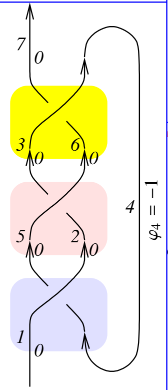
Conventions. T, T_1, and T_2 are indeterminates and $T_3 := T_1T_2$.

Preparation. Draw an n-crossing knot K as a diagram D as on the right: all crossings face up, and the edges are marked with a running index $k \in \{1, \dots, 2n+1\}$ and with rotation numbers φ_k.



Model T Traffic Rules. Cars always drive forward. When a car crosses over a sign-s bridge it goes through with (algebraic) probability $T^s \sim 1$, but falls off with probability $1 - T^s \sim 0$. At the very end, cars fall off and disappear. On various edges *traffic counters* are placed. See also [Jo, LTW].

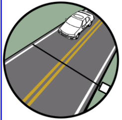


image credits: diamondtraffic.com

$p = 1 - T^s$

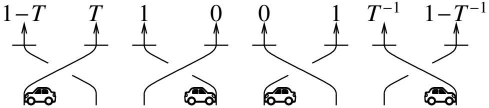
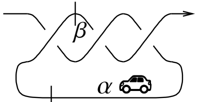


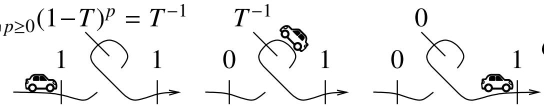
image credits: Dall-E

Definition. The *traffic function* $G = (g_{\alpha\beta})$ (also, the *Green function* or the *two-point function*) is the reading of a traffic counter at β, if car traffic is injected at α (if $\alpha = \beta$, the counter is *after* the injection point). There are also model-T_ν traffic functions $(g_{\nu\alpha\beta})$ for $\nu = 1, 2, 3$.



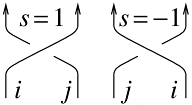
Example.

$\sum_{p\geq 0}(1-T)^p = T^{-1}$

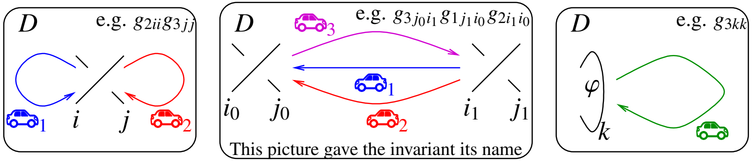


$$G = \begin{pmatrix} 1 & T^{-1} & 1 \\ 0 & T^{-1} & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

Theorem [BV3]. With $c = (s, i, j)$, $c_0 = (s_0, i_0, j_0)$, and $c_1 = (s_1, i_1, j_1)$ denoting crossings, there is a quadratic $F_1(c) \in \mathbb{Q}(T_\nu)[g_{\nu\alpha\beta} : \alpha, \beta \in \{i, j\}]$, a cubic $F_2(c_0, c_1) \in \mathbb{Q}(T_\nu)[g_{\nu\alpha\beta} : \alpha, \beta \in \{i_0, j_0, i_1, j_1\}]$, and a linear $F_3(\varphi, k)$ such that θ is a knot invariant:



$$\theta(D) := \underbrace{\Delta_1\Delta_2\Delta_3}_{\text{normalization, see below}} \left(\sum_c F_1(c) + \sum_{c_0, c_1} F_2(c_0, c_1) + \sum_k F_3(\varphi_k, k) \right),$$



These pictures should remind you of Feynman diagrams!

Δ_ν is the normalized Alexander polynomial at T_ν

F_1, F_2, and F_3 are below