Pensieve header: Fixing the bug with R2c.

```
In[∘]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\ICERM-2305"];
```

```
In[∘]:= Once[<< KnotTheory`];
```

Loading KnotTheory` version of October 29, 2024, 10:29:52.1301.
Read more at http://katlas.org/wiki/KnotTheory.

Utilities. The step function, algebraic numbers, canonical forms.

```
In[∘]:= θ[x_] /; NumericQ[x] := UnitStep[x]
```

```
In[∘]:= ω2[v_][p_] := Module[{q = Expand[p], n, c},
    If[q === 0, 0, c = Coefficient[q, ω, n = Exponent[q, ω]];
     c v^n + ω2[v][q - c (ω + ω^-1)^n]]];
```

```
In[∘]:= sign[ℰ_] := Module[{n, d, v, p, rs, e, k},
   {n, d} = NumeratorDenominator[ℰ];
   {n, d} /= ω^(Exponent[n,ω]/2+Exponent[n,ω,Min]/2);
   p = Factor[ω2[v]@n * ω2[v]@d /. v → 4 u^2 - 2];
   rs = Solve[p == 0, u, Reals];
   If[rs === {}, Sign[p /. u → 0],
    rs = Union@(u /. rs);
    Sign[(-1)^(e=Exponent[p,u]) Coefficient[p, u, e]] + Sum[
      k = 0; While[(d = RootReduce[∂_{u,++k} p /. u → r]) == 0];
      If[EvenQ[k], 0, 2 Sign[d]] * θ[u - r],
      {r, rs}]
   ]
  ]
```

```
In[∘]:= SetAttributes[B, Orderless];
CF[b_B] := RotateLeft[#, First@Ordering[#] - 1] & /@ DeleteCases[b, {}]
```

```
In[∘]:= CF[ℰ_] := Module[{γs = Union@Cases[ℰ, γ_ | γ̄_, ∞]},
   Total[CoefficientRules[ℰ, γs] /. (ps_ → c_) :> Factor[c] Times @@ γs^ps] ]
```

```
In[∘]:= CF[{}] = {};
CF[C_List] := Module[{γs = Union@Cases[C, γ_, ∞], γ},
   CF /@ DeleteCases[0][
     RowReduce[Table[∂_γ r, {r, C}, {γ, γs}]].γs] ]
```

```
In[∘]:= (ℰ_)* := ℰ /. {γ̄ → γ, γ → γ̄, ω → ω^-1, c_Complex ⧴ c*};
        r_Rule⁺ := {r, r*}
```

```
In[∘]:= RulesOf[γi_ + rest_.] := (γi → -rest)⁺;
        CF[PQ[𝒞_, q_]] := Module[{n𝒞 = CF[𝒞]},
          PQ[n𝒞, CF[q /. Union @@ RulesOf /@ n𝒞]] ]
```

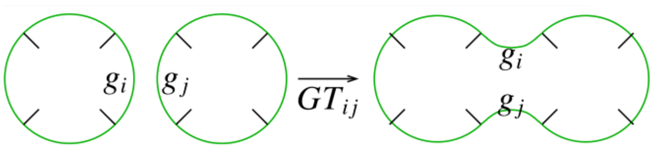
```
In[∘]:= CF[Σb_[σ_, pq_]] := ΣCF[b][σ, CF[pq]]
```

Pretty-Printing.

```
In[∘]:= Format[Σb_B[σ_, PQ[𝒞_, q_]]] := Module[{γs},
          γs = γ# & /@ Join @@ b;
          Column[{TraditionalForm@σ,
            TableForm[Join[
              Prepend[""] /@ Table[TraditionalForm[∂c r], {r, 𝒞}, {c, γs}],
              {Prepend[""][
                Join @@ (b /. {l_, m___, r_} ⧴ {DisplayForm@RowBox[{"(", l}],
                      m, DisplayForm@RowBox[{r, ")"}]}) /. i_Integer ⧴ γi ]},
              MapThread[Prepend, {Table[TraditionalForm[∂r,c q], {r, γs*}, {c, γs}], γs*}]
             ], TableAlignments → Center]
          }, Center] ];
```

The Face-Centric Core.

```
In[∘]:= Σb1_[σ1_, PQ[𝒞1_, q1_]] ⊕ Σb2_[σ2_, PQ[𝒞2_, q2_]] ^:=
          CF@ΣJoin[b1,b2][σ1 + σ2, PQ[𝒞1 ⋃ 𝒞2, q1 + q2]];
```



GT for Gap Touch:

```
In[∘]:= GTi_,j_@ΣB[{li___,i_,ri___},{lj___,j_,rj___},bs___][σ_, PQ[𝒞_, q_]] :=
         CF@ΣB[{ri,li,j,rj,lj,i},bs][σ, PQ[𝒞 ⋃ {γi - γj}, q]]
```

cor·don (kôr′dn)

n.

1. A line of people, military posts, or ships stationed around an area to enclose or guard it: *a police cordon.*

2. A rope, line, tape, or similar border stretched around an area, usually by the police, indicating that access is restricted.

\par\vskip 1mm\par\Cordon

In[∘]:=
```
Cordon_i_@Σ_B[{li___,i_,ri___},bs___][σ_, PQ[C_, q_]] :=
 Module[{ϕ = ∂_{γ_i} C, λ = ∂_{γ̄_i,γ_i} q, nσ = σ, nC, nq, p},
  {p} = FirstPosition[(# =!= 0) & /@ ϕ, True, {0}];
  {nC, nq} = Which[
    p > 0, {C, q} /. (γ_i → -C⟦p⟧ / ϕ⟦p⟧)^+ /. (γ_i → 0)^+,
    λ =!= 0, (nσ += sign[λ]; {C, q /. (γ_i → -(∂_{γ̄_i} q) / λ)^+ /. (γ_i → 0)^+}),
    λ === 0, {C ⋃ {∂_{γ̄_i} q}, q /. (γ_i → 0)^+}];
  CF@Σ_B[Most@{ri,li},bs][nσ, PQ[nC, nq] /. (ϒ_Last@{ri,li} → ϒ_First@{ri,li})^+] ]
```

Strand Operations. *c* for contract, mc for magnetic contract:

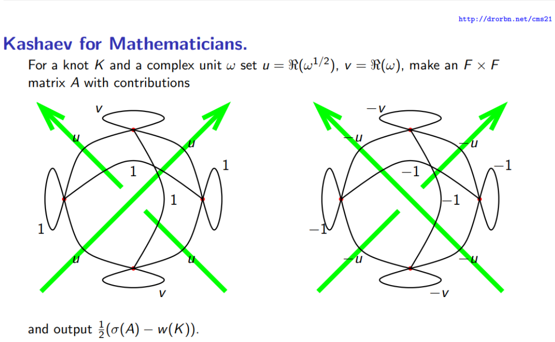
In[∘]:=
```
c_i_,j_@t : Σ_B[{li___,i_,ri___},{___,j_,___},___][__] := t // GT_j,First@{ri,li} // Cordon_j
```

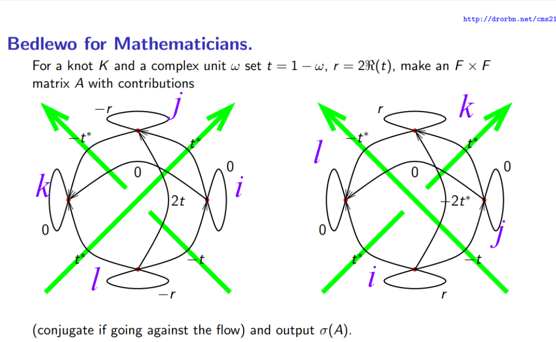
In[∘]:=
```
c_i_,j_@t : Σ_B[{___,i_,j_,___},___][__] := Cordon_j@t
c_i_,j_@t : Σ_B[{j_,___,i_},___][__] := Cordon_j@t
c_i_,j_@t : Σ_B[{___,j_,i_,___},___][__] := Cordon_i@t
c_i_,j_@t : Σ_B[{i_,___,j_},___][__] := Cordon_i@t
```

In[∘]:=
```
mc[E_] := E //.
  t : Σ_B[{___,i_,___},{___,j_,___},___][__] | Σ_B[{___,i_,j_,___},___][__] | Σ_B[{j_,___,i_},___][__] /;
   i + j == 0 :> c_i,j@t
```

The Crossings (and empty strands).

In[∘]:=
```
Kas@P_i_,j_ := CF@Σ_B[{i,j}][0, PQ[{}, 0]];
TL@P_i_,j_ := CF@Σ_B[{i,j}][0, PQ[{}, 0]]
```





In[∘]:=
```
Kas[x : X[i_, j_, k_, l_]] := Kas@If[PositiveQ[x], X_{-i,j,k,-l}, X̄_{-j,k,l,-i}];
Kas[(x : X | X̄)_{fs__}] := Module[{v = 2 u^2 - 1, p, γs, m},
  γs = γ_# & /@ {fs}; p = (x === X);
  m = If[p, {{v, u, 1, u}, {u, 1, u, 1}, {1, u, v, u}, {u, 1, u, 1}}, -{{v, u, 1, u}, {u, 1, u, 1}, {1, u, v, u}, {u, 1, u, 1}}];
  CF@Σ_{B[{fs}]}[If[p, -1, 1], PQ[{}, γs*.m.γs]]]
```

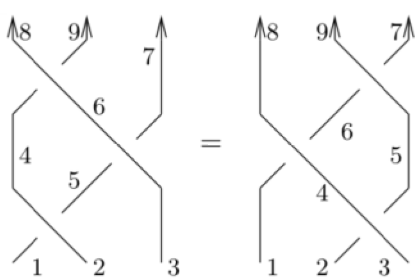
In[∘]:=
```
TL[x : X[i_, j_, k_, l_]] := TL@If[PositiveQ[x], X_{-i,j,k,-l}, X̄_{-j,k,l,-i}];
TL[(x : X | X̄)_{fs__}] := Module[{t = 1 - ω, r, γs, m},
  r = t + t*; γs = γ_# & /@ {fs};
  m = If[x === X,
    {{-r, -t, 2 t, t*}, {-t*, 0, t*, 0}, {2 t*, t, -r, -t*}, {t, 0, -t, 0}}, {{r, -t, -2 t*, t*}, {-t*, 0, t*, 0}, {-2 t, t, r, -t*}, {t, 0, -t, 0}}];
  CF@Σ_{B[{fs}]}[0, PQ[{}, γs*.m.γs]]]
```

\par{\bf\red Evaluation on Tangles and Knots.}

In[∘]:=
```
Kas[K_] := Fold[mc[#1 ⊕ #2] &, Σ_{B[]}[0, PQ[{}, 0]], List @@ (Kas /@ PD@K)];
KasSig[K_] := Expand[Kas[K][[1]] / 2]
```

In[∘]:=
```
TL[K_] := Fold[mc[#1 ⊕ #2] &, Σ_{B[]}[0, PQ[{}, 0]], List @@ (TL /@ PD@K)] /.
   θ[c_ + u] /; Abs[c] ≥ 1 :→ θ[c];
TLSig[K_] := TL[K][[1]]
```

Reidemeister 3



In[∘]:=
```
R3L = PD[X_{-2,5,4,-1}, X_{-3,7,6,-5},
    X_{-6,9,8,-4}];
R3R = PD[X_{-3,5,4,-2}, X_{-4,6,8,-1},
    X_{-5,7,9,-6}];
{TL@R3L == TL@R3R, Kas@R3L == Kas@R3R}
```

Out[∘]=
```
{True, True}
```

In[∘]:=
```
TL@R3L
```

Out[∘]=

$$
\begin{array}{c|cccccc}
 & \gamma_{-3} & \gamma_{7} & \gamma_{9} & \gamma_{8} & \gamma_{-1} & \gamma_{-2} \\
\bar\gamma_{-3} & \frac{\omega^2+1}{\omega} & \omega-1 & -2\,\omega & 2 & 0 & -\frac{\omega+1}{\omega} \\
\bar\gamma_{7} & -\frac{\omega-1}{\omega} & 0 & \frac{\omega-1}{\omega} & 0 & 0 & 0 \\
\bar\gamma_{9} & -\frac{2}{\omega} & 1-\omega & \frac{\omega^2+1}{\omega} & -\frac{\omega+1}{\omega} & 0 & \frac{2}{\omega} \\
\bar\gamma_{8} & 2 & 0 & -\omega-1 & \frac{\omega^2+1}{\omega} & -\frac{\omega-1}{\omega} & -\frac{2}{\omega} \\
\bar\gamma_{-1} & 0 & 0 & 0 & \omega-1 & 0 & 1-\omega \\
\bar\gamma_{-2} & -\omega-1 & 0 & 2\,\omega & -2\,\omega & \frac{\omega-1}{\omega} & \frac{\omega^2+1}{\omega}
\end{array}^{-1}
$$

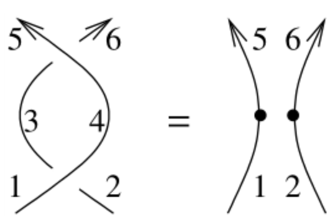
In[∘]:=
```
Kas@R3L
```

Out[∘]=

$$2\,\ominus\left(u-\frac{1}{2}\right)-2\,\ominus\left(u+\frac{1}{2}\right)-2$$

$$
\begin{array}{c|cccccc}
 & \gamma_{-3} & \gamma_{7} & \gamma_{9} & \gamma_{8} & \gamma_{-1} & \gamma_{-2} \\
\bar\gamma_{-3} & \frac{2u^2\,(4u^2-3)}{(2u-1)\,(2u+1)} & \frac{u\,(4u^2-3)}{(2u-1)\,(2u+1)} & -\frac{1}{(2u-1)\,(2u+1)} & -\frac{2u}{(2u-1)\,(2u+1)} & -\frac{1}{(2u-1)\,(2u+1)} & \frac{u\,(4u^2-}{(2u-1)\,(2} \\
\bar\gamma_{7} & \frac{u\,(4u^2-3)}{(2u-1)\,(2u+1)} & \frac{2\,(2u^2-1)}{(2u-1)\,(2u+1)} & \frac{u\,(4u^2-3)}{(2u-1)\,(2u+1)} & -\frac{1}{(2u-1)\,(2u+1)} & -\frac{2u}{(2u-1)\,(2u+1)} & -\frac{1}{(2u-1)\,(} \\
\bar\gamma_{9} & -\frac{1}{(2u-1)\,(2u+1)} & \frac{u\,(4u^2-3)}{(2u-1)\,(2u+1)} & \frac{2u^2\,(4u^2-3)}{(2u-1)\,(2u+1)} & \frac{u\,(4u^2-3)}{(2u-1)\,(2u+1)} & -\frac{1}{(2u-1)\,(2u+1)} & -\frac{2u}{(2u-1)\,(} \\
\bar\gamma_{8} & -\frac{2u}{(2u-1)\,(2u+1)} & -\frac{1}{(2u-1)\,(2u+1)} & \frac{u\,(4u^2-3)}{(2u-1)\,(2u+1)} & \frac{2u^2\,(4u^2-3)}{(2u-1)\,(2u+1)} & \frac{u\,(4u^2-3)}{(2u-1)\,(2u+1)} & -\frac{1}{(2u-1)\,(} \\
\bar\gamma_{-1} & -\frac{1}{(2u-1)\,(2u+1)} & -\frac{2u}{(2u-1)\,(2u+1)} & -\frac{1}{(2u-1)\,(2u+1)} & \frac{u\,(4u^2-3)}{(2u-1)\,(2u+1)} & \frac{2\,(2u^2-1)}{(2u-1)\,(2u+1)} & \frac{u\,(4u^2-}{(2u-1)\,(2} \\
\bar\gamma_{-2} & \frac{u\,(4u^2-3)}{(2u-1)\,(2u+1)} & -\frac{1}{(2u-1)\,(2u+1)} & -\frac{2u}{(2u-1)\,(2u+1)} & -\frac{1}{(2u-1)\,(2u+1)} & \frac{u\,(4u^2-3)}{(2u-1)\,(2u+1)} & \frac{2u^2\,(4u^2}{(2u-1)\,(2}
\end{array}
$$

Reidemeister 2b



In[∘]:= $\text{TL@PD}\left[X_{-2,4,3,-1},\ \overline{X}_{-4,6,5,-3}\right]$

Out[∘]=

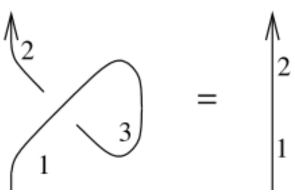
$$\begin{array}{ccccc} & & 0 & & \\ & 1 & 0 & -1 & 0 \\ & (\Upsilon_{-2} & \Upsilon_6 & \Upsilon_5 & \Upsilon_{-1}) \\ \overline{\Upsilon}_{-2} & 0 & 0 & 0 & 0 \\ \overline{\Upsilon}_6 & 0 & 0 & 0 & 0 \\ \overline{\Upsilon}_5 & 0 & 0 & 0 & 0 \\ \overline{\Upsilon}_{-1} & 0 & 0 & 0 & 0 \end{array}$$

In[∘]:= $\{\text{TL@PD}\left[X_{-2,4,3,-1},\ \overline{X}_{-4,6,5,-3}\right] == \text{GT}_{5,-2}\text{@TL@PD}[P_{-1,5},\ P_{-2,6}],$
$\text{Kas@PD}\left[X_{-2,4,3,-1},\ \overline{X}_{-4,6,5,-3}\right] == \text{GT}_{5,-2}\text{@Kas@PD}[P_{-1,5},\ P_{-2,6}]\}$

Out[∘]=

{True, True}

Reidemeister 1

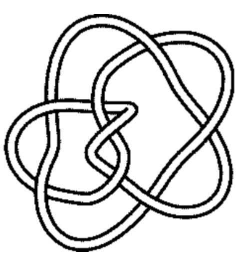


In[∘]:= $\{\text{TL@PD}[X_{-3,3,2,-1}] == \text{TL@}P_{-1,2},$
$\text{Kas@PD}[X_{-3,3,2,-1}] == \text{Kas@}P_{-1,2}\}$

Out[∘]=

{True, True}

A Knot



In[∘]:= f = TLSig[Knot[8, 5]]

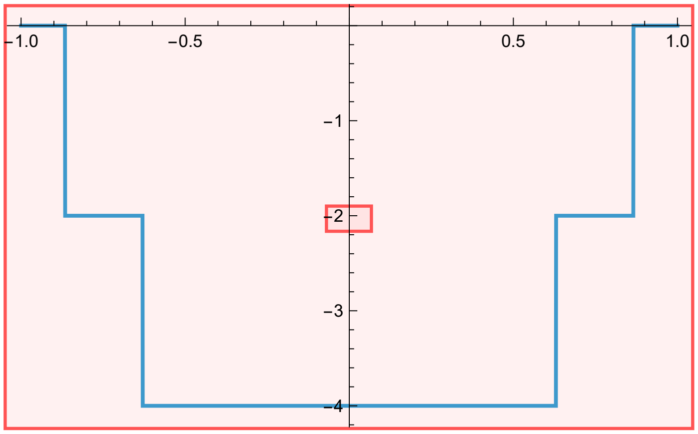
KnotTheory: Loading precomputed data in PD4Knots`.

Out[∘]=

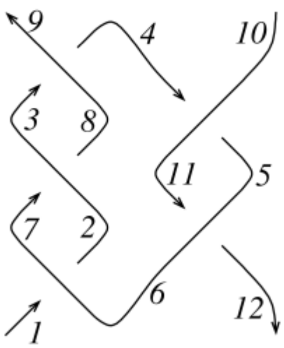
$$2\,\theta\left[-\frac{\sqrt{3}}{2}+u\right]-2\,\theta\left[\frac{\sqrt{3}}{2}+u\right]-2\,\theta\left[u-\boxed{-0.630\ldots}\right]+2\,\theta\left[u-\boxed{0.630\ldots}\right]$$

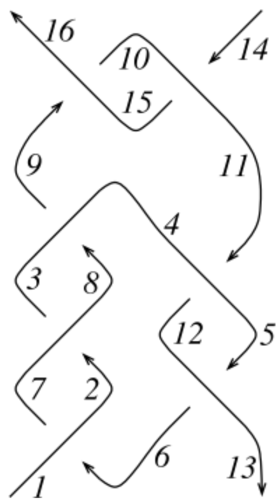
In[ ]:= `Plot[f, {u, -1, 1}]`

Out[ ]=



Some Tangles





In[ ]:=

$$T1 = PD\left[\overline{X}_{-6,2,7,-1}, \overline{X}_{-2,8,3,-7}, \overline{X}_{-8,4,9,-3}, X_{-11,6,12,-5}, X_{-4,11,5,-10}\right];$$

$$T2 = PD\left[X_{-6,2,7,-1}, X_{-2,8,3,-7}, X_{-8,4,9,-3}, \overline{X}_{-12,6,13,-5}, \overline{X}_{-4,12,5,-11}, \overline{X}_{-10,15,11,-14}, \overline{X}_{-15,10,16,-9}\right];$$

In[∘]:= `Column@{TL[T1], Kas[T1]}`

Out[∘]=

$$-2\,\theta\left(u-\frac{\sqrt{3}}{2}\right)+2\,\theta\left(u+\frac{\sqrt{3}}{2}\right)-1$$

$$\begin{array}{c|cccc}
 & \Upsilon_{-10} & \Upsilon_{9} & \Upsilon_{-1} & \Upsilon_{12} \\
\hline
\overline{\Upsilon}_{-10} & 0 & 1-\omega & 0 & \omega-1 \\
\overline{\Upsilon}_{9} & \frac{\omega-1}{\omega} & \frac{2\,\omega}{\omega^2-\omega+1} & -\frac{\omega-1}{\omega} & -\frac{2\,\omega}{\omega^2-\omega+1} \\
\overline{\Upsilon}_{-1} & 0 & \omega-1 & 0 & 1-\omega \\
\overline{\Upsilon}_{12} & -\frac{\omega-1}{\omega} & -\frac{2\,\omega}{\omega^2-\omega+1} & \frac{\omega-1}{\omega} & \frac{2\,\omega}{\omega^2-\omega+1}
\end{array}$$

$$-2\,\theta\left(u-\frac{\sqrt{3}}{2}\right)+2\,\theta\left(u+\frac{\sqrt{3}}{2}\right)-1$$

$$\begin{array}{c|cccc}
 & \Upsilon_{-10} & \Upsilon_{9} & \Upsilon_{-1} & \Upsilon_{12} \\
\hline
\overline{\Upsilon}_{-10} & 2\,(u-1)\,(u+1)\,\left(4\,u^2-3\right) & 0 & -2\,(u-1)\,(u+1)\,\left(4\,u^2-3\right) & 0 \\
\overline{\Upsilon}_{9} & 0 & \frac{1}{2\,\left(4\,u^2-3\right)} & 0 & -\frac{1}{2\,\left(4\,u^2-3\right)} \\
\overline{\Upsilon}_{-1} & -2\,(u-1)\,(u+1)\,\left(4\,u^2-3\right) & 0 & 2\,(u-1)\,(u+1)\,\left(4\,u^2-3\right) & 0 \\
\overline{\Upsilon}_{12} & 0 & -\frac{1}{2\,\left(4\,u^2-3\right)} & 0 & \frac{1}{2\,\left(4\,u^2-3\right)}
\end{array}$$

In[∘]:= `Column@{TL[T2], Kas[T2]}`

Out[∘]=

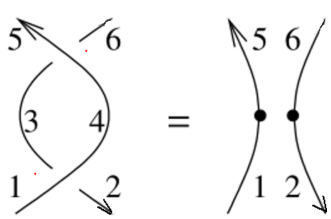
$$0$$

$$\begin{array}{c|cccc}
 & \Upsilon_{-14} & \Upsilon_{16} & \Upsilon_{-1} & \Upsilon_{13} \\
\hline
\overline{\Upsilon}_{-14} & 0 & 1-\omega & 0 & \omega-1 \\
\overline{\Upsilon}_{16} & \frac{\omega-1}{\omega} & -\frac{2\,(\omega-1)^2\,\omega}{\omega^4-3\,\omega^3+5\,\omega^2-3\,\omega+1} & -\frac{\omega-1}{\omega} & \frac{2\,(\omega-1)^2\,\omega}{\omega^4-3\,\omega^3+5\,\omega^2-3\,\omega+1} \\
\overline{\Upsilon}_{-1} & 0 & \omega-1 & 0 & 1-\omega \\
\overline{\Upsilon}_{13} & -\frac{\omega-1}{\omega} & \frac{2\,(\omega-1)^2\,\omega}{\omega^4-3\,\omega^3+5\,\omega^2-3\,\omega+1} & \frac{\omega-1}{\omega} & -\frac{2\,(\omega-1)^2\,\omega}{\omega^4-3\,\omega^3+5\,\omega^2-3\,\omega+1}
\end{array}$$

$$1$$

$$\begin{array}{c|cccc}
 & \Upsilon_{-14} & \Upsilon_{16} & \Upsilon_{-1} & \Upsilon_{13} \\
\hline
\overline{\Upsilon}_{-14} & \frac{1}{2}\left(-16\,u^4+28\,u^2-13\right) & 0 & \frac{1}{2}\left(16\,u^4-28\,u^2+13\right) & 0 \\
\overline{\Upsilon}_{16} & 0 & -\frac{2\,(u-1)\,(u+1)}{16\,u^4-28\,u^2+13} & 0 & \frac{2\,(u-1)\,(u+1)}{16\,u^4-28\,u^2+13} \\
\overline{\Upsilon}_{-1} & \frac{1}{2}\left(16\,u^4-28\,u^2+13\right) & 0 & \frac{1}{2}\left(-16\,u^4+28\,u^2-13\right) & 0 \\
\overline{\Upsilon}_{13} & 0 & \frac{2\,(u-1)\,(u+1)}{16\,u^4-28\,u^2+13} & 0 & -\frac{2\,(u-1)\,(u+1)}{16\,u^4-28\,u^2+13}
\end{array}$$

Reidemeister 2c



In[∘]:= $\mathtt{Kas@PD}\left[\overline{X}_{-1,2,4,-3},\ X_{-6,5,3,-4}\right] == \mathtt{GT}_{5,2}\mathtt{@Kas@PD}\left[P_{-1,5},\ P_{-6,2}\right]$

Out[∘]=

True

In[∘]:= $\left\{\text{TL@PD}\left[\overline{X}_{-1,2,4,-3}, X_{-6,5,3,-4}\right], \text{TL@PD}\left[X_{-1,2,4,-3}, \overline{X}_{-6,5,3,-4}\right]\right\}$

Out[∘]=

$$\left\{\begin{array}{ccccc} & & 0 & & \\ & (\gamma_{-6} & \gamma_5 & \gamma_{-1} & \gamma_2) \\ \bar\gamma_{-6} & 0 & \omega-1 & 0 & 1-\omega \\ \bar\gamma_5 & -\frac{\omega-1}{\omega} & 0 & \frac{\omega-1}{\omega} & 0 \\ \bar\gamma_{-1} & 0 & 1-\omega & 0 & \omega-1 \\ \bar\gamma_2 & \frac{\omega-1}{\omega} & 0 & -\frac{\omega-1}{\omega} & 0 \end{array}, \begin{array}{ccccc} & & 0 & & \\ & (\gamma_{-6} & \gamma_5 & \gamma_{-1} & \gamma_2) \\ \bar\gamma_{-6} & 0 & \omega-1 & 0 & 1-\omega \\ \bar\gamma_5 & -\frac{\omega-1}{\omega} & 0 & \frac{\omega-1}{\omega} & 0 \\ \bar\gamma_{-1} & 0 & 1-\omega & 0 & \omega-1 \\ \bar\gamma_2 & \frac{\omega-1}{\omega} & 0 & -\frac{\omega-1}{\omega} & 0 \end{array}\right\}$$

In[∘]:= $\text{TL@PD}[P_{-1,5}, P_{-6,2}]$

Out[∘]=

$$\begin{array}{ccccc} & & 0 & & \\ & (\gamma_{-6} & \gamma_2) & (\gamma_{-1} & \gamma_5) \\ \bar\gamma_{-6} & 0 & 0 & 0 & 0 \\ \bar\gamma_2 & 0 & 0 & 0 & 0 \\ \bar\gamma_{-1} & 0 & 0 & 0 & 0 \\ \bar\gamma_5 & 0 & 0 & 0 & 0 \end{array}$$

In[∘]:= $\text{TL@PD}\left[\overline{X}_{-1,2,4,-3}, X_{-6,5,3,-4}\right] == GT_{5,2}\text{@TL@PD}[P_{-1,5}, P_{-6,2}]$

Out[∘]=

$$\begin{array}{ccccc} & & 0 & & \\ & (\gamma_{-6} & \gamma_5 & \gamma_{-1} & \gamma_2) \\ \bar\gamma_{-6} & 0 & \omega-1 & 0 & 1-\omega \\ \bar\gamma_5 & -\frac{\omega-1}{\omega} & 0 & \frac{\omega-1}{\omega} & 0 \\ \bar\gamma_{-1} & 0 & 1-\omega & 0 & \omega-1 \\ \bar\gamma_2 & \frac{\omega-1}{\omega} & 0 & -\frac{\omega-1}{\omega} & 0 \end{array} == \begin{array}{ccccc} & & 0 & & \\ & 0 & -1 & 0 & 1 \\ & (\gamma_{-6} & \gamma_5 & \gamma_{-1} & \gamma_2) \\ \bar\gamma_{-6} & 0 & 0 & 0 & 0 \\ \bar\gamma_5 & 0 & 0 & 0 & 0 \\ \bar\gamma_{-1} & 0 & 0 & 0 & 0 \\ \bar\gamma_2 & 0 & 0 & 0 & 0 \end{array}$$

That’s a fail for TL!