

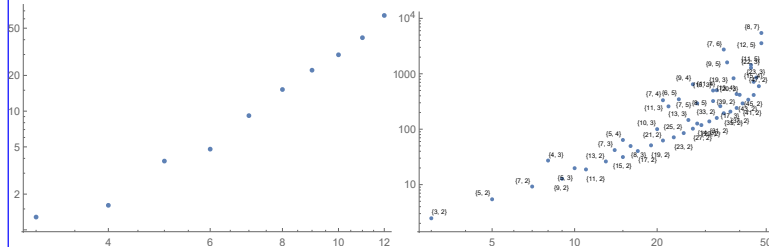


The Dogma is Wrong

Abstract. It has long been known that there are knot invariants associated to semi-simple Lie algebras, and there has long been a dogma as for how to extract them: “quantize and use representation theory”. We present an alternative and better procedure: “centrally extend, approximate by solvable, and learn how to re-order exponentials in a universal enveloping algebra”. While equivalent to the old invariants via a complicated process, our invariants are in practice stronger, faster to compute (poly-time vs. exp-time), and clearly carry topological information.

KiW 43 Abstract ($\omega\epsilon\beta$ /kiw). Whether or not you like the formulas on this page, they describe the strongest truly computable knot invariant we know.

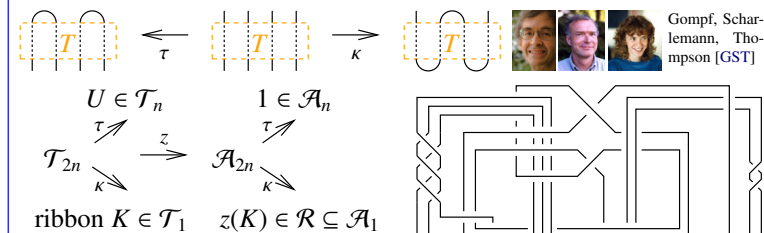
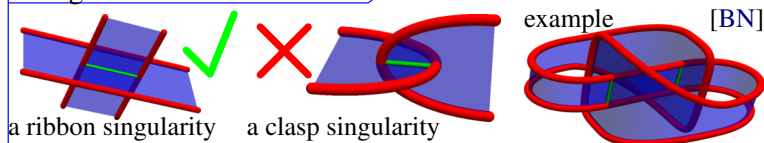
Experimental Analysis ($\omega\epsilon\beta$ /Exp). Log-log plots of computation time (sec) vs. crossing number, for all knots with up to 12 crossings (mean times) and for all torus knots with up to 48 crossings:



Power. On the 250 knots with at most 10 crossings, the pair (ω, ρ_1) attains 250 distinct values, while (Khovanov, HOMFLY-PT) attains only 249 distinct values. To 11 crossings the numbers are (802, 788, 772) and to 12 they are (2978, 2883, 2786).

Genus. Up to 12 crossings, always ρ_1 is symmetric under $t \leftrightarrow t^{-1}$. With ρ_1^+ denoting the positive-degree part of ρ_1 , always $\deg \rho_1^+ \leq 2g - 1$, where g is the 3-genus of K (equality for 2530 knots). This gives a lower bound on g in terms of ρ_1 (conjectural, but undoubtedly true). This bound is often weaker than the Alexander bound, yet for 10 of the 12-xing Alexander failures it does give the right answer.

Ribbon Knots.



[Vo]: Works with $\mathcal{R} := \kappa(\tau^{-1}(1))$ for Alexander!
 $A^+ = -t^8 + 2t^7 - t^6 - 2t^4 + 5t^3 - 2t^2 - 7t + 13$
 $\rho_1^+ = 5t^{15} - 18t^{14} + 33t^{13} - 32t^{12} + 2t^{11} + 42t^{10} - 62t^9 - 8t^8 + 166t^7 - 242t^6 +$
 Faster is better, leaner is meaner! $108t^5 + 132t^4 - 226t^3 + 148t^2 - 11t - 36$

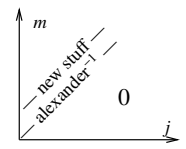
Ordering Symbols. \odot (*poly* | *specs*) plants the variables of *poly* in $S(\oplus \mathfrak{g})$ on several tensor copies of $\mathcal{U}(\mathfrak{g})$ according to *specs*. E.g., $\odot(a_1^3 y_1 a_2 e^{y_3} x_3^2 | x_3 a_1 \otimes y_1 y_3 a_2) = x^9 a^3 \otimes y e^y a \in \mathcal{U}(\mathfrak{g}) \otimes \mathcal{U}(\mathfrak{g})$

This enables the description of elements of $\hat{\mathcal{U}}(\mathfrak{g})^{\otimes S}$ using commutative polynomials / power series.

Theorem ([BNG], conjectured [MM], elucidated [Ro1]). Let $J_d(K)$ be the coloured Jones polynomial of K , in the d -dimensional representation of sl_2 . Writing

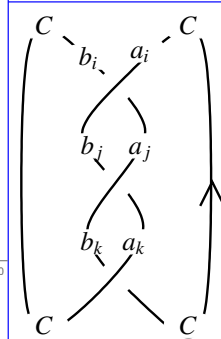
$$\left. \frac{(q^{1/2} - q^{-1/2})J_d(K)}{q^{d/2} - q^{-d/2}} \right|_{q=e^h} = \sum_{j,m \geq 0} a_{jm}(K) d^j h^m,$$

“below diagonal” coefficients vanish, $a_{jm}(K) = 0$ if $j > m$, and “on diagonal” coefficients give the inverse of the Alexander polynomial: $(\sum_{m=0}^{\infty} a_{mm}(K) h^m) \cdot \omega(K)(e^h) = 1$.



“Above diagonal” we have **Rozansky’s Theorem** [Ro3, (1.2)]:

$$J_d(K)(q) = \frac{q^d - q^{-d}}{(q - q^{-1})\omega(K)(q^d)} \left(1 + \sum_{k=1}^{\infty} \frac{(q-1)^k \rho_k(K)(q^d)}{\omega^{2k}(K)(q^d)} \right).$$



The Yang-Baxter Technique. Given an algebra U (typically $\hat{\mathcal{U}}(\mathfrak{g})$ or $\hat{\mathcal{U}}_q(\mathfrak{g})$) and elements

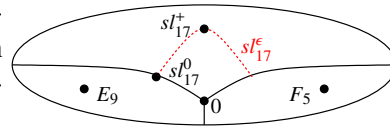
$$R = \sum a_i \otimes b_i \in U \otimes U \quad \text{and} \quad C \in U,$$

$$\text{form} \quad Z = \sum_{i,j,k} C a_i b_j a_k C^2 b_i a_j b_k C.$$

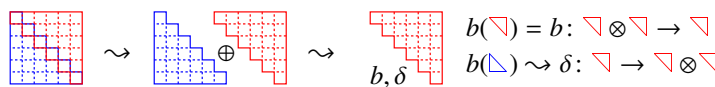
Problem. Extract information from Z .
 The Dogma. Use representation theory. In principle finite, but *slow*.

The Loyal Opposition. For certain algebras, work in a homomorphic poly-dimensional “space of formulas”. $m_k^{ij} \circlearrowleft \{F_S\} \xrightarrow{\mathbb{E}} \{U^{\otimes S}\} \circlearrowright m_k^{ij}$

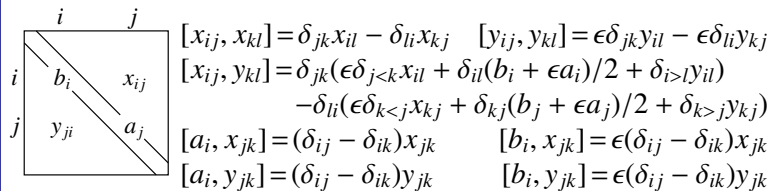
The (fake) moduli of Lie algebras on V , a quadratic variety in $(V^*)^{\otimes 2} \otimes V$ is on the right. We care about $sl_{17}^\epsilon := sl_{17}^\epsilon / (\epsilon^{k+1} = 0)$.



Recomposing gl_n . Half is enough! $gl_n \oplus \mathfrak{a}_n = \mathcal{D}(\nabla, b, \delta)$:



Now define $g_n^\epsilon := \mathcal{D}(\nabla, b, \epsilon\delta)$. Schematically, this is $[\nabla, \nabla] = \nabla$, $[\Delta, \Delta] = \epsilon\Delta$, and $[\nabla, \Delta] = \Delta + \epsilon\nabla$. In detail, it is



The Main sl_2 Theorem. Let $g^\epsilon = \langle t, y, a, x \rangle / ([t, \cdot] = 0, [a, x] = x, [a, y] = -y, [x, y] = t - 2\epsilon a)$ and let $g_k = g^\epsilon / (\epsilon^{k+1} = 0)$. The g_k -invariant of any S -component tangle K can be written in the form

$Z(K) = \odot(\omega e^{L+Q+P} : \otimes_{i \in S} y_i a_i x_i)$, where ω is a scalar (a rational function in the variables t_i and their exponentials $T_i := e^{t_i}$), where $L = \sum l_{ij} t_i a_j$ is a quadratic in t_i and a_j with integer coefficients l_{ij} , where $Q = \sum q_{ij} y_i x_j$ is a quadratic in the variables y_i and x_j with scalar coefficients q_{ij} , and where P is a polynomial in $(\epsilon, y_i, a_i, x_i)$ (with scalar coefficients) whose ϵ^d -term is of degree at most $2d + 2$ in $\{y_i, \sqrt{a_i}, x_i\}$. Furthermore, after setting $t_i = t$ and $T_i = T$ for all i , the invariant $Z(K)$ is poly-time computable.

The PBW Problem. In $\mathcal{U}(g^\epsilon)$, bring $Z = y^3 a^2 x^2 \cdot y^2 a^2 x$ to yax -order. In other words, find $g \in \mathbb{Z}[\epsilon, t, y, a, x]$ such that $Z = \mathbb{O}(f = y_1^3 y_2^2 a_1^2 a_2^2 x_1^2 x_2 : y_1 a_1 x_1 y_2 a_2 x_2) = \mathbb{O}(g : yax)$.

Solution, Part 1. In $\hat{\mathcal{U}}(g^\epsilon)$ we have

$$X_{\tau_1, \eta_1, \alpha_1, \xi_1, \tau_2, \eta_2, \alpha_2, \xi_2} := e^{\tau_1 t} e^{\eta_1 y} e^{\alpha_1 a} e^{\xi_1 x} e^{\tau_2 t} e^{\eta_2 y} e^{\alpha_2 a} e^{\xi_2 x} \\ = e^{\tau t} e^{\eta y} e^{\alpha a} e^{\xi x} =: Y_{\tau, \eta, \alpha, \xi},$$

where τ, η, α, ξ are ugly functions of $\tau_1, \eta_i, \alpha_i, \xi_i$:

$$\tau = \tau_1 + \tau_2 - \frac{\log(1 - \epsilon \eta_2 \xi_1)}{\epsilon} = \tau_1 + \tau_2 + \eta_2 \xi_1 + \frac{\epsilon}{2} \eta_2^2 \xi_1^2 + \dots, \\ \eta = \eta_1 + \frac{e^{-\alpha_1} \eta_2}{(1 - \epsilon \eta_2 \xi_1)} = \eta_1 + e^{-\alpha_1} \eta_2 + \epsilon e^{-\alpha_1} \eta_2^2 \xi_1 + \dots, \\ \alpha = \alpha_1 + \alpha_2 + 2 \log(1 - \epsilon \eta_2 \xi_1) = \alpha_1 + \alpha_2 - 2 \epsilon \eta_2 \xi_1 + \dots, \\ \xi = \frac{e^{-\alpha_2} \xi_1}{(1 - \epsilon \eta_2 \xi_1)} + \xi_2 = e^{-\alpha_2} \xi_1 + \xi_2 + \epsilon e^{-\alpha_2} \eta_2 \xi_1^2 + \dots$$

Note 1. This defines a mapping $\Phi: \mathbb{R}_{\tau_1, \eta_1, \alpha_1, \xi_1, \tau_2, \eta_2, \alpha_2, \xi_2}^8 \rightarrow \mathbb{R}_{\tau, \eta, \alpha, \xi}^4$

Proof. g^ϵ has a 2D representation ρ :

$$\rho t = \begin{pmatrix} 1 & \theta \\ \theta & 1 \end{pmatrix}; \quad \rho y = \begin{pmatrix} \theta & \theta \\ -\epsilon & \theta \end{pmatrix}; \\ \rho a = \begin{pmatrix} (1 + 1/\epsilon) / 2 & \theta \\ \theta & -(1 - 1/\epsilon) / 2 \end{pmatrix}; \quad \rho x = \begin{pmatrix} \theta & 1 \\ \theta & \theta \end{pmatrix};$$

$$\text{Simplify} \{ \rho a \cdot \rho x - \rho x \cdot \rho a = \rho x, \quad \rho a \cdot \rho y - \rho y \cdot \rho a = -\rho y, \\ \rho x \cdot \rho y - \rho y \cdot \rho x = \rho t - 2 \epsilon \rho a \}$$

{ True, True, True }

It is enough to verify the desired identity in ρ :

ME = MatrixExp;

Simplify[

$$\text{ME}[\tau_1 \rho t] \cdot \text{ME}[\eta_1 \rho y] \cdot \text{ME}[\alpha_1 \rho a] \cdot \text{ME}[\xi_1 \rho x] \cdot \text{ME}[\tau_2 \rho t] \cdot \\ \text{ME}[\eta_2 \rho y] \cdot \text{ME}[\alpha_2 \rho a] \cdot \text{ME}[\xi_2 \rho x] = \\ \text{ME}[\tau \rho t] \cdot \text{ME}[\eta \rho y] \cdot \text{ME}[\alpha \rho a] \cdot \text{ME}[\xi \rho x] / . \\ \left\{ \tau \theta \rightarrow -\frac{\log[1 - \epsilon \eta_2 \xi_1]}{\epsilon} + \tau_1 + \tau_2, \quad \eta \theta \rightarrow \eta_1 + \frac{e^{-\alpha_1} \eta_2}{1 - \epsilon \eta_2 \xi_1}, \right. \\ \left. \alpha \theta \rightarrow 2 \text{Log}[1 - \epsilon \eta_2 \xi_1] + \alpha_1 + \alpha_2, \quad \xi \theta \rightarrow \frac{e^{-\alpha_2} \xi_1}{1 - \epsilon \eta_2 \xi_1} + \xi_2 \right\}$$

True

Solution, Part 2. But now, with $D_f = f(z \mapsto \partial_z) = \partial_{\eta_1}^3 \partial_{\alpha_1}^2 \partial_{\xi_1}^2 \partial_{\eta_2}^2 \partial_{\alpha_2}^2 \partial_{\xi_2}^2$,

$$Z = D_f X_{\tau_1, \eta_1, \alpha_1, \xi_1, \tau_2, \eta_2, \alpha_2, \xi_2} \Big|_{y_S=0} = D_f Y_{\tau, \eta, \alpha, \xi} \Big|_{y_S=0} \\ = \mathbb{O} \left(D_f e^{\tau t} e^{\eta y} e^{\alpha a} e^{\xi x} \Big|_{y_S=0} : yax \right) = \mathbb{O}(g : yax) :$$

$$\text{Expand} \left[\partial_{\{\eta_1, 3\}} \partial_{\{\alpha_1, 2\}} \partial_{\{\xi_1, 2\}} \partial_{\{\eta_2, 2\}} \partial_{\{\alpha_2, 2\}} \partial_{\{\xi_2, 1\}} \text{Exp} \left[\right. \right. \\ \left. \left. \left(-\frac{\log[1 - \epsilon \eta_2 \xi_1]}{\epsilon} + \tau_1 + \tau_2 \right) t + \left(\eta_1 + \frac{e^{-\alpha_1} \eta_2}{1 - \epsilon \eta_2 \xi_1} \right) y + \right. \right. \\ \left. \left. (2 \text{Log}[1 - \epsilon \eta_2 \xi_1] + \alpha_1 + \alpha_2) a + \left(\frac{e^{-\alpha_2} \xi_1}{1 - \epsilon \eta_2 \xi_1} + \xi_2 \right) x \right. \right. \\ \left. \left. \right] / . (\tau | \eta | \alpha | \xi)_{1|2} \rightarrow \theta \right]$$

$$2 a^4 t^2 x y^3 + 4 t x^2 y^4 - 16 a t x^2 y^4 + 24 a^2 t x^2 y^4 - 16 a^3 t x^2 y^4 + \\ 4 a^4 t x^2 y^4 + 16 x^3 y^5 - 32 a x^3 y^5 + 24 a^2 x^3 y^5 - 8 a^3 x^3 y^5 + a^4 x^3 y^5 + \\ 2 a^4 t x y^3 \epsilon - 8 a^5 t x y^3 \epsilon + 8 x^2 y^4 \epsilon - 40 a x^2 y^4 \epsilon + 80 a^2 x^2 y^4 \epsilon - \\ 80 a^3 x^2 y^4 \epsilon + 40 a^4 x^2 y^4 \epsilon - 8 a^5 x^2 y^4 \epsilon - 4 a^5 x y^3 \epsilon^2 + 8 a^6 x y^3 \epsilon^2$$

Note 2. Replacing $f \rightarrow D_f$ (and likewise $g \rightarrow D_g$), we find that $D_g = \Phi_* D_f$.

Note 3. The two great evils of mathematics are non-commutativity and non-linearity. We traded one for the other.

Note 4. We could have done similarly with $e^{\tau_1 t} e^{\eta_1 y} e^{\alpha_1 a} e^{\xi_1 x} = e^{\tau t + \eta y + \alpha a + \xi x}$, and with $S(e^{\tau_1 t} e^{\eta_1 y} e^{\alpha_1 a} e^{\xi_1 x})$, $\Delta(e^{\tau_1 t} e^{\eta_1 y} e^{\alpha_1 a} e^{\xi_1 x})$, $\prod_{i=1}^5 e^{\tau_i t} e^{\eta_i y} e^{\alpha_i a} e^{\xi_i x}$.

Fact. $R_{12} \rightarrow \exp(\partial_{\tau_1} \partial_{\alpha_2} + \partial_{y_1} \partial_{x_2})(1 + \sum_{d \geq 1} \epsilon^d p_d)$, where the p_d are computable polynomials of a-priori bounded degrees.

Moral. We need to understand the pushforwards via maps like Φ of (formally ∞ -order) “differential operators at 0”, that in themselves are perturbed Gaussians. This turns out to be the same problem as “0-dimensional QFT” (except no integration is ever needed), and if $\epsilon^{k+1} = 0$, it is explicitly soluble.

References.

[BN] D. Bar-Natan, *Polynomial Time Knot Polynomial*, research proposal for the 2017 Killam Fellowship, [oeß/K17](#).
[BNG] D. Bar-Natan and S. Garoufalidis, *On the Melvin-Morton-Rozansky conjecture*, *Invent. Math.* **125** (1996) 103–133.
[BV1] D. Bar-Natan and R. van der Veen, *A Polynomial Time Knot Polynomial*, [arXiv:1708.04853](#).
[BV2] D. Bar-Natan and R. van der Veen, *Poly-Time Knot Polynomials Via Solvable Approximations*, in preparation.
[GST] R. E. Gompf, M. Scharlemann, and A. Thompson, *Fibered Knots and Potential Counterexamples to the Property 2R and Slice-Ribbon Conjectures*, *Geom. and Topol.* **14** (2010) 2305–2347, [arXiv:1103.1601](#).
[MM] P. M. Melvin and H. R. Morton, *The coloured Jones function*, *Commun. Math. Phys.* **169** (1995) 501–520.
[Ov] A. Overbay, *Perturbative Expansion of the Colored Jones Polynomial*, University of North Carolina PhD thesis, [oeß/Ov](#).
[Ro1] L. Rozansky, *A contribution of the trivial flat connection to the Jones polynomial and Witten’s invariant of 3d manifolds, I*, *Comm. Math. Phys.* **175-2** (1996) 275–296, [arXiv:hep-th/9401061](#).
[Ro2] L. Rozansky, *The Universal R-Matrix, Burau Representation and the Melvin-Morton Expansion of the Colored Jones Polynomial*, *Adv. Math.* **134-1** (1998) 1–31, [arXiv:q-alg/9604005](#).
[Ro3] L. Rozansky, *A Universal U(1)-RCC Invariant of Links and Rationality Conjecture*, [arXiv:math/0201139](#).
[Vo] H. Vo, University of Toronto Ph.D. thesis, in preparation.

dog·ma (dôg·mə, dôg·) The Free Dictionary, [oeß/TFD](#)

n. pl. dog·mas or dog·ma·ta (-mə-tə)

1. A doctrine or a corpus of doctrines relating to matters such as morality and faith, set forth in an authoritative manner by a religion.
2. A principle or statement of ideas, or a group of such principles or statements, especially when considered to be authoritative or accepted uncritically: *“Much education consists in the instilling of unfounded dogmas in place of a spirit of inquiry” (Bertrand Russell).*

diagram	n_k^t Alexander’s ω^+ Today’s / Rozansky’s ρ_k^+	genus / ribbon unknotting number / amphicheiral	diagram	n_k^t Alexander’s ω^+ Today’s / Rozansky’s ρ_k^+	genus / ribbon unknotting number / amphicheiral
	0_1^a 0	1 0 / ✓		3_1^a t	t - 1 1 / ✗
	4_1^a 0	3 - t 1 / ✗		5_1^a $2t^3 + 3t$	t ² - t + 1 2 / ✗
	5_2^a $5t - 4$	2t - 3 1 / ✗		6_1^a t - 4	5 - 2t 1 / ✓































































diagram	n'_k Alexander's ω^+ Today's / Rozansky's ρ_1^+	genus / ribbon unknotting number / amphicheiral	diagram	n'_k Alexander's ω^+ Today's / Rozansky's ρ_1^+	genus / ribbon unknotting number / amphicheiral
	6_2^a $-t^2 + 3t - 3$ $t^3 - 4t^2 + 4t - 4$	2 / ✗ 1 / ✗		6_3^a $t^2 - 3t + 5$ 0	2 / ✗ 1 / ✓
	7_1^a $t^3 - t^2 + t - 1$ $3t^5 + 5t^3 + 6t$	3 / ✗ 3 / ✗		7_2^a $3t - 5$ $14t - 16$	1 / ✗ 1 / ✗
	7_3^a $2t^2 - 3t + 3$ $-9t^3 + 8t^2 - 16t + 12$	2 / ✗ 2 / ✗		7_4^a $4t - 7$ $32 - 24t$	1 / ✗ 2 / ✗
	7_5^a $2t^2 - 4t + 5$ $9t^3 - 16t^2 + 29t - 28$	2 / ✗ 2 / ✗		7_6^a $-t^2 + 5t - 7$ $t^3 - 8t^2 + 19t - 20$	2 / ✗ 1 / ✗
	7_7^a $t^2 - 5t + 9$ $8 - 3t$	2 / ✗ 1 / ✗		8_1^a $7 - 3t$ $5t - 16$	1 / ✗ 1 / ✗
	8_2^a $-t^3 + 3t^2 - 3t + 3$ $2t^5 - 8t^4 + 10t^3 - 12t^2 + 13t - 12$	3 / ✗ 2 / ✗		8_3^a $9 - 4t$ 0	1 / ✗ 2 / ✓
	8_4^a $-2t^2 + 5t - 5$ $3t^3 - 8t^2 + 6t - 4$	2 / ✗ 2 / ✗		8_5^a $-t^3 + 3t^2 - 4t + 5$ $-2t^5 + 8t^4 - 13t^3 + 20t^2 - 22t + 24$	3 / ✗ 2 / ✗
	8_6^a $-2t^2 + 6t - 7$ $5t^3 - 20t^2 + 28t - 32$	2 / ✗ 2 / ✗		8_7^a $t^3 - 3t^2 + 5t - 5$ $-t^5 + 4t^4 - 10t^3 + 12t^2 - 13t + 12$	3 / ✗ 1 / ✗
	8_8^a $2t^2 - 6t + 9$ $-t^3 + 4t^2 - 12t + 16$	2 / ✓ 2 / ✗		8_9^a $-t^3 + 3t^2 - 5t + 7$ 0	3 / ✓ 1 / ✓
	8_{10}^a $t^3 - 3t^2 + 6t - 7$ $-t^5 + 4t^4 - 11t^3 + 16t^2 - 21t + 20$	3 / ✗ 2 / ✗		8_{11}^a $-2t^2 + 7t - 9$ $5t^3 - 24t^2 + 39t - 44$	2 / ✗ 1 / ✗
	8_{12}^a $t^2 - 7t + 13$ 0	2 / ✗ 2 / ✓		8_{13}^a $2t^2 - 7t + 11$ $-t^3 + 4t^2 - 14t + 20$	2 / ✗ 1 / ✗
	8_{14}^a $-2t^2 + 8t - 11$ $5t^3 - 28t^2 + 57t - 68$	2 / ✗ 1 / ✗		8_{15}^a $3t^2 - 8t + 11$ $21t^3 - 64t^2 + 120t - 140$	2 / ✗ 2 / ✗
	8_{16}^a $t^3 - 4t^2 + 8t - 9$ $t^5 - 6t^4 + 17t^3 - 28t^2 + 35t - 36$	3 / ✗ 2 / ✗		8_{17}^a $-t^3 + 4t^2 - 8t + 11$ 0	3 / ✗ 1 / ✓
	8_{18}^a $-t^3 + 5t^2 - 10t + 13$ 0	3 / ✗ 2 / ✓		8_{19}^a $t^3 - t^2 + 1$ $-3t^5 - 4t^2 - 3t$	3 / ✗ 3 / ✗
	8_{20}^a $t^2 - 2t + 3$ $4t - 4$	2 / ✓ 1 / ✗		8_{21}^a $-t^2 + 4t - 5$ $t^3 - 8t^2 + 16t - 20$	2 / ✗ 1 / ✗
	9_1^a $t^4 - t^3 + t^2 - t + 1$ $4t^7 + 7t^5 + 9t^3 + 10t$	4 / ✗ 4 / ✗		9_2^a $4t - 7$ $30t - 40$	1 / ✗ 1 / ✗
	9_3^a $2t^3 - 3t^2 + 3t - 3$ $-13t^5 + 12t^4 - 25t^3 + 20t^2 - 32t + 24$	3 / ✗ 3 / ✗		9_4^a $3t^2 - 5t + 5$ $23t^3 - 28t^2 + 46t - 44$	2 / ✗ 2 / ✗
	9_5^a $6t - 11$ $100 - 65t$	1 / ✗ 2 / ✗		9_6^a $2t^3 - 4t^2 + 5t - 5$ $13t^5 - 24t^4 + 45t^3 - 52t^2 + 68t - 64$	3 / ✗ 3 / ✗
	9_7^a $3t^2 - 7t + 9$ $23t^3 - 56t^2 + 99t - 108$	2 / ✗ 2 / ✗		9_8^a $-2t^2 + 8t - 11$ $3t^3 - 16t^2 + 29t - 28$	2 / ✗ 2 / ✗
	9_9^a $2t^3 - 4t^2 + 6t - 7$ $13t^5 - 24t^4 + 55t^3 - 72t^2 + 98t - 96$	3 / ✗ 3 / ✗		9_{10}^a $4t^2 - 8t + 9$ $-40t^3 + 72t^2 - 114t + 120$	2 / ✗ 2,3 / ✗
	9_{11}^a $-t^3 + 5t^2 - 7t + 7$ $-2t^5 + 16t^4 - 41t^3 + 52t^2 - 66t + 64$	3 / ✗ 2 / ✗		9_{12}^a $-2t^2 + 9t - 13$ $5t^3 - 36t^2 + 84t - 100$	2 / ✗ 1 / ✗
	9_{13}^a $4t^2 - 9t + 11$ $-40t^3 + 92t^2 - 154t + 168$	2 / ✗ 2,3 / ✗		9_{14}^a $2t^2 - 9t + 15$ $-t^3 + 8t^2 - 35t + 60$	2 / ✗ 1 / ✗
	9_{15}^a $-2t^2 + 10t - 15$ $-5t^3 + 40t^2 - 108t + 136$	2 / ✗ 2 / ✗		9_{16}^a $2t^3 - 5t^2 + 8t - 9$ $-13t^5 + 36t^4 - 80t^3 + 120t^2 - 161t + 168$	3 / ✗ 3 / ✗
	9_{17}^a $t^3 - 5t^2 + 9t - 9$ $t^5 - 8t^4 + 23t^3 - 32t^2 + 28t - 24$	3 / ✗ 2 / ✗		9_{18}^a $4t^2 - 10t + 13$ $40t^3 - 108t^2 + 193t - 220$	2 / ✗ 2 / ✗
	9_{19}^a $2t^2 - 10t + 17$ $t^3 - 8t^2 + 20t - 24$	2 / ✗ 1 / ✗		9_{20}^a $-t^3 + 5t^2 - 9t + 11$ $2t^5 - 16t^4 + 47t^3 - 84t^2 + 117t - 124$	3 / ✗ 2 / ✗
	9_{21}^a $-2t^2 + 11t - 17$ $-5t^3 + 44t^2 - 127t + 164$	2 / ✗ 1 / ✗		9_{22}^a $t^3 - 5t^2 + 10t - 11$ $-t^5 + 8t^4 - 24t^3 + 38t^2 - 40t + 36$	3 / ✗ 1 / ✗
	9_{23}^a $4t^2 - 11t + 15$ $40t^3 - 128t^2 + 243t - 288$	2 / ✗ 2 / ✗		9_{24}^a $-t^3 + 5t^2 - 10t + 13$ $-4t^2 + 16t - 20$	3 / ✗ 1 / ✗
	9_{25}^a $-3t^2 + 12t - 17$ $12t^3 - 70t^2 + 153t - 188$	2 / ✗ 2 / ✗		9_{26}^a $t^3 - 5t^2 + 11t - 13$ $-t^5 + 8t^4 - 31t^3 + 64t^2 - 85t + 92$	3 / ✗ 1 / ✗
	9_{27}^a $-t^3 + 5t^2 - 11t + 15$ $t^3 - 8t^2 + 24t - 32$	3 / ✓ 1 / ✗		9_{28}^a $t^3 - 5t^2 + 12t - 15$ $t^5 - 8t^4 + 30t^3 - 68t^2 + 105t - 120$	3 / ✗ 1 / ✗
	9_{29}^a $t^3 - 5t^2 + 12t - 15$ $t^5 - 8t^4 + 26t^3 - 48t^2 + 59t - 56$	3 / ✗ 2 / ✗		9_{30}^a $-t^3 + 5t^2 - 12t + 17$ $2t^3 - 10t^2 + 25t - 32$	3 / ✗ 1 / ✗
	9_{31}^a $t^3 - 5t^2 + 13t - 17$ $t^5 - 8t^4 + 33t^3 - 80t^2 + 132t - 152$	3 / ✗ 2 / ✗		9_{32}^a $t^3 - 6t^2 + 14t - 17$ $-t^5 + 10t^4 - 42t^3 + 94t^2 - 133t + 148$	3 / ✗ 2 / ✗

diagram	n'_k Alexander's ω^+ Today's / Rozansky's ρ_1^+	genus / ribbon unknotting number / amphicheiral	diagram	n'_k Alexander's ω^+ Today's / Rozansky's ρ_1^+	genus / ribbon unknotting number / amphicheiral
	$9a_{33}$ $-t^3 + 6t^2 - 14t + 19$ $t^3 - 10t^2 + 30t - 40$	3 / ✗ 1 / ✗		$9a_{34}$ $-t^3 + 6t^2 - 16t + 23$ $3t^3 - 18t^2 + 43t - 56$	3 / ✗ 1 / ✗
	$9a_{35}$ $7t - 13$ $90t - 144$	1 / ✗ 2, 3 / ✗		$9a_{36}$ $-t^3 + 5t^2 - 8t + 9$ $-2t^5 + 16t^4 - 44t^3 + 66t^2 - 87t + 88$	3 / ✗ 2 / ✗
	$9a_{37}$ $2t^2 - 11t + 19$ $t^3 - 8t^2 + 22t - 28$	2 / ✗ 2 / ✗		$9a_{38}$ $5t^2 - 14t + 19$ $62t^3 - 204t^2 + 382t - 452$	2 / ✗ 2, 3 / ✗
	$9a_{39}$ $-3t^2 + 14t - 21$ $-12t^3 + 84t^2 - 210t + 268$	2 / ✗ 1 / ✗		$9a_{40}$ $t^3 - 7t^2 + 18t - 23$ $t^5 - 12t^4 + 57t^3 - 144t^2 + 229t - 264$	3 / ✗ 2 / ✗
	$9a_{41}$ $3t^2 - 12t + 19$ $3t^3 - 20t^2 + 70t - 108$	2 / ✓ 2 / ✗		$9a_{42}$ $-t^2 + 2t - 1$ $-t^3 + 2t^2 + t - 4$	2 / ✗ 1 / ✗
	$9a_{43}$ $-t^3 + 3t^2 - 2t + 1$ $-2t^5 + 8t^4 - 7t^3 + 2t^2 - 5t + 4$	3 / ✗ 2 / ✗		$9a_{44}$ $t^2 - 4t + 7$ $-2t^2 + 9t - 12$	2 / ✗ 1 / ✗
	$9a_{45}$ $-t^2 + 6t - 9$ $t^3 - 14t^2 + 47t - 60$	2 / ✗ 1 / ✗		$9a_{46}$ $5 - 2t$ $3t - 12$	1 / ✓ 2 / ✗
	$9a_{47}$ $t^3 - 4t^2 + 6t - 5$ $-t^5 + 6t^4 - 15t^3 + 16t^2 - 10t + 12$	3 / ✗ 2 / ✗		$9a_{48}$ $-t^2 + 7t - 11$ $-t^3 + 12t^2 - 42t + 52$	2 / ✗ 2 / ✗
	$9a_{49}$ $3t^2 - 6t + 7$ $-21t^3 + 38t^2 - 61t + 60$	2 / ✗ 3 / ✗		$10a_1$ $9 - 4t$ $14t - 40$	1 / ✗ 1 / ✗
	$10a_2$ $-t^4 + 3t^3 - 3t^2 + 3t - 3$ $3t^7 - 12t^6 + 16t^5 - 20t^4 + 24t^3 - 24t^2 + 27t - 24$	4 / ✗ 3 / ✗		$10a_3$ $13 - 6t$ $11t - 28$	1 / ✓ 2 / ✗
	$10a_4$ $-3t^2 + 7t - 7$ $4t^3 - 8t^2 + t + 8$	2 / ✗ 2 / ✗		$10a_5$ $t^4 - 3t^3 + 5t^2 - 5t + 5$ $-2t^7 + 8t^6 - 20t^5 + 28t^4 - 36t^3 + 36t^2 - 39t + 36$	4 / ✗ 2 / ✗
	$10a_6$ $-2t^3 + 6t^2 - 7t + 7$ $9t^5 - 36t^4 + 56t^3 - 72t^2 + 81t - 84$	3 / ✗ 3 / ✗		$10a_7$ $-3t^2 + 11t - 15$ $14t^3 - 72t^2 + 135t - 160$	2 / ✗ 1 / ✗
	$10a_8$ $-2t^3 + 5t^2 - 5t + 5$ $7t^5 - 20t^4 + 23t^3 - 28t^2 + 26t - 24$	3 / ✗ 2 / ✗		$10a_9$ $-t^4 + 3t^3 - 5t^2 + 7t - 7$ $-t^7 + 4t^6 - 10t^5 + 20t^4 - 25t^3 + 28t^2 - 28t + 28$	4 / ✗ 1 / ✗
	$10a_{10}$ $3t^2 - 11t + 17$ $-5t^3 + 24t^2 - 71t + 100$	2 / ✗ 1 / ✗		$10a_{11}$ $-4t^2 + 11t - 13$ $16t^3 - 52t^2 + 68t - 72$	2 / ✗ 2, 3 / ✗
	$10a_{12}$ $2t^3 - 6t^2 + 10t - 11$ $-5t^5 + 20t^4 - 50t^3 + 72t^2 - 89t + 92$	3 / ✗ 2 / ✗		$10a_{13}$ $2t^2 - 13t + 23$ $t^3 - 12t^2 + 51t - 84$	2 / ✗ 2 / ✗
	$10a_{14}$ $-2t^3 + 8t^2 - 12t + 13$ $9t^5 - 52t^4 + 119t^3 - 180t^2 + 225t - 236$	3 / ✗ 2 / ✗		$10a_{15}$ $2t^3 - 6t^2 + 9t - 9$ $-3t^5 + 12t^4 - 24t^3 + 24t^2 - 17t + 12$	3 / ✗ 2 / ✗
	$10a_{16}$ $-4t^2 + 12t - 15$ $-16t^3 + 56t^2 - 76t + 80$	2 / ✗ 2 / ✗		$10a_{17}$ $t^4 - 3t^3 + 5t^2 - 7t + 9$ 0	4 / ✗ 1 / ✓
	$10a_{18}$ $-4t^2 + 14t - 19$ $16t^3 - 68t^2 + 121t - 140$	2 / ✗ 1 / ✗		$10a_{19}$ $2t^3 - 7t^2 + 11t - 11$ $3t^5 - 16t^4 + 35t^3 - 40t^2 + 30t - 24$	3 / ✗ 2 / ✗
	$10a_{20}$ $-3t^2 + 9t - 11$ $14t^3 - 56t^2 + 88t - 104$	2 / ✗ 2 / ✗		$10a_{21}$ $-2t^3 + 7t^2 - 9t + 9$ $9t^5 - 44t^4 + 80t^3 - 104t^2 + 121t - 124$	3 / ✗ 2 / ✗
	$10a_{22}$ $-2t^3 + 6t^2 - 10t + 13$ $-t^5 + 4t^4 - 10t^3 + 24t^2 - 37t + 44$	3 / ✓ 2 / ✗		$10a_{23}$ $2t^3 - 7t^2 + 13t - 15$ $-5t^5 + 24t^4 - 67t^3 + 108t^2 - 137t + 144$	3 / ✗ 1 / ✗
	$10a_{24}$ $-4t^2 + 14t - 19$ $24t^3 - 116t^2 + 221t - 268$	2 / ✗ 2 / ✗		$10a_{25}$ $-2t^3 + 8t^2 - 14t + 17$ $9t^5 - 52t^4 + 131t^3 - 232t^2 + 314t - 344$	3 / ✗ 2 / ✗
	$10a_{26}$ $-2t^3 + 7t^2 - 13t + 17$ $-t^5 + 4t^4 - 10t^3 + 28t^2 - 49t + 60$	3 / ✗ 1 / ✗		$10a_{27}$ $2t^3 - 8t^2 + 16t - 19$ $5t^5 - 28t^4 + 87t^3 - 164t^2 + 229t - 252$	3 / ✗ 1 / ✗
	$10a_{28}$ $4t^2 - 13t + 19$ $-8t^3 + 36t^2 - 100t + 136$	2 / ✗ 2 / ✗		$10a_{29}$ $t^3 - 7t^2 + 15t - 17$ $t^5 - 12t^4 + 52t^3 - 104t^2 + 124t - 128$	3 / ✗ 2 / ✗
	$10a_{30}$ $-4t^2 + 17t - 25$ $24t^3 - 148t^2 + 345t - 440$	2 / ✗ 1 / ✗		$10a_{31}$ $4t^2 - 14t + 21$ $-4t^2 + 9t - 12$	2 / ✗ 1 / ✗
	$10a_{32}$ $-2t^3 + 8t^2 - 15t + 19$ $t^5 - 4t^4 + 13t^3 - 40t^2 + 78t - 96$	3 / ✗ 1 / ✗		$10a_{33}$ $4t^2 - 16t + 25$ 0	2 / ✗ 1 / ✓
	$10a_{34}$ $3t^2 - 9t + 13$ $-5t^3 + 20t^2 - 52t + 68$	2 / ✗ 2 / ✗		$10a_{35}$ $2t^2 - 12t + 21$ $-t^3 + 12t^2 - 47t + 76$	2 / ✓ 2 / ✗
	$10a_{36}$ $-3t^2 + 13t - 19$ $14t^3 - 88t^2 + 208t - 264$	2 / ✗ 2 / ✗		$10a_{37}$ $4t^2 - 13t + 19$ 0	2 / ✗ 2 / ✓
	$10a_{38}$ $-4t^2 + 15t - 21$ $24t^3 - 128t^2 + 270t - 336$	2 / ✗ 2 / ✗		$10a_{39}$ $-2t^3 + 8t^2 - 13t + 15$ $9t^5 - 52t^4 + 125t^3 - 204t^2 + 263t - 280$	3 / ✗ 2 / ✗
	$10a_{40}$ $2t^3 - 8t^2 + 17t - 21$ $-5t^5 + 28t^4 - 89t^3 + 176t^2 - 258t + 288$	3 / ✗ 2 / ✗		$10a_{41}$ $t^3 - 7t^2 + 17t - 21$ $t^5 - 12t^4 + 54t^3 - 120t^2 + 157t - 164$	3 / ✗ 2 / ✗
	$10a_{42}$ $-t^3 + 7t^2 - 19t + 27$ $2t^3 - 8t^2 + 11t - 12$	3 / ✓ 1 / ✗		$10a_{43}$ $-t^3 + 7t^2 - 17t + 23$ 0	3 / ✗ 2 / ✓
	$10a_{44}$ $t^3 - 7t^2 + 19t - 25$ $t^5 - 12t^4 + 56t^3 - 140t^2 + 220t - 248$	3 / ✗ 1 / ✗		$10a_{45}$ $-t^3 + 7t^2 - 21t + 31$ 0	3 / ✗ 2 / ✓

diagram	n'_k Alexander's ω^+ Today's / Rozansky's ρ_1^+	genus / ribbon unknotting number / amphicheiral	diagram	n'_k Alexander's ω^+ Today's / Rozansky's ρ_1^+	genus / ribbon unknotting number / amphicheiral
	10_{46}^a $-t^4 + 3t^3 - 4t^2 + 5t - 5$ $-3t^7 + 12t^6 - 21t^5 + 34t^4 - 43t^3 + 52t^2 - 55t + 56$	4 / ✗ 3 / ✗		10_{47}^a $t^4 - 3t^3 + 6t^2 - 7t + 7$ $-2t^7 + 8t^6 - 23t^5 + 38t^4 - 56t^3 + 60t^2 - 68t + 64$	4 / ✗ 2, 3 / ✗
	10_{48}^a $t^4 - 3t^3 + 6t^2 - 9t + 11$ $t^5 - 2t^4 + 2t^3 - 3t + 4$	4 / ✓ 2 / ✗		10_{49}^a $3t^3 - 8t^2 + 12t - 13$ $30t^5 - 94t^4 + 196t^3 - 292t^2 + 372t - 392$	3 / ✗ 3 / ✗
	10_{50}^a $-2t^3 + 7t^2 - 11t + 13$ $-9t^5 + 44t^4 - 94t^3 + 150t^2 - 186t + 200$	3 / ✗ 2 / ✗		10_{51}^a $2t^3 - 7t^2 + 15t - 19$ $-5t^5 + 24t^4 - 73t^3 + 134t^2 - 194t + 212$	3 / ✗ 2, 3 / ✗
	10_{52}^a $2t^3 - 7t^2 + 13t - 15$ $-3t^5 + 16t^4 - 37t^3 + 50t^2 - 49t + 44$	3 / ✗ 2 / ✗		10_{53}^a $6t^2 - 18t + 25$ $93t^3 - 346t^2 + 680t - 828$	2 / ✗ 2, 3 / ✗
	10_{54}^a $2t^3 - 6t^2 + 10t - 11$ $-3t^5 + 12t^4 - 24t^3 + 26t^2 - 21t + 16$	3 / ✗ 2, 3 / ✗		10_{55}^a $5t^2 - 15t + 21$ $66t^3 - 246t^2 + 488t - 596$	2 / ✗ 2 / ✗
	10_{56}^a $-2t^3 + 8t^2 - 14t + 17$ $-9t^5 + 52t^4 - 133t^3 + 234t^2 - 312t + 340$	3 / ✗ 2 / ✗		10_{57}^a $2t^3 - 8t^2 + 18t - 23$ $-5t^5 + 28t^4 - 93t^3 + 194t^2 - 300t + 340$	3 / ✗ 2 / ✗
	10_{58}^a $3t^2 - 16t + 27$ $3t^3 - 28t^2 + 94t - 140$	2 / ✗ 2 / ✗		10_{59}^a $t^3 - 7t^2 + 18t - 23$ $-t^5 + 12t^4 - 55t^3 + 128t^2 - 181t + 196$	3 / ✗ 1 / ✗
	10_{60}^a $-t^3 + 7t^2 - 20t + 29$ $5t^5 - 40t^4 + 122t^3 - 176$	3 / ✗ 1 / ✗		10_{61}^a $-2t^3 + 5t^2 - 6t + 7$ $-7t^5 + 20t^4 - 27t^3 + 36t^2 - 35t + 36$	3 / ✗ 2, 3 / ✗
	10_{62}^a $t^4 - 3t^3 + 6t^2 - 8t + 9$ $-2t^7 + 8t^6 - 23t^5 + 40t^4 - 63t^3 + 76t^2 - 89t + 88$	4 / ✗ 2 / ✗		10_{63}^a $5t^2 - 14t + 19$ $66t^3 - 220t^2 + 416t - 496$	2 / ✗ 2 / ✗
	10_{64}^a $-t^4 + 3t^3 - 6t^2 + 10t - 11$ $-t^7 + 4t^6 - 11t^5 + 24t^4 - 37t^3 + 52t^2 - 60t + 64$	4 / ✗ 2 / ✗		10_{65}^a $2t^3 - 7t^2 + 14t - 17$ $-5t^5 + 24t^4 - 71t^3 + 124t^2 - 169t + 180$	3 / ✗ 2 / ✗
	10_{66}^a $3t^3 - 9t^2 + 16t - 19$ $30t^5 - 112t^4 + 279t^3 - 480t^2 + 662t - 724$	3 / ✗ 3 / ✗		10_{67}^a $-4t^2 + 16t - 23$ $24t^3 - 140t^2 + 312t - 392$	2 / ✗ 2 / ✗
	10_{68}^a $4t^2 - 14t + 21$ $8t^3 - 40t^2 + 117t - 164$	2 / ✗ 2 / ✗		10_{69}^a $t^3 - 7t^2 + 21t - 29$ $-t^5 + 12t^4 - 68t^3 + 212t^2 - 397t + 476$	3 / ✗ 2 / ✗
	10_{70}^a $t^3 - 7t^2 + 16t - 19$ $-t^5 + 12t^4 - 53t^3 + 114t^2 - 146t + 152$	3 / ✗ 2 / ✗		10_{71}^a $-t^3 + 7t^2 - 18t + 25$ $t^3 - 2t^2 - t + 4$	3 / ✗ 1 / ✗
	10_{72}^a $-2t^3 + 9t^2 - 16t + 19$ $-9t^5 + 60t^4 - 167t^3 + 298t^2 - 410t + 448$	3 / ✗ 2 / ✗		10_{73}^a $t^3 - 7t^2 + 20t - 27$ $t^5 - 12t^4 + 65t^3 - 194t^2 + 350t - 416$	3 / ✗ 1 / ✗
	10_{74}^a $-4t^2 + 16t - 23$ $24t^3 - 136t^2 + 290t - 360$	2 / ✗ 2 / ✗		10_{75}^a $-t^3 + 7t^2 - 19t + 27$ $-4t^3 + 36t^2 - 117t + 172$	3 / ✓ 2 / ✗
	10_{76}^a $-2t^3 + 7t^2 - 12t + 15$ $-9t^5 + 44t^4 - 104t^3 + 184t^2 - 245t + 272$	3 / ✗ 2, 3 / ✗		10_{77}^a $2t^3 - 7t^2 + 14t - 17$ $-5t^5 + 24t^4 - 71t^3 + 132t^2 - 189t + 208$	3 / ✗ 2, 3 / ✗
	10_{78}^a $-t^3 + 7t^2 - 16t + 21$ $2t^5 - 24t^4 + 105t^3 - 244t^2 + 390t - 448$	3 / ✗ 2 / ✗		10_{79}^a $t^4 - 3t^3 + 7t^2 - 12t + 15$ 0	4 / ✗ 2, 3 / ✓
	10_{80}^a $3t^3 - 9t^2 + 15t - 17$ $30t^5 - 112t^4 + 260t^3 - 426t^2 + 568t - 616$	3 / ✗ 3 / ✗		10_{81}^a $-t^3 + 8t^2 - 20t + 27$ 0	3 / ✗ 2 / ✓
	10_{82}^a $-t^4 + 4t^3 - 8t^2 + 12t - 13$ $t^7 - 6t^6 + 19t^5 - 42t^4 + 64t^3 - 78t^2 + 84t - 84$	4 / ✗ 1 / ✗		10_{83}^a $2t^3 - 9t^2 + 19t - 23$ $-5t^5 + 34t^4 - 110t^3 + 214t^2 - 301t + 332$	3 / ✗ 2 / ✗
	10_{84}^a $2t^3 - 9t^2 + 20t - 25$ $-5t^5 + 34t^4 - 116t^3 + 246t^2 - 373t + 424$	3 / ✗ 1 / ✗		10_{85}^a $t^4 - 4t^3 + 8t^2 - 10t + 11$ $2t^7 - 12t^6 + 36t^5 - 68t^4 + 101t^3 - 124t^2 + 138t - 140$	4 / ✗ 2 / ✗
	10_{86}^a $-2t^3 + 9t^2 - 19t + 25$ $-t^5 + 6t^4 - 21t^3 + 58t^2 - 105t + 128$	3 / ✗ 2 / ✗		10_{87}^a $-2t^3 + 9t^2 - 18t + 23$ $-t^5 + 6t^4 - 23t^3 + 66t^2 - 125t + 152$	3 / ✓ 2 / ✗
	10_{88}^a $-t^3 + 8t^2 - 24t + 35$ 0	3 / ✗ 1 / ✓		10_{89}^a $t^3 - 8t^2 + 24t - 33$ $t^5 - 14t^4 + 83t^3 - 264t^2 + 495t - 596$	3 / ✗ 2 / ✗
	10_{90}^a $-2t^3 + 8t^2 - 17t + 23$ $-t^5 + 6t^4 - 21t^3 + 54t^2 - 93t + 112$	3 / ✗ 2 / ✗		10_{91}^a $t^4 - 4t^3 + 9t^2 - 14t + 17$ $t^5 - 2t^4 + 2t^3 - 3t + 4$	4 / ✗ 1 / ✗
	10_{92}^a $-2t^3 + 10t^2 - 20t + 25$ $-9t^5 + 68t^4 - 216t^3 + 428t^2 - 622t + 696$	3 / ✗ 2 / ✗		10_{93}^a $2t^3 - 8t^2 + 15t - 17$ $3t^5 - 18t^4 + 43t^3 - 58t^2 + 55t - 48$	3 / ✗ 2 / ✗
	10_{94}^a $-t^4 + 4t^3 - 9t^2 + 14t - 15$ $-t^7 + 6t^6 - 20t^5 + 46t^4 - 76t^3 + 102t^2 - 115t + 120$	4 / ✗ 2 / ✗		10_{95}^a $2t^3 - 9t^2 + 21t - 27$ $-5t^5 + 32t^4 - 114t^3 + 248t^2 - 384t + 436$	3 / ✗ 1 / ✗
	10_{96}^a $-t^3 + 7t^2 - 22t + 33$ $-7t^3 + 50t^2 - 147t + 212$	3 / ✗ 2 / ✗		10_{97}^a $-5t^2 + 22t - 33$ $-37t^3 + 242t^2 - 603t + 788$	2 / ✗ 2 / ✗
	10_{98}^a $-2t^3 + 9t^2 - 18t + 23$ $9t^5 - 60t^4 + 177t^3 - 348t^2 + 501t - 564$	3 / ✗ 2 / ✗		10_{99}^a $t^4 - 4t^3 + 10t^2 - 16t + 19$ 0	4 / ✓ 2 / ✓
	10_{100}^a $t^4 - 4t^3 + 9t^2 - 12t + 13$ $2t^7 - 12t^6 + 39t^5 - 80t^4 + 128t^3 - 164t^2 + 192t - 196$	4 / ✗ 2, 3 / ✗		10_{101}^a $7t^2 - 21t + 29$ $-129t^3 + 480t^2 - 942t + 1148$	2 / ✗ 2, 3 / ✗
	10_{102}^a $-2t^3 + 8t^2 - 16t + 21$ $-t^5 + 6t^4 - 19t^3 + 50t^2 - 89t + 108$	3 / ✗ 1 / ✗		10_{103}^a $2t^3 - 8t^2 + 17t - 21$ $5t^5 - 30t^4 + 93t^3 - 178t^2 + 254t - 280$	3 / ✗ 3 / ✗
	10_{104}^a $t^4 - 4t^3 + 9t^2 - 15t + 19$ $t^5 - 2t^4 + 2t^3 - 3t + 4$	4 / ✗ 1 / ✗		10_{105}^a $t^3 - 8t^2 + 22t - 29$ $-t^5 + 14t^4 - 71t^3 + 184t^2 - 292t + 332$	3 / ✗ 2 / ✗
	10_{106}^a $-t^4 + 4t^3 - 9t^2 + 15t - 17$ $-t^7 + 6t^6 - 20t^5 + 48t^4 - 82t^3 + 114t^2 - 134t + 140$	4 / ✗ 2 / ✗		10_{107}^a $-t^3 + 8t^2 - 22t + 31$ $2t^3 - 8t^2 + 13t - 16$	3 / ✗ 1 / ✗

diagram	n'_k Alexander's ω^+ Today's / Rozansky's ρ_1^+	genus / ribbon unknotting number / amphicheiral	diagram	n'_k Alexander's ω^+ Today's / Rozansky's ρ_1^+	genus / ribbon unknotting number / amphicheiral
	10^a_{108} $2t^3 - 8t^2 + 14t - 15$ $-3t^5 + 18t^4 - 41t^3 + 50t^2 - 40t + 32$	3 / ✗ 2 / ✗		10^a_{109} $t^4 - 4t^3 + 10t^2 - 17t + 21$ 0	4 / ✗ 2 / ✓
	10^a_{110} $t^3 - 8t^2 + 20t - 25$ $t^5 - 14t^4 + 69t^3 - 160t^2 + 219t - 236$	3 / ✗ 2 / ✗		10^a_{111} $-2t^3 + 9t^2 - 17t + 21$ $-9t^5 + 60t^4 - 171t^3 + 316t^2 - 436t + 480$	3 / ✗ 2 / ✗
	10^a_{112} $-t^4 + 5t^3 - 11t^2 + 17t - 19$ $t^7 - 8t^6 + 29t^5 - 68t^4 + 115t^3 - 152t^2 + 175t - 180$	4 / ✗ 2 / ✗		10^a_{113} $2t^3 - 11t^2 + 26t - 33$ $-5t^5 + 42t^4 - 167t^3 + 394t^2 - 623t + 720$	3 / ✗ 1 / ✗
	10^a_{114} $-2t^3 + 10t^2 - 21t + 27$ $t^5 - 8t^4 + 30t^3 - 78t^2 + 140t - 168$	3 / ✗ 1 / ✗		10^a_{115} $-t^3 + 9t^2 - 26t + 37$ 0	3 / ✗ 2 / ✓
	10^a_{116} $-t^4 + 5t^3 - 12t^2 + 19t - 21$ $t^7 - 8t^6 + 30t^5 - 74t^4 + 132t^3 - 184t^2 + 217t - 228$	4 / ✗ 2 / ✗		10^a_{117} $2t^3 - 10t^2 + 24t - 31$ $-5t^5 + 38t^4 - 144t^3 + 330t^2 - 522t + 600$	3 / ✗ 2 / ✗
	10^a_{118} $t^4 - 5t^3 + 12t^2 - 19t + 23$ 0	4 / ✗ 1 / ✓		10^a_{119} $-2t^3 + 10t^2 - 23t + 31$ $-t^5 + 6t^4 - 26t^3 + 86t^2 - 175t + 220$	3 / ✗ 1 / ✗
	10^a_{120} $8t^2 - 26t + 37$ $166t^3 - 692t^2 + 1433t - 1788$	2 / ✗ 2, 3 / ✗		10^a_{121} $2t^3 - 11t^2 + 27t - 35$ $5t^5 - 42t^4 + 167t^3 - 396t^2 + 634t - 732$	3 / ✗ 2 / ✗
	10^a_{122} $-2t^3 + 11t^2 - 24t + 31$ $-t^5 + 8t^4 - 34t^3 + 104t^2 - 211t + 264$	3 / ✗ 2 / ✗		10^a_{123} $t^4 - 6t^3 + 15t^2 - 24t + 29$ 0	4 / ✓ 2 / ✓
	10^a_{124} $t^4 - t^3 + t - 1$ $-4t^7 - 6t^4 - 4t^2 - 6t$	4 / ✗ 4 / ✗		10^a_{125} $t^3 - 2t^2 + 2t - 1$ $-t^5 + 2t^4 - 2t^3 + 3t - 4$	3 / ✗ 2 / ✗
	10^a_{126} $t^3 - 2t^2 + 4t - 5$ $t^5 - 2t^4 + 10t^3 - 12t^2 + 22t - 20$	3 / ✗ 2 / ✗		10^a_{127} $-t^3 + 4t^2 - 6t + 7$ $2t^5 - 14t^4 + 32t^3 - 52t^2 + 67t - 72$	3 / ✗ 2 / ✗
	10^a_{128} $2t^3 - 3t^2 + t + 1$ $-13t^5 + 12t^4 - 3t^3 - 10t^2 - 9t + 12$	3 / ✗ 3 / ✗		10^a_{129} $2t^2 - 6t + 9$ $-t^3 - 2t^2 + 14t - 20$	2 / ✓ 1 / ✗
	10^a_{130} $2t^2 - 4t + 5$ $t^3 - 2t^2 + 19t - 24$	2 / ✗ 2 / ✗		10^a_{131} $-2t^2 + 8t - 11$ $5t^3 - 38t^2 + 87t - 112$	2 / ✗ 1 / ✗
	10^a_{132} $t^2 - t + 1$ $2t^2 + 5t - 4$	2 / ✗ 1 / ✗		10^a_{133} $-t^2 + 5t - 7$ $t^3 - 14t^2 + 37t - 48$	2 / ✗ 1 / ✗
	10^a_{134} $2t^3 - 4t^2 + 4t - 3$ $-13t^5 + 24t^4 - 33t^3 + 30t^2 - 41t + 40$	3 / ✗ 3 / ✗		10^a_{135} $3t^2 - 9t + 13$ $t^3 - 6t^2 + 18t - 24$	2 / ✗ 2 / ✗
	10^a_{136} $-t^2 + 4t - 5$ $-t^3 + 4t^2 - 2t - 4$	2 / ✗ 1 / ✗		10^a_{137} $t^2 - 6t + 11$ $-4t^2 + 24t - 44$	2 / ✓ 1 / ✗
	10^a_{138} $t^3 - 5t^2 + 8t - 7$ $-t^5 + 8t^4 - 22t^3 + 24t^2 - 11t + 8$	3 / ✗ 2 / ✗		10^a_{139} $t^4 - t^3 + 2t - 3$ $-4t^7 - 12t^4 + 5t^3 - 4t^2 - 16t + 12$	4 / ✗ 4 / ✗
	10^a_{140} $t^2 - 2t + 3$ $8t - 8$	2 / ✓ 2 / ✗		10^a_{141} $-t^3 + 3t^2 - 4t + 5$ $t^3 - 8t^2 + 16t - 20$	3 / ✗ 1 / ✗
	10^a_{142} $2t^3 - 3t^2 + 2t - 1$ $-13t^5 + 12t^4 - 13t^3 + 4t^2 - 17t + 12$	3 / ✗ 3 / ✗		10^a_{143} $t^3 - 3t^2 + 6t - 7$ $t^5 - 4t^4 + 15t^3 - 28t^2 + 45t - 48$	3 / ✗ 1 / ✗
	10^a_{144} $-3t^2 + 10t - 13$ $10t^3 - 44t^2 + 80t - 96$	2 / ✗ 2 / ✗		10^a_{145} $t^2 + t - 3$ $2t^3 + 8t^2 + 6t - 8$	2 / ✗ 2 / ✗
	10^a_{146} $2t^2 - 8t + 13$ $t^3 - 8t^2 + 21t - 28$	2 / ✗ 1 / ✗		10^a_{147} $-2t^2 + 7t - 9$ $-3t^3 + 12t^2 - 15t + 12$	2 / ✗ 1 / ✗
	10^a_{148} $t^3 - 3t^2 + 7t - 9$ $t^5 - 4t^4 + 18t^3 - 36t^2 + 62t - 68$	3 / ✗ 2 / ✗		10^a_{149} $-t^3 + 5t^2 - 9t + 11$ $2t^5 - 18t^4 + 55t^3 - 104t^2 + 149t - 164$	3 / ✗ 2 / ✗
	10^a_{150} $-t^3 + 4t^2 - 6t + 7$ $-2t^5 + 12t^4 - 26t^3 + 38t^2 - 45t + 44$	3 / ✗ 2 / ✗		10^a_{151} $t^3 - 4t^2 + 10t - 13$ $-t^5 + 6t^4 - 21t^3 + 42t^2 - 66t + 72$	3 / ✗ 2 / ✗
	10^a_{152} $t^4 - t^3 - t^2 + 4t - 5$ $4t^7 - 7t^5 + 18t^4 - 7t^3 - 12t^2 + 45t - 52$	4 / ✗ 4 / ✗		10^a_{153} $t^3 - t^2 - t + 3$ $t^5 - 2t^4 + t^3 + 2t^2 - t$	3 / ✓ 2 / ✗
	10^a_{154} $t^3 - 4t + 7$ $-3t^5 - 6t^4 + 13t^3 - 47t + 68$	3 / ✗ 3 / ✗		10^a_{155} $-t^3 + 3t^2 - 5t + 7$ $-2t^3 + 12t^2 - 22t + 28$	3 / ✓ 2 / ✗
	10^a_{156} $t^3 - 4t^2 + 8t - 9$ $t^5 - 6t^4 + 19t^3 - 30t^2 + 33t - 32$	3 / ✗ 1 / ✗		10^a_{157} $-t^3 + 6t^2 - 11t + 13$ $-2t^5 + 22t^4 - 78t^3 + 148t^2 - 218t + 240$	3 / ✗ 2 / ✗
	10^a_{158} $-t^3 + 4t^2 - 10t + 15$ $2t^2 - 7t + 12$	3 / ✗ 2 / ✗		10^a_{159} $t^3 - 4t^2 + 9t - 11$ $t^5 - 6t^4 + 26t^3 - 60t^2 + 98t - 112$	3 / ✗ 1 / ✗
	10^a_{160} $-t^3 + 4t^2 - 4t + 3$ $-2t^5 + 12t^4 - 20t^3 + 14t^2 - 16t + 12$	3 / ✗ 2 / ✗		10^a_{161} $t^3 - 2t + 3$ $3t^5 + 6t^4 - 3t^3 + 4t^2 + 14t - 12$	3 / ✗ 3 / ✗
	10^a_{162} $-3t^2 + 9t - 11$ $10t^3 - 38t^2 + 58t - 68$	2 / ✗ 2 / ✗		10^a_{163} $t^3 - 5t^2 + 12t - 15$ $-t^5 + 8t^4 - 30t^3 + 62t^2 - 89t + 96$	3 / ✗ 1, 2 / ✗
	10^a_{164} $3t^2 - 11t + 17$ $t^3 - 10t^2 + 29t - 40$	2 / ✗ 1 / ✗		10^a_{165} $-2t^2 + 10t - 15$ $-5t^3 + 50t^2 - 146t + 196$	2 / ✗ 2 / ✗