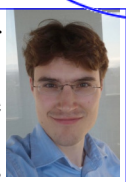
Homework

OMG, thanks!

ωεβ:=http://drorbn.net/ld26

Abstract. I'll start with a review of my recent paper with van der Veen, "A Fast, Strong, Topologically Meaningful, and Fun Knot Invariant", and then assign some homework.



van der Veen

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A: Much of what I'll say follows earlier work by Roznansky, Kricker, Garoufalidis and Ohtsuki.

A

G

O & Θ

Fast

program

300 string example w/ reading guide.

Strong

1. Table
2. Conway & K-T.

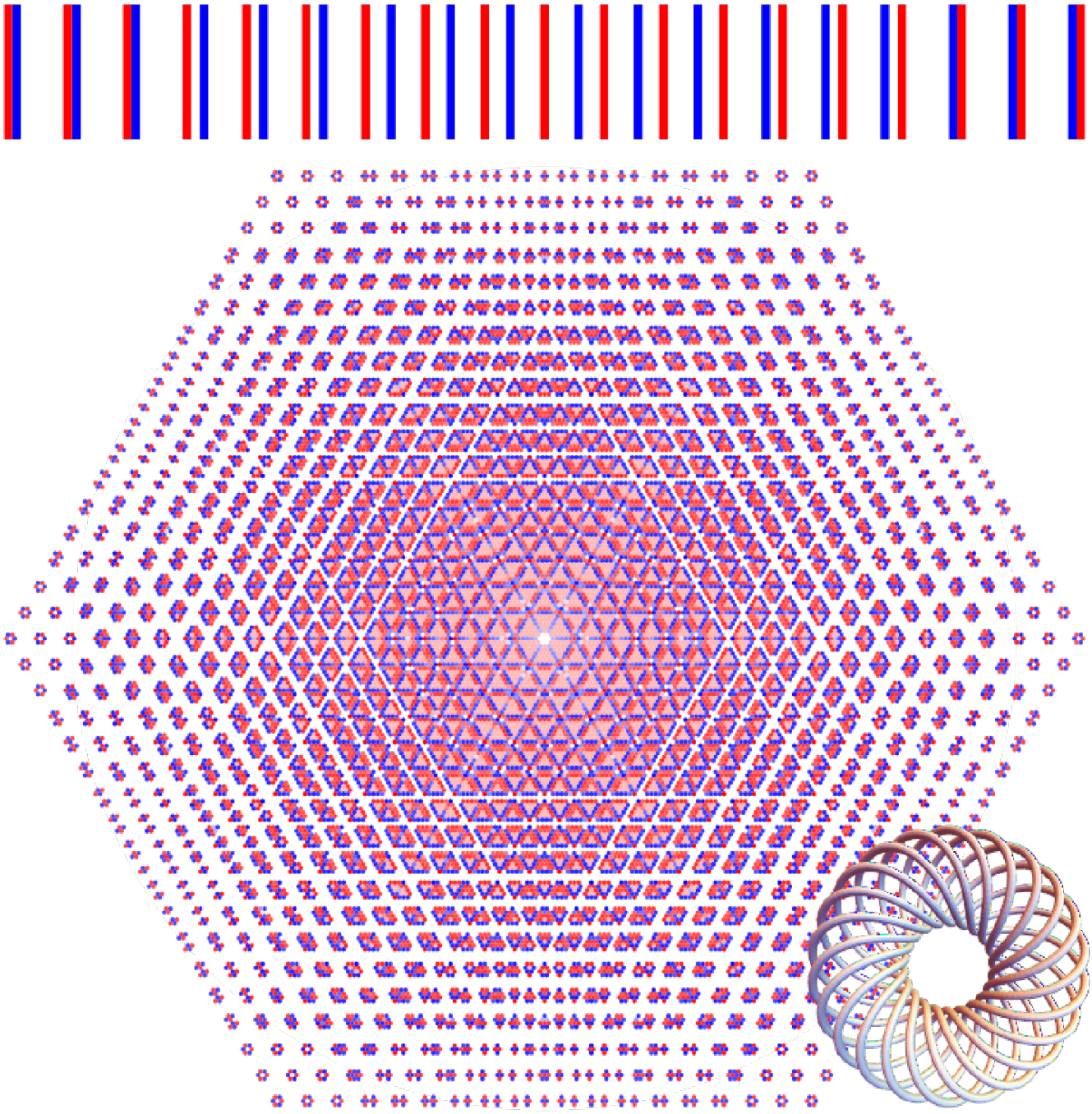
Top manifold

Fun

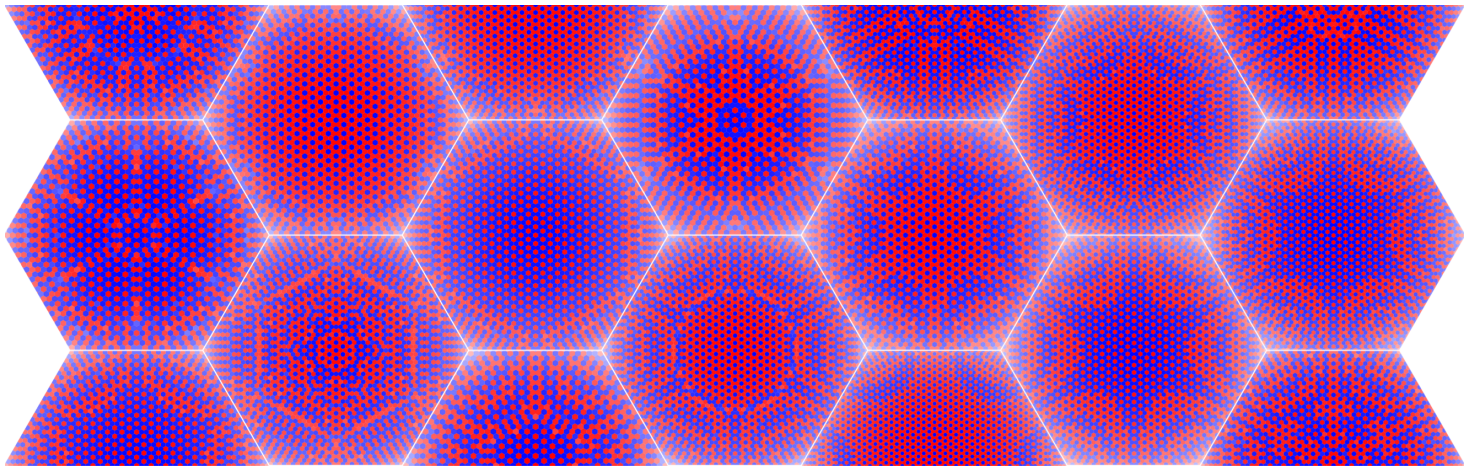
Add Rolfsen.

Add a pretzel.

The 132-crossing torus knot $T_{22/7}$: (many more at ωεβ/TK)



Random knots from [DHOEBL], with 51 – 68 crossings: (many more at ωεβ/DK)



Homework Task 1. *Prove the hexagonal symmetry of $\theta(K)$, and that $\theta(K) = \theta(-K) = -\theta(\bar{K})$.*
That's harder than it seems! The formulas don't naively show any of that. Alexander has a palindromic symmetry — it is invariant under $T \to T^{-1}$. Proving this took a few years, and the proof starting from the Wirtinger presentation is quite involved.

Homework Task 2. *Explain the "Chladni patterns". Are there "dominant parts" of θ that can be computed in isolation?*

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Homework Task 3. *Prove the genus bound and the fibered condition.*

Homework Task 4. *Find a formula for $\Theta(K)$ that starts from a Seifert surface of K. Better if it is completely 3D! Maybe it's about configuration space integrals / chopstick towers?*

Homework Task 5. *Is there an intrinsic theory of finite type invariants for Seifert surfaces? Does its gr map to functions on H_1?*

Homework Task 6. *In general, find a formula for Θ corresponding to each known formula for the Alexander polynomial.*

Homework Task 7. *Write up the integration story.*

Homework Task 8. *Prove that Θ is equal to the two-loop contribution to the Kontsevich integral.*

Homework Task 9. *Complete and write up the $\mathfrak{gl}$ story.*

Homework Task 10. *Understand Chern-Simons theory with gauge group $\mathfrak{gl}$.*

Homework Task 11. *What happens to representation theory as $\epsilon \to 0$?*

Homework Task 12. *Does Θ extend to knots in $\mathbb{Z}HS/\mathbb{Q}HS$?*

Homework Task 13. *Is there a surgery formula for Θ?*

Homework Task 14. *Find a ribbon condition satisfied by Θ.*

Homework Task 15. *Carthago delenda est? Must Θ be categorified?*

References.

[DHOEBL] N. Dunfield, A. Hirani, M. Obeidin, A. Ehrenberg, S. Bhattacharya, D. Lei, and others, *Random Knots: A Preliminary Report,* lecture notes at ωεβ/DHOEBL. Also a data file at ωεβ/DD.