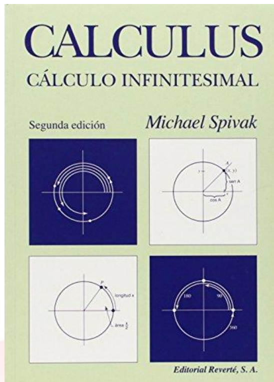
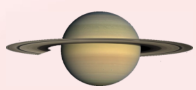
Do Not Turn Over Until Instructed



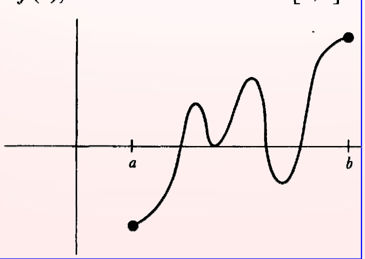


for every $\varepsilon > 0$ there is $\delta > 0$ such that, for all x,
if $0 < |x - a| < \delta$, then $|f(x) - f(a)| < \varepsilon$.

If f and g are continuous at a, then

(1) $f + g$ is continuous at a,
(2) $f \cdot g$ is continuous at a.

If f is continuous on $[a, b]$ and $f(a) < 0 < f(b)$, then there is some x in $[a, b]$ such that $f(x) = 0$.

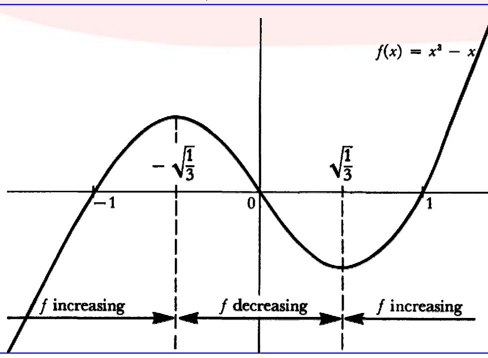


$y = x^3 - x$

$y' = 3x^2 - 1$

$= (\sqrt{3}x + 1)(\sqrt{3}x - 1)$

$= \begin{cases} > 0 & x > \sqrt{1/3} \\ < 0 & -\sqrt{1/3} < x < \sqrt{1/3} \\ > 0 & x < -\sqrt{1/3} \end{cases}$



If f is integrable on $[a, b]$ and $f = g'$ for some function g, then

$$\int_a^b f = g(b) - g(a).$$

Tweets Tweets & replies

Dror Bar-Natan @drorbarnatan · 2 Apr 2013
π=a/b, f(x)=x^n(a-bx)^n/n!, n large => 0<V=∫(0,π)f(x)sin(x)dx<1. Repeated integration by parts & f(x)=f(π-x) => V∈Z. So π is irrational.

1

Suppose that f is a function for which

$$f'(a), \ldots, f^{(n)}(a)$$

all exist. Let

$$a_k = \frac{f^{(k)}(a)}{k!}, \quad 0 \le k \le n,$$

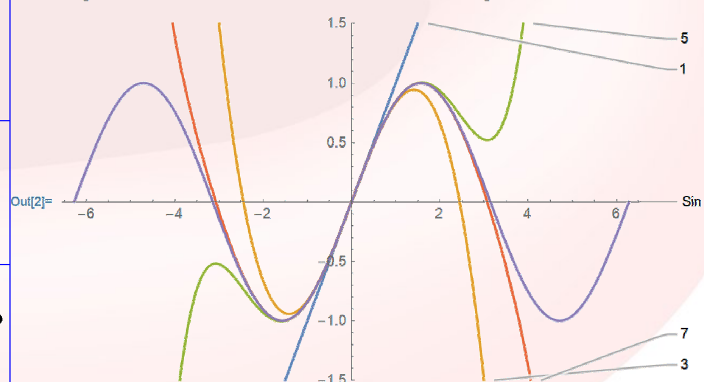
and define
$P_{n,a}(x) = a_0 + a_1(x - a) + \cdots + a_n(x - a)^n$.
Then

$$\lim_{x \to a} \frac{f(x) - P_{n,a}(x)}{(x - a)^n} = 0.$$

In[1]:= $a_{k\_} := \begin{cases} (-1)^{(k-1)/2} / k! & \text{OddQ}[k] \\ 0 & \text{EvenQ}[k] \end{cases}$;

```
Plot[Evaluate@Append[
    Table[Labeled[Sum[a_k x^k, {k, 0, n}], n], {n, {1, 3, 5, 7}}],
    Labeled[Sin[x], Sin]
  ], {x, -2 π, 2 π}, PlotRange → {-1.5, 1.5}]
```

Out[2]=

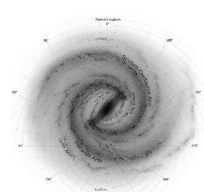


In[3]:= `Column@Table[k → N[a_k 157^k], {k, {0, 3, 9, 13, 29, 35, 157, 223, 457}}]`

Out[3]=
$0 \to 0.$
$3 \to -644982.$
$9 \to 1.59711 \times 10^{14}$
$13 \to 5.65477 \times 10^{18}$
$29 \to 5.42689 \times 10^{32}$
$35 \to -6.95433 \times 10^{36}$
$157 \to 4.86366 \times 10^{66}$
$223 \to -1.94045 \times 10^{61}$
$457 \to 4.87404 \times 10^{-16}$

In[4]:= $\left\{N\left[\sum_{k=0}^{457} a_k 157^k\right], \sum_{k=0}^{457} N\left[a_k 157^k\right]\right\}$

Out[4]= $\{-0.0795485, 5.10624 \times 10^{50}\}$



In[5]:= `N@Sin[157]`

Out[5]= -0.0795485

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