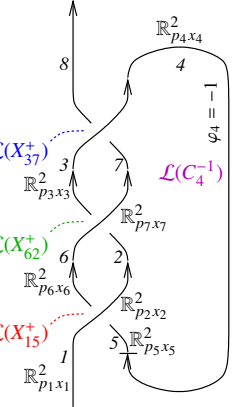


The $sl_2^{\epsilon^2}$ Example. With T an indeterminate and with $\epsilon^2 = 0$:



$$\longrightarrow Z = \oint_{\mathbb{R}_{p_i x_i}^{14} \text{ measure on } \mathbb{R} \text{ is } (2\pi)^{-1/2} \cdot \text{standard}} \mathcal{L}(X_{15}^+) \mathcal{L}(X_{62}^+) \mathcal{L}(X_{37}^+) \mathcal{L}(C_4^{-1})$$

where $\mathcal{L}(X_{ij}^s) = T^{s/2} e^{L(X_{ij}^s)}$ and $\mathcal{L}(C_i^\varphi) = T^{\varphi/2} e^{L(C_i^\varphi)}$, and

$$L(X_{ij}^s) = x_i(p_{i+1} - p_i) + x_j(p_{j+1} - p_j) + (T^s - 1)x_i(p_{i+1} - p_{j+1}) + \frac{\epsilon s}{2} \left(x_i(p_i - p_j) \left(\begin{matrix} (T^s - 1)x_i p_j \\ + 2(1 - x_j p_j) \end{matrix} \right) - 1 \right)$$

$$L(C_i^\varphi) = x_i(p_{i+1} - p_i) + \epsilon \varphi (1/2 - x_i p_i)$$

So $Z = T \oint e^{L(\odot)} dp_1 \dots dp_7 dx_1 \dots dx_7$, where $L(\odot) = \sum_{i=1}^7 x_i(p_{i+1} - p_i) + (T-1)(x_1(p_2 - p_6) + x_6(p_7 - p_3) + x_3(p_4 - p_8)) + \frac{\epsilon}{2} \left(\begin{matrix} x_1(p_1 - p_5) ((T-1)x_1 p_5 + 2(1 - x_5 p_5)) - 1 \\ + x_6(p_6 - p_2) ((T-1)x_6 p_2 + 2(1 - x_2 p_2)) - 1 \\ + x_3(p_3 - p_7) ((T-1)x_3 p_7 + 2(1 - x_7 p_7)) - 1 \\ + 2x_4 p_4 - 1 \end{matrix} \right)$,

and so $Z = (T - 1 + T^{-1})^{-1} \exp\left(\epsilon \cdot \frac{(T-2+T^{-1})(T+T^{-1})}{(T-1+T^{-1})^2}\right) = \Delta^{-1} \exp\left(\epsilon \cdot \frac{(T-2+T^{-1})\rho_1}{\Delta^2}\right)$. Here Δ is the Alexander polynomial and ρ_1 is the Rozansky-Overbay polynomial [Ro1, Ro2, Ro3, Ov, BV1, BV2]. It is a reduction of θ .

The $sl_3^{\epsilon^2}$ Example [BV3]. Here we have two formal variables T_1 and T_2 , we set $T_3 := T_1 T_2$, we integrate over 6 variables for each edge: $p_{1i}, p_{2i}, p_{3i}, x_{1i}, x_{2i}$, and x_{3i} , with the Lagrangian given by:

$$\mathcal{L}[\mathbf{X}_{i,j}, [\mathbf{s}_-]] := T_3^s \mathbb{E}[\text{CF@Plus}[\sum_{v=1}^3 (\mathbf{x}_{vi} (\mathbf{p}_{vi}^+ - \mathbf{p}_{vi}) + \mathbf{x}_{vj} (\mathbf{p}_{vj}^+ - \mathbf{p}_{vj}) + (\mathbf{T}_v^s - 1) \mathbf{x}_{vi} (\mathbf{p}_{vi}^+ - \mathbf{p}_{vj}^+)), (\mathbf{T}_1^s - 1) \mathbf{p}_{3j} \mathbf{x}_{1i} (\mathbf{T}_2^s \mathbf{x}_{2i} - \mathbf{x}_{2j}), \in \mathbf{s} (\mathbf{T}_3^s - 1) \mathbf{p}_{1j} (\mathbf{p}_{2i} - \mathbf{p}_{2j}) \mathbf{x}_{3i} / (\mathbf{T}_2^s - 1), \in \mathbf{s} (1/2 + \mathbf{T}_2^s \mathbf{p}_{1i} \mathbf{p}_{2j} \mathbf{x}_{1i} \mathbf{x}_{2i} - \mathbf{p}_{1i} \mathbf{p}_{2j} \mathbf{x}_{1i} \mathbf{x}_{2j} - \mathbf{p}_{3i} \mathbf{x}_{3i} - (\mathbf{T}_2^s - 1) \mathbf{p}_{2j} \mathbf{p}_{3i} \mathbf{x}_{2i} \mathbf{x}_{3i} + (\mathbf{T}_3^s - 1) \mathbf{p}_{2j} \mathbf{p}_{3j} \mathbf{x}_{2i} \mathbf{x}_{3i} + 2 \mathbf{p}_{2j} \mathbf{p}_{3i} \mathbf{x}_{2j} \mathbf{x}_{3i} + \mathbf{p}_{1i} \mathbf{p}_{3j} \mathbf{x}_{1i} \mathbf{x}_{3j} - \mathbf{p}_{2i} \mathbf{p}_{3j} \mathbf{x}_{2i} \mathbf{x}_{3j} - \mathbf{T}_2^s \mathbf{p}_{2j} \mathbf{p}_{3j} \mathbf{x}_{2i} \mathbf{x}_{3j} + ((\mathbf{T}_1^s - 1) \mathbf{p}_{1j} \mathbf{x}_{1i} (\mathbf{T}_2^{2s} \mathbf{p}_{2j} \mathbf{x}_{2i} - \mathbf{T}_2^s \mathbf{p}_{2j} \mathbf{x}_{2j} - (\mathbf{T}_2^s + 1) (\mathbf{T}_3^s - 1) \mathbf{p}_{3j} \mathbf{x}_{3i} + \mathbf{T}_2^s \mathbf{p}_{3j} \mathbf{x}_{3j}) + (\mathbf{T}_3^s - 1) \mathbf{p}_{3j} \mathbf{x}_{3i} (1 - \mathbf{T}_2^s \mathbf{p}_{1i} \mathbf{x}_{1i} + \mathbf{p}_{2i} \mathbf{x}_{2j} + (\mathbf{T}_2^s - 2) \mathbf{p}_{2j} \mathbf{x}_{2j})) / (\mathbf{T}_2^s - 1)]]]$$

$$\mathcal{L}[\mathbf{C}_i, [\varphi_-]] := T_3^\varphi \mathbb{E}[\sum_{v=1}^3 \mathbf{x}_{vi} (\mathbf{p}_{vi}^+ - \mathbf{p}_{vi}) + \epsilon \varphi (\mathbf{p}_{3i} \mathbf{x}_{3i} - 1/2)]$$

Theorem. Here, $Z = \frac{1}{\Delta_1 \Delta_2 \Delta_3} \exp\left(\epsilon \frac{\theta}{\Delta_1 \Delta_2 \Delta_3}\right)$.