Computation without Representation

Thanks for inviting me to Hefei! ωεβ:=http://drorbn.net/hef18/

Follows Rozansky [Ro1, Ro2, Ro3] and Overbay [Ov], joint with van der Veen. More at [BV] and at ωεβ/talks.

Abstract. A major part of "quantum topology" is the definition and computation of various knot invariants by carrying out computations in quantum groups. Traditionally these computations are carried out "in a representation", but this is very slow: one has to use tensor powers of these representations, and the dimensions of powers grow exponentially fast.
In my talk, I will describe a direct method for carrying out such computations without having to choose a representation and explain why in many ways the results are better and faster. The two key points we use are a technique for composing infinite-order "perturbed Gaussian" differential operators, and the little-known fact that every semi-simple Lie algebra can be approximated by solvable Lie algebras, where computations are easier.

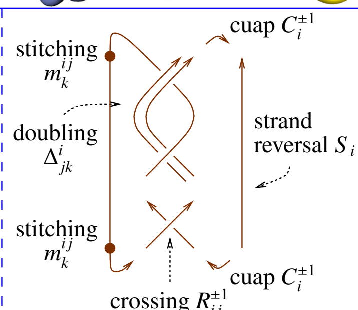
Knotted Candies

ωεβ/kc

A Knot Theory Portfolio.

- Has operations $\sqcup$, m^{ij}_k, Δ^i_{jk}, S_i.
- All tangloids are generated by $R^{\pm1}$ and $C^{\pm1}$ (so "easy" to produce invariants).
- Makes some knot properties ("genus", "ribbon") become "definable".

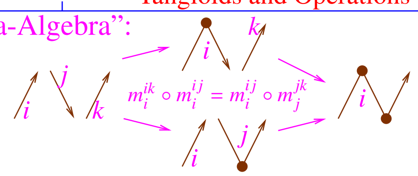
(more to say, but not now).



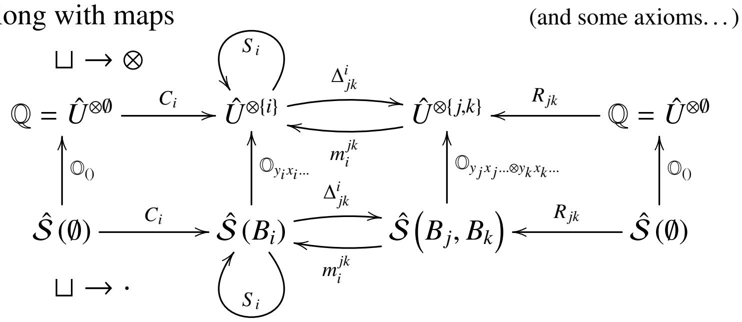
Tangloids and Operations

Tangloids make a "Hopf Meta-Algebra":

(and my original plan was to say more about it)



A "Quantum Group" Portfolio consists of an vector space U along with maps (and some axioms...)



PBW Bases. The U's we care about always have "Poincaré-Birkhoff-Witt" bases; there is some finite set $B = \{y, x, \dots\}$ of "generators" and isomorphisms $\mathbb{O}_{y,x,\dots}\colon \hat{\mathcal{S}}(B) \to U$ defined by "ordering monomials" to some fixed $y, x, \dots$ order. The quantum group portfolio now becomes a "symmetric algebra" portfolio, or a "power series" portfolio.

Operations are Objects.

★ $B^* := \{z^*_i = \zeta_i\colon z_i \in B\}$,

$\langle z_i^m, \zeta_i^n \rangle = \delta_{mn} n!$,

$\left\langle \prod z_i^{m_i}, \prod \zeta_i^{n_i} \right\rangle = \prod \delta_{m_in_i} n_i!$,

in general, for $f \in \mathcal{S}(z_i)$ and $g \in \mathcal{S}(\zeta_i)$,

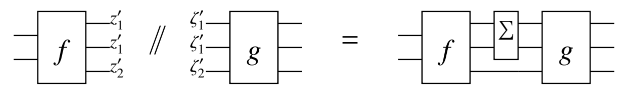
$\langle f, g\rangle = f(\partial_{\zeta_i})g\big|_{\zeta_i=0} = g(\partial_{z_i})f\big|_{z_i=0}$.

$$f \in \operatorname{Hom}_{\mathbb{Q}}(\mathcal{S}(B) \to \mathcal{S}(B')) = \mathcal{S}(B)^* \otimes \mathcal{S}(B') \overset{\star}{=} \mathcal{S}(B^*) \otimes \mathcal{S}(B') = \mathcal{S}(B^* \sqcup B') \ni \tilde{f} \in \mathbb{Q}[\zeta_i, z'_i]$$

The Composition Law. If

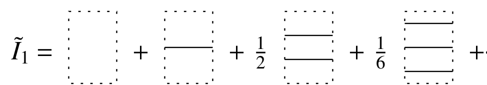
$$\mathcal{S}(B) \xrightarrow[\tilde{f}\in\mathbb{Q}[\![\zeta_i, z'_j]\!]]{f} \mathcal{S}(B') \xrightarrow[\tilde{g}\in\mathbb{Q}[\![\zeta'_j, z''_k]\!]]{g} \mathcal{S}(B'')$$

then $\widetilde{(f/\!\!/g)} = \widetilde{(g \circ f)} = \left(\tilde{g}\big|_{\zeta'_j \to \partial_{z'_j}} \tilde{f}\right)_{z'_j=0} = \left(\tilde{f}\big|_{z'_j \to \partial_{\zeta'_j}} \tilde{g}\right)_{\zeta'_j=0}$:



Examples

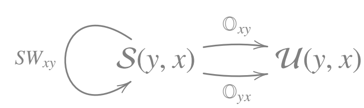
1. The 1-variable identity map $I\colon \mathcal{S}(z) \to \mathcal{S}(z)$ is given by $\tilde{I}_1 = \mathbb{e}^{z\zeta}$ and the n-variable one by $\tilde{I}_n = \mathbb{e}^{z_1\zeta_1 + \cdots + z_n\zeta_n}$:



Proposition. If $F\colon \mathcal{S}(B) \to \mathcal{S}(B')$ is linear, then $\tilde{F} = F\left(\exp\left(\sum_{z_i \in B} \zeta_i z_i\right)\right)$ (in the 1-variable case, $= \sum F(z^n)\frac{1}{n!}\zeta^n$).

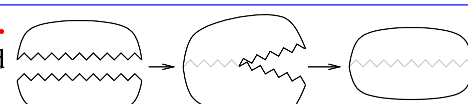
More Examples

2. The "$z_i \to z_j$ variable rename map $\sigma^i_j\colon \mathcal{S}(z_i) \to \mathcal{S}(z_j)$" becomes $\tilde{\sigma}^i_j = \mathbb{e}^{z_j\zeta_i}$, and it's easy to rename several variables simultaneously.
3. The "archetypal multiplication map $m^{ij}_k\colon \mathcal{S}(z_i, z_j) \to \mathcal{S}(z_k)$" has $\tilde{m} = \mathbb{e}^{z_k(\zeta_i + \zeta_j)}$.
4. The "archetypal coproduct $\Delta^i_{jk}\colon \mathcal{S}(z_i) \to \mathcal{S}(z_j, z_k)$", given by $z_i \to z_j + z_k$ or $\Delta z = z \otimes 1 + 1 \otimes z$, has $\tilde{\Delta} = \mathbb{e}^{(z_j+z_k)\zeta_i}$.
5. R-matrices tend to have terms of the form $\mathbb{e}_q^{\hbar y_1 x_2} \in \mathcal{U}_q \otimes \mathcal{U}_q$. The "baby R-matrix" is $\tilde{R} = \mathbb{e}^{\hbar yx} \in \mathcal{S}(y, x)$.
6. The "Weyl form of the canonical commutation relations" states that if $[y, x] = t$ is a scalar, then $\mathbb{e}^{\xi x}\mathbb{e}^{\eta y} = \mathbb{e}^{\eta y}\mathbb{e}^{\xi x}\mathbb{e}^{-\eta\xi t}$. Thus with



we have $\widetilde{SW}_{xy} = \mathbb{e}^{\eta y + \xi x - \eta\xi t}$.

The Zipping Issue. (between unbound and bound lies half-zipped).



Zipping. If $P(\zeta^j, z_i)$ is a polynomial, or whenever otherwise convergent, set

$$\left\langle P(\zeta^j, z_i)\right\rangle_{(\zeta^j)} = P\left(\partial_{z_j}, z_i\right)\Big|_{z_i=0}.$$

(E.g., if $P = \sum a_{nm}\zeta^n z^m$ then $\langle P\rangle_\zeta = \sum a_{nm}\, \partial_z^n z^m\big|_{z=0} = \sum n! a_{nn}$).

Implementation. ωεβ/Zip

```
z* = ζ; ζ* = z; Zip{}[P_] := P;
Zip{ζ_,ζs___}[P_] :=
  (Expand[P // Zip{ζs}] /. f_. ζ^d_. :> ∂{ζ*,d}f) /. ζ* → 0
{Zip{ζ}[ζ^2 e^(δ z^2)], Zip{ζ}[ζ^4 e^(δ z^2)]}
```
{2 δ, 12 δ^2}

The Zipping / Contraction Theorem. If P has a finite ζ-degree and the y's and the q's are "small" then

$$\left\langle P(z_i,\zeta^j)\mathbb{e}^{c+\eta^i z_i + y_j\zeta^j + q^i_j z_i\zeta^j}\right\rangle_{(\zeta^j)} = \det(\tilde q)\left\langle P(z_i,\zeta^j)\mathbb{e}^{c+\eta^i z_i}\Big|_{z_i\to\tilde q^k_i(z_k+y_k)}\right\rangle_{(\zeta^j)}$$

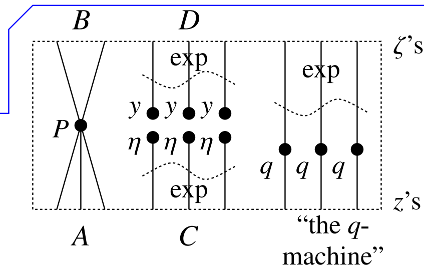
where $\tilde q$ is the inverse matrix of $1-q$: $(\delta^i_j - q^i_j)\tilde q^j_k = \delta^i_k$.

Exponential Reservoirs.. The true Hilbert hotel is exp! Remove one x from an "exponential reservoir" of x's and you are left with the same exponential reservoir:

$$\mathbb{e}^x = \left[\ldots + \frac{xxxxx}{120} + \ldots\right] \xrightarrow{\partial_x} \left[\ldots + \frac{x\!\!\!/xxxx}{120} + \ldots\right] = (\mathbb{e}^x)' = \mathbb{e}^x,$$

and if you let each element choose left or right, you get twice the same reservoir:

$$\mathbb{e}^x \xrightarrow{x\to x_l+x_r} \mathbb{e}^{x_l+x_r} = \mathbb{e}^{x_l}\mathbb{e}^{x_r}.$$



A Graphical Proof. Glue top to bottom on the right, in all possible ways. Several scenarios occur:

1. Start at A, go through the q-machine $k\geq 0$ times, stop at B. Get $\left\langle P\left(\sum_{k\geq 0} q^k z,\zeta\right)\right\rangle = \langle P(\tilde q z,\zeta)\rangle$.
2. Loop through the q-machine and swallow your own tail. Get $\exp\left(\sum q^k/k\right) = \exp(-\log(1-q)) = \tilde q$.
3. ...

By the reservoir splitting principle, these scenarios contribute multiplicatively. □

Implementation. ωεβ/Zip

```
𝔼 /: Zip_{ζs_List}@𝔼[Q_, P_] :=      (* 𝔼[Q,P] means e^Q P *)
  Module[{ζ, z, zs, c, ys, ηs, qt, zrule, Q1, Q2},
   zs = Table[ζ*, {ζ, ζs}];
   c = Q /. Alternatives @@ (ζs ⋃ zs) → 0;
   ys = Table[∂_ζ (Q /. Alternatives @@ zs → 0), {ζ, ζs}];
   ηs = Table[∂_z (Q /. Alternatives @@ ζs → 0), {z, zs}];
   qt = Inverse@Table[Kδ_{z,ζ*} - ∂_{z,ζ}Q, {ζ, ζs}, {z, zs}];
   zrule = Thread[zs → qt.(zs + ys)];
   Q1 = c + ηs.zs /. zrule; Q2 = Q1 /. Alternatives @@ zs → 0;
   Simplify /@ 𝔼[Q2, Det[qt] e^{-Q2} Zip_{ζs}[e^{Q1} (P /. zrule)]] ];
```

Fuller program & testing suite: ωεβ/mm, ωεβ/port.

The Real Thing. In the algebra QU_ϵ (explained later), over $\mathbb{Q}[[\hbar]]$ using the $yaxt$ order, $T=\mathbb{e}^{\hbar t}$, $\bar T = T^{-1}$, $\mathcal{A}=\mathbb{e}^{\alpha}$, and $\bar{\mathcal{A}} = \mathcal{A}^{-1}$, we have

$$\tilde R_{ij} = \mathbb{e}^{\hbar(y_ix_j - t_ia_j)}\left(1+\epsilon\hbar\left(a_ia_j - \hbar^2y_i^2x_j^2/4\right)+O(\epsilon^2)\right)$$

in $\mathcal{S}(B_i,B_j)$, and in $\mathcal{S}(B_1^*,B_2^*,B)$ we have

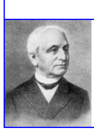
$$\tilde m = \mathbb{e}^{(\alpha_1+\alpha_2)a+\eta_2\xi_1(1-T)/\hbar+(\xi_1\bar{\mathcal{A}}_2+\xi_2)x+(\eta_1+\eta_2\bar{\mathcal{A}}_1)y}\left(1+\epsilon\lambda+O(\epsilon^2)\right),$$

where $\lambda = 2a\eta_2\xi_1T + \eta_2^2\xi_1^2\left(3T^2-4T+1\right)/4\hbar - \eta_2\xi_1^2(3T-1)x\bar{\mathcal{A}}_2/2$
$-\eta_2^2\xi_1(3T-1)y\bar{\mathcal{A}}_1/2 + 2\eta_2\xi_1xy\hbar\bar{\mathcal{A}}_1\bar{\mathcal{A}}_2$.

Finally,

$$\tilde\Delta = \mathbb{e}^{\tau(t_1+t_1)+\eta(y_1+T_1y_2)+\alpha(a_1+a_2)+\xi(x_1+x_2)}\left(1+O(\epsilon)\right)\in\mathcal{S}(B^*,B_1,B_2),$$

and $\tilde S = \mathbb{e}^{-\tau t-\alpha a-\eta\xi(1-\bar T)\mathcal{A}/\hbar - \bar T\eta y\mathcal{A}-\xi x\mathcal{A}}\left(1+O(\epsilon)\right)\in\mathcal{S}(B^*,B)$.



"God created the knots, all else in topology is the work of mortals."

Leopold Kronecker (modified)

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Real Zipping is a minor mess, and is done in two phases:

	τa-phase		ξy-phase	
ζ-like variables	τ	a	ξ	y
z-like variables	t	α	x	η

Already at $\epsilon=0$ we get the best known formulas for the Alexander polynomial!

Generic Docility. A "docile perturbed Gaussian" in the variables $(z_i)_{i\in S}$ over the ring R is an expression of the form

$$\mathbb{e}^{q^{ij}z_iz_j}P = \mathbb{e}^{q^{ij}z_iz_j}\left(\sum_{k\geq 0}\epsilon^kP_k\right),$$

where all coefficients are in R and where P is a "docile series": $\deg P_k \leq 4k$.

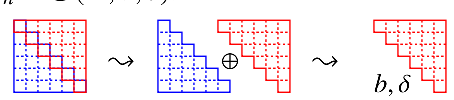
Our Docility. In the case of QU_ϵ, all invariants and operations are of the form $\mathbb{e}^{L+Q}P$, where

- L is a quadratic of the form $\sum l_{z\zeta}z\zeta$, where z runs over $\{t_i,\alpha_i\}_{i\in S}$ and ζ over $\{\tau_i,a_i\}_{i\in S}$, with integer coefficients $l_{z\zeta}$.
- Q is a quadratic for the form $\sum q_{z\zeta}z\zeta$, where z runs over $\{x_i,\eta_i\}_{i\in S}$ and ζ over $\{\xi_i,y_i\}_{i\in S}$, with coefficients $q_{z\zeta}$ in the ring R_S of rational functions in $\{T_i,\mathcal{A}_i\}_{i\in S}$.
- P is a docile power series in $\{y_i,a_i,x_i,\eta_i,\xi_i\}_{i\in S}$ with coefficients in R_S, and where $\deg(y_i,a_i,x_i,\eta_i,\xi_i)=(1,2,1,1,1)$.

Docililty Matters! The rank of the space of docile series to ϵ^k is polynomial in the number of variables $|S|$. !!!!!

- At $\epsilon^2=0$ we get the Rozansky-Overbay [Ro1, Ro2, Ro3, Ov] invariant, which is stronger than HOMFLY-PT polynomial and Khovanov homology taken together!
- In general, get "higher diagonals in the Melvin-Morton-Rozansky expansion of the coloured Jones polynomial" [MM, BNG], but why spoil something good?

Solvable Approximation. In gl_n, half is enough! Indeed $gl_n\oplus\mathfrak{a}_n = \mathcal{D}(◹, b, \delta)$:



$b(◹) = b$: ◹⊗◹ → ◹
$b(◺) \rightsquigarrow \delta$: ◹ → ◹⊗◹

Now define $gl_n^\epsilon := \mathcal{D}(◹, b, \epsilon\delta)$. Schematically, this is $[◹,◹]=◹$, $[◺,◺]=\epsilon◺$, and $[◹,◺]=◺+\epsilon◹$. The same process works for all semi-simple Lie algebras, and at $\epsilon^{k+1}=0$ always yields a solvable Lie algebra.

CU and QU. Starting from sl_2, get $CU_\epsilon = \langle y,a,x,t\rangle/([t,-]=0,\ [a,y]=-y,\ [a,x]=x,\ [x,y]=2\epsilon a - t)$. Quantize using standard tools (I'm sorry) and get $QU_\epsilon=\langle y,a,x,t\rangle/([t,-]=0,\ [a,y]=-y,\ [a,x]=x,\ xy-\mathbb{e}^{\hbar\epsilon}yx=(1-T\mathbb{e}^{-2\hbar\epsilon a})/\hbar)$.

References.

[BNG] D. Bar-Natan and S. Garoufalidis, *On the Melvin-Morton-Rozansky conjecture,* Invent. Math. **125** (1996) 103–133.

[BV] D. Bar-Natan and R. van der Veen, *A Polynomial Time Knot Polynomial,* arXiv:1708.04853.

[MM] P. M. Melvin and H. R. Morton, *The coloured Jones function,* Commun. Math. Phys. **169** (1995) 501–520.

[Ov] A. Overbay, *Perturbative Expansion of the Colored Jones Polynomial,* University of North Carolina PhD thesis, ωεβ/Ov.

[Ro1] L. Rozansky, *A contribution of the trivial flat connection to the Jones polynomial and Witten's invariant of 3d manifolds, I,* Comm. Math. Phys. **175-2** (1996) 275–296, arXiv:hep-th/9401061.

[Ro2] L. Rozansky, *The Universal R-Matrix, Burau Representation and the Melvin-Morton Expansion of the Colored Jones Polynomial,* Adv. Math. **134-1** (1998) 1–31, arXiv:q-alg/9604005.

[Ro3] L. Rozansky, *A Universal U(1)-RCC Invariant of Links and Rationality Conjecture,* arXiv:math/0201139.

The Algebras H and H^*. Let $q = e^{\hbar\epsilon\gamma}$ and set $H = \langle a, x\rangle/([a,x] = \gamma x)$ with

$$A = e^{-\hbar\epsilon a}, \quad xA = qAx, \quad S_H(a,A,x) = (-a, A^{-1}, -A^{-1}x),$$
$$\Delta_H(a,A,x) = (a_1 + a_2, A_1A_2, x_1 + A_1x_2)$$

and dual $H^* = \langle b, y\rangle/([b,y] = -\epsilon y)$ with

$$B = e^{-\hbar\gamma b}, \quad By = qyB, \quad S_{H^*}(b,B,y) = (-b, B^{-1}, -yB^{-1}),$$
$$\Delta_{H^*}(b,B,y) = (b_1 + b_2, B_1B_2, y_1B_2 + y_2).$$

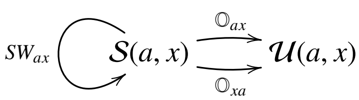
Pairing by $(a,x)^* = (b,y)$ ($\Rightarrow \langle B, A\rangle = q$) making $\langle y^l b^i, a^j x^k\rangle = \delta_{ij}\delta_{kl} j![k]_q!$ so $R = \sum \frac{y^k b^j \otimes a^j x^k}{j![k]_q!}$.

The Algebra QU. Using the Drinfel'd double procedure, $QU_{\gamma,\epsilon} := H^{*cop} \otimes H$ with $(\phi f)(\psi g) = \langle \psi_1 S^{-1} f_3\rangle\langle\psi_3, f_1\rangle(\phi\psi_2)(f_2 g)$ and

$$S(y,b,a,x) = (-B^{-1}y, -b, -a, -A^{-1}x),$$
$$\Delta(y,b,a,x) = (y_1 + y_2B_1, b_1 + b_2, a_1 + a_2, x_1 + A_1x_2).$$

Note also that $t := \epsilon a - \gamma b$ is central and can replace b, and set $QU = QU_\epsilon = QU_{1,\epsilon}$.

The 2D Lie Algebra. One may show* that if $[a,x] = \gamma x$ then $e^{\xi x} e^{\alpha a} = e^{\alpha a} e^{e^{-\gamma\alpha}\xi x}$. Ergo with



we have $\widetilde{SW}_{ax} = e^{\alpha a + e^{-\gamma\alpha}\xi x}$.

* Indeed $xa = (a-\gamma)x$ thus $xa^n = (a-\gamma)^n x$ thus $xe^{\alpha a} = e^{\alpha(a-\gamma)}x = e^{-\gamma\alpha}e^{\alpha a}x$ thus $x^n e^{\alpha a} = e^{\alpha a}(e^{-\gamma\alpha})^n x^n$ thus $e^{\xi x}e^{\alpha a} = e^{\alpha a}e^{e^{-\gamma\alpha}\xi x}$.

Faddeev's Formula (In as much as we can tell, first appeared without proof in Faddeev [Fa], rediscovered and proven in Quesne [Qu], and again with easier proof, in Zagier [Za]). With $[n]_q := \frac{q^n-1}{q-1}$, with $[n]_q! := [1]_q[2]_q\cdots[n]_q$ and with $e_q^x := \sum_{n\geq 0}\frac{x^n}{[n]_q!}$, we have

$$\log e_q^x = \sum_{k\geq 1}\frac{(1-q)^k x^k}{k(1-q^k)} = x + \frac{(1-q)^2x^2}{2(1-q^2)} + \ldots.$$

Proof. We have that $e_q^x = \frac{e_q^{qx} - e_q^x}{qx - x}$ ("the q-derivative of e_q^x is itself"), and hence $e_q^{qx} = (1 + (1-q)x)e_q^x$, and

$$\log e_q^{qx} = \log(1 + (1-q)x) + \log e_q^x.$$

Writing $\log e_q^x = \sum_{k\geq 1} a_k x^k$ and comparing powers of x, we get $q^k a_k = -(1-q)^k/k + a_k$, or $a_k = \frac{(1-q)^k}{k(1-q^k)}$. □

A Partial To Do List.

- Complete all "docility" arguments by identifying a "contained" docile substructure.
- Understand denominators and get rid of them.
- See if much can be gained by including P in the exponential: $e^{L+Q}P \rightsquigarrow e^{L+Q+P}$?
- Clean the program and make it more efficient.
- Run it for all small knots and links, at $k = 2, 3$.
- Really understand the cuap C.
- Understand the centre and figure out how to read the output.
- Extend to sl_3 and beyond.
- Prove a genus bound and a Seifert formula.
- Obtain "Gauss-Gassner formulas" (ωεβ/NCSU).
- Relate with Melvin-Morton-Rozansky and with Rozansky-Overbay.
- Understand the braid group representations that arise.
- Find a topological interpretation. The Garoufalidis-Rozansky "loop expansion" [GR]?
- Figure out the action of the Cartan automorphism.
- Disprove the ribbon-slice conjecture!
- Figure out the action of the Weyl group.
- Do everything at the "arrow diagram" level of finite-type invariants of (rotational) virtual tangles.
- What else can you do with the "solvable approximations"?
- And with the "Gaussian zip and bind" technology?

Further References.

[GR] S. Garoufalidis and L. Rozansky, *The Loop Exapnsion of the Kontsevich Integral, the Null-Move, and S-Equivalence,* arXiv:math.GT/0003187.

[Fa] L. Faddeev, *Modular Double of a Quantum Group,* arXiv:math/9912078.

[Qu] C. Quesne, *Jackson's q-Exponential as the Exponential of a Series,* arXiv:math-ph/0305003.

[Za] D. Zagier, *The Dilogarithm Function,* in Cartier, Moussa, Julia, and Vanhove (eds) *Frontiers in Number Theory, Physics, and Geometry II.* Springer, Berlin, Heidelberg, and ωεβ/Za.

A Full Implementation. ωεβ/Full

Utilities

```
CF[sd_SeriesData] := MapAt[CF, sd, 3];
CF[ℰ_] := ExpandDenominator@ExpandNumerator@Together[
    Expand[ℰ] //. 𝕖^x_ 𝕖^y_ :→ 𝕖^(x+y) /. 𝕖^x_ :→ 𝕖^CF[x]];
Kδ /: Kδ_{i_,j_} := If[i === j, 1, 0];
𝔼 /: 𝔼[L1_, Q1_, P1_] ≡ 𝔼[L2_, Q2_, P2_] :=
  CF[L1 == L2] ∧ CF[Q1 == Q2] ∧ CF[Normal[P1 - P2] == 0];
𝔼 /: 𝔼[L1_, Q1_, P1_] 𝔼[L2_, Q2_, P2_] :=
  𝔼[L1 + L2, Q1 + Q2, P1 * P2];
𝔼[L_, Q_, P_]_{$k_} := 𝔼[L, Q, Series[Normal@P, {ϵ, 0, $k}]];
```

Zip and Bind

```
{t*, b*, y*, a*, x*, z*} = {τ, β, η, α, ξ, ζ};
{τ*, β*, η*, α*, ξ*, ζ*} = {t, b, y, a, x, z};
(u_i_)* := (u*)_i;
collect[sd_SeriesData, ζ_] :=
  MapAt[collect[#, ζ] &, sd, 3];
collect[ℰ_, ζ_] := Collect[ℰ, ζ];
Zip_{}[P_] := P; Zip_{ζ_,ζs___}[P_] :=
  (collect[P // Zip_{ζs}, ζ] /. f_. ζ^d_. :→ ∂_{ζ*,d} f) /. ζ* → 0
QZip_{ζs_List}@𝔼[L_, Q_, P_] :=
  Module[{ζ, z, zs, c, ys, ηs, qt, zrule, Q1, Q2},
    zs = Table[ζ*, {ζ, ζs}];
    c = Q /. Alternatives @@ (ζs ⋃ zs) → 0;
    ys = Table[∂_ζ (Q /. Alternatives @@ zs → 0), {ζ, ζs}];
    ηs = Table[∂_z (Q /. Alternatives @@ ζs → 0), {z, zs}];
    qt = Inverse@Table[Kδ_{z,ζ*} - ∂_{z,ζ} Q, {ζ, ζs}, {z, zs}];
    zrule = Thread[zs → qt.(zs + ys)];
    Q2 = (Q1 = c + ηs.zs /. zrule) /. Alternatives @@ zs → 0;
    CF /@ 𝔼[L, Q2, Det[qt] 𝕖^(-Q2) Zip_{ζs}[𝕖^Q1 (P /. zrule)]]];
U2l = {B_{i_}^p_. → 𝕖^(-p ħ γ b_i), B^p_. → 𝕖^(-p ħ γ b), T_{i_}^p_. → 𝕖^(p ħ t_i),
    T^p_. → 𝕖^(p ħ t), 𝒜_{i_}^p_. → 𝕖^(p γ α_i), 𝒜^p_. → 𝕖^(p γ α)};
l2U = {𝕖^(c_. b_{i_} + d_.) :→ B_i^(-c/(ħ γ)) 𝕖^d, 𝕖^(c_. b + d_.) :→ B^(-c/(ħ γ)) 𝕖^d,
    𝕖^(c_. t_{i_} + d_.) :→ T_i^(c/ħ) 𝕖^d, 𝕖^(c_. t + d_.) :→ T^(c/ħ) 𝕖^d,
    𝕖^(c_. α_{i_} + d_.) :→ 𝒜_i^(c/γ) 𝕖^d, 𝕖^(c_. α + d_.) :→ 𝒜^(c/γ) 𝕖^d,
    𝕖^ℰ_ :→ 𝕖^(Expand@ℰ)};
```

```
LZip_{ζs_List}@𝔼[L_, Q_, P_] :=
  Module[{ζ, z, zs, c, ys, ηs, lt, zrule, L1, L2, Q1, Q2},
   zs = Table[ζ*, {ζ, ζs}];
   c = L /. Alternatives @@ (ζs ⋃ zs) → 0;
   ys = Table[∂_ζ (L /. Alternatives @@ zs → 0), {ζ, ζs}];
   ηs = Table[∂_z (L /. Alternatives @@ ζs → 0), {z, zs}];
   lt = Inverse@Table[Kδ_{z,ζ*} - ∂_{z,ζ} L, {ζ, ζs}, {z, zs}];
   zrule = Thread[zs → lt.(zs + ys)];
   L2 = (L1 = c + ηs.zs /. zrule) /. Alternatives @@ zs → 0;
   Q2 = (Q1 = Q /. U2l /. zrule) /. Alternatives @@ zs → 0;
   CF /@ 𝔼[L2, Q2, Det[lt] e^{-L2-Q2}
       Zip_{ζs}[e^{L1+Q1} (P /. U2l /. zrule)]] //. l2U];
B_{}[L_, R_] := L R;
B_{{is__}}[L_𝔼, R_𝔼] := Module[{n}, Times[
      L /. Table[(v : b | B | t | T | a | x | y)_i → v_{n@i}, {i, {is}}],
      R /. Table[(v : β | τ | α | 𝒜 | ξ | η)_i → v_{n@i}, {i, {is}}]
     ] // LZip_{Join@@Table[{β_{n@i},τ_{n@i},a_{n@i}},{i,{is}}]} //
    QZip_{Join@@Table[{ξ_{n@i},y_{n@i}},{i,{is}}]} ];
B_{is___}[L_, R_] := B_{{is}}[L, R];
```

𝔼 morphisms with domain and range.

```
B_{is_List}[𝔼_{d1_→r1_}[L1_, Q1_, P1_], 𝔼_{d2_→r2_}[L2_, Q2_, P2_]] :=
  𝔼_{(d1⋃Complement[d2,is])→(r2⋃Complement[r1,is])} @@
   B_{is}[𝔼[L1, Q1, P1], 𝔼[L2, Q2, P2]];
𝔼_{d1_→r1_}[L1_, Q1_, P1_] // 𝔼_{d2_→r2_}[L2_, Q2_, P2_] :=
  B_{r1⋂d2}[𝔼_{d1→r1}[L1, Q1, P1], 𝔼_{d2→r2}[L2, Q2, P2]];
𝔼_{d1_→r1_}[L1_, Q1_, P1_] ≡ 𝔼_{d2_→r2_}[L2_, Q2_, P2_] ^:=
  (d1 == d2) ∧ (r1 == r2) ∧ (𝔼[L1, Q1, P1] ≡ 𝔼[L2, Q2, P2]);
𝔼_{d1_→r1_}[L1_, Q1_, P1_] 𝔼_{d2_→r2_}[L2_, Q2_, P2_] ^:=
  𝔼_{(d1⋃d2)→(r1⋃r2)} @@ (𝔼[L1, Q1, P1] 𝔼[L2, Q2, P2]);
𝔼_{d_→r_}[L_, Q_, P_]_{$k_} := 𝔼_{d→r} @@ 𝔼[L, Q, P]_{$k};
𝔼_[ℰ___][i_] := {ℰ}⟦i⟧;
```

“Define” code

```
SetAttributes[Define, HoldAll];
Define[def_, defs__] := (Define[def]; Define[defs];);
Define[op_{is__} = ℰ_] :=
 Module[{SD, ii, jj, kk, isp, nis, nisp, sis},
  Block[{i, j, k},
   ReleaseHold[Hold[
      SD[op_{nisp,$k_Integer}, Block[{i, j, k}, op_{isp,$k} = ℰ;
        op_{nis,$k}]];
      SD[op_{isp}, op_{{is},$k}]; SD[op_{sis__}, op_{{sis}}];
     ] /. {SD → SetDelayed,
      isp → {is} /. {i → i_, j → j_, k → k_},
      nis → {is} /. {i → ii, j → jj, k → kk},
      nisp → {is} /. {i → ii_, j → jj_, k → kk_}
     }] ]]
```

The Fundamental Tensors

```
Define[am_{i,j→k} = 𝔼_{{i,j}→{k}}[(α_i + α_j) a_k, (e^{-γ α_j} ξ_i + ξ_j) x_k, 1]_{$k},
 bm_{i,j→k} = 𝔼_{{i,j}→{k}}[(β_i + β_j) b_k, (η_i + η_j) y_k, e^{(e^{-ϵ β_i} - 1) η_j y_k}]_{$k}]
Define[R_{i,j} =
  𝔼_{{}→{i,j}}[ħ a_j b_i, ħ x_j y_i, e^(Sum_{k=2}^{$k+1} (1 - e^{γ ϵ ħ})^k (ħ y_i x_j)^k / (k (1 - e^{k γ ϵ ħ})))]_{$k}]
Define[R̄_{i,j} = 𝔼_{{}→{i,j}}[-ħ a_j b_i, -ħ x_j y_i / B_i,
   1 + If[$k == 0, 0, (R̄_{{i,j},$k-1})_{$k}[3] -
      (((R̄_{{i,j},0})_{$k} R_{1,2} (R̄_{{3,4},$k-1})_{$k}) // (bm_{i,1→i} am_{j,2→j}) //
        (bm_{i,3→i} am_{j,4→j})) [3]]],
 P_{i,j} = 𝔼_{{i,j}→{}}[β_i α_j / ħ, η_i ξ_j / ħ,
   1 + If[$k == 0, 0, (P_{{i,j},$k-1})_{$k}[3] -
      (R_{1,2} // ((P_{{1,j},0})_{$k} (P_{{i,2},$k-1})_{$k}))[3]]]]
Define[aS_j = R̄_{i,j} ~ B_i ~ P_{i,j},
 aS̄_i = 𝔼_{{i}→{i}}[-a_i α_i, -x_i 𝒜_i ξ_i,
   1 + If[$k == 0, 0, (aS̄_{{i},$k-1})_{$k}[3] -
      ((aS̄_{{i},0})_{$k} ~ B_i ~ aS_i ~ B_i ~ (aS̄_{{i},$k-1})_{$k}) [3]]]]
Define[bS_i = R_{i,1} ~ B_1 ~ aS_1 ~ B_1 ~ P_{i,1},
 bS̄_i = R_{i,1} ~ B_1 ~ aS̄_1 ~ B_1 ~ P_{i,1},
 aΔ_{i→j,k} = (R_{1,j} R_{2,k}) // bm_{1,2→3} // P_{3,i},
 bΔ_{i→j,k} = (R_{j,1} R_{k,2}) // am_{1,2→3} // P_{i,3}]
Define[
 dm_{i,j→k} =
  (𝔼_{{i,j}→{i,j}}[β_i b_i + α_j a_j, η_i y_i + ξ_j x_j, 1]
      (aΔ_{i→1,2} // aΔ_{2→2,3} // aS̄_3) (bΔ_{j→-1,-2} // bΔ_{-2→-2,-3})) //
   (P_{-1,3} P_{-3,1} am_{2,j→k} bm_{i,-2→k}),
 dS_i = 𝔼_{{i}→{1,2}}[β_i b_1 + α_i a_2, η_i y_1 + ξ_i x_2, 1] // (bS̄_1 aS_2) //
   dm_{2,1→i},
 dΔ_{i→j,k} = (bΔ_{i→3,1} aΔ_{i→2,4}) // (dm_{3,4→k} dm_{1,2→j})]
Define[C_i = 𝔼_{{}→{i}}[0, 0, B_i^{1/2} e^{-ħ ϵ a_i/2}]_{$k},
 C̄_i = 𝔼_{{}→{i}}[0, 0, B_i^{-1/2} e^{ħ ϵ a_i/2}]_{$k},
 Kink_i = (R_{1,3} C̄_2) // dm_{1,2→1} // dm_{1,3→i},
 K̄ink_i = (R̄_{1,3} C_2) // dm_{1,2→1} // dm_{1,3→i}]
Define[
 b2t_i = 𝔼_{{i}→{i}}[α_i a_i - β_i t_i / γ, ξ_i x_i + η_i y_i, e^{ϵ β_i a_i/γ}]_{$k},
 t2b_i = 𝔼_{{i}→{i}}[α_i a_i - τ_i γ b_i, ξ_i x_i + η_i y_i, e^{ϵ τ_i a_i}]_{$k}]
Define[kR_{i,j} = R_{i,j} // (b2t_i b2t_j) /. t_{i|j} → t,
 kR̄_{i,j} = R̄_{i,j} // (b2t_i b2t_j) /. {t_{i|j} → t, T_{i|j} → T},
 km_{i,j→k} = (t2b_i t2b_j) // dm_{i,j→k} //
   b2t_k /. {t_k → t, T_k → T, τ_{i|j} → 0},
 kC_i = C_i // b2t_i /. T_i → T, kC̄_i = C̄_i // b2t_i /. T_i → T,
 kKink_i = Kink_i // b2t_i /. {t_i → t, T_i → T},
 kK̄ink_i = K̄ink_i // b2t_i /. {t_i → t, T_i → T}]
```

The Trefoil

```
$k = 2; Z = kR_{1,5} kR_{6,2} kR_{3,7} kC̄_4 kK̄ink_8 kK̄ink_9 kK̄ink_10;
Do[Z = Z ~ B_{1,r} ~ km_{1,r→1}, {r, 2, 10}];
Simplify /@ Z /. v_1 :→ v
```

$$\mathbb{E}_{\{\}\to\{1\}}\Big[0,\ 0,\ \frac{T}{1-T+T^2}+\frac{1}{\left(1-T+T^2\right)^3}T\hbar\Big(2a\left(-1+T-T^3+T^4\right)+T\left(-1+2T-3T^2+2T^3\right)\gamma-2\left(1+T^3\right)xy\gamma\hbar\Big)\epsilon+\frac{1}{2\left(1-T+T^2\right)^5}T\hbar^2\Big(4a^2\left(1-T+T^2\right)^2\left(1+T-6T^2+T^3+T^4\right)+4a\left(1-T+T^2\right)\gamma\left(T\left(2-5T+8T^2-7T^3-2T^4+2T^5\right)-2\left(-1-2T+5T^2-4T^3+T^4+2T^5\right)xy\hbar\right)+\gamma^2\left(T\left(1-2T+4T^2-2T^3+6T^5-11T^6+4T^7\right)+4\left(-1+2T+T^3+T^4+2T^6-T^7\right)xy\hbar+6\left(1-T+T^2\right)^2\left(1+3T+T^2\right)x^2y^2\hbar^2\right)\Big)\epsilon^2+O[\epsilon]^3\Big]$$

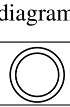
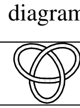
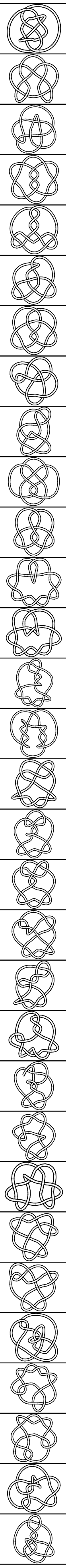
diagram	n_k^t Alexander's ω^+ Today's / Rozansky's ρ_1^+	genus / ribbon unknotting number / amphicheiral	diagram	n_k^t Alexander's ω^+ Today's / Rozansky's ρ_1^+	genus / ribbon unknotting number / amphicheiral
	0_1^a 1 0	0 / ✔ 0 / ✔		3_1^a $t-1$ t	1 / ✘ 1 / ✘

diagram	n_k^t	Alexander's ω^+ Today's / Rozansky's ρ_1^+	genus / ribbon unknotting number / amphicheiral	diagram	n_k^t	Alexander's ω^+ Today's / Rozansky's ρ_1^+	genus / ribbon unknotting number / amphicheiral
	4_1^a	$3-t$ 0	1 / ✘ 1 / ✔		5_1^a	t^2-t+1 $2t^3+3t$	2 / ✘ 2 / ✘
	5_2^a	$2t-3$ $5t-4$	1 / ✘ 1 / ✘		6_1^a	$5-2t$ $t-4$	1 / ✔ 1 / ✘
	6_2^a	$-t^2+3t-3$ t^3-4t^2+4t-4	2 / ✘ 1 / ✘		6_3^a	t^2-3t+5 0	2 / ✘ 1 / ✔
	7_1^a	t^3-t^2+t-1 $3t^5+5t^3+6t$	3 / ✘ 3 / ✘		7_2^a	$3t-5$ $14t-16$	1 / ✘ 1 / ✘
	7_3^a	$2t^2-3t+3$ $-9t^3+8t^2-16t+12$	2 / ✘ 2 / ✘		7_4^a	$4t-7$ $32-24t$	1 / ✘ 2 / ✘
	7_5^a	$2t^2-4t+5$ $9t^3-16t^2+29t-28$	2 / ✘ 2 / ✘		7_6^a	$-t^2+5t-7$ $t^3-8t^2+19t-20$	2 / ✘ 1 / ✘
	7_7^a	t^2-5t+9 $8-3t$	2 / ✘ 1 / ✘		8_1^a	$7-3t$ $5t-16$	1 / ✘ 1 / ✘
	8_2^a	$-t^3+3t^2-3t+3$ $2t^5-8t^4+10t^3-12t^2+13t-12$	3 / ✘ 2 / ✘		8_3^a	$9-4t$ 0	1 / ✘ 2 / ✔
	8_4^a	$-2t^2+5t-5$ $3t^3-8t^2+6t-4$	2 / ✘ 2 / ✘		8_5^a	$-t^3+3t^2-4t+5$ $-2t^5+8t^4-13t^3+20t^2-22t+24$	3 / ✘ 2 / ✘
	8_6^a	$-2t^2+6t-7$ $5t^3-20t^2+28t-32$	2 / ✘ 2 / ✘		8_7^a	t^3-3t^2+5t-5 $-t^5+4t^4-10t^3+12t^2-13t+12$	3 / ✘ 1 / ✘
	8_8^a	$2t^2-6t+9$ $-t^3+4t^2-12t+16$	2 / ✔ 2 / ✘		8_9^a	$-t^3+3t^2-5t+7$ 0	3 / ✔ 1 / ✔
	8_{10}^a	t^3-3t^2+6t-7 $-t^5+4t^4-11t^3+16t^2-21t+20$	3 / ✘ 2 / ✘		8_{11}^a	$-2t^2+7t-9$ $5t^3-24t^2+39t-44$	2 / ✘ 1 / ✘
	8_{12}^a	$t^2-7t+13$ 0	2 / ✘ 2 / ✔		8_{13}^a	$2t^2-7t+11$ $-t^3+4t^2-14t+20$	2 / ✘ 1 / ✘
	8_{14}^a	$-2t^2+8t-11$ $5t^3-28t^2+57t-68$	2 / ✘ 1 / ✘		8_{15}^a	$3t^2-8t+11$ $21t^3-64t^2+120t-140$	2 / ✘ 2 / ✘
	8_{16}^a	t^3-4t^2+8t-9 $t^5-6t^4+17t^3-28t^2+35t-36$	3 / ✘ 2 / ✘		8_{17}^a	$-t^3+4t^2-8t+11$ 0	3 / ✘ 1 / ✔
	8_{18}^a	$-t^3+5t^2-10t+13$ 0	3 / ✘ 2 / ✔		8_{19}^n	t^3-t^2+1 $-3t^5-4t^2-3t$	3 / ✘ 3 / ✘
	8_{20}^n	t^2-2t+3 $4t-4$	2 / ✔ 1 / ✘		8_{21}^n	$-t^2+4t-5$ $t^3-8t^2+16t-20$	2 / ✘ 1 / ✘
	9_1^a	$t^4-t^3+t^2-t+1$ $4t^7+7t^5+9t^3+10t$	4 / ✘ 4 / ✘		9_2^a	$4t-7$ $30t-40$	1 / ✘ 1 / ✘
	9_3^a	$2t^3-3t^2+3t-3$ $-13t^5+12t^4-25t^3+20t^2-32t+24$	3 / ✘ 3 / ✘		9_4^a	$3t^2-5t+5$ $23t^3-28t^2+46t-44$	2 / ✘ 2 / ✘
	9_5^a	$6t-11$ $100-65t$	1 / ✘ 2 / ✘		9_6^a	$2t^3-4t^2+5t-5$ $13t^5-24t^4+45t^3-52t^2+68t-64$	3 / ✘ 3 / ✘
	9_7^a	$3t^2-7t+9$ $23t^3-56t^2+99t-108$	2 / ✘ 2 / ✘		9_8^a	$-2t^2+8t-11$ $3t^3-16t^2+29t-28$	2 / ✘ 2 / ✘
	9_9^a	$2t^3-4t^2+6t-7$ $13t^5-24t^4+55t^3-72t^2+98t-96$	3 / ✘ 3 / ✘		9_{10}^a	$4t^2-8t+9$ $-40t^3+72t^2-114t+120$	2 / ✘ 2, 3 / ✘
	9_{11}^a	$-t^3+5t^2-7t+7$ $-2t^5+16t^4-41t^3+52t^2-66t+64$	3 / ✘ 2 / ✘		9_{12}^a	$-2t^2+9t-13$ $5t^3-36t^2+84t-100$	2 / ✘ 1 / ✘
	9_{13}^a	$4t^2-9t+11$ $-40t^3+92t^2-154t+168$	2 / ✘ 2, 3 / ✘		9_{14}^a	$2t^2-9t+15$ $-t^3+8t^2-35t+60$	2 / ✘ 1 / ✘
	9_{15}^a	$-2t^2+10t-15$ $-5t^3+40t^2-108t+136$	2 / ✘ 2 / ✘		9_{16}^a	$2t^3-5t^2+8t-9$ $-13t^5+36t^4-80t^3+120t^2-161t+168$	3 / ✘ 3 / ✘
	9_{17}^a	t^3-5t^2+9t-9 $t^5-8t^4+23t^3-32t^2+28t-24$	3 / ✘ 2 / ✘		9_{18}^a	$4t^2-10t+13$ $40t^3-108t^2+193t-220$	2 / ✘ 2 / ✘
	9_{19}^a	$2t^2-10t+17$ $t^3-8t^2+20t-24$	2 / ✘ 1 / ✘		9_{20}^a	$-t^3+5t^2-9t+11$ $2t^5-16t^4+47t^3-84t^2+117t-124$	3 / ✘ 2 / ✘
	9_{21}^a	$-2t^2+11t-17$ $-5t^3+44t^2-127t+164$	2 / ✘ 1 / ✘		9_{22}^a	$t^3-5t^2+10t-11$ $-t^5+8t^4-24t^3+38t^2-40t+36$	3 / ✘ 1 / ✘
	9_{23}^a	$4t^2-11t+15$ $40t^3-128t^2+243t-288$	2 / ✘ 2 / ✘		9_{24}^a	$-t^3+5t^2-10t+13$ $-4t^2+16t-20$	3 / ✘ 1 / ✘
	9_{25}^a	$-3t^2+12t-17$ $12t^3-70t^2+153t-188$	2 / ✘ 2 / ✘		9_{26}^a	$t^3-5t^2+11t-13$ $-t^5+8t^4-31t^3+64t^2-85t+92$	3 / ✘ 1 / ✘
	9_{27}^a	$-t^3+5t^2-11t+15$ $t^3-8t^2+24t-32$	3 / ✔ 1 / ✘		9_{28}^a	$t^3-5t^2+12t-15$ $t^5-8t^4+30t^3-68t^2+105t-120$	3 / ✘ 1 / ✘



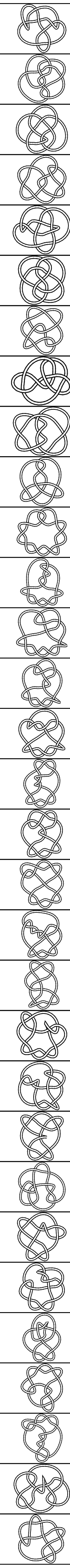
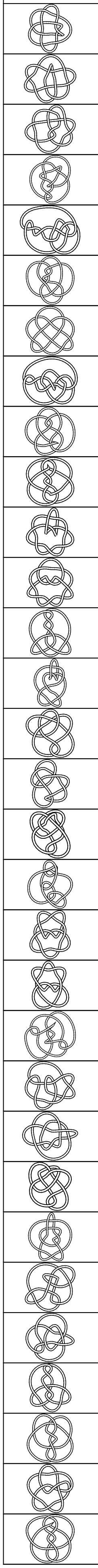


diagram	n_k^t Alexander's ω^+ Today's / Rozansky's ρ_1^+	genus / ribbon unknotting number / amphicheiral	diagram	n_k^t Alexander's ω^+ Today's / Rozansky's ρ_1^+	genus / ribbon unknotting number / amphicheiral
	9^a_{29} $t^3-5t^2+12t-15$ $t^5-8t^4+26t^3-48t^2+59t-56$	3 / ✘ 2 / ✘		9^a_{30} $-t^3+5t^2-12t+17$ $2t^3-10t^2+25t-32$	3 / ✘ 1 / ✘
	9^a_{31} $t^3-5t^2+13t-17$ $t^5-8t^4+33t^3-80t^2+132t-152$	3 / ✘ 2 / ✘		9^a_{32} $t^3-6t^2+14t-17$ $-t^5+10t^4-42t^3+94t^2-133t+148$	3 / ✘ 2 / ✘
	9^a_{33} $-t^3+6t^2-14t+19$ $t^3-10t^2+30t-40$	3 / ✘ 1 / ✘		9^a_{34} $-t^3+6t^2-16t+23$ $3t^3-18t^2+43t-56$	3 / ✘ 1 / ✘
	9^a_{35} $7t-13$ $90t-144$	1 / ✘ 2, 3 / ✘		9^a_{36} $-t^3+5t^2-8t+9$ $-2t^5+16t^4-44t^3+66t^2-87t+88$	3 / ✘ 2 / ✘
	9^a_{37} $2t^2-11t+19$ $t^3-8t^2+22t-28$	2 / ✘ 2 / ✘		9^a_{38} $5t^2-14t+19$ $62t^3-204t^2+382t-452$	2 / ✘ 2, 3 / ✘
	9^a_{39} $-3t^2+14t-21$ $-12t^3+84t^2-210t+268$	2 / ✘ 1 / ✘		9^a_{40} $t^3-7t^2+18t-23$ $t^5-12t^4+57t^3-144t^2+229t-264$	3 / ✘ 2 / ✘
	9^a_{41} $3t^2-12t+19$ $3t^3-20t^2+70t-108$	2 / ✔ 2 / ✘		9^n_{42} $-t^2+2t-1$ $-t^3+2t^2+t-4$	2 / ✘ 1 / ✘
	9^n_{43} $-t^3+3t^2-2t+1$ $-2t^5+8t^4-7t^3+2t^2-5t+4$	3 / ✘ 2 / ✘		9^n_{44} t^2-4t+7 $-2t^2+9t-12$	2 / ✘ 1 / ✘
	9^n_{45} $-t^2+6t-9$ $t^3-14t^2+47t-60$	2 / ✘ 1 / ✘		9^n_{46} $5-2t$ $3t-12$	1 / ✔ 2 / ✘
	9^n_{47} t^3-4t^2+6t-5 $-t^5+6t^4-15t^3+16t^2-10t+12$	3 / ✘ 2 / ✘		9^n_{48} $-t^2+7t-11$ $-t^3+12t^2-42t+52$	2 / ✘ 2 / ✘
	9^n_{49} $3t^2-6t+7$ $-21t^3+38t^2-61t+60$	2 / ✘ 3 / ✘		10^a_1 $9-4t$ $14t-40$	1 / ✘ 1 / ✘
	10^a_2 $-t^4+3t^3-3t^2+3t-3$ $3t^7-12t^6+16t^5-20t^4+24t^3-24t^2+27t-24$	4 / ✘ 3 / ✘		10^a_3 $13-6t$ $11t-28$	1 / ✔ 2 / ✘
	10^a_4 $-3t^2+7t-7$ $4t^3-8t^2+t+8$	2 / ✘ 2 / ✘		10^a_5 $t^4-3t^3+5t^2-5t+5$ $-2t^7+8t^6-20t^5+28t^4-36t^3+36t^2-39t+36$	4 / ✘ 2 / ✘
	10^a_6 $-2t^3+6t^2-7t+7$ $9t^5-36t^4+56t^3-72t^2+81t-84$	3 / ✘ 3 / ✘		10^a_7 $-3t^2+11t-15$ $14t^3-72t^2+135t-160$	2 / ✘ 1 / ✘
	10^a_8 $-2t^3+5t^2-5t+5$ $7t^5-20t^4+23t^3-28t^2+26t-24$	3 / ✘ 2 / ✘		10^a_9 $-t^4+3t^3-5t^2+7t-7$ $-t^7+4t^6-10t^5+20t^4-25t^3+28t^2-28t+28$	4 / ✘ 1 / ✘
	10^a_{10} $3t^2-11t+17$ $-5t^3+24t^2-71t+100$	2 / ✘ 1 / ✘		10^a_{11} $-4t^2+11t-13$ $16t^3-52t^2+68t-72$	2 / ✘ 2, 3 / ✘
	10^a_{12} $2t^3-6t^2+10t-11$ $-5t^5+20t^4-50t^3+72t^2-89t+92$	3 / ✘ 2 / ✘		10^a_{13} $2t^2-13t+23$ $t^3-12t^2+51t-84$	2 / ✘ 2 / ✘
	10^a_{14} $-2t^3+8t^2-12t+13$ $9t^5-52t^4+119t^3-180t^2+225t-236$	3 / ✘ 2 / ✘		10^a_{15} $2t^3-6t^2+9t-9$ $-3t^5+12t^4-24t^3+24t^2-17t+12$	3 / ✘ 2 / ✘
	10^a_{16} $-4t^2+12t-15$ $-16t^3+56t^2-76t+80$	2 / ✘ 2 / ✘		10^a_{17} $t^4-3t^3+5t^2-7t+9$ 0	4 / ✘ 1 / ✔
	10^a_{18} $-4t^2+14t-19$ $16t^3-68t^2+121t-140$	2 / ✘ 1 / ✘		10^a_{19} $2t^3-7t^2+11t-11$ $3t^5-16t^4+35t^3-40t^2+30t-24$	3 / ✘ 2 / ✘
	10^a_{20} $-3t^2+9t-11$ $14t^3-56t^2+88t-104$	2 / ✘ 2 / ✘		10^a_{21} $-2t^3+7t^2-9t+9$ $9t^5-44t^4+80t^3-104t^2+121t-124$	3 / ✘ 2 / ✘
	10^a_{22} $-2t^3+6t^2-10t+13$ $-t^5+4t^4-10t^3+24t^2-37t+44$	3 / ✔ 2 / ✘		10^a_{23} $2t^3-7t^2+13t-15$ $-5t^5+24t^4-67t^3+108t^2-137t+144$	3 / ✘ 1 / ✘
	10^a_{24} $-4t^2+14t-19$ $24t^3-116t^2+221t-268$	2 / ✘ 2 / ✘		10^a_{25} $-2t^3+8t^2-14t+17$ $9t^5-52t^4+131t^3-232t^2+314t-344$	3 / ✘ 2 / ✘
	10^a_{26} $-2t^3+7t^2-13t+17$ $-t^5+4t^4-10t^3+28t^2-49t+60$	3 / ✘ 1 / ✘		10^a_{27} $2t^3-8t^2+16t-19$ $5t^5-28t^4+87t^3-164t^2+229t-252$	3 / ✘ 1 / ✘
	10^a_{28} $4t^2-13t+19$ $-8t^3+36t^2-100t+136$	2 / ✘ 2 / ✘		10^a_{29} $t^3-7t^2+15t-17$ $t^5-12t^4+52t^3-104t^2+124t-128$	3 / ✘ 2 / ✘
	10^a_{30} $-4t^2+17t-25$ $24t^3-148t^2+345t-440$	2 / ✘ 1 / ✘		10^a_{31} $4t^2-14t+21$ $-4t^2+9t-12$	2 / ✘ 1 / ✘
	10^a_{32} $-2t^3+8t^2-15t+19$ $t^5-4t^4+13t^3-40t^2+78t-96$	3 / ✘ 1 / ✘		10^a_{33} $4t^2-16t+25$ 0	2 / ✘ 1 / ✔
	10^a_{34} $3t^2-9t+13$ $-5t^3+20t^2-52t+68$	2 / ✘ 2 / ✘		10^a_{35} $2t^2-12t+21$ $-t^3+12t^2-47t+76$	2 / ✔ 2 / ✘
	10^a_{36} $-3t^2+13t-19$ $14t^3-88t^2+208t-264$	2 / ✘ 2 / ✘		10^a_{37} $4t^2-13t+19$ 0	2 / ✘ 2 / ✔
	10^a_{38} $-4t^2+15t-21$ $24t^3-128t^2+270t-336$	2 / ✘ 2 / ✘		10^a_{39} $-2t^3+8t^2-13t+15$ $9t^5-52t^4+125t^3-204t^2+263t-280$	3 / ✘ 2 / ✘
	10^a_{40} $2t^3-8t^2+17t-21$ $-5t^5+28t^4-89t^3+176t^2-258t+288$	3 / ✘ 2 / ✘		10^a_{41} $t^3-7t^2+17t-21$ $t^5-12t^4+54t^3-120t^2+157t-164$	3 / ✘ 2 / ✘

n_k^t	Alexander's ω^+	genus / ribbon	Today's / Rozansky's ρ_1^+	unknotting number / amphicheiral
10^a_{42}	$-t^3+7t^2-19t+27$	3 / ✔	$2t^3-8t^2+11t-12$	1 / ✘
10^a_{43}	$-t^3+7t^2-17t+23$	3 / ✘	0	2 / ✔
10^a_{44}	$t^3-7t^2+19t-25$	3 / ✘	$t^5-12t^4+56t^3-140t^2+220t-248$	1 / ✘
10^a_{45}	$-t^3+7t^2-21t+31$	3 / ✘	0	2 / ✔
10^a_{46}	$-t^4+3t^3-4t^2+5t-5$	4 / ✘	$-3t^7+12t^6-21t^5+34t^4-43t^3+52t^2-55t+56$	3 / ✘
10^a_{47}	$t^4-3t^3+6t^2-7t+7$	4 / ✘	$-2t^7+8t^6-23t^5+38t^4-56t^3+60t^2-68t+64$	2, 3 / ✘
10^a_{48}	$t^4-3t^3+6t^2-9t+11$	4 / ✔	$t^5-2t^4+2t^3-3t+4$	2 / ✘
10^a_{49}	$3t^3-8t^2+12t-13$	3 / ✘	$30t^5-94t^4+196t^3-292t^2+372t-392$	3 / ✘
10^a_{50}	$-2t^3+7t^2-11t+13$	3 / ✘	$-9t^5+44t^4-94t^3+150t^2-186t+200$	2 / ✘
10^a_{51}	$2t^3-7t^2+15t-19$	3 / ✘	$-5t^5+24t^4-73t^3+134t^2-194t+212$	2, 3 / ✘
10^a_{52}	$2t^3-7t^2+13t-15$	3 / ✘	$-3t^5+16t^4-37t^3+50t^2-49t+44$	2 / ✘
10^a_{53}	$6t^2-18t+25$	2 / ✘	$93t^3-346t^2+680t-828$	2, 3 / ✘
10^a_{54}	$2t^3-6t^2+10t-11$	3 / ✘	$-3t^5+12t^4-24t^3+26t^2-21t+16$	2, 3 / ✘
10^a_{55}	$5t^2-15t+21$	2 / ✘	$66t^3-246t^2+488t-596$	2 / ✘
10^a_{56}	$-2t^3+8t^2-14t+17$	3 / ✘	$-9t^5+52t^4-133t^3+234t^2-312t+340$	2 / ✘
10^a_{57}	$2t^3-8t^2+18t-23$	3 / ✘	$-5t^5+28t^4-93t^3+194t^2-300t+340$	2 / ✘
10^a_{58}	$3t^2-16t+27$	2 / ✘	$3t^3-28t^2+94t-140$	2 / ✘
10^a_{59}	$t^3-7t^2+18t-23$	3 / ✘	$-t^5+12t^4-55t^3+128t^2-181t+196$	1 / ✘
10^a_{60}	$-t^3+7t^2-20t+29$	3 / ✘	$5t^3-40t^2+122t-176$	1 / ✘
10^a_{61}	$-2t^3+5t^2-6t+7$	3 / ✘	$-7t^5+20t^4-27t^3+36t^2-35t+36$	2, 3 / ✘
10^a_{62}	$t^4-3t^3+6t^2-8t+9$	4 / ✘	$-2t^7+8t^6-23t^5+40t^4-63t^3+76t^2-89t+88$	2 / ✘
10^a_{63}	$5t^2-14t+19$	2 / ✘	$66t^3-220t^2+416t-496$	2 / ✘
10^a_{64}	$-t^4+3t^3-6t^2+10t-11$	4 / ✘	$-t^7+4t^6-11t^5+24t^4-37t^3+52t^2-60t+64$	2 / ✘
10^a_{65}	$2t^3-7t^2+14t-17$	3 / ✘	$-5t^5+24t^4-71t^3+124t^2-169t+180$	2 / ✘
10^a_{66}	$3t^3-9t^2+16t-19$	3 / ✘	$30t^5-112t^4+279t^3-480t^2+662t-724$	3 / ✘
10^a_{67}	$-4t^2+16t-23$	2 / ✘	$24t^3-140t^2+312t-392$	2 / ✘
10^a_{68}	$4t^2-14t+21$	2 / ✘	$8t^3-40t^2+117t-164$	2 / ✘
10^a_{69}	$t^3-7t^2+21t-29$	3 / ✘	$-t^5+12t^4-68t^3+212t^2-397t+476$	2 / ✘
10^a_{70}	$t^3-7t^2+16t-19$	3 / ✘	$-t^5+12t^4-53t^3+114t^2-146t+152$	2 / ✘
10^a_{71}	$-t^3+7t^2-18t+25$	3 / ✘	t^3-2t^2-t+4	1 / ✘
10^a_{72}	$-2t^3+9t^2-16t+19$	3 / ✘	$-9t^5+60t^4-167t^3+298t^2-410t+448$	2 / ✘
10^a_{73}	$t^3-7t^2+20t-27$	3 / ✘	$t^5-12t^4+65t^3-194t^2+350t-416$	1 / ✘
10^a_{74}	$-4t^2+16t-23$	2 / ✘	$24t^3-136t^2+290t-360$	2 / ✘
10^a_{75}	$-t^3+7t^2-19t+27$	3 / ✔	$-4t^3+36t^2-117t+172$	2 / ✘
10^a_{76}	$-2t^3+7t^2-12t+15$	3 / ✘	$-9t^5+44t^4-104t^3+184t^2-245t+272$	2, 3 / ✘
10^a_{77}	$2t^3-7t^2+14t-17$	3 / ✘	$-5t^5+24t^4-71t^3+132t^2-189t+208$	2, 3 / ✘
10^a_{78}	$-t^3+7t^2-16t+21$	3 / ✘	$2t^5-24t^4+105t^3-244t^2+390t-448$	2 / ✘
10^a_{79}	$t^4-3t^3+7t^2-12t+15$	4 / ✘	0	2, 3 / ✔
10^a_{80}	$3t^3-9t^2+15t-17$	3 / ✘	$30t^5-112t^4+260t^3-426t^2+568t-616$	3 / ✘
10^a_{81}	$-t^3+8t^2-20t+27$	3 / ✘	0	2 / ✔
10^a_{82}	$-t^4+4t^3-8t^2+12t-13$	4 / ✘	$t^7-6t^6+19t^5-42t^4+64t^3-78t^2+84t-84$	1 / ✘
10^a_{83}	$2t^3-9t^2+19t-23$	3 / ✘	$-5t^5+34t^4-110t^3+214t^2-301t+332$	2 / ✘
10^a_{84}	$2t^3-9t^2+20t-25$	3 / ✘	$-5t^5+34t^4-116t^3+246t^2-373t+424$	1 / ✘
10^a_{85}	$t^4-4t^3+8t^2-10t+11$	4 / ✘	$2t^7-12t^6+36t^5-68t^4+101t^3-124t^2+138t-140$	2 / ✘
10^a_{86}	$-2t^3+9t^2-19t+25$	3 / ✘	$-t^5+6t^4-21t^3+58t^2-105t+128$	2 / ✘
10^a_{87}	$-2t^3+9t^2-18t+23$	3 / ✔	$-t^5+6t^4-23t^3+66t^2-125t+152$	2 / ✘
10^a_{88}	$-t^3+8t^2-24t+35$	3 / ✘	0	1 / ✔
10^a_{89}	$t^3-8t^2+24t-33$	3 / ✘	$t^5-14t^4+83t^3-264t^2+495t-596$	2 / ✘
10^a_{90}	$-2t^3+8t^2-17t+23$	3 / ✘	$-t^5+6t^4-21t^3+54t^2-93t+112$	2 / ✘
10^a_{91}	$t^4-4t^3+9t^2-14t+17$	4 / ✘	$t^5-2t^4+2t^3-3t+4$	1 / ✘
10^a_{92}	$-2t^3+10t^2-20t+25$	3 / ✘	$-9t^5+68t^4-216t^3+428t^2-622t+696$	2 / ✘
10^a_{93}	$2t^3-8t^2+15t-17$	3 / ✘	$3t^5-18t^4+43t^3-58t^2+55t-48$	2 / ✘
10^a_{94}	$-t^4+4t^3-9t^2+14t-15$	4 / ✘	$-t^7+6t^6-20t^5+46t^4-76t^3+102t^2-115t+120$	2 / ✘
10^a_{95}	$2t^3-9t^2+21t-27$	3 / ✘	$-5t^5+32t^4-114t^3+248t^2-384t+436$	1 / ✘
10^a_{96}	$-t^3+7t^2-22t+33$	3 / ✘	$-7t^3+50t^2-147t+212$	2 / ✘
10^a_{97}	$-5t^2+22t-33$	2 / ✘	$-37t^3+242t^2-603t+788$	2 / ✘
10^a_{98}	$-2t^3+9t^2-18t+23$	3 / ✘	$9t^5-60t^4+177t^3-348t^2+501t-564$	2 / ✘
10^a_{99}	$t^4-4t^3+10t^2-16t+19$	4 / ✔	0	2 / ✔
10^a_{100}	$t^4-4t^3+9t^2-12t+13$	4 / ✘	$2t^7-12t^6+39t^5-80t^4+128t^3-164t^2+192t-196$	2, 3 / ✘
10^a_{101}	$7t^2-21t+29$	2 / ✘	$-129t^3+480t^2-942t+1148$	2, 3 / ✘
10^a_{102}	$-2t^3+8t^2-16t+21$	3 / ✘	$-t^5+6t^4-19t^3+50t^2-89t+108$	1 / ✘
10^a_{103}	$2t^3-8t^2+17t-21$	3 / ✘	$5t^5-30t^4+93t^3-178t^2+254t-280$	3 / ✘



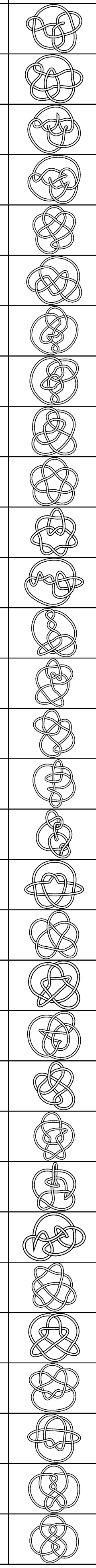


diagram	n^t_k	Alexander's ω^+	genus / ribbon	Today's / Rozansky's ρ^+_1	unknotting number / amphicheiral
	10^a_{104}	$t^4 - 4t^3 + 9t^2 - 15t + 19$	4 / ✗	$t^5 - 2t^4 + 2t^3 - 3t + 4$	1 / ✗
	10^a_{105}	$t^3 - 8t^2 + 22t - 29$	3 / ✗	$-t^5 + 14t^4 - 71t^3 + 184t^2 - 292t + 332$	2 / ✗
	10^a_{106}	$-t^4 + 4t^3 - 9t^2 + 15t - 17$	4 / ✗	$-t^7 + 6t^6 - 20t^5 + 48t^4 - 82t^3 + 114t^2 - 134t + 140$	2 / ✗
	10^a_{107}	$-t^3 + 8t^2 - 22t + 31$	3 / ✗	$2t^3 - 8t^2 + 13t - 16$	1 / ✗
	10^a_{108}	$2t^3 - 8t^2 + 14t - 15$	3 / ✗	$-3t^5 + 18t^4 - 41t^3 + 50t^2 - 40t + 32$	2 / ✗
	10^a_{109}	$t^4 - 4t^3 + 10t^2 - 17t + 21$	4 / ✗	0	2 / ✓
	10^a_{110}	$t^3 - 8t^2 + 20t - 25$	3 / ✗	$t^5 - 14t^4 + 69t^3 - 160t^2 + 219t - 236$	2 / ✗
	10^a_{111}	$-2t^3 + 9t^2 - 17t + 21$	3 / ✗	$-9t^5 + 60t^4 - 171t^3 + 316t^2 - 436t + 480$	2 / ✗
	10^a_{112}	$-t^4 + 5t^3 - 11t^2 + 17t - 19$	4 / ✗	$t^7 - 8t^6 + 29t^5 - 68t^4 + 115t^3 - 152t^2 + 175t - 180$	2 / ✗
	10^a_{113}	$2t^3 - 11t^2 + 26t - 33$	3 / ✗	$-5t^5 + 42t^4 - 167t^3 + 394t^2 - 623t + 720$	1 / ✗
	10^a_{114}	$-2t^3 + 10t^2 - 21t + 27$	3 / ✗	$t^5 - 8t^4 + 30t^3 - 78t^2 + 140t - 168$	1 / ✗
	10^a_{115}	$-t^3 + 9t^2 - 26t + 37$	3 / ✗	0	2 / ✓
	10^a_{116}	$-t^4 + 5t^3 - 12t^2 + 19t - 21$	4 / ✗	$t^7 - 8t^6 + 30t^5 - 74t^4 + 132t^3 - 184t^2 + 217t - 228$	2 / ✗
	10^a_{117}	$2t^3 - 10t^2 + 24t - 31$	3 / ✗	$-5t^5 + 38t^4 - 144t^3 + 330t^2 - 522t + 600$	2 / ✗
	10^a_{118}	$t^4 - 5t^3 + 12t^2 - 19t + 23$	4 / ✗	0	1 / ✓
	10^a_{119}	$-2t^3 + 10t^2 - 23t + 31$	3 / ✗	$-t^5 + 6t^4 - 26t^3 + 86t^2 - 175t + 220$	1 / ✗
	10^a_{120}	$8t^2 - 26t + 37$	2 / ✗	$166t^3 - 692t^2 + 1433t - 1788$	2, 3 / ✗
	10^a_{121}	$2t^3 - 11t^2 + 27t - 35$	3 / ✗	$5t^5 - 42t^4 + 167t^3 - 396t^2 + 634t - 732$	2 / ✗
	10^a_{122}	$-2t^3 + 11t^2 - 24t + 31$	3 / ✗	$-t^5 + 8t^4 - 34t^3 + 104t^2 - 211t + 264$	2 / ✗
	10^a_{123}	$t^4 - 6t^3 + 15t^2 - 24t + 29$	4 / ✓	0	2 / ✓
	10^n_{124}	$t^4 - t^3 + t - 1$	4 / ✗	$-4t^7 - 6t^4 - 4t^2 - 6t$	4 / ✗
	10^n_{125}	$t^3 - 2t^2 + 2t - 1$	3 / ✗	$-t^5 + 2t^4 - 2t^3 + 3t - 4$	2 / ✗
	10^n_{126}	$t^3 - 2t^2 + 4t - 5$	3 / ✗	$t^5 - 2t^4 + 10t^3 - 12t^2 + 22t - 20$	2 / ✗
	10^n_{127}	$-t^3 + 4t^2 - 6t + 7$	3 / ✗	$2t^5 - 14t^4 + 32t^3 - 52t^2 + 67t - 72$	2 / ✗
	10^n_{128}	$2t^3 - 3t^2 + t + 1$	3 / ✗	$-13t^5 + 12t^4 - 3t^3 - 10t^2 - 9t + 12$	3 / ✗
	10^n_{129}	$2t^2 - 6t + 9$	2 / ✓	$-t^3 - 2t^2 + 14t - 20$	1 / ✗
	10^n_{130}	$2t^2 - 4t + 5$	2 / ✗	$t^3 - 2t^2 + 19t - 24$	2 / ✗
	10^n_{131}	$-2t^2 + 8t - 11$	2 / ✗	$5t^3 - 38t^2 + 87t - 112$	1 / ✗
	10^n_{132}	$t^2 - t + 1$	2 / ✗	$2t^2 + 5t - 4$	1 / ✗
	10^n_{133}	$-t^2 + 5t - 7$	2 / ✗	$t^3 - 14t^2 + 37t - 48$	1 / ✗
	10^n_{134}	$2t^3 - 4t^2 + 4t - 3$	3 / ✗	$-13t^5 + 24t^4 - 33t^3 + 30t^2 - 41t + 40$	3 / ✗
	10^n_{135}	$3t^2 - 9t + 13$	2 / ✗	$t^3 - 6t^2 + 18t - 24$	2 / ✗
	10^n_{136}	$-t^2 + 4t - 5$	2 / ✗	$-t^3 + 4t^2 - 2t - 4$	1 / ✗
	10^n_{137}	$t^2 - 6t + 11$	2 / ✓	$-4t^2 + 24t - 44$	1 / ✗
	10^n_{138}	$t^3 - 5t^2 + 8t - 7$	3 / ✗	$-t^5 + 8t^4 - 22t^3 + 24t^2 - 11t + 8$	2 / ✗
	10^n_{139}	$t^4 - t^3 + 2t - 3$	4 / ✗	$-4t^7 - 12t^4 + 5t^3 - 4t^2 - 16t + 12$	4 / ✗
	10^n_{140}	$t^2 - 2t + 3$	2 / ✓	$8t - 8$	2 / ✗
	10^n_{141}	$-t^3 + 3t^2 - 4t + 5$	3 / ✗	$t^3 - 8t^2 + 16t - 20$	1 / ✗
	10^n_{142}	$2t^3 - 3t^2 + 2t - 1$	3 / ✗	$-13t^5 + 12t^4 - 13t^3 + 4t^2 - 17t + 12$	3 / ✗
	10^n_{143}	$t^3 - 3t^2 + 6t - 7$	3 / ✗	$t^5 - 4t^4 + 15t^3 - 28t^2 + 45t - 48$	1 / ✗
	10^n_{144}	$-3t^2 + 10t - 13$	2 / ✗	$10t^3 - 44t^2 + 80t - 96$	2 / ✗
	10^n_{145}	$t^2 + t - 3$	2 / ✗	$2t^3 + 8t^2 + 6t - 8$	2 / ✗
	10^n_{146}	$2t^2 - 8t + 13$	2 / ✗	$t^3 - 8t^2 + 21t - 28$	1 / ✗
	10^n_{147}	$-2t^2 + 7t - 9$	2 / ✗	$-3t^3 + 12t^2 - 15t + 12$	1 / ✗
	10^n_{148}	$t^3 - 3t^2 + 7t - 9$	3 / ✗	$t^5 - 4t^4 + 18t^3 - 36t^2 + 62t - 68$	2 / ✗
	10^n_{149}	$-t^3 + 5t^2 - 9t + 11$	3 / ✗	$2t^5 - 18t^4 + 55t^3 - 104t^2 + 149t - 164$	2 / ✗
	10^n_{150}	$-t^3 + 4t^2 - 6t + 7$	3 / ✗	$-2t^5 + 12t^4 - 26t^3 + 38t^2 - 45t + 44$	2 / ✗
	10^n_{151}	$t^3 - 4t^2 + 10t - 13$	3 / ✗	$-t^5 + 6t^4 - 21t^3 + 42t^2 - 66t + 72$	2 / ✗
	10^n_{152}	$t^4 - t^3 - t^2 + 4t - 5$	4 / ✗	$4t^7 - 7t^5 + 18t^4 - 7t^3 - 12t^2 + 45t - 52$	4 / ✗
	10^n_{153}	$t^3 - t^2 - t + 3$	3 / ✓	$t^5 - 2t^4 + t^3 + 2t^2 - t$	2 / ✗
	10^n_{154}	$t^3 - 4t + 7$	3 / ✗	$-3t^5 - 6t^4 + 13t^3 - 47t + 68$	3 / ✗
	10^n_{155}	$-t^3 + 3t^2 - 5t + 7$	3 / ✓	$-2t^3 + 12t^2 - 22t + 28$	2 / ✗
	10^n_{156}	$t^3 - 4t^2 + 8t - 9$	3 / ✗	$t^5 - 6t^4 + 19t^3 - 30t^2 + 33t - 32$	1 / ✗
	10^n_{157}	$-t^3 + 6t^2 - 11t + 13$	3 / ✗	$-2t^5 + 22t^4 - 78t^3 + 148t^2 - 218t + 240$	2 / ✗
	10^n_{158}	$-t^3 + 4t^2 - 10t + 15$	3 / ✗	$2t^2 - 7t + 12$	2 / ✗
	10^n_{159}	$t^3 - 4t^2 + 9t - 11$	3 / ✗	$t^5 - 6t^4 + 26t^3 - 60t^2 + 98t - 112$	1 / ✗
	10^n_{160}	$-t^3 + 4t^2 - 4t + 3$	3 / ✗	$-2t^5 + 12t^4 - 20t^3 + 14t^2 - 16t + 12$	2 / ✗
	10^n_{161}	$t^3 - 2t + 3$	3 / ✗	$3t^5 + 6t^4 - 3t^3 + 4t^2 + 14t - 12$	3 / ✗
	10^n_{162}	$-3t^2 + 9t - 11$	2 / ✗	$10t^3 - 38t^2 + 58t - 68$	2 / ✗
	10^n_{163}	$t^3 - 5t^2 + 12t - 15$	3 / ✗	$-t^5 + 8t^4 - 30t^3 + 62t^2 - 89t + 96$	1, 2 / ✗
	10^n_{164}	$3t^2 - 11t + 17$	2 / ✗	$t^3 - 10t^2 + 29t - 40$	1 / ✗
	10^n_{165}	$-2t^2 + 10t - 15$	2 / ✗	$-5t^3 + 50t^2 - 146t + 196$	2 / ✗