| Dror Bar-Natan: Talks: Pitzer-250308: ωεβ:=http://drorbn.net/pi25 | Dream. There is a similar perturbed Gaussian integral formula for θ , but with integration over $6H_1(\Sigma)$. The quadratic Q will |
|---|--|
| Abstract Given a knot K with a Seifert surface Σ I dream | be the same as in the Seifert-Alexander formula (but repeated 3 |
| that the well-known Seifert linking form Q a quadratic form on | times, for each T_{y}). The perturbation P_{ϵ} will be given by low- |
| $H_1(\Sigma)$ has plenty docide local perturbations P such that the for- | degree finite type invariants of curves on Σ (possibly also depen- |
| mal Gaussian integrals of $exp(O \pm P)$ are invariants of K | dent on the intersection points of such curves, or on other infor- |
| In my talk I will explain what the above means, why this dream | mation coming from Σ). |
| is ob so sweet, and why it is in fact closer to a plan than to a | Evidence. Experimentally (vet undeniably), deg θ is bounded by |
| delusion | the genus of Σ . How else could such a genus bound arise? Further |
| delusion. Joint with Roland van del Veen. | very strong evidence comes from the conjectural (vet undenjable) |
| The Seifert-Alexander Formula. With | understanding of θ as the two-loop contribution to the Kontsevich |
| $P, Q \in H_1(\Sigma),$ | integral [Oh] and/or as the "solvable approximation" of the uni- |
| $Q(P,G) = T^{1/2} lk(P^+,G) - T^{-1/2} lk(P,G^+)$ | versal <i>sl</i> ₃ invariant [BN1, BV2]. |
| $\Delta(K) = \det(Q)$ | Why so sweet? It will allow us to prove the aforementioned ge- |
| $\int dn dx \exp Q(n x) \doteq \det(Q)^{-1}$ | nus bound and likely, the hexagonal symmetry. Sweeter and dre- |
| $\int_{2H_1(\Sigma)}^{u_P u_R \text{ only } (p, u)} d\theta d(\underline{v})$ | amier, it may allow us to say something about ribbon knots! |
| (where = means "ignoring silly factors"). | |
| Perturbed Gaussian Integration. We say | |
| that $P_{\epsilon} \in \epsilon \mathbb{Q}[x_1, \dots, x_n][\epsilon]$ is <i>M</i> -docile (for | |
| some $M: \mathbb{N} \to \mathbb{N}$) if for every monomial m From Mexico City, tariffs exemption | |
| in P_{ϵ} we have $\deg_{x_1,\dots,x_n}(m) \leq M(\deg_{\epsilon}(m))$. | |
| Theorem (Feynman). If Q is a quadratic in x_1, \ldots, x_n and P_{ϵ} is | What's "local"? How will we compute? The Bedlewo Alexan- |
| docile, set $Z_{\epsilon} = \int_{\mathbb{R}^n} dx_1 \cdots x_n \exp(Q + P_{\epsilon})$. Then every coeffi- | der formula: Let F be the faces of a knot diagram. Make an $F \times F$ |
| cient in the ϵ -expansion of Z_{ϵ} is computable in polynomial time | matrix A by adding for each crossing contributions |
| in <i>n</i> . in fact, (0^{-1}) (0^{-1}) | $_{k}$ $\begin{pmatrix} -1 & -1 & 2 & 0 \end{pmatrix}$ $_{k}$ $\begin{pmatrix} 1 & -1 & 0 & 0 \end{pmatrix}$ |
| $\Lambda^{1/2} Z_{\epsilon} \doteq \left(\exp Q^{-1}(\partial_{x}), \exp P_{\epsilon} \right) = \qquad $ | $\begin{bmatrix} \mathbf{n} & \mathbf{n} \\ \mathbf{n} \\ \mathbf{n} & \mathbf{n} \\ \mathbf{n} $ |
| $\epsilon \left(\frac{1}{2} - \frac{1}{2$ | l / j / 0 = 1 - 10 $l / j / -2 = 1 = 10$ |
| | $(1 \ 0 \ -1 \ 0)$ $(1 \ 0 \ -1 \ 0)$ |
| $\theta(T, 1)$ is like that! With $\epsilon^2 = 0$, P_{ϵ} P_{ϵ} | at rows / columns (i i k l) Then $\Lambda = \det' \left((T^{1/2}A - T^{-1/2}A)/2 \right)$ |
| | (1, 1, 1, 1) |
| | |
| $\mathbb{R}^2_{p_{4}x_4} \longrightarrow Z \doteq \oint_{\mathcal{T}_{F}} \underbrace{\mathcal{L}(X^+_{15})}_{\mathcal{L}(X^+_{62})} \underbrace{\mathcal{L}(X^+_{37})}_{\mathcal{L}(C^{-1}_{4})} $ | |
| $Z \doteq \oint_{2E = \mathbb{R}^{14}_{p_{1}x_{1}}} (X_{15}^{+}) \mathcal{L}(X_{62}^{+}) \mathcal{L}(C_{4}^{-1})$ | $ \longrightarrow \longrightarrow$ |
| $Z \doteq \oint_{2E = \mathbb{R}_{p_i x_i}^{14}} \underbrace{\mathbb{L}(X_{15}^+) \mathcal{L}(X_{62}^+) \mathcal{L}(X_{37}^+) \mathcal{L}(C_4^{-1})}_{\mathbb{I}}$ where $\mathcal{L}(X_{ij}^s) \doteq e^{\mathcal{L}(X_{ij}^s)}, \mathcal{L}(C_i^{\varphi}) \doteq e^{\mathcal{L}(C_i^{\varphi})},$ | $(\text{the Seifert algorithm by Emily Redelmeier}) \xrightarrow{\text{(H)}} \xrightarrow{(H)}} \xrightarrow{\text{(H)}} \xrightarrow{\text{(H)}} \xrightarrow{\text{(H)}} \xrightarrow{\text{(H)}} \xrightarrow{\text{(H)}} \xrightarrow{\text{(H)}} \xrightarrow{(H)}} \xrightarrow{(H)} \xrightarrow{(H)} \xrightarrow{(H)}} \xrightarrow{(H)} \xrightarrow{(H)} \xrightarrow{(H)} \xrightarrow{(H)}} \xrightarrow{(H)} \xrightarrow{(H)} \xrightarrow{(H)} \xrightarrow{(H)} \xrightarrow{(H)} \xrightarrow{(H)} \xrightarrow{(H)} \xrightarrow{(H)} \xrightarrow{(H)} \xrightarrow{(H)}} \xrightarrow{(H)} ($ |
| $Z \doteq \oint_{2E = \mathbb{R}^{14}_{p_i x_i}} \mathcal{L}(X^+_{15}) \mathcal{L}(X^+_{37}) \mathcal{L}(C^{-1}_{4})$ where $\mathcal{L}(X^s_{ij}) \doteq e^{\mathcal{L}(X^s_{ij})}, \mathcal{L}(C^{\varphi}_i) \doteq e^{\mathcal{L}(C^{\varphi}_i)},$ $\mathcal{L}(X^s_{ij}) = x_i(p_{i+1} - p_i) + x_j(p_{j+1} - p_j)$ | the Seifert algorithm by Emily Redelmeier) Expect the like for θ ! Expect more like θ ! Topology first! Resist |
| $Z \doteq \oint_{2E = \mathbb{R}^{14}_{p_{1}x_{1}}} \mathcal{L}(X_{5}^{+}) \mathcal{L}(X_{62}^{+}) \mathcal{L}(X_{37}^{+}) \mathcal{L}(C_{4}^{-1})$ where $\mathcal{L}(X_{ij}^{s}) \doteq e^{\mathcal{L}(X_{ij}^{s})}, \mathcal{L}(C_{i}^{\varphi}) \doteq e^{\mathcal{L}(C_{i}^{\varphi})},$ $\mathcal{L}(X_{ij}^{s}) = x_{i}(p_{i+1} - p_{i}) + x_{j}(p_{j+1} - p_{j})$ $+ (T^{s} - 1)x_{i}(p_{i+1} - p_{i+1})$ | (the Seifert algorithm by Emily Redelmeier) Expect the like for θ ! Expect more like θ ! Topology first! Resist the tyranny of quantum algebra! |
| $Z \doteq \oint_{2E = \mathbb{R}_{p_{i}x_{i}}^{l_{4}}} \underbrace{\mathcal{L}(X_{15}^{+}) \mathcal{L}(X_{62}^{+}) \mathcal{L}(X_{37}^{+}) \mathcal{L}(C_{4}^{-1})}_{\mathbb{R}_{p_{3}x_{3}}^{2}} \xrightarrow{\mathcal{L}(C_{4}^{-1})}_{\mathbb{R}_{p_{7}x_{7}}^{2}} \qquad $ | (the Seifert algorithm by Emily Redelmeier) Expect the like for θ ! Expect more like θ ! Topology first! Resist the tyranny of quantum algebra! |
| $Z \doteq \oint_{2E = \mathbb{R}^{14}_{p_i x_i}} \mathcal{L}(X^+_{15}) \mathcal{L}(X^+_{62}) \mathcal{L}(X^+_{37}) \mathcal{L}(C^{-1}_{4})$ where $\mathcal{L}(X^s_{ij}) \doteq e^{\mathcal{L}(X^s_{ij})}, \mathcal{L}(C^{\varphi}_i) \doteq e^{\mathcal{L}(C^{\varphi}_i)},$ $\mathcal{L}(X^s_{ij}) = x_i(p_{i+1} - p_i) + x_j(p_{j+1} - p_j) + (T^s - 1)x_i(p_{i+1} - p_{j+1}) + \frac{\epsilon s}{2} \left(x_i(p_i - p_j) \left(\frac{(T^s - 1)x_ip_j}{2} + 2(1 - r_i) \right) - 1 \right) \right)$ | the Seifert algorithm by Emily Redelmeier) Expect the like for θ ! Expect more like θ ! Topology first! Resist the tyranny of quantum algebra! |
| $Z \doteq \oint_{2E = \mathbb{R}_{p_{i}x_{i}}^{14}} \underbrace{\mathcal{L}(X_{15}^{+}) \mathcal{L}(X_{62}^{+}) \mathcal{L}(X_{37}^{+}) \mathcal{L}(C_{4}^{-1})}_{\mathbb{R}_{p_{3}x_{3}}^{2}} \xrightarrow{\mathcal{L}(C_{4}^{-1})}_{\mathbb{R}_{p_{5}x_{6}}^{2}} \xrightarrow{\mathcal{L}(C_{4}^{-1})}_{\mathbb{R}_{p_{5}x_{7}}^{2}} \xrightarrow{\mathcal{L}(C_{4}^{-1})}_{$ | (the Seifert algorithm by Emily Redelmeier) Expect the like for θ ! Expect more like θ ! Topology first! Resist the tyranny of quantum algebra! |
| $Z \doteq \oint_{2E = \mathbb{R}_{p_{i}x_{i}}^{l_{4}}} \underbrace{\mathcal{L}(X_{15}^{+}) \mathcal{L}(X_{62}^{+}) \mathcal{L}(X_{37}^{+}) \mathcal{L}(C_{4}^{-1})}_{\mathbb{R}_{p_{i}x_{i}}^{2}} \\ \text{where } \mathcal{L}(X_{ij}^{s}) \doteq e^{\mathcal{L}(X_{ij}^{s})}, \mathcal{L}(C_{i}^{\varphi}) \doteq e^{\mathcal{L}(C_{i}^{\varphi})}, \\ \mathcal{L}(X_{ij}^{s}) \xrightarrow{3} \mathcal{L}(C_{4}^{-1}) \\ \mathcal{L}(X_{ij}^{s}) \xrightarrow{3} \mathcal{L}(C_{4}^{-1}) \\ \mathbb{R}_{p_{3}x_{3}}^{2} \mathcal{L}(C_{4}^{-1}) \\ \mathbb{R}_{p_{6}x_{6}}^{2} \xrightarrow{3} \mathbb{R}_{p_{7}x_{7}}^{2} \\ \mathbb{R}_{p_{7}x_{7}}^{2} \xrightarrow{3} \mathbb{R}_{p_{7}x_{7}}^{2} \\ \mathbb{R}_{p_{7}x_{7}^{2} \xrightarrow{3} \mathbb{R}_{p_{7}x_{7}}^{2} \\ \mathbb{R}_{p_{7}x_{7}}^{2} \xrightarrow{3} \mathbb{R}_{p_{7}x_{7}}^{2} \\ \mathbb{R}_{p_{7}x_{7}^{2} \xrightarrow{3} \mathbb{R}_{p_{7}x_{7}}^{2} \\ \mathbb{R}_{p_{7}x_{7}^{2} \xrightarrow{3} \mathbb{R}_{p_{7}x_{7}}^{2} \\ \mathbb{R}_{p_{7}x_{7}^{2} \xrightarrow{3} \mathbb{R}_{p_{7}x_{7}}^{2} \\ \mathbb{R}_{p_{7}x_{7}^{2} \xrightarrow{3} \mathbb{R}_{p_{7}x_{7}^{2} \\ \mathbb{R}_{p_{7}x_{7}^{2} \xrightarrow{3} \mathbb{R}_{p_{7}x_{7}^{2} \\ \mathbb{R}_{p_{7}x_{7}^{2} \\$ | $\begin{array}{c} \text{(I + I + I + I)} \\ \text{(I + I + I)} \\ \text{(I + Seifert algorithm by Emily Redelmeier)} \\ \text{Expect the like for } \theta! \\ \text{Expect the like for } \theta! \\ \text{Expect more like } \theta! \\ \text{Topology first! Resist the tyranny of quantum algebra!} \\ \hline \end{array}$ |
| $Z \doteq \oint_{2E = \mathbb{R}_{p_{i}x_{i}}^{14}} \mathcal{L}(X_{15}^{+}) \mathcal{L}(X_{62}^{+}) \mathcal{L}(X_{37}^{+}) \mathcal{L}(C_{4}^{-1})$ where $\mathcal{L}(X_{ij}^{s}) \doteq e^{\mathcal{L}(X_{ij}^{s})}, \mathcal{L}(C_{i}^{\varphi}) \doteq e^{\mathcal{L}(C_{i}^{\varphi})},$ $\mathcal{L}(X_{51}^{*}) \stackrel{3}{=} \int_{\mathbb{R}_{p_{5}x_{5}}}^{\mathbb{R}_{p_{7}x_{7}}} \mathcal{L}(C_{4}^{-1})$ where $\mathcal{L}(X_{ij}^{s}) \doteq e^{\mathcal{L}(X_{ij}^{s})}, \mathcal{L}(C_{i}^{\varphi}) \doteq e^{\mathcal{L}(C_{i}^{\varphi})},$ $\mathcal{L}(X_{52}^{*}) \stackrel{3}{=} \int_{\mathbb{R}_{p_{5}x_{5}}}^{\mathbb{R}_{p_{7}x_{7}}} \mathcal{L}(C_{4}^{-1})$ $+ \frac{\epsilon_{s}}{2} \left(x_{i}(p_{i} - p_{j}) \left(\frac{(T^{s} - 1)x_{i}p_{j}}{+2(1 - x_{j}p_{j})} \right) - 1 \right)$ $\mathcal{L}(C_{i}^{\varphi}) = x_{i}(p_{i+1} - p_{i}) + \epsilon\varphi(1/2 - x_{i}p_{i})$ $\theta(T_{1}, T_{2})$ is likewise, with harder formulas | $ \begin{array}{c} \text{ (If } I = I = I) \\ \text{ (If } I = I) \\ \text$ |
| $Z \doteq \oint_{2E = \mathbb{R}_{p_{1}x_{i}}^{14}} \underbrace{\mathcal{L}(X_{15}^{+}) \mathcal{L}(X_{62}^{+}) \mathcal{L}(X_{37}^{+}) \mathcal{L}(C_{4}^{-1})}_{\mathbb{R}_{p_{3}x_{3}}^{2}} \xrightarrow{\mathcal{L}(C_{4}^{-1})}_{\mathbb{R}_{p_{3}x_{3}}^{2}} \xrightarrow{\mathcal{L}(C_{4}^{-1})}_{\mathbb{R}_{p_{4}x_{3}}^{2}} \xrightarrow{\mathcal{L}(C_{4}^{-1})}_{\mathbb{R}_{p_{4}x_{3}}^{2}} \xrightarrow{\mathcal{L}(C_{4}^{-1})}_{\mathbb{R}_{p_{4}x_{3}}^{2}} \xrightarrow{\mathcal{L}(C_{4}^{-1})}_{\mathbb{R}_{p_{4}x_{3}}^{2}} \xrightarrow{\mathcal{L}(C_{4}^{-1})}_{\mathbb{R}_{p_{4}x_{3}}^{2}} \xrightarrow{\mathcal{L}(C_{4}^{-1})}_{\mathbb{R}_{p_{4}x_{3}}^{2}} \xrightarrow{\mathcal{L}(C_{4}^{-1})}_{$ | $ \begin{array}{c} (1 \\ (1 \\ (1 \\ (1 \\ (1 \\ (1 \\ (1 \\ (1 $ |
| $Z \doteq \oint_{2E=\mathbb{R}_{p_{i}x_{i}}^{l_{4}}} \mathcal{L}(X_{15}^{+}) \mathcal{L}(X_{62}^{+}) \mathcal{L}(X_{37}^{+}) \mathcal{L}(C_{4}^{-1})}$ where $\mathcal{L}(X_{ij}^{s}) \doteq e^{\mathcal{L}(X_{ij}^{s})}, \mathcal{L}(C_{i}^{\varphi}) \doteq e^{\mathcal{L}(C_{i}^{\varphi})},$ $\mathcal{L}(X_{ij}^{s}) \stackrel{3}{=} \sum_{p_{3}x_{3}} \mathcal{L}(C_{4}^{-1})$ $\mathcal{L}(X_{ij}^{s}) \stackrel{3}{=} x_{i}(p_{i+1} - p_{i}) + x_{j}(p_{j+1} - p_{j})$ $\mathcal{L}(X_{ij}^{s}) \stackrel{3}{=} x_{i}(p_{i+1} - p_{i}) \begin{pmatrix} (T^{s} - 1)x_{i}p_{j} \\ +2(1 - x_{j}p_{j}) \end{pmatrix} - 1 \end{pmatrix}$ $\mathcal{L}(C_{i}^{\varphi}) \stackrel{3}{=} x_{i}(p_{i+1} - p_{i}) + \epsilon\varphi(1/2 - x_{i}p_{i})$ $\theta(T_{1}, T_{2}) \text{ is likewise, with harder formulas and integration over 6E.$ Right. The 132-crossing torus knot $T_{22}x_{i}$ (more at $\varphi_{i}\beta/TK$) | $ \begin{array}{c} \text{ (if } I = I = I) \\ \text{ (if } I = I) \\ \text$ |
| $Z \doteq \oint_{2E=\mathbb{R}_{p_{i}x_{i}}^{14}} \mathcal{L}(X_{15}^{+}) \mathcal{L}(X_{62}^{+}) \mathcal{L}(X_{37}^{+}) \mathcal{L}(C_{4}^{-1})$ where $\mathcal{L}(X_{ij}^{s}) \doteq e^{\mathcal{L}(X_{ij}^{s})}, \mathcal{L}(C_{i}^{\varphi}) \doteq e^{\mathcal{L}(C_{i}^{\varphi})},$ $\mathcal{L}(X_{ij}^{s}) \stackrel{3}{=} \sum_{p_{2}x_{3}} \mathcal{L}(C_{4}^{-1})$ $\mathcal{L}(X_{ij}^{s}) \stackrel{3}{=} \sum_{p_{2}x_{3}} \mathcal{L}(C_{4}^{s}) \stackrel{3}{=} \sum_{p_{2}x_{3}} \mathcal{L}(C_{4}^{s})$ $\mathcal{L}(X_{ij}^{s}) \stackrel{3}{=} \sum_{p_{2}x_{3}} \mathcal{L}(C_{4}^{s}) \stackrel{3}{=} \sum_{p_{2}x_{3}} \mathcal{L}(C_{4}^{s})$ $\mathcal{L}(X_{ij}^{s}) \stackrel{3}{=} \sum_{p_{2}x_{3}} \mathcal{L}(C_{4}^{s}) \stackrel{3}{=} \sum_{p_{2}x_{3}} \mathcal{L}(C_{4}^{s}) \stackrel{3}{=} \sum_{p_{2}x_{3}} \mathcal{L}(C_{1}^{s}) \stackrel{3}{=} \sum_{p_{2}x_{3}} \mathcal{L}(C_{1}^{s})$ | the Seifert algorithm by Emily Redelmeier) Expect the like for θ ! Expect more like θ ! Topology first! Resist the tyranny of quantum algebra! |
| $Z \doteq \oint_{2E = \mathbb{R}_{p_{1}x_{i}}^{14}} \underbrace{\mathcal{L}(X_{15}^{+})}_{2E = \mathbb{R}_{p_{1}x_{i}}^{14}} = e^{L(X_{15}^{+})} \underbrace{\mathcal{L}(X_{37}^{+})}_{2E = \mathbb{R}_{p_{1}x_{i}}^{14}} \\ \text{where } \underbrace{\mathcal{L}(X_{ij}^{s})}_{2E = \mathbb{R}_{p_{1}x_{i}}^{14}} = e^{L(X_{ij}^{\varphi})}, \underbrace{\mathcal{L}(C_{i}^{\varphi})}_{2E = \mathbb{R}_{p_{1}x_{i}}^{14}} \\ \text{where } \underbrace{\mathcal{L}(X_{ij}^{s})}_{2E = \mathbb{R}_{p_{1}x_{i}}^{14}} = e^{L(X_{ij}^{\varphi})}, \underbrace{\mathcal{L}(C_{i}^{\varphi})}_{2E = \mathbb{R}_{p_{1}x_{i}}^{14}} \\ \underbrace{\mathcal{L}(X_{ij}^{s})}_{2E = \mathbb{R}_{p_{1}x_{i}}^{14}} = \underbrace{\mathcal{L}(C_{i}^{\varphi})}_{2E = \mathbb{R}_{p_{1}x_{i}}^{14}} \\ \underbrace{\mathcal{L}(X_{ij}^{s})}_{2E = \mathbb{R}_{p_{1}x_{i}}} \underbrace{\mathcal{L}(C_{i}^{s})}_{2E = \mathbb{R}_{p_{1}x_{i}}^{14}} = \underbrace{\mathcal{L}(C_{i}^{\varphi})}_{2E = \mathbb{R}_{p_{1}x_{i}}^{14}} \\ \underbrace{\mathcal{L}(X_{ij}^{s})}_{2E = \mathbb{R}_{p_{1}x_{i}}} \underbrace{\mathcal{L}(C_{i}^{s})}_{2E = \mathbb{R}_{p_{1}x_{i}}^{14}} \\ \underbrace{\mathcal{L}(X_{ij}^{s})}_{2E = \mathbb{R}_{p_{1}x_{i}}} \underbrace{\mathcal{L}(C_{i}^{s})}_{2E = \mathbb{R}_{p_{1}x_{i}}} = x_{i}(p_{i+1} - p_{i}) + x_{j}(p_{j+1} - p_{j}) \\ + \underbrace{\mathcal{L}(X_{ij}^{s})}_{2E = \mathbb{R}_{p_{2}x_{i}}} \underbrace{\mathcal{L}(X_{ij}^{s})}_{2E = \mathbb{R}_{p_{1}x_{i}}} \underbrace{\mathcal{L}(X_{ij}^{s})}_{2E = \mathbb{R}_{p_{1}x_{i}} \underbrace{\mathcal{L}(X_{ij}^{s})}_{2E = \mathbb{R}_{p_{1}x_{i}} \underbrace{\mathcal{L}(X_{ij}^{s})}_{2E = \mathbb{R}_{p_{1}x_{i}}} \underbrace{\mathcal{L}(X_{$ | $ \begin{array}{c} \text{ (If } I = I = I = I = I = I = I = I = I = I$ |
| $Z \doteq \oint_{2E=\mathbb{R}_{p_{i}x_{i}}^{14}} \mathcal{L}(X_{15}^{+}) \mathcal{L}(X_{62}^{+}) \mathcal{L}(X_{37}^{+}) \mathcal{L}(C_{4}^{-1})$ where $\mathcal{L}(X_{ij}^{s}) \doteq \mathbb{C}^{L(X_{ij}^{s})}, \mathcal{L}(C_{i}^{\varphi}) \doteq \mathbb{C}^{L(C_{i}^{\varphi})}, \mathcal{L}(C_{4}^{\varphi})$ $U(X_{ij}^{s}) = \mathbb{C}^{L(X_{ij}^{s})}, \mathcal{L}(C_{i}^{\varphi}) \doteq \mathbb{C}^{L(C_{i}^{\varphi})}, \mathcal{L}(X_{ij}^{\varphi}) = \mathbb{C}^{L(C_{ij}^{\varphi})}, \mathcal{L}(X_{ij}^{s}) = \mathbb{C}^{L(X_{ij}^{s})}, \mathcal{L}(C_{ij}^{\varphi}) = \mathbb{C}^{L(X_{ij}^{s})}, \mathbb{C}^{L(X_{ij}^{s})} = \mathbb{C}^{L(X_{ij}^{s}$ | the Seifert algorithm by Emily Redelmeier) Expect the like for θ ! Expect more like θ ! Topology first! Resist the tyranny of quantum algebra! |
| $Z = \oint_{2E=\mathbb{R}_{p_{i}x_{i}}} \mathcal{L}(X_{5}^{+}) \mathcal{L}(X_{62}^{+}) \mathcal{L}(X_{37}^{+}) \mathcal{L}(C_{4}^{-1})$ where $\mathcal{L}(X_{ij}^{s}) \doteq e^{\mathcal{L}(X_{ij}^{s})}, \mathcal{L}(C_{i}^{\varphi}) \doteq e^{\mathcal{L}(C_{i}^{\varphi})}, \mathcal{L}(X_{62}^{\varphi}) = e^{\mathcal{L}(C_{i}^{\varphi})}, \mathcal{L}(X_{ij}^{\varphi}) = e^{\mathcal{L}(C_{i}^{\varphi})}, \mathcal{L}(X_{ij}^{s}) = x_{i}(p_{i+1} - p_{i}) + x_{j}(p_{j+1} - p_{j}) + (T^{s} - 1)x_{i}(p_{i+1} - p_{j+1}), \mathcal{L}(X_{ij}^{s}) = x_{i}(p_{i} - p_{j}) \begin{pmatrix} (T^{s} - 1)x_{i}(p_{i+1} - p_{j+1}) + e^{S} \\ \mathcal{L}(X_{ij}^{s}) = x_{i}(p_{i+1} - p_{i}) + e^{S} (1 - x_{j}p_{j}) \end{pmatrix} = 1 \end{pmatrix}$ $\mathcal{L}(C_{i}^{\varphi}) = x_{i}(p_{i+1} - p_{i}) + e^{\varphi(1/2 - x_{i}p_{i})}, \mathcal{L}(C_{i}^{\varphi}) = x_{i}(p_{i+1} - p_{i}) + e^{\varphi(1/2 - x_{i}p_{i})}, \mathcal{L}(C_{i}^{\varphi}) = x_{i}(p_{i+1} - p_{i}) + e^{\varphi(1/2 - x_{i}p_{i})}, \mathcal{L}(T_{i}, T_{2})$ is likewise, with harder formulas and integration over 6E. Right. The 132-crossing torus knot $T_{22/7}$ (more at $\omega \epsilon \beta/TK$). Below. Random knots from [DHOEBL], with 101-115 crossings (more at $\omega \epsilon \beta/DK$). | the Seifert algorithm by Emily Redelmeier) Expect the like for θ ! Expect more like θ ! Topology first! Resist the tyranny of quantum algebra! |
| $ Z \doteq \oint_{2E = \mathbb{R}_{p_{1}x_{1}}^{14}} \mathcal{L}(X_{52}^{+}) \mathcal{L}(X_{52}^{+}) \mathcal{L}(X_{37}^{+}) \mathcal{L}(C_{4}^{-1}) $ where $\mathcal{L}(X_{ij}^{s}) \doteq e^{\mathcal{L}(X_{ij}^{s})}, \mathcal{L}(C_{i}^{\varphi}) \doteq e^{\mathcal{L}(C_{i}^{\varphi})}, $ $ \mathcal{L}(X_{ij}^{s}) = \sum_{i=1}^{k} \sum_{j=1}^{k} \sum_{$ | $ \begin{array}{c} \text{ are (if } if $ |
| $Z \doteq \oint_{2E = \mathbb{R}_{p_{1}x_{1}}^{14}} \mathcal{L}(X_{5}^{+}) \mathcal{L}(X_{62}^{+}) \mathcal{L}(X_{37}^{+}) \mathcal{L}(C_{4}^{-1})$ where $\mathcal{L}(X_{ij}^{s}) \doteq e^{\mathcal{L}(X_{ij}^{s})}, \mathcal{L}(C_{i}^{\varphi}) \doteq e^{\mathcal{L}(C_{i}^{\varphi})}, \mathcal{L}(X_{5}^{\varphi}) = e^{\mathcal{L}(C_{i}^{\varphi})}, \mathcal{L}(X_{5}^{\varphi}) = e^{\mathcal{L}(C_{i}^{\varphi})}, \mathcal{L}(X_{5}^{e}) = e^{\mathcal{L}(Z_{i}^{\varphi})}, \mathcal{L}(X_{5}^{e}) = e^$ | $ \begin{array}{c} \text{ are (if } if $ |
| $Z \doteq \oint_{2E = \mathbb{R}_{p_{2}x_{4}}^{14}} \underbrace{\mathbb{L}(X_{15}^{+})}_{2E = \mathbb{R}_{p_{1}x_{1}}^{14}} = \mathbb{L}(X_{15}^{+}) \underbrace{\mathbb{L}(X_{37}^{+})}_{2E = \mathbb{R}_{p_{1}x_{1}}^{14}} = \mathbb{L}(X_{15}^{0}) \underbrace{\mathbb{L}(X_{15}^{0})}_{2E = \mathbb{R}_{p_{1}x_{1}}^{14}} = \mathbb{L}(X_{15}^{0}) \underbrace{\mathbb{L}(X_{15}^{0})}_{2E = \mathbb{R}_{p_{1}x_{1}}^{14}} = \mathbb{L}(X_{15}^{0}) \underbrace{\mathbb{L}(X_{15}^{0})}_{12E = \mathbb{L}_{p_{1}x_{1}}^{14}} = \mathbb{L}(X_{15}^{0}) \underbrace{\mathbb{L}(X_{15}^{0})}_{12E = \mathbb{L}_{p_{1}x_{1}}^{12}} = \mathbb{L}(X_{15}^{0}) \underbrace{\mathbb{L}(X_{15}^{0})}_{12E = \mathbb{L}(X_{15}^{0})} = \mathbb{L}(X_{15}^{0}) \underbrace{\mathbb{L}$ | $ \begin{array}{c} \text{ are ((1 + 1 + 1))} \\ \text{ be if er algorithm by Emily Redelmeier)} \\ \text{Expect the like for } \theta! \\ \text{Expect the like for } \theta! \\ \text{Expect the like of } \theta! \\ Expe$ |
| $Z \doteq \oint_{2E \in \mathbb{R}_{p_{1}x_{1}}^{l_{4}}} \mathcal{L}(X_{15}^{+}) \mathcal{L}(X_{62}^{+}) \mathcal{L}(X_{37}^{+}) \mathcal{L}(C_{4}^{-1})$ where $\mathcal{L}(X_{ij}^{s}) \doteq \mathbb{C}^{L(X_{ij}^{s})}, \mathcal{L}(C_{i}^{\varphi}) \doteq \mathbb{C}^{L(C_{i}^{\varphi})}, \mathcal{L}(C_{4}^{\varphi}), \mathcal{L}(X_{ij}^{\varphi}) = \mathbb{C}^{L(C_{ij}^{\varphi})}, \mathcal{L}(X_{ij}^{\varphi}) = \mathbb{C}^{L(C_{ij}^{\varphi})}, \mathcal{L}(X_{ij}^{\varphi}) = \mathbb{C}^{L(C_{ij}^{\varphi})}, \mathcal{L}(X_{ij}^{s}) = x_{i}(p_{i+1} - p_{i}) + x_{j}(p_{j+1} - p_{j}) + (T^{s} - 1)x_{i}(p_{i+1} - p_{j+1}), \mathcal{L}(X_{ij}^{s}) = x_{i}(p_{i+1} - p_{i}) \left((T^{s} - 1)x_{i}p_{j} - 1 \right) + \frac{\varepsilon}{2} \left(x_{i}(p_{i} - p_{j}) \left((T^{s} - 1)x_{i}p_{j} - 1 \right) + \mathcal{L}(C_{i}^{\varphi}) = x_{i}(p_{i+1} - p_{i}) + \varepsilon\varphi(1/2 - x_{i}p_{i}) + \mathcal{L}(C_{i}^{\varphi}) = x_{i}(p_{i+1} - p_{i}) + \varepsilon\varphi(1/2 - x_{i}p_{i}) + \mathcal{L}(C_{i}^{\varphi}) = x_{i}(p_{i+1} - p_{i}) + \varepsilon\varphi(1/2 - x_{i}p_{i}) + \mathcal{L}(T_{i}^{s} - 1)x_{i}p_{i} + 2 - x_{i}p_{i}) + \mathcal{L}(T_{i}^{s} - 1)x_{i}p_{i} + 2 - x_{i}p_{i}) + \mathcal{L}(C_{i}^{\varphi}) = x_{i}(p_{i+1} - p_{i}) + \varepsilon\varphi(1/2 - x_{i}p_{i}) + \mathcal{L}(T_{i}^{s} - 1)x_{i}p_{i} + 2 - x_{i}p_{i})$ | the Seifert algorithm by Emily Redelmeier) Expect the like for θ ! Expect more like θ ! Topology first! Resist the tyranny of quantum algebra! |
| $Z \doteq \oint_{2E = \mathbb{R}_{p_{i}x_{i}}^{l_{4}}} \mathcal{L}(X_{5}^{+}) \mathcal{L}(X_{5}^{+}) \mathcal{L}(X_{5}^{+}) \mathcal{L}(C_{4}^{-1})$ where $\mathcal{L}(X_{ij}^{s}) \doteq \mathbb{C}^{L(X_{ij}^{s})}, \mathcal{L}(C_{i}^{\varphi}) \doteq \mathbb{C}^{L(C_{i}^{\varphi})}, \mathcal{L}(C_{4}^{\varphi}), \mathcal{L}(X_{5j}^{\varphi}) = \mathbb{C}^{L(X_{ij}^{s})}, \mathcal{L}(C_{i}^{\varphi}) = \mathbb{C}^{L(C_{i}^{\varphi})}, \mathcal{L}(X_{ij}^{s}) = x_{i}(p_{i+1} - p_{i}) + x_{j}(p_{j+1} - p_{j}) + (T^{s} - 1)x_{i}(p_{i+1} - p_{j+1}), \mathcal{L}(X_{ij}^{s}) = x_{i}(p_{i+1} - p_{i}) + (T^{s} - 1)x_{i}(p_{i+1} - p_{j+1}) + \frac{\epsilon_{s}}{2} \left(x_{i}(p_{i} - p_{j}) \left(\frac{(T^{s} - 1)x_{i}p_{j}}{+2(1 - x_{j}p_{j})}\right) - 1\right) \right)$ $\mathcal{L}(C_{i}^{\varphi}) = x_{i}(p_{i+1} - p_{i}) + \epsilon\varphi(1/2 - x_{i}p_{i}), \mathcal{L}(T_{i}, T_{2})$ is likewise, with harder formulas and integration over $6E$. Right. The 132-crossing torus knot $T_{22/7}$ (more at $\omega \epsilon \beta/TK$). Below. Random knots from [DHOEBL], with 101-115 crossings (more at $\omega \epsilon \beta/DK$). | the seifert algorithm by Emily Redelmeier) Expect the like for θ ! Expect more like θ ! Topology first! Resist the tyranny of quantum algebra! |
| $Z \doteq \oint_{2E=\mathbb{R}_{p_{i}r_{i}}} \mathcal{L}(X_{5}^{+}) \mathcal{L}(X_{6}^{+}) \mathcal{L}(X_{5}^{+}) \mathcal{L}(C_{4}^{-1})$ where $\mathcal{L}(X_{ij}^{s}) \doteq \mathbb{e}^{\mathcal{L}(X_{15}^{s})}, \mathcal{L}(C_{i}^{\varphi}) \doteq \mathbb{e}^{\mathcal{L}(C_{i}^{\varphi})}, \mathcal{L}(C_{4}^{-1})$ $(X_{52}^{+}) \stackrel{\mathcal{L}}{\underset{k}{\overset{p}{\atop}}_{p_{3}x_{3}}} \stackrel{\mathcal{L}(C_{4}^{-1})}{\underset{k}{\overset{p}{\atop}}_{p_{5}x_{5}}} \stackrel{\mathcal{L}(X_{ij}^{s})}{\underset{k}{\overset{p}{\atop}}_{p_{5}x_{5}}} = x_{i}(p_{i+1} - p_{i}) + x_{j}(p_{j+1} - p_{j}) + (T^{s} - 1)x_{i}(p_{i+1} - p_{j+1}))$ $(X_{52}^{+}) \stackrel{\mathcal{L}}{\underset{k}{\overset{p}{\atop}}_{p_{5}x_{5}}} \stackrel{\mathcal{L}}{\underset{k}{\atop}}_{p_{5}x_{5}} \stackrel{\mathcal{L}}{\underset{k}{\overset{p}{\atop}}_{p_{5}x_{5}}} \stackrel{\mathcal{L}}{\underset{k}{\atop}} \stackrel{\mathcal{L}}{\underset{k}{\overset{p}{\atop}}_{p_{5}x_{5}}} \stackrel{\mathcal{L}}{\underset{k}{\atop}} \stackrel{\mathcal{L}}{\underset{k}{\atop}} \stackrel{\mathcal{L}}{\underset{k}{\underset{k}{\atop}}} \stackrel{\mathcal{L}}{\underset{k}{\atop}} \stackrel{\mathcal{L}}{\underset{k}} \stackrel{\mathcal{L}}{\underset{k}{\atop}} \stackrel{\mathcal{L}}}{\underset$ | (the Seifert algorithm by Emily Redelincier) Expect the like for θ ! Expect more like θ ! Topology first! Resist the tyranny of quantum algebra! |
| $Z \doteq \oint_{2E=\mathbb{R}_{p_{i}x_{i}}} \mathcal{L}(X_{15}^{+}) \mathcal{L}(X_{62}^{+}) \mathcal{L}(X_{37}^{+}) \mathcal{L}(C_{4}^{-1})$ where $\mathcal{L}(X_{ij}^{s}) \doteq \mathbb{e}^{\mathcal{L}(X_{ij}^{s})}, \mathcal{L}(C_{i}^{\varphi}) \doteq \mathbb{e}^{\mathcal{L}(C_{i}^{\varphi})}, \mathcal{L}(C_{4}^{-1})$ $U(X_{ij}^{s}) = \mathcal{R}_{p_{3}x_{3}}^{2} \mathcal{L}(C_{4}^{-1})$ $U(X_{ij}^{s}) = \mathcal{R}_{p_{3}x_{3}}^{2} \mathcal{L}(C_{4}^{-1})$ $U(X_{ij}^{s}) = x_{i}(p_{i+1} - p_{i}) + x_{j}(p_{j+1} - p_{j}) + (T^{s} - 1)x_{i}(p_{i+1} - p_{j+1})$ $+ \frac{\epsilon s}{2} \left(x_{i}(p_{i} - p_{j}) \left(\frac{(T^{s} - 1)x_{i}p_{j}}{+2(1 - x_{j}p_{j})} \right) - 1 \right)$ $U(C_{i}^{\varphi}) = x_{i}(p_{i+1} - p_{i}) + \epsilon \varphi(1/2 - x_{i}p_{i})$ $\theta(T_{1}, T_{2}) \text{ is likewise, with harder formulas and integration over 6E.$ Right. The 132-crossing torus knot $T_{22/7}$ (more at $\omega \epsilon \beta/TK$). Below. Random knots from [DHOEBL], with 101-115 crossings (more at $\omega \epsilon \beta/DK$). | the Seifert algorithm by Emily Redelmeier) Expect the like for θ ! Expect more like θ ! Topology first! Resist the tyranny of quantum algebra! |
| $Z \doteq \oint_{2E=\mathbb{R}_{p,x_{1}}} \mathcal{L}(X_{5}^{+}) \mathcal{L}(X_{62}^{+}) \mathcal{L}(X_{57}^{+}) \mathcal{L}(C_{4}^{-1})$ where $\mathcal{L}(X_{ij}^{s}) \doteq e^{\mathcal{L}(X_{ij}^{s})}, \mathcal{L}(C_{i}^{\varphi}) \doteq e^{\mathcal{L}(C_{i}^{\varphi})}, \mathcal{L}(C_{4}^{-1})$ $(X_{ij}^{s}) \stackrel{\mathcal{L}}{\rightarrow} \mathcal{L}(C_{4}^{s}) \stackrel{\mathcal{L}}{\rightarrow} \mathcal{L}(C_{ij}^{s}) \mathcal{$ | the Seifert algorithm by Emily Redelmeier) Expect the like for θ ! Expect more like θ ! Topology first! Resist the tyranny of quantum algebra! |
| $Z = \oint_{2E=\mathbb{R}_{p_{i_{i_{i}}}}^{L}} \mathcal{L}(X_{52}^{+}) \mathcal{L}(X_{52}^{+}) \mathcal{L}(X_{53}^{+}) \mathcal{L}(C_{4}^{-1})}$ where $\mathcal{L}(X_{i_{j}}^{s}) \doteq e^{\mathcal{L}(X_{i_{j}}^{s})} \mathcal{L}(C_{i}^{\phi}) \doteq e^{\mathcal{L}(C_{i}^{\phi})}$, $\mathcal{L}(X_{i_{j}}^{s}) \stackrel{\mathcal{L}}{=} e^{\mathcal{L}(X_{i_{j}}^{s})} \mathcal{L}(C_{i}^{\phi}) = e^{\mathcal{L}(C_{i}^{\phi})}$, $\mathcal{L}(X_{i_{j}}^{s}) \doteq e^{\mathcal{L}(X_{i_{j}}^{s})} \mathcal{L}(C_{i}^{\phi}) = e^{\mathcal{L}(C_{i}^{\phi})}$, $\mathcal{L}(X_{i_{j}}^{s}) \doteq e^{\mathcal{L}(X_{i_{j}}^{s})} \mathcal{L}(C_{i}^{\phi}) = e^{\mathcal{L}(C_{i}^{\phi})}$, $\mathcal{L}(X_{i_{j}}^{s}) = x_{i}(p_{i+1} - p_{i}) + x_{j}(p_{j+1} - p_{j})$, $\mathcal{L}(X_{i_{j}}^{s}) = x_{i}(p_{i+1} - p_{i}) + e\varphi(1/2 - x_{i_{j}}p_{i})$, $\mathcal{L}(C_{i}^{\phi}) = x_{i}(p_{i+1} - p_{i}) + e\varphi(1/2 - x_{i_{j}}p_{i})$, $\mathcal{H}(T_{1}, T_{2})$ is likewise, with harder formulas and integration over 6E. Right. The 132-crossing torus knot $T_{22/7}$ (more at $\omega \epsilon \beta/T K$). Below. Random knots from [DHOEBL], with 101-115 crossings (more at $\omega \epsilon \beta/D K$). | the Seifert algorithm by Emily Redelmeier) Expect the like for θ ! Expect more like θ ! Topology first! Resist the tyranny of quantum algebra! |



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Theorem. With $c = (s, i, j), c_0 = (s_0, i_0, j_0),$ $\uparrow s = 1$ and $c_1 = (s_1, i_1, j_1)$ denoting crossings, there is a quadratic $R_{11}(c) \in \mathbb{Q}(T_{\nu})[g_{\nu\alpha\beta} : \alpha, \beta \in \{i, j\}], (i)$ a cubic $R_{12}(c_0, c_1) \in \mathbb{Q}(T_{\nu})[g_{\nu\alpha\beta} : \alpha, \beta \in \{i_0, j_0, i_1, j_1\}]$, and a **Conjecture 2.** On classical (non-virtual) knots, θ always has helinear $\Gamma_1(\varphi, k)$ such that the following is a knot invariant:



If these pictures remind you of Feynman diagrams, it's because they are Feynman diagrams [BN2]

Lemma 1. The traffic function $g_{\alpha\beta}$ is a "relative invariant":



Lemma 2. With $k^+ := k + 1$, the "g-rules" hold near a crossing c = (s, i, j):

 $g_{j\beta} = g_{j^+\beta} + \delta_{j\beta}$ $g_{i\beta} = T^s g_{i^+\beta} + (1 - T^s) g_{j^+\beta} + \delta_{i\beta}$ $g_{2n^+,\beta} = \delta_{2n^+,\beta}$ $g_{\alpha i^+} = T^s g_{\alpha i} + \delta_{\alpha i^+}$ $g_{\alpha j^+} = g_{\alpha j} + (1 - T^s)g_{\alpha i} + \delta_{\alpha j^+}$ $g_{\alpha,1} = \delta_{\alpha,1}$ **Corollary 1.** *G* is easily computable, for AG = I (= *GA*), with *A* [DHOEBL] N. Dunfield, A. Hirani, M. Obeidin, A. Ehrenberg, S. Bhattacharythe $(2n+1)\times(2n+1)$ identity matrix with additional contributions:

$$c = (s, i, j) \mapsto \frac{A}{\operatorname{row} i} \frac{\operatorname{col} i}{-T^s} \frac{\operatorname{col} j}{T^s - 1}$$

row $i = 0$

For the trefoil example, we have:

$$A = \begin{pmatrix} 1 & -T & 0 & 0 & T - 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -T & 0 & 0 & T - 1 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & T - 1 & 0 & 1 & -T & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \end{pmatrix},$$

$$G = \begin{pmatrix} 1 & T & 1 & T & 1 & T & 1 \\ 0 & 1 & \frac{1}{T^2 - T + 1} & \frac{T}{T^2 - T + 1} & \frac{T}{T^2 - T + 1} & \frac{T^2}{T^2 - T + 1} & 1 \\ 0 & 0 & \frac{1}{T^2 - T + 1} & \frac{T}{T^2 - T + 1} & \frac{T}{T^2 - T + 1} & \frac{T^2}{T^2 - T + 1} & 1 \\ 0 & 0 & \frac{1 - T}{T^2 - T + 1} & \frac{1}{T^2 - T + 1} & \frac{1}{T^2 - T + 1} & \frac{T}{T^2 - T + 1} & 1 \\ 0 & 0 & \frac{1 - T}{T^2 - T + 1} & -\frac{(T - 1)T}{T^2 - T + 1} & \frac{1}{T^2 - T + 1} & \frac{T}{T^2 - T - T + 1} & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Note. The Alexander polynomial Δ is given by

 $\Delta = T^{(-\varphi - w)/2} \det(A),$ with $\varphi = \sum_k \varphi_k, w = \sum_c s$. We also set $\Delta_{\nu} \coloneqq \Delta(T_{\nu})$ for $\nu = 1, 2, 3$.

Questions, Conjectures, Expectations, Dreams.

What's the relationship between Θ and the Question 1. Garoufalidis-Kashaev invariants [GK, GL]?

xagonal (D_6) symmetry.

Conjecture 3. θ is the ϵ^1 contribution to the "solvable approximation" of the *sl*₃ universal invariant, obtained by running the quantization machinery on the double $\mathcal{D}(\mathfrak{b}, b, \epsilon \delta)$, where \mathfrak{b} is the Borel subalgebra of sl_3 , b is the bracket of b, and δ the cobracket. See [BV2, BN1, Sch]

Conjecture 4. θ is equal to the "two-loop contribution to the Kontsevich Integral", as studied by Garoufalidis, Rozansky, Kricker, and in great detail by Ohtsuki [GR, Ro1, Ro2, Ro3, Kr, Oh].

Fact 5. θ has a perturbed Gaussian integral formula, with integration carried out over over a space 6E, consisting of 6 copies of the space of edges of a knot diagram D. See [BN2].

Conjecture 6. For any knot K, its genus g(K) is bounded by the T_1 -degree of θ : MK/~/deg (K)/~ Z (K) > d(K)

Conjecture 7. $\theta(K)$ has another perturbed Gaussian integral formula, with integration carried out over over the space $6H_1$, consisting of 6 copies of $H_1(\Sigma)$, where Σ is a Seifert surface for K.

Expectation 8. There are many further invariants like θ , given by Green function formulas and/or Gaussian integration formulas. One or two of them may be stronger than θ and as computable.

Dream 9. These invariants can be explained by something less foreign than semisimple Lie algebras.

Dream 10. θ will have something to say about ribbon knots.

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rhs = DSum[{1, i, j}, {1, i⁺, k}, {1, j⁺, k⁺}, {s, m, n}] //. $gR_{1,i,j} \cup gR_{1,i^+,k} \cup gR_{1,j^+,k^+}$; Simplify[lhs == rhs] □ True

The Main Program



The Trefoil, Conway, and Kinoshita-Terasaka



(Note that the genus of the Conway knot appears to be bigger than the genus of Kinoshita-Terasaka)

Some Torus Knots

© TKs = {{13, 2}, {17, 3}, {13, 5}, {7, 6}}; GraphicsRow[PolyPlot[@[TorusKnot@@ #]] & /@ TKs] GraphicsRow[TubePlot[TorusKnot@@#] & /@TKs]

