```
CF[ℰ_] := Expand@Collect[ℰ, g__, F] /. F → Factor;
T_3 = T_1 T_2;

F_1[{s_, i_, j_}] := CF[
  s (1 / 2 - g_{3ii} + T_2^s g_{1ii} g_{2ji} - g_{1ii} g_{2jj} - (T_2^s - 1) g_{2ji} g_{3ii} + 2 g_{2jj} g_{3ii} -
      (1 - T_3^s) g_{2ji} g_{3ji} - g_{2ii} g_{3jj} - T_2^s g_{2ji} g_{3jj} + g_{1ii} g_{3jj} +
      ((T_1^s - 1) g_{1ji} (T_2^{2 s} g_{2ji} - T_2^s g_{2jj} + T_2^s g_{3jj}) +
          (T_3^s - 1) g_{3ji} (1 - T_2^s g_{1ii} + g_{2ij} + (T_2^s - 2) g_{2jj} - (T_1^s - 1) (T_2^s + 1) g_{1ji})) /
       (T_2^s - 1))]
F_2[{s0_, i0_, j0_}, {s1_, i1_, j1_}] :=
 CF[s1 (T_1^{s0} - 1) (T_2^{s1} - 1)^{-1} (T_3^{s1} - 1) g_{1,j1,i0} g_{3,j0,i1}
    ((T_2^{s0} g_{2,i1,i0} - g_{2,i1,j0}) - (T_2^{s0} g_{2,j1,i0} - g_{2,j1,j0}))]
F_3[φ_, k_] = φ g_{3kk} - φ / 2;

Θ[K_] := Θ[K] = Module[{X, φ, n, A, Δ, G, ev, θ, k, k1, k2},
    {X, φ} = Rot[K]; n = Length[X]; A = IdentityMatrix[2 n + 1];
    Cases[X, {s_, i_, j_} ⧴ (A⟦{i, j}, {i + 1, j + 1}⟧ += {{-T^s, T^s - 1}, {0, -1}})];
    Δ = T^((-Total[φ] - Total[X⟦All,1⟧]) / 2) Det[A];
    G = Inverse[A];
    ev[ℰ_] := Factor[ℰ /. g_{ν_,α_,β_} ⧴ (G⟦α, β⟧ /. T → T_ν)];
    θ = ev[Sum[F_1[X⟦k⟧], {k, n}]];
    θ += ev[Sum[F_2[X⟦k1⟧, X⟦k2⟧], {k1, n}, {k2, n}]];
    θ += ev[Sum[F_3[φ⟦k⟧, k], {k, Length@φ}]];
    Factor@{Δ, (Δ /. T → T_1) (Δ /. T → T_2) (Δ /. T → T_3) θ}
   ];
```