Task. Define $\mathrm{Exp}_{U_i,k}[\xi, P]$ which computes $e^{\xi\,\mathbb{O}(P)}$ to $\epsilon^k$ in the algebra $U_i$, where $\xi$ is a scalar, $X$ is $x_i$ or $y_i$, and $P$ is an $\epsilon$-dependent near-docile element, giving the answer in $\mathbb{E}$-form. Should satisfy

$U$ @ $\mathrm{Exp}_{U_i,k}[\xi, P]$ == $\mathbb{S}_U\left[e^{\xi x},\ x \to \mathbb{O}(P)\right]$.

Methodology. If $P_0 := P_{\epsilon=0}$ and $e^{\xi\,\mathbb{O}(P)} = \mathbb{O}\left(e^{\xi P_0}\,F(\xi)\right)$, then $F(\xi = 0) = 1$ and we have:

$$\mathbb{O}\left(e^{\xi P_0}(P_0\,F(\xi) + \partial_\xi F\right) = \mathbb{O}\left(\partial_\xi e^{\xi P_0}\,F(\xi)\right) = \qquad .$$

$$\partial_\xi \mathbb{O}\left(e^{\xi P_0}\,F(\xi)\right) = \partial_\xi e^{\xi\,\mathbb{O}(P)} = e^{\xi\,\mathbb{O}(P)}\,\mathbb{O}(P) = \mathbb{O}\left(e^{\xi P_0}\,F(\xi)\right)\mathbb{O}(P)$$

This is an ODE for $F$. Setting inductively $F_k = F_{k-1} + \epsilon^k\,\varphi$ we find that $F_0 = 1$ and solve for $\varphi$.