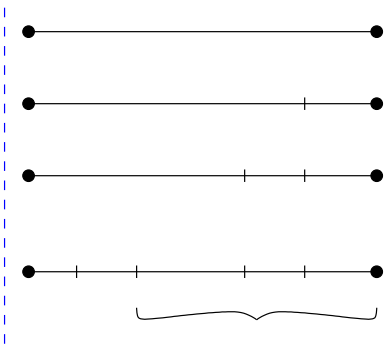

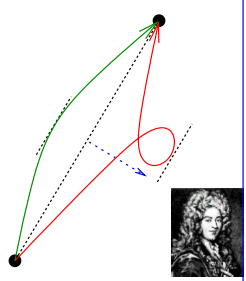

Theorem: $\pi$ is irrational.

Proof: Assume $\pi = a/b$ and consider the polynomial $P(x) = \frac{x^n(a-bx)^n}{n!}$ For $n$ quite large. Clearly $P(x)$ is posi tive yet small, hence $I = \int_0^\pi P(x)\sin x\,dx$ satisfies $0 <$ $I < 1$. On the other hand, repeated integration by parts shows that $I = (\text{boundary terms}) \pm \int P^{(2n+1)}(x)\cos x\,dx$. The second term is 0 because $P$ is a polynomial of degree $2n$, and the first term is an integer for clearly $P^{(k)}(0)$ is always an integer, for $P(\pi - x) = P(x)$ hence same is true for $P^{(k)}(\pi)$ and for sin & cos of 0 & $\pi$ are all integers. Ergo $I$ is an integer between 0 and 1, and these are rare indeed. □