Pensieve header: Implementing ρ_1.

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Preliminaries

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This is Rho1.nb of http://drorbn.net/j22/ap.

In[ ]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\Geneva-2206"];
```

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In[ ]:=
```
Once[<< KnotTheory`; << Rot.m];
```

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```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
```

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```
Loading Rot.m from http://drorbn.net/j22/ap to compute rotation numbers.
```

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The Program

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In[ ]:=
```
R_1[s_, i_, j_] := s (g_{j i} (g_{j+1,j} + g_{j,j+1} - g_{i j}) - g_{i i} (g_{j,j+1} - 1) - 1 / 2);
ρ[K_] := Module[{Cs, φ, n, A, s, i, j, k, Δ, G, ρ1},
   {Cs, φ} = Rot[K]; n = Length[Cs];
   A = IdentityMatrix[2 n + 1];
   Cases[Cs, {s_, i_, j_} :> (A⟦{i, j}, {i + 1, j + 1}⟧ += ( -T^s  T^s - 1
                                                            0     -1    ))];
   Δ = T^((-Total[φ]-Total[Cs⟦All,1⟧])/2) Det[A];
   G = Inverse[A];
   ρ1 = Sum_{k=1}^{n} R_1 @@ Cs⟦k⟧ - Sum_{k=1}^{2 n} φ⟦k⟧ (g_{kk} - 1 / 2);
   Factor@{Δ, Δ^2 ρ1 /. g_{α_,β_} :> G⟦α, β⟧}];
```

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The First Few Knots

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In[ ]:= `Table[K → ρ[K], {K, AllKnots[{3, 6}]}]`

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KnotTheory: Loading precomputed data in PD4Knots`.

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Out[ ]= $\{\text{Knot}[3,1]\to\{\frac{1-T+T^2}{T},\frac{(-1+T)^2(1+T^2)}{T^2}\},\ \text{Knot}[4,1]\to\{-\frac{1-3T+T^2}{T},0\},$

$\text{Knot}[5,1]\to\{\frac{1-T+T^2-T^3+T^4}{T^2},\frac{(-1+T)^2(1+T^2)(2+T^2+2T^4)}{T^4}\},$

$\text{Knot}[5,2]\to\{\frac{2-3T+2T^2}{T},\frac{(-1+T)^2(5-4T+5T^2)}{T^2}\},$

$\text{Knot}[6,1]\to\{-\frac{(-2+T)(-1+2T)}{T},\frac{(-1+T)^2(1-4T+T^2)}{T^2}\},$

$\text{Knot}[6,2]\to\{-\frac{1-3T+3T^2-3T^3+T^4}{T^2},\frac{(-1+T)^2(1-4T+4T^2-4T^3+4T^4-4T^5+T^6)}{T^4}\},$

$\text{Knot}[6,3]\to\{\frac{1-3T+5T^2-3T^3+T^4}{T^2},0\}\}$

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\def\nbpdfText#1{\vskip -3mm\[\includegraphics[width=0.4\linewidth]{#1}\quad p=1-T^s \]}

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tex

\def\nbpdfText#1{\vskip 1mm\par\noindent\includegraphics{#1}}

tex

\needspace{2in}

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Fast!

tex

\[\resizebox{\linewidth}{!}{\import{../Waco-2203/}{GST48-Marked.pdf_t}} \]

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In[∘]:=
$$\text{Timing@}\rho[\text{EPD}[X_{14,1}, \overline{X}_{2,29}, X_{3,40}, X_{43,4}, \overline{X}_{26,5}, X_{6,95}, X_{96,7}, X_{13,8}, \overline{X}_{9,28}, X_{10,41}, X_{42,11}, \overline{X}_{27,12}, X_{30,15}, \overline{X}_{16,61}, \overline{X}_{17,72}, \overline{X}_{18,83}, X_{19,34}, \overline{X}_{89,20}, \overline{X}_{21,92}, \overline{X}_{79,22}, \overline{X}_{68,23}, \overline{X}_{57,24}, \overline{X}_{25,56}, X_{62,31}, X_{73,32}, X_{84,33}, \overline{X}_{50,35}, X_{36,81}, X_{37,70}, X_{38,59}, \overline{X}_{39,54}, X_{44,55}, X_{58,45}, X_{69,46}, X_{80,47}, X_{48,91}, X_{90,49}, X_{51,82}, \overline{X}_{52,71}, X_{53,60}, \overline{X}_{63,74}, \overline{X}_{64,85}, \overline{X}_{76,65}, \overline{X}_{87,66}, \overline{X}_{67,94}, \overline{X}_{75,86}, \overline{X}_{88,77}, \overline{X}_{78,93}]]$$

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Out[∘]=
$$\Big\{86.2031, \Big\{-\frac{(-1+2T-T^2-T^3+2T^4-T^5+T^8)(-1+T^3-2T^4+T^5+T^6-2T^7+T^8)}{T^8},$$
$$\frac{1}{T^{16}}(-1+T)^2(5-18T+33T^2-32T^3+2T^4+42T^5-62T^6-8T^7+166T^8-242T^9+108T^{10}+132T^{11}-226T^{12}+148T^{13}-11T^{14}-36T^{15}-11T^{16}+148T^{17}-226T^{18}+132T^{19}+108T^{20}-242T^{21}+166T^{22}-8T^{23}-62T^{24}+42T^{25}+2T^{26}-32T^{27}+33T^{28}-18T^{29}+5T^{30})\Big\}\Big\}$$

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Strong!

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```
{NumberOfKnots[{3, 12}],
 Length@Union@Table[ρ[K], {K, AllKnots[{3, 12}]}],
 Length@Union@Table[{HOMFLYPT[K], Kh[K]}, {K, AllKnots[{3, 12}]}]}
```

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Out[∘]= {2977, 2882, 2785}

In[∘]:= `2977 - {2882, 2785}`

Out[∘]= {95, 192}

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So the pair (Δ,ρ_1) attains 2,882 distinct values on the 2,977 prime knots with up to 12 crossings (a deficit of 95), whereas the pair (HOMFLYPT, Khovanov Homology) attains only 2,785 distinct values on the same knots (a deficit of 192).

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\def\nbpdfText#1{\vskip 1mm\par\noindent\includegraphics[width=\linewidth]{#1}}

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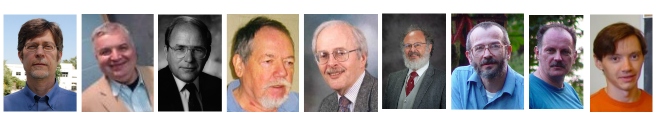

Hoste Ocneanu Millett Freyd Lickorish Yetter Przytycki Traczyk Khovanov

tex

\def\nbpdfText#1{\vskip 1mm\par\noindent\includegraphics{#1}}

Invariance under R3

exec

```
nb2tex$TeXFileName = "Invariance.tex";
```

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In[∘]:=
```
δ_{i_,j_} := If[i == j, 1, 0];
gRules_{s_,i_,j_} := {g_{iβ_} :> δ_{iβ} + T^s g_{i+1,β} + (1 - T^s) g_{j+1,β},
   g_{jβ_} :> δ_{jβ} + g_{j+1,β}, g_{α_,i} :> T^-s (g_{α,i+1} - δ_{α,i+1}),
   g_{α_j} :> g_{α,j+1} - (1 - T^s) g_{αi} - δ_{α,j+1}}
```

Proof of Reidemeister 3:

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In[∘]:=
```
lhs = R_1[1, 20, 30] + R_1[1, 10, 31] + R_1[1, 11, 21] //. gRules_{1,20,30} ⋃ gRules_{1,10,31} ⋃ gRules_{1,11,21};
rhs = R_1[1, 10, 20] + R_1[1, 11, 30] + R_1[1, 21, 31] //. gRules_{1,10,20} ⋃ gRules_{1,11,30} ⋃ gRules_{1,21,31};
Simplify[lhs == rhs]
```

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Out[∘]= True

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Next comes Reid1, where we use results from an earlier example:

In[∘]:= $\begin{pmatrix} 1 & T^{-1} & 1 \\ 0 & T^{-1} & 1 \\ 0 & 0 & 1 \end{pmatrix}$ // Inverse // MatrixForm

Out[∘]//MatrixForm=

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & T & -T \\ 0 & 0 & 1 \end{pmatrix}$$

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In[∘]:= R₁[1, 2, 1] - 1 (g₂₂ - 1 / 2) /. $g_{\alpha\_,\beta\_} \rightarrow \begin{pmatrix} 1 & T^{-1} & 1 \\ 0 & T^{-1} & 1 \\ 0 & 0 & 1 \end{pmatrix}$〚α, β〛

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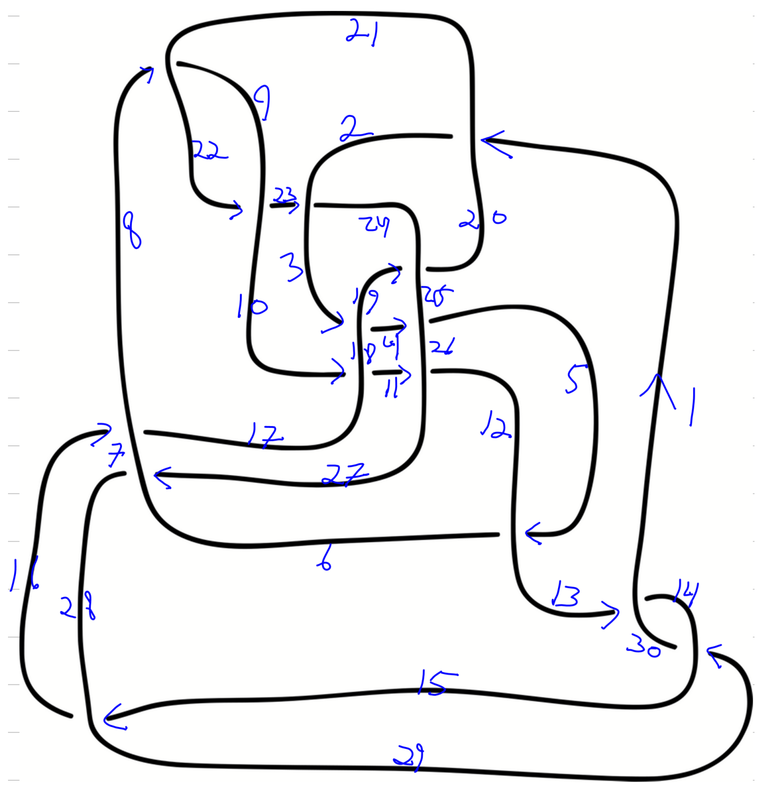
Out[∘]= $\frac{1}{T^2} - \frac{1}{T} - \frac{-1 + \frac{1}{T}}{T}$

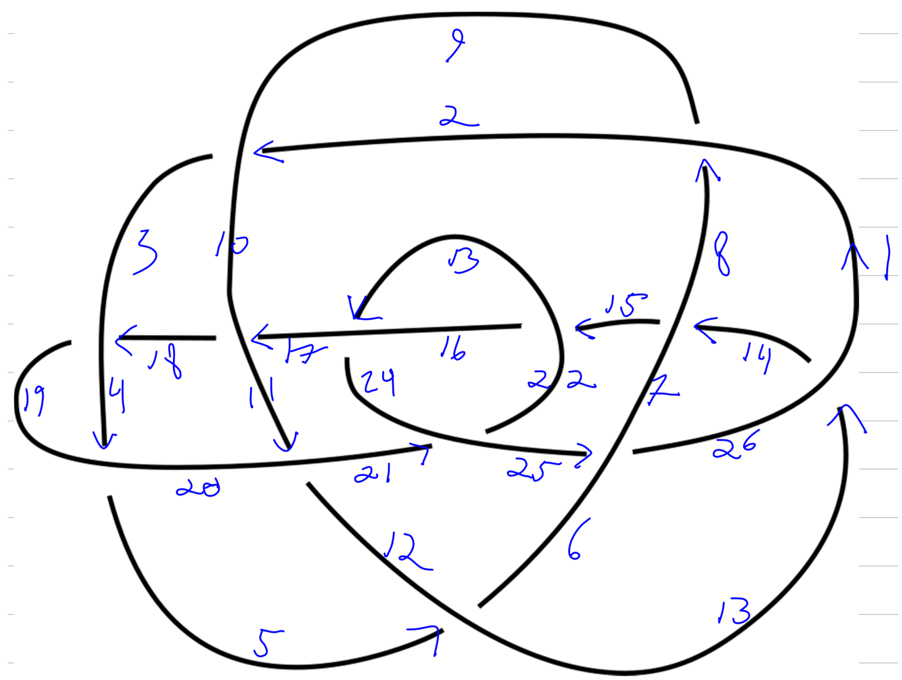
tex

Invariance under the other moves is proven similarly.



In[ ]:= $\texttt{Timing@}\rho\left[\texttt{EPD}\left[X_{20,1}, \overline{X}_{18,3}, X_{25,4}, \overline{X}_{12,5}, X_{21,8}, \overline{X}_{17,10}, X_{26,11}, \overline{X}_{30,13}, \overline{X}_{28,15}, \overline{X}_{7,16}, X_{24,19}, X_{9,22}, X_{2,23}, X_{6,27}, \overline{X}_{14,29}\right]\right]$

Out[ ]= {1.21875, {1, 0}}



In[∘]:= Timing@ρ[EPD[$\overline{X}_{9,2}$, $\overline{X}_{19,4}$, $X_{12,5}$, $\overline{X}_{1,8}$, $\overline{X}_{20,11}$, $X_{26,13}$, $X_{7,14}$, $X_{22,15}$, $\overline{X}_{10,17}$, $\overline{X}_{3,18}$, $X_{24,21}$, $X_{16,23}$, $\overline{X}_{6,25}$]]

Out[∘]= {0.703125, {1, 0}}

In[∘]:= K = PD[X[4, 2, 5, 1], X[2, 6, 3, 5], X[6, 4, 7, 3]];

In[∘]:= {Cs, r} = List @@ RVK[K]

Set: Lists {Cs, r} and {PD[X[4, 2, 5, 1], X[2, 6, 3, 5], X[6, 4, 7, 3]]} are not the same shape.

Out[∘]= {PD[X[4, 2, 5, 1], X[2, 6, 3, 5], X[6, 4, 7, 3]]}

In[∘]:= n = Length[Cs]

Out[∘]= 0

In[∘]:= A = IdentityMatrix[2 n + 1]

Out[∘]= {{1}}

In[∘]:= Do[{s, i, j} = c; A⟦{i, j}, {i + 1, j + 1}⟧ = $\begin{pmatrix} -T^s & T^s - 1 \\ 0 & -1 \end{pmatrix}$, {c, Cs}]

Do: Iterator {c, Cs} does not have appropriate bounds.

Out[∘]= Do[{s, i, j} = c; A⟦{i, j}, {i + 1, j + 1}⟧ = {{$-T^s$, $T^s - 1$}, {0, -1}}, {c, Cs}]

In[∘]:= A // MatrixForm

Out[∘]//MatrixForm=

(1)

In[∘]:= A // MatrixForm // TeXForm

Out[∘]//TeXForm=

```
\left(
\begin{array}{c}
 1 \\
\end{array}
\right)
```

In[∘]:= Δ = $T^{(-\text{Total}[r]-\text{Total}[\text{First}/@\text{Cs}])/2}$ Det[A]

Out[∘]= $T^{\frac{1}{2}(-\text{Total}[\text{Cs}]-\text{Total}[r])}$

In[∘]:= G = Inverse[A];

In[∘]:= G // MatrixForm

Out[∘]//MatrixForm=

(1)

In[∘]:= G // Simplify // MatrixForm

Out[∘]//MatrixForm=

(1)

In[ ]:= G // Simplify // MatrixForm // TeXForm

Out[ ]//TeXForm=

```
\left(
\begin{array}{c}
 1 \\
\end{array}
\right)
```