2 + 84t - 84$	1 / ✘
	10^a_{83}	$2t^3 - 9t^2 + 19t - 23$	3 / ✘	$-5t^5 + 34t^4 - 110t^3 + 214t^2 - 301t + 332$	2 / ✘
	10^a_{84}	$2t^3 - 9t^2 + 20t - 25$	3 / ✘	$-5t^5 + 34t^4 - 116t^3 + 246t^2 - 373t + 424$	1 / ✘
	10^a_{85}	$t^4 - 4t^3 + 8t^2 - 10t + 11$	4 / ✘	$2t^7 - 12t^6 + 36t^5 - 68t^4 + 101t^3 - 124t^2 + 138t - 140$	2 / ✘
	10^a_{86}	$-2t^3 + 9t^2 - 19t + 25$	3 / ✘	$-t^5 + 6t^4 - 21t^3 + 58t^2 - 105t + 128$	2 / ✘
	10^a_{87}	$-2t^3 + 9t^2 - 18t + 23$	3 / ✔	$-t^5 + 6t^4 - 23t^3 + 66t^2 - 125t + 152$	2 / ✘
	10^a_{88}	$-t^3 + 8t^2 - 24t + 35$	3 / ✘	0	1 / ✔
	10^a_{89}	$t^3 - 8t^2 + 24t - 33$	3 / ✘	$t^5 - 14t^4 + 83t^3 - 264t^2 + 495t - 596$	2 / ✘
	10^a_{90}	$-2t^3 + 8t^2 - 17t + 23$	3 / ✘	$-t^5 + 6t^4 - 21t^3 + 54t^2 - 93t + 112$	2 / ✘
	10^a_{91}	$t^4 - 4t^3 + 9t^2 - 14t + 17$	4 / ✘	$t^5 - 2t^4 + 2t^3 - 3t + 4$	1 / ✘
	10^a_{92}	$-2t^3 + 10t^2 - 20t + 25$	3 / ✘	$-9t^5 + 68t^4 - 216t^3 + 428t^2 - 622t + 696$	2 / ✘
	10^a_{93}	$2t^3 - 8t^2 + 15t - 17$	3 / ✘	$3t^5 - 18t^4 + 43t^3 - 58t^2 + 55t - 48$	2 / ✘
	10^a_{94}	$-t^4 + 4t^3 - 9t^2 + 14t - 15$	4 / ✘	$-t^7 + 6t^6 - 20t^5 + 46t^4 - 76t^3 + 102t^2 - 115t + 120$	2 / ✘
	10^a_{95}	$2t^3 - 9t^2 + 21t - 27$	3 / ✘	$-5t^5 + 32t^4 - 114t^3 + 248t^2 - 384t + 436$	1 / ✘
	10^a_{96}	$-t^3 + 7t^2 - 22t + 33$	3 / ✘	$-7t^3 + 50t^2 - 147t + 212$	2 / ✘
	10^a_{97}	$-5t^2 + 22t - 33$	2 / ✘	$-37t^3 + 242t^2 - 603t + 788$	2 / ✘
	10^a_{98}	$-2t^3 + 9t^2 - 18t + 23$	3 / ✘	$9t^5 - 60t^4 + 177t^3 - 348t^2 + 501t - 564$	2 / ✘
	10^a_{99}	$t^4 - 4t^3 + 10t^2 - 16t + 19$	4 / ✔	0	2 / ✔
	10^a_{100}	$t^4 - 4t^3 + 9t^2 - 12t + 13$	4 / ✘	$2t^7 - 12t^6 + 39t^5 - 80t^4 + 128t^3 - 164t^2 + 192t - 196$	2, 3 / ✘
	10^a_{101}	$7t^2 - 21t + 29$	2 / ✘	$-129t^3 + 480t^2 - 942t + 1148$	2, 3 / ✘
	10^a_{102}	$-2t^3 + 8t^2 - 16t + 21$	3 / ✘	$-t^5 + 6t^4 - 19t^3 + 50t^2 - 89t + 108$	1 / ✘
	10^a_{103}	$2t^3 - 8t^2 + 17t - 21$	3 / ✘	$5t^5 - 30t^4 + 93t^3 - 178t^2 + 254t - 280$	3 / ✘
	10^a_{104}	$t^4 - 4t^3 + 9t^2 - 15t + 19$	4 / ✘	$t^5 - 2t^4 + 2t^3 - 3t + 4$	1 / ✘
	10^a_{105}	$t^3 - 8t^2 + 22t - 29$	3 / ✘	$-t^5 + 14t^4 - 71t^3 + 184t^2 - 292t + 332$	2 / ✘

diagram	n_k^t	Alexander's ω^+ / Today's / Rozansky's ρ_1^+	genus / ribbon	unknotting number / amphicheiral
	10^a_{106}	$-t^4+4t^3-9t^2+15t-17$ / $-t^7+6t^6-20t^5+48t^4-82t^3+114t^2-134t+140$	4 / ✘	2 / ✘
	10^a_{107}	$-t^3+8t^2-22t+31$ / $2t^3-8t^2+13t-16$	3 / ✘	1 / ✘
	10^a_{108}	$2t^3-8t^2+14t-15$ / $-3t^5+18t^4-41t^3+50t^2-40t+32$	3 / ✘	2 / ✘
	10^a_{109}	$t^4-4t^3+10t^2-17t+21$ / 0	4 / ✘	2 / ✔
	10^a_{110}	$t^3-8t^2+20t-25$ / $t^5-14t^4+69t^3-160t^2+219t-236$	3 / ✘	2 / ✘
	10^a_{111}	$-2t^3+9t^2-17t+21$ / $-9t^5+60t^4-171t^3+316t^2-436t+480$	3 / ✘	2 / ✘
	10^a_{112}	$-t^4+5t^3-11t^2+17t-19$ / $t^7-8t^6+29t^5-68t^4+115t^3-152t^2+175t-180$	4 / ✘	2 / ✘
	10^a_{113}	$2t^3-11t^2+26t-33$ / $-5t^5+42t^4-167t^3+394t^2-623t+720$	3 / ✘	1 / ✘
	10^a_{114}	$-2t^3+10t^2-21t+27$ / $t^5-8t^4+30t^3-78t^2+140t-168$	3 / ✘	1 / ✘
	10^a_{115}	$-t^3+9t^2-26t+37$ / 0	3 / ✘	2 / ✔
	10^a_{116}	$-t^4+5t^3-12t^2+19t-21$ / $t^7-8t^6+30t^5-74t^4+132t^3-184t^2+217t-228$	4 / ✘	2 / ✘
	10^a_{117}	$2t^3-10t^2+24t-31$ / $-5t^5+38t^4-144t^3+330t^2-522t+600$	3 / ✘	2 / ✘
	10^a_{118}	$t^4-5t^3+12t^2-19t+23$ / 0	4 / ✘	1 / ✔
	10^a_{119}	$-2t^3+10t^2-23t+31$ / $-t^5+6t^4-26t^3+86t^2-175t+220$	3 / ✘	1 / ✘
	10^a_{120}	$8t^2-26t+37$ / $166t^3-692t^2+1433t-1788$	2 / ✘	2, 3 / ✘
	10^a_{121}	$2t^3-11t^2+27t-35$ / $5t^5-42t^4+167t^3-396t^2+634t-732$	3 / ✘	2 / ✘
	10^a_{122}	$-2t^3+11t^2-24t+31$ / $-t^5+8t^4-34t^3+104t^2-211t+264$	3 / ✘	2 / ✘
	10^a_{123}	$t^4-6t^3+15t^2-24t+29$ / 0	4 / ✔	2 / ✔
	10^n_{124}	t^4-t^3+t-1 / $-4t^7-6t^4-4t^2-6t$	4 / ✘	4 / ✘
	10^n_{125}	t^3-2t^2+2t-1 / $-t^5+2t^4-2t^3+3t-4$	3 / ✘	2 / ✘
	10^n_{126}	t^3-2t^2+4t-5 / $t^5-2t^4+10t^3-12t^2+22t-20$	3 / ✘	2 / ✘
	10^n_{127}	$-t^3+4t^2-6t+7$ / $2t^5-14t^4+32t^3-52t^2+67t-72$	3 / ✘	2 / ✘
	10^n_{128}	$2t^3-3t^2+t+1$ / $-13t^5+12t^4-3t^3-10t^2-9t+12$	3 / ✘	3 / ✘
	10^n_{129}	$2t^2-6t+9$ / $-t^3-2t^2+14t-20$	2 / ✔	1 / ✘
	10^n_{130}	$2t^2-4t+5$ / $t^3-2t^2+19t-24$	2 / ✘	2 / ✘
	10^n_{131}	$-2t^2+8t-11$ / $5t^3-38t^2+87t-112$	2 / ✘	1 / ✘
	10^n_{132}	t^2-t+1 / $2t^2+5t-4$	2 / ✘	1 / ✘
	10^n_{133}	$-t^2+5t-7$ / $t^3-14t^2+37t-48$	2 / ✘	1 / ✘
	10^n_{134}	$2t^3-4t^2+4t-3$ / $-13t^5+24t^4-33t^3+30t^2-41t+40$	3 / ✘	3 / ✘
	10^n_{135}	$3t^2-9t+13$ / $t^3-6t^2+18t-24$	2 / ✘	2 / ✘
	10^n_{136}	$-t^2+4t-5$ / $-t^3+4t^2-2t-4$	2 / ✘	1 / ✘
	10^n_{137}	$t^2-6t+11$ / $-4t^2+24t-44$	2 / ✔	1 / ✘
	10^n_{138}	t^3-5t^2+8t-7 / $-t^5+8t^4-22t^3+24t^2-11t+8$	3 / ✘	2 / ✘
	10^n_{139}	t^4-t^3+2t-3 / $-4t^7-12t^4+5t^3-4t^2-16t+12$	4 / ✘	4 / ✘
	10^n_{140}	t^2-2t+3 / $8t-8$	2 / ✔	2 / ✘
	10^n_{141}	$-t^3+3t^2-4t+5$ / $t^3-8t^2+16t-20$	3 / ✘	1 / ✘
	10^n_{142}	$2t^3-3t^2+2t-1$ / $-13t^5+12t^4-13t^3+4t^2-17t+12$	3 / ✘	3 / ✘
	10^n_{143}	t^3-3t^2+6t-7 / $t^5-4t^4+15t^3-28t^2+45t-48$	3 / ✘	1 / ✘
	10^n_{144}	$-3t^2+10t-13$ / $10t^3-44t^2+80t-96$	2 / ✘	2 / ✘
	10^n_{145}	t^2+t-3 / $2t^3+8t^2+6t-8$	2 / ✘	2 / ✘
	10^n_{146}	$2t^2-8t+13$ / $t^3-8t^2+21t-28$	2 / ✘	1 / ✘
	10^n_{147}	$-2t^2+7t-9$ / $-3t^3+12t^2-15t+12$	2 / ✘	1 / ✘
	10^n_{148}	t^3-3t^2+7t-9 / $t^5-4t^4+18t^3-36t^2+62t-68$	3 / ✘	2 / ✘
	10^n_{149}	$-t^3+5t^2-9t+11$ / $2t^5-18t^4+55t^3-104t^2+149t-164$	3 / ✘	2 / ✘
	10^n_{150}	$-t^3+4t^2-6t+7$ / $-2t^5+12t^4-26t^3+38t^2-45t+44$	3 / ✘	2 / ✘
	10^n_{151}	$t^3-4t^2+10t-13$ / $-t^5+6t^4-21t^3+42t^2-66t+72$	3 / ✘	2 / ✘
	10^n_{152}	$t^4-t^3-t^2+4t-5$ / $4t^7-7t^5+18t^4-7t^3-12t^2+45t-52$	4 / ✘	4 / ✘
	10^n_{153}	t^3-t^2-t+3 / $t^5-2t^4+t^3+2t^2-t$	3 / ✔	2 / ✘
	10^n_{154}	t^3-4t+7 / $-3t^5-6t^4+13t^3-47t+68$	3 / ✘	3 / ✘
	10^n_{155}	$-t^3+3t^2-5t+7$ / $-2t^3+12t^2-22t+28$	3 / ✔	2 / ✘
	10^n_{156}	t^3-4t^2+8t-9 / $t^5-6t^4+19t^3-30t^2+33t-32$	3 / ✘	1 / ✘
	10^n_{157}	$-t^3+6t^2-11t+13$ / $-2t^5+22t^4-78t^3+148t^2-218t+240$	3 / ✘	2 / ✘
	10^n_{158}	$-t^3+4t^2-10t+15$ / $2t^2-7t+12$	3 / ✘	2 / ✘
	10^n_{159}	$t^3-4t^2+9t-11$ / $t^5-6t^4+26t^3-60t^2+98t-112$	3 / ✘	1 / ✘
	10^n_{160}	$-t^3+4t^2-4t+3$ / $-2t^5+12t^4-20t^3+14t^2-16t+12$	3 / ✘	2 / ✘
	10^n_{161}	t^3-2t+3 / $3t^5+6t^4-3t^3+4t^2+14t-12$	3 / ✘	3 / ✘
	10^n_{162}	$-3t^2+9t-11$ / $10t^3-38t^2+58t-68$	2 / ✘	2 / ✘
	10^n_{163}	$t^3-5t^2+12t-15$ / $-t^5+8t^4-30t^3+62t^2-89t+96$	3 / ✘	1, 2 / ✘
	10^n_{164}	$3t^2-11t+17$ / $t^3-10t^2+29t-40$	2 / ✘	1 / ✘
	10^n_{165}	$-2t^2+10t-15$ / $-5t^3+50t^2-146t+196$	2 / ✘	2 / ✘