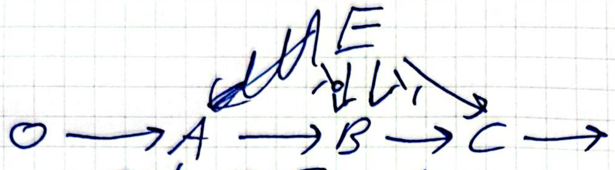
http://drorbn.net/ge24

Title. Secondary Operations, Emergent Knots, and the Goldman-Turaev Lie bialgebra. (~~Just like Guillaume,~~ my ~~talk will have 3 parts~~)

Apologies: Mostly language Thu: Mostly tofu.

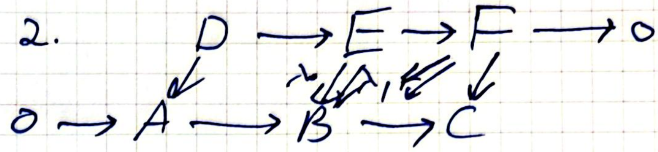
Part I Secondary operations Goal: Emergent.

1.



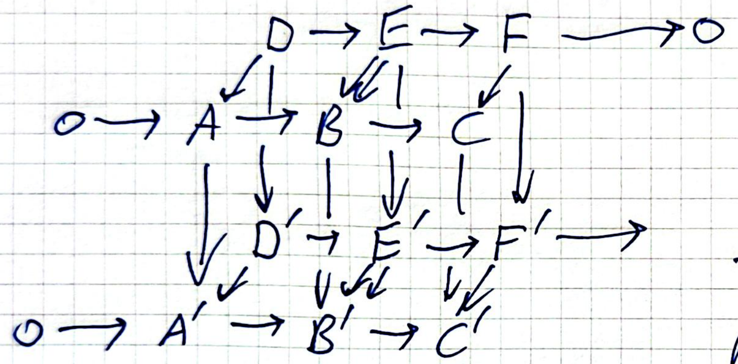
get $\eta: E \to A$

2.



get $\eta: F \to A$

3. Naturality



If all the squares commute, then the η–η' square commutes too.

For us, the top layer will be K, the bottom $\operatorname{gr} K = A$, and the vertical arrows are some knot-theoretic expansion.

Part II Emergent knots in $\Sigma \times I$ [If Σ = (xxx), $\Sigma \times I$ is PDS_n]

$K(S)$: l.c. of knots in $\Sigma \times I$, with skeleton S.

$K^1(S)$: the span of knots w/ one S-S double point, where $\times\!\!\!\!\bullet = \times_1 - \times$

$K^2(S)$: . . . w/ 2 double points.

$K/K^1 = K^{gh}$ = homotopy classes $K^{gh}(|\bigcirc) = \pi_1 \otimes |\pi_1|$ (there is also a framed variant)

$\mathcal{K}^{1/2} = \mathcal{K}^{em}$ = "emergent knots"

$\mathcal{K}^{1/2} = \mathcal{K}^1/\mathcal{K}^2 \to \mathcal{K}^{em} \longrightarrow \mathcal{K}^{gh} \longrightarrow 0$

[from now on, Σ = (× × ×) D \ {pts}]

Example 1. $S = \bigcirc_1 \bigcirc_2$ λ_0 : 1 above 2
λ_1 : 2 above 1.

$\mathcal{K}^{em}(\bigcirc\bigcirc) \longrightarrow \mathcal{K}^{gh}(\bigcirc\bigcirc) = \langle\pi_1\rangle \otimes \langle\pi_1\rangle$

η: The generalized Goldman bracket

$\lambda_0 \downarrow \downarrow \lambda_1$

$\mathcal{K}^{1/2}(\bigcirc\bigcirc) \to \mathcal{K}^{em}(\bigcirc\bigcirc)$

~~homotopy~~ maps

8 $\to \Sigma$ up to homotopy

Example 2 S = ⌒ λ_0 : ascending
λ_1 : descending.

ext $\eta : \pi_1 \to \pi_1$ "generalized co-action"

Part II.5 $\mathcal{K}_H := \mathbb{Q}[b] \otimes \mathcal{K}$ / (crossing relation) $= b$)((deg 1)

Prop $\hat{b} : \mathcal{K}^{gh}_{(H)} \longrightarrow \mathcal{K}^{1/2}_H$ is an isomorphism.

Pf Surjectivity is obvious, so we need $\Psi : \mathcal{K}^{1/2}_H \to \mathcal{K}^{gh}$
s.t. $\hat{b} /\!/ \Psi = I_{\mathcal{K}^{gh}}$

$\Psi : b \cdot K \mapsto K$ $\Psi : b^0 K \mapsto \frac{1}{2} \sum_{x \in K} (-1)^x K|_{x \to)(}$

Satisfies R2, R3
Satisfies H.

Part III Go-Tu

$$\mathcal{K}^{cm}(\bigcirc\!\bigcirc) \longrightarrow \mathcal{K}^{gh}(\bigcirc\bigcirc) \cong |\pi_1| \otimes |\pi_1|$$

$\lambda_{10} \downarrow \downarrow \lambda_{11}$ The Goldman bracket

$$|\pi_1| = \mathcal{K}^{gh}(\bigcirc) \xrightarrow{\hat{b}} \mathcal{K}^{1/2}(\bigcirc\bigcirc) \longrightarrow \mathcal{K}^{cm}(\bigcirc\bigcirc)$$

$$\mathcal{K}^{cm}(\cap) \longrightarrow \mathcal{K}^{gh}(\cap) \cong \pi_1$$

$\lambda_{20} \downarrow \downarrow \lambda_{21}$ asc / desc The "co-action"

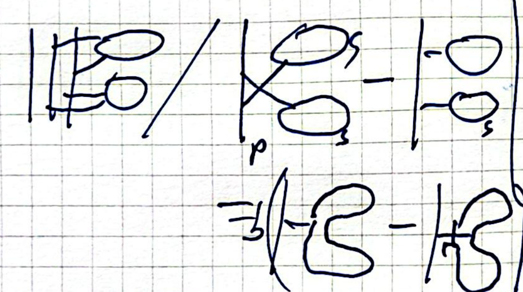
$$\pi_1 \otimes |\pi_1| \cong \mathcal{K}^{gh}\left(\begin{smallmatrix}\circ\\ \cap\end{smallmatrix}\right) \xrightarrow{\hat{b}} \mathcal{K}^{1/2}(\cap) \longrightarrow \mathcal{K}^{cm}(\cap)$$

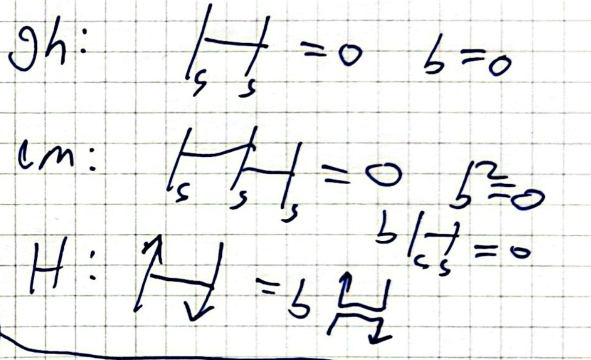
Part IV Take gr of everything

$\mathcal{A}\left(\begin{smallmatrix}\circ\\ \cap\end{smallmatrix}\right) = \langle$  $\rangle / 4T$

$$\mathcal{A}^{gh}_{(H)}\left(\begin{smallmatrix}\circ\\ \cap\end{smallmatrix}\right) = A \otimes |A|$$

w/ $A = FA\langle x_1 \dots x_n\rangle$

$\mathcal{A}^{cm}_{H}(\bigcirc\bigcirc) =$ 



$$\mathcal{A}^{cm}(\bigcirc\bigcirc) \to \mathcal{A}^{gh}\left(\begin{smallmatrix}\circ\\ \circ\end{smallmatrix}\right) = |A| \otimes |A|$$

$$|A| = \mathcal{A}^{gh}(\bigcirc) \xrightarrow{\hat{b}} \mathcal{A}^{cm}(\bigcirc\bigcirc)$$

similarly of the co-action...

Comments

② HW: Do the same for $\mathcal{K}^w / \mathcal{A}^w$

① I'm not happy! I think we are still missing something small, perhaps an extension of the KI