Everything around sl^ϵ_{2+} is **DoPeGDO**. So what?

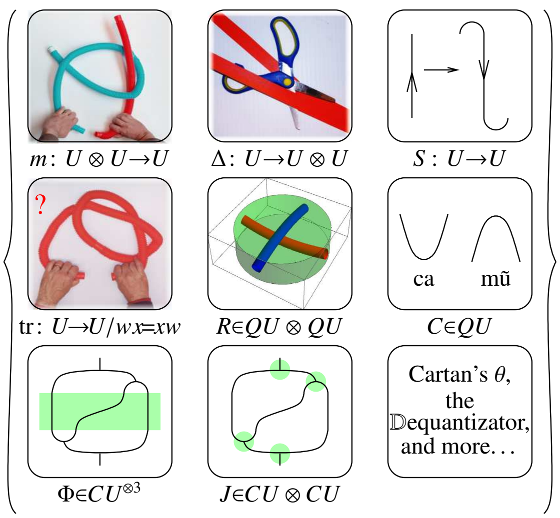
Thanks for inviting me to Da Nang!
ωεβ:=http://drorbn.net/v19/
More at ωεβ/talks

Continues Rozansky [Ro1, Ro2, Ro3] and Overbay [Ov], joint with van der Veen [BV].

Abstract. I'll explain what "everything around" means: classical and quantum m, Δ, S, tr, R, C, and θ, as well as P, Φ, J, $\mathbb{D}$, and more, and all of their compositions. What **DoPeGDO** means: the category of Docile Perturbed Gaussian Differential Operators. And what sl^ϵ_{2+} means: a solvable approximation of the semi-simple Lie algebra sl_2.

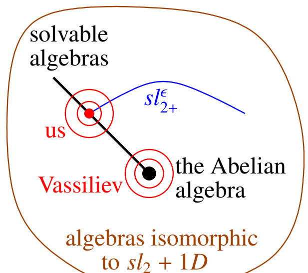
Knot theorists should rejoice because all this leads to very powerful and well-behaved poly-time-computable knot invariants. Quantum algebraists should rejoice because it's a realistic playground for testing complicated equations and theories.

Conventions. 1. For a set A, let $z_A := \{z_i\}_{i\in A}$ and let $\zeta_A := \{z_i^* = \zeta_i\}_{i\in A}$.†1 2. Everything converges!



Less Abstract

4D Metrized Lie Algebras



DoPeGDO := The category with objects finite sets†2 and mor($A \to B$):

$$\{\mathcal{F} = \omega \exp(Q + P)\} \subset \mathbb{Q}[\![\zeta_A, z_B]\!]$$

Where: • ω is a scalar.†3 • Q is a "small" quadratic in $\zeta_A \cup z_B$.†4 • P is a "docile perturbation": $P = \sum_{k\geq 1}\epsilon^k P^{(k)}$, where $\deg P^{(k)} \leq 2k+2$.†5
• Compositions:†6

$$\mathcal{F}/\!/\mathcal{G} = \mathcal{G}\circ\mathcal{F} := \left(\mathcal{G}|_{\zeta_i\to\partial_{z_i}}\mathcal{F}\right)_{z_i=0} = \left(\mathcal{F}|_{z_i\to\partial_{\zeta_i}}\mathcal{G}\right)_{\zeta_i=0}.$$

Cool! $(V^*)^{\otimes\Sigma} \otimes V^{\otimes S}$ explodes; the ranks of quadratics and bounded-degree polynomials grow slowly!†7

Representation theory is over-rated!

Our Algebras. Let $sl^\epsilon_{2+} := L\langle y,b,a,x\rangle$ subject to $[a,x]=x$, $[b,y]=-\epsilon y$, $[a,b]=0$, $[a,y]=-y$, $[b,x]=\epsilon x$, and $[x,y]=\epsilon a+b$. So $t := \epsilon a - b$ is central and if $\exists\epsilon^{-1}$, $sl^\epsilon_{2+}/\langle t\rangle \cong sl_2$.
U is either $CU = \hat{\mathcal{U}}(sl^\epsilon_{2+})$ or $QU = \mathcal{U}_\hbar(sl^\epsilon_{2+}) = A\langle y,b,a,x\rangle$ with $[a,x]=x$, $[b,y]=-\epsilon y$, $[a,b]=0$, $[a,y]=-y$, $[b,x]=\epsilon x$, and $xy - qyx = (1-AB)/\hbar$, where $q = \mathbb{e}^{\hbar\epsilon}$, $A = \mathbb{e}^{-\hbar\epsilon a}$, and $B = \mathbb{e}^{-\hbar b}$. Set also $T = A^{-1}B = \mathbb{e}^{\hbar t}$.
The Quantum Leap. Also decree that in QU,

$$\Delta(y,b,a,x) = (y_1 + B_1y_2, b_1+b_2, a_1+a_2, x_1+A_1x_2),$$
$$S(y,b,a,x) = (-B^{-1}y, -b, -a, -A^{-1}x),$$

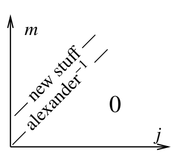
and $R = \sum \hbar^{j+k}y^kb^j \otimes a^jx^k/j![k]_q!$.

Mid-Talk Debts. • What is this good for in quantum algebra?
- In knot theory?
- How does the "inclusion" $\mathcal{D}$: Hom($U^{\otimes\Sigma} \to U^{\otimes S}$) $\rightsquigarrow$ **DoPeGDO** work?
- Proofs that everything around sl^ϵ_{2+} really is **DoPeGDO**.
- Relations with prior art.
- The rest of the "compositions" story.

Theorem ([BG], conjectured [MM], elucidated [Ro1]). Let $J_d(K)$ be the coloured Jones polynomial of K, in the d-dimensional representation of sl_2. Writing

$$\left.\frac{(q^{1/2}-q^{-1/2})J_d(K)}{q^{d/2}-q^{-d/2}}\right|_{q=e^\hbar} = \sum_{j,m\geq 0} a_{jm}(K)d^j\hbar^m,$$

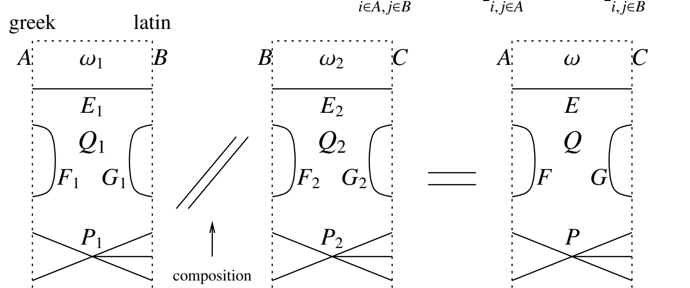
"below diagonal" coefficients vanish, $a_{jm}(K) = 0$ if $j > m$, and "on diagonal" coefficients give the inverse of the Alexander polynomial: $\left(\sum_{m=0}^\infty a_{mm}(K)\hbar^m\right)\cdot\omega(K)(e^\hbar) = 1$.



"Above diagonal" we have Rozansky's Theorem [Ro3, (1.2)]:

$$J_d(K)(q) = \frac{q^d - q^{-d}}{(q-q^{-1})\omega(K)(q^d)}\left(1 + \sum_{k=1}^\infty \frac{(q-1)^k\rho_k(K)(q^d)}{\omega^{2k}(K)(q^d)}\right).$$

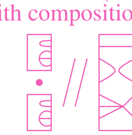
Compositions (1). In mor($A\to B$), $Q = \sum_{i\in A, j\in B} E_{ij}\zeta_i z_j + \frac{1}{2}\sum_{i,j\in A} F_{ij}\zeta_i\zeta_j + \frac{1}{2}\sum_{i,j\in B} G_{ij}z_iz_j$



Where • $\omega = \omega_1\omega_2\det(I - F_2G_1)^{-1}$.
- $E = E_1(I - F_2G_1)^{-1}E_2$.
- $F = F_1 + E_1F_2(I - G_1F_2)^{-1}E_1^T$.
- $G = G_2 + E_2^TG_1(I - F_2G_1)^{-1}E_2$.
- P is computed using "connected Feynman diagrams" or as the solution of a messy PDE (yet we're still in algebra!).

Alexander lives here

One abstraction level up from tangles!
{tangles} → {tangle}
with compositions:



DoPeGDO Footnotes. †1. Each variable has a "weight"∈ {0, 1, 2}, and always wt z_i + wt $\zeta_i = 2$.
†2. Really, "weight-graded finite sets" $A = A_0 \sqcup A_1 \sqcup A_2$.
†3. Really, a power series in the weight-0 variables†9.
†4. The weight of Q must be 2, so it decomposes as $Q = Q_{20} + Q_{11}$. The coefficients of Q_{20} are rational numbers while the coefficients of Q_{11} may be weight-0 power series†9.
†5. Setting wt $\epsilon = -2$, the weight of P is ≤ 2 (so the powers of the weight-0 variables are not constrained†9).
†6. There's also an obvious product
mor($A_1 \to B_1$) × mor($A_2 \to B_2$) → mor($A_1 \sqcup A_2 \to B_1 \sqcup B_2$).
†7. That is, if the weight-0 variables are ignored. Otherwise more care is needed yet the conclusion remains.
†8. Hom($U^{\otimes\Sigma} \to U^{\otimes S}$) $\rightsquigarrow$ mor($\{\eta_i,\beta_i,\tau_i,\alpha_i,\xi_i\}_{i\in\Sigma} \to \{y_i,b_i,t_i,a_i,x_i\}_{i\in S}$), where wt($\eta_i,\xi_i,y_i,x_i$) = 1 and wt($\beta_i,\tau_i,\alpha_i;b_i,t_i,a_i$) = (2, 2, 0; 0, 0, 2).
†9. For tangle invariants the weight-0 power series are always rational functions in the exponentials of the weight-0 variables (for knots: just one variable).

$\mathcal{D}\colon \operatorname{Hom}(U^{\otimes\Sigma} \to U^{\otimes S}) \to \mathbb{Q}[\![\eta_\Sigma, \beta_\Sigma, \alpha_\Sigma, \xi_\Sigma, y_S, b_S, a_S, x_S]\!]$. The PBW theorem for CU (always in the *ybax* order), or its quantum analog for QU, say that if $U = CU$ or QU then $U^{\otimes S}$ is isomorphic as a vector space to $\mathbb{Q}[\![y_i, b_i, a_i, x_i]\!]_{i\in S}$; so it is enough to understand $\operatorname{Hom}(\mathbb{Q}[\![z_A]\!] \to \mathbb{Q}[\![z_B]\!])$ for finite sets A and B. Using the pairing

$$\langle z_i^m, \zeta_j^n\rangle = \partial_{\zeta_i}^m \zeta_j^n\Big|_{\zeta_A\to 0} = \delta_{ij}\delta_{mn} n!,$$

we get a map

$$\mathcal{D}\colon \operatorname{Hom}(\mathbb{Q}[\![z_A]\!] \to \mathbb{Q}[\![z_B]\!]) \cong \mathbb{Q}[\![z_A]\!]^* \otimes \mathbb{Q}[\![z_B]\!] \cong \mathbb{Q}[\![\zeta_A]\!] \otimes \mathbb{Q}[\![z_B]\!] \cong \mathbb{Q}[\![\zeta_A, z_B]\!]$$

Example. $\mathcal{D}(id\colon \mathbb{Q}[\![z]\!] \to \mathbb{Q}[\![z]\!]) = \mathbb{e}^{\zeta z}$. Indeed,

$$\langle z^n, \mathbb{e}^{\zeta z}\rangle = \left\langle z^n, \sum_m \frac{(\zeta z)^m}{m!}\right\rangle = \sum_m \frac{z^m}{m!}\delta_{mn} n! = z^n.$$

Example. $\mathcal{D}(id\colon U \to U) = \mathbb{e}^{\eta y + \beta b + \alpha a + \xi x}$.

Claim. Assuming convergence, if $F \in \operatorname{Hom}(\mathbb{Q}[\![z_A]\!] \to \mathbb{Q}[\![z_B]\!])$, $G \in \operatorname{Hom}(\mathbb{Q}[\![z_B]\!] \to \mathbb{Q}[\![z_C]\!])$, $\mathcal{F} = \mathcal{D}(F)$, and $\mathcal{G} = \mathcal{D}(G)$, then

$$\mathcal{D}(F/\!\!/G) = \left(\mathcal{F}|_{z_i\to\partial_{\zeta_i}}\mathcal{G}\right)_{\zeta_i=0}.$$

And so the title of the talk finally makes sense!

Other GDOs. **Claim.** If $L\colon \mathbb{Q}[\![z_A]\!] \to \mathbb{Q}[\![z_B]\!]$ is linear, then $\mathcal{D}(L) = L\left(\mathbb{e}^{\sum_{i\in A}\zeta_i z_i}\right)$. **Proof.** Exercise.

Example. Let $c\Delta^i_{jk}\colon CU^{\otimes\{i\}} \to CU^{\otimes\{j,k\}}$ be the standard co-product, given by $c\Delta^i_{jk}(y_i, b_i, a_i, x_i) = (y_j + y_k, b_j + b_k, a_j + a_k, x_j + x_k)$. Then

$$\mathcal{D}(c\Delta^i_{jk}) = c\Delta^i_{jk}(\mathbb{e}^{\eta_i y_i + \beta_i b_i + \alpha_i a_i + \xi_i x_i}) = \mathbb{e}^{\eta_i(y_j+y_k) + \beta_i(b_j+b_k) + \alpha_i(a_j+a_k) + \xi_i(x_j+x_k)}.$$

Example. The standard commutative product m^{ij}_k of polynomials is given by $z_i, z_j \to z_k$. Hence $\mathcal{D}(m^{ij}_k) = m^{ij}_k(\mathbb{e}^{\zeta_i z_i + \zeta_j z_j}) = \mathbb{e}^{(\zeta_i+\zeta_j)z_k}$.

$$\begin{array}{ccc} \mathbb{Q}[\![z]\!]_i \otimes \mathbb{Q}[\![z]\!]_j & \xrightarrow{m^{ij}_k} & \mathbb{Q}[\![z]\!]_k \\ \| & & \| \\ \mathbb{Q}[\![z_i, z_j]\!] & \xrightarrow{m^{ij}_k} & \mathbb{Q}[\![z_k]\!] \end{array}$$

A real DoPeGDO Example. Let $cm^{ij}_k\colon CU_i \otimes CU_j \to CU_k$ be "classical multiplication" for sl^ϵ_{2+}, and let $\mathbb{O}_i\colon \mathbb{Q}[\![y_i, b_i, a_i, x_i]\!] \to CU_i$ be the PBW ordering map.

$$\begin{array}{ccc} CU_i \otimes CU_j & \xrightarrow{cm^{ij}_k} & CU_k \\ \uparrow{\scriptstyle \mathbb{O}_{i,j}} & & \uparrow{\scriptstyle \mathbb{O}_k} \\ \mathbb{Q}[\![y_i, b_i, a_i, x_i, y_j, b_j, a_j, x_j]\!] & & \mathbb{Q}[\![y_k, b_k, a_k, x_k]\!] \end{array}$$

Claim. Let

$$\Lambda = \left(\eta_i + \frac{e^{-\alpha_i - \epsilon\beta_i}\eta_j}{1 + \epsilon\eta_j\xi_i}\right) y_k + \left(\beta_i + \beta_j + \frac{\log\left(1 + \epsilon\eta_j\xi_i\right)}{\epsilon}\right) b_k + \left(\alpha_i + \alpha_j + \log\left(1 + \epsilon\eta_j\xi_i\right)\right) a_k + \left(\frac{e^{-\alpha_j - \epsilon\beta_j}\xi_i}{1 + \epsilon\eta_j\xi_i} + \xi_j\right) x_k$$

Then $\mathbb{e}^{\eta_i y_i + \beta_i b_i + \alpha_i a_i + \xi_i x_i + \eta_j y_j + \beta_j b_j + \alpha_j a_j + \xi_j x_j} /\!\!/ \mathbb{O}_{i,j} /\!\!/ cm^{ij}_k = \mathbb{e}^\Lambda /\!\!/ \mathbb{O}_k$, and hence $\mathcal{D}(cm^{ij}_k) = \mathbb{e}^\Lambda$ and cm^{ij}_k is DoPeGDO.

Proof. We compute in a faithful 2D representation ρ of CU: (ωεβ/cm)

```
HL[ℰ_] := Style[ℰ, Background → If[TrueQ@ℰ, ■, ■]];
{ρy = (0 0; ϵ 0), ρb = (0 0; 0 -ϵ), ρa = (1 0; 0 0), ρx = (0 1; 0 0)};
HL /@ {ρa.ρx - ρx.ρa == ρx, ρa.ρy - ρy.ρa == -ρy,
  ρb.ρy - ρy.ρb == -ϵ ρy, ρb.ρx - ρx.ρb == ϵ ρx,
  ρx.ρy - ρy.ρx == ρb + ϵ ρa}
{True, True, True, True, True}
HL@Simplify@With[{𝔼 = MatrixExp},
   𝔼[ηi ρy].𝔼[βi ρb].𝔼[αi ρa].𝔼[ξi ρx].𝔼[ηj ρy].𝔼[βj ρb].
    𝔼[αj ρa].𝔼[ξj ρx] ==
   𝔼[∂yk Λ ρy].𝔼[∂bk Λ ρb].𝔼[∂ak Λ ρa].𝔼[∂xk Λ ρx]]
True
Series[Λ, {ϵ, 0, 1}]
(ak (αi + αj) + yk (ηi + e^-αi ηj) +
   bk (βi + βj + ηj ξi) + xk (e^-αj ξi + ξj)) +
 (ak ηj ξi - 1/2 bk ηj^2 ξi^2 - e^-αi yk ηj (βi + ηj ξi) -
   e^-αj xk ξi (βj + ηj ξi)) ϵ + O[ϵ]^2
```

(Shame, but this technique fails for QU).

Claim. In QU, R is DoPeGDO.
Proof. Recall that with $q = \mathbb{e}^{\hbar\epsilon}$,

$$R = \sum \hbar^{j+k} y^k b^j \otimes a^j x^k / j! [k]_q! = \mathbb{O}\left(\mathbb{e}^{\hbar b_1 a_2}\mathbb{e}_q^{\hbar y_1 x_2}\right).$$

Now expand $\mathbb{e}_q^{\hbar y_1 x_2}$ in powers of ϵ using:
Faddeev's Formula (In as much as we can tell, first appeared without proof in Faddeev [Fa], rediscovered and proven in Quesne [Qu], and again with easier proof, in Zagier [Za]). With $[n]_q := \frac{q^n - 1}{q - 1}$, with $[n]_q! := [1]_q[2]_q\cdots[n]_q$ and with $\mathbb{e}_q^x := \sum_{n\geq 0}\frac{x^n}{[n]_q!}$, we have

$$\log \mathbb{e}_q^x = \sum_{k\geq 1}\frac{(1-q)^k x^k}{k(1-q^k)} = x + \frac{(1-q)^2x^2}{2(1-q^2)} + \dots.$$

Proof. We have that $\mathbb{e}_q^x = \frac{\mathbb{e}_q^{qx} - \mathbb{e}_q^x}{qx - x}$ ("the q-derivative of $\mathbb{e}_q^x$ is itself"), and hence $\mathbb{e}_q^{qx} = (1 + (1-q)x)\mathbb{e}_q^x$, and

$$\log \mathbb{e}_q^{qx} = \log(1 + (1-q)x) + \log \mathbb{e}_q^x.$$

Writing $\log \mathbb{e}_q^x = \sum_{k\geq 1} a_k x^k$ and comparing powers of x, we get $q^k a_k = -(1-q)^k/k + a_k$, or $a_k = \frac{(1-q)^k}{k(1-q^k)}$. □

Compositions (2). Recall that with all indices i running in some set B,

$$\mathcal{F}/\!\!/\mathcal{G} = \left(\mathcal{F}|_{z_i\to\partial_{\zeta_i}}\mathcal{G}\right)_{\zeta_i=0} = \mathbb{e}^{\sum\partial_{z_i}\partial_{\zeta_i}}(\mathcal{F}\mathcal{G})\Big|_{z_i=\zeta_i=0},$$

so in general we wish to understand

$$[F\colon \mathcal{E}]_B := \mathbb{e}^{\frac12\sum_{i,j\in B}F_{ij}\partial_{z_i}\partial_{z_j}}\mathcal{E} \quad\text{and}\quad \langle F\colon \mathcal{E}\rangle_B := [F\colon \mathcal{E}]_B|_{z_B\to 0},$$

where $\mathcal{E}$ is a docile perturbed Gaussian. The following lemma allows us to restrict to the case where $\mathcal{E}$ has no B-B quadratic part:

Lemma 1. With convergences left to the reader,

$$\left\langle F\colon \mathcal{E}\mathbb{e}^{\frac12\sum_{i,j\in B}G_{ij}z_iz_j}\right\rangle_B = \det(1 - GF)^{-1/2}\left\langle F(1 - GF)^{-1}\colon \mathcal{E}\right\rangle_B.$$

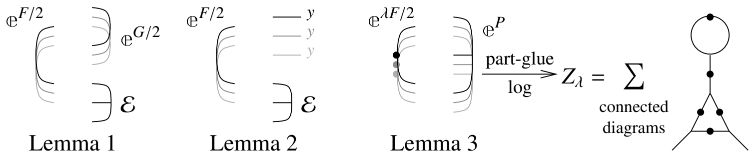
The next lemma dispatches the case where $\mathcal{E}$ has a B-linear part:

Lemma 2. $\left\langle F\colon \mathcal{E}\mathbb{e}^{\sum_{i\in B}y_iz_i}\right\rangle_B = \mathbb{e}^{\frac12\sum_{i,j\in B}F_{ij}y_iy_j}\left\langle F\colon \mathcal{E}|_{z_B\to z_B + Fy_B}\right\rangle_B.$

Finally, we deal with the docile perturbation case:

Lemma 3. With an extra variable λ, $Z_\lambda := \log[\lambda F : \mathbb{e}^P]_B$ satisfies and is determined by the following PDE / IVP:

$$Z_0 = P \quad\text{and}\quad \partial_\lambda Z_\lambda = \frac{1}{2}\sum_{i,j\in B} F_{ij}\left(\partial_{z_i}\partial_{z_j}Z_\lambda + (\partial_{z_i}Z_\lambda)(\partial_{z_j}Z_\lambda)\right).$$



Warning. Some implementation details match earlier versions of the theory.

The "Speedy" Engine

ωεβ/engine

Internal Utilities

Canonical Form:

```
CCF[ℰ_] :=
  PP_CCF@ExpandDenominator@
    ExpandNumerator@PP_Together@Together[PP_Exp[
        Expand[ℰ] //. e^x_ e^y_ :→ e^(x+y) /. e^x_ :→ e^CCF[x]]];
CF[ℰ_List] := CF /@ ℰ;
CF[sd_SeriesData] := MapAt[CF, sd, 3];
CF[ℰ_] := PP_CF@Module[
    {vs = Cases[ℰ, (y | b | t | a | x | η | β | τ | α | ξ)_, ∞] ⋃
        {y, b, t, a, x, η, β, τ, α, ξ}},
    Total[CoefficientRules[Expand[ℰ], vs] /.
      (ps_ → c_) :→ CCF[c] (Times @@ vs^ps)]
   ];
CF[ℰ_𝔼] := CF /@ ℰ;
CF[𝔼_sp___[ℰs___]] := CF /@ 𝔼_sp[ℰs];
```

The Kronecker δ:

```
Kδ /: Kδ_{i_,j_} := If[i === j, 1, 0];
```

Equality, multiplication, and degree-adjustment of perturbed Gaussians; $\mathbb{E}[L, Q, P]$ stands for $\mathbb{e}^{L+Q} P$:

```
𝔼 /: 𝔼[L1_, Q1_, P1_] ≡ 𝔼[L2_, Q2_, P2_] :=
  CF[L1 == L2] ∧ CF[Q1 == Q2] ∧ CF[Normal[P1 - P2] == 0];
𝔼 /: 𝔼[L1_, Q1_, P1_] × 𝔼[L2_, Q2_, P2_] :=
  𝔼[L1 + L2, Q1 + Q2, P1 * P2];
𝔼[L_, Q_, P_]_$k_ := 𝔼[L, Q, Series[Normal@P, {ϵ, 0, $k}]];
```

Zip and Bind

Variables and their duals:

```
{t*, b*, y*, a*, x*, z*} = {τ, β, η, α, ξ, ζ};
{τ*, β*, η*, α*, ξ*, ζ*} = {t, b, y, a, x, z};
(u_i_)* := (u*)_i;
```

```
U2l = {B_i_^p_. → e^(-p ħ γ b_i), B^p_. → e^(-p ħ γ b), T_i_^p_. → e^(p ħ t_i),
    T^p_. → e^(p ħ t), 𝒜_i_^p_. → e^(p γ α_i), 𝒜^p_. → e^(p γ α)};
l2U = {e^(c_. b_i_ + d_.) :→ B_i^(-c/(ħ γ)) e^d, e^(c_. b + d_.) :→ B^(-c/(ħ γ)) e^d,
    e^(c_. t_i_ + d_.) :→ T_i^(c/ħ) e^d, e^(c_. t + d_.) :→ T^(c/ħ) e^d,
    e^(c_. α_i_ + d_.) :→ 𝒜_i^(c/γ) e^d, e^(c_. α + d_.) :→ 𝒜^(c/γ) e^d,
    e^ℰ_ :→ e^(Expand@ℰ)};
```

```
D_b[f_] := ∂_b f - ħ γ B ∂_B f;  D_b_i_[f_] := ∂_b_i f - ħ γ B_i ∂_B_i f;
D_t[f_] := ∂_t f + ħ T ∂_T f;  D_t_i_[f_] := ∂_t_i f + ħ T_i ∂_T_i f;
D_α[f_] := ∂_α f + γ 𝒜 ∂_𝒜 f;  D_α_i_[f_] := ∂_α_i f + γ 𝒜_i ∂_𝒜_i f;
D_v_[f_] := ∂_v f;  D_{v_,0}[f_] := f;  D_{}[f_] := f;
D_{v_,n_Integer}[f_] := D_v[D_{v,n-1}[f]];
D_{l_List,ls___}[f_] := D_{ls}[D_l[f]];
```

Finite Zips:

```
collect[sd_SeriesData, ζ_] :=
  MapAt[collect[#, ζ] &, sd, 3];
collect[ℰ_, ζ_] := PP_Collect@Collect[ℰ, ζ];
Zip_{}[P_] := P;
Zip_ζs_[Ps_List] := Zip_ζs /@ Ps;
Zip_{ζ_,ζs___}[P_] := PP_Zip[
  (collect[P // Zip_{ζs}, ζ] /. f_. ζ^d_. :→ (D_{ζ*,d}[f])) /.
    ζ* → 0 /. ((ζ* /. {b → B, t → T, α → 𝒜}) → 1)]
```

QZip implements the "Q-level zips" on $\mathbb{E}(L, Q, P) = \mathbb{e}^{L+Q} P(\epsilon)$. Such zips regard the L variables as scalars.

```
QZip_ζs_List@𝔼[L_, Q_, P_] :=
  PP_QZip@Module[{ζ, z, zs, c, ys, ηs, qt, zrule, ζrule, out},
    zs = Table[ζ*, {ζ, ζs}];
    c = CF[Q /. Alternatives @@ (ζs ⋃ zs) → 0];
    ys = CF@Table[∂_ζ (Q /. Alternatives @@ zs → 0),
        {ζ, ζs}];
    ηs = CF@Table[∂_z (Q /. Alternatives @@ ζs → 0), {z, zs}];
    qt = CF@Inverse@Table[Kδ_{z,ζ*} - ∂_{z,ζ}Q, {ζ, ζs}, {z, zs}];
    zrule = Thread[zs → CF[qt.(zs + ys)]];
    ζrule = Thread[ζs → ζs + ηs.qt];
    CF /@ 𝔼[L, c + ηs.qt.ys,
      Det[qt] Zip_ζs[P /. (zrule ⋃ ζrule)]] ];
```

Upper to lower and lower to Upper:

```
U2l = {B_i_^p_. → e^(-p ħ γ b_i), B^p_. → e^(-p ħ γ b), T_i_^p_. → e^(p ħ t_i),
    T^p_. → e^(p ħ t), 𝒜_i_^p_. → e^(p γ α_i), 𝒜^p_. → e^(p γ α)};
l2U = {e^(c_. b_i_ + d_.) :→ B_i^(-c/(ħ γ)) e^d, e^(c_. b + d_.) :→ B^(-c/(ħ γ)) e^d,
    e^(c_. t_i_ + d_.) :→ T_i^(c/ħ) e^d, e^(c_. t + d_.) :→ T^(c/ħ) e^d,
    e^(c_. α_i_ + d_.) :→ 𝒜_i^(c/γ) e^d, e^(c_. α + d_.) :→ 𝒜^(c/γ) e^d,
    e^ℰ_ :→ e^(Expand@ℰ)};
```

LZip implements the "L-level zips" on $\mathbb{E}(L, Q, P) = P\mathbb{e}^{L+Q}$. Such zips regard all of $P\mathbb{e}^Q$ as a single"P". Here the z's are b and α and the ζ's are β and a.

```
LZip_ζs_List@𝔼[L_, Q_, P_] :=
  PP_LZip@Module[{ζ, z, zs, Zs, c, ys, ηs, lt, zrule,
     Zrule, ζrule, Q1, EEQ, EQ},
    zs = Table[ζ*, {ζ, ζs}];
    Zs = zs /. {b → B, t → T, α → 𝒜};
    c = L /. Alternatives @@ (ζs ⋃ zs) → 0;
    ys = Table[∂_ζ (L /. Alternatives @@ zs → 0), {ζ, ζs}];
    ηs = Table[∂_z (L /. Alternatives @@ ζs → 0), {z, zs}];
    lt = Inverse@Table[Kδ_{z,ζ*} - ∂_{z,ζ}L, {ζ, ζs}, {z, zs}];
    zrule = Thread[zs → lt.(zs + ys)];
    Zrule = Join[zrule,
      zrule /.
       r_Rule :→ ((U = r[[1]] /. {b → B, t → T, α → 𝒜}) →
          (U /. U2l /. r //. l2U))];
    ζrule = Thread[ζs → ζs + ηs.lt];
    Q1 = Q /. (Zrule ⋃ ζrule);
    EEQ[ps___] :=
     EEQ[ps] =
      PP_"EEQ"@(CF[e^-Q1 D_Thread[{zs,{ps}}][e^Q1]] /.
          {Alternatives @@ zs → 0, Alternatives @@ Zs → 1});
    CF@𝔼[c + ηs.lt.ys,
      Q1 /. {Alternatives @@ zs → 0, Alternatives @@ Zs → 1},
      Det[lt]
       (Zip_ζs[(EQ @@ zs) (P /. (Zrule ⋃ ζrule))] /.
          Derivative[ps___][EQ][___] :→ EEQ[ps] /.
         _EQ → 1) ]];
```

```
B{}[L_, R_] := L R;
B{is__}[L_𝔼, R_𝔼] := PP_B@Module[{n},
    Times[
      L /. Table[(v : b | B | t | T | a | x | y)_i → v_{n@i},
        {i, {is}}],
      R /. Table[(v : β | τ | α | 𝒜 | ξ | η)_i → v_{n@i}, {i, {is}}]
    ] // LZip_{Join@@Table[{β_{n@i}, τ_{n@i}, a_{n@i}}, {i, {is}}]} //
   QZip_{Join@@Table[{ξ_{n@i}, y_{n@i}}, {i, {is}}]} ];
B_{is___}[L_, R_] := B_{{is}}[L, R];
```

𝔼 morphisms with domain and range.

```
B_{is_List}[𝔼_{d1_→r1_}[L1_, Q1_, P1_], 𝔼_{d2_→r2_}[L2_, Q2_, P2_]] :=
  𝔼_{(d1⋃Complement[d2,is])→(r2⋃Complement[r1,is])} @@
   B_{is}[𝔼[L1, Q1, P1], 𝔼[L2, Q2, P2]];
𝔼_{d1_→r1_}[L1_, Q1_, P1_] // 𝔼_{d2_→r2_}[L2_, Q2_, P2_] :=
  B_{r1⋂d2}[𝔼_{d1→r1}[L1, Q1, P1], 𝔼_{d2→r2}[L2, Q2, P2]];
𝔼_{d1_→r1_}[L1_, Q1_, P1_] ≡ 𝔼_{d2_→r2_}[L2_, Q2_, P2_] ^:=
  (d1 == d2) ∧ (r1 == r2) ∧ (𝔼[L1, Q1, P1] ≡ 𝔼[L2, Q2, P2]);
𝔼_{d1_→r1_}[L1_, Q1_, P1_] 𝔼_{d2_→r2_}[L2_, Q2_, P2_] ^:=
  𝔼_{(d1⋃d2)→(r1⋃r2)} @@ (𝔼[L1, Q1, P1] × 𝔼[L2, Q2, P2]);
𝔼_{d_→r_}[L_, Q_, P_]_{$k_} := 𝔼_{d→r} @@ 𝔼[L, Q, P]_{$k};
𝔼_[ℰ___][i_] := {ℰ}[[i]];
```

“Define” Code

Define[lhs = rhs, ...] defines the lhs to be rhs, except that rhs is computed only once for each value of \$k. Fancy Mathematica not for the faint of heart. Most readers should ignore.

```
SetAttributes[Define, HoldAll];
Define[def_, defs__] := (Define[def]; Define[defs];);
Define[op_{is__} = ℰ_] :=
 Module[{SD, ii, jj, kk, isp, nis, nisp, sis},
  Block[{i, j, k},
   ReleaseHold[Hold[
      SD[op_{nisp,$k_Integer}, PP_Boot@Block[{i, j, k}, op_{isp,$k} = ℰ;
         op_{nis,$k}]];
      SD[op_{isp}, op_{{is},$k}]; SD[op_{sis__}, op_{{sis}}];
     ] /. {SD → SetDelayed,
      isp → {is} /. {i → i_, j → j_, k → k_},
      nis → {is} /. {i → ii, j → jj, k → kk},
      nisp → {is} /. {i → ii_, j → jj_, k → kk_}
     }] ]]
```

The Objects

ωεβ/objects

Symmetric Algebra Objects

```
sm_{i_,j_→k_} := 𝔼_{{i,j}→{k}}[b_k (β_i + β_j) + t_k (τ_i + τ_j) + a_k (α_i + α_j),
   y_k (η_i + η_j) + x_k (ξ_i + ξ_j), 1];
sϒ_{i_→j_,k_,l_,m_} := 𝔼_{{i}→{j,k,l,m}}[β_i b_k + τ_i t_k + α_i a_l,
   η_i y_j + ξ_i x_m, 1];
sΔ_{i_→j_,k_} := 𝔼_{{i}→{j,k}}[β_i (b_j + b_k) + τ_i (t_j + t_k) + α_i (a_j + a_k),
   η_i (y_j + y_k) + ξ_i (x_j + x_k), 1];
sS_{i_} := 𝔼_{{i}→{i}}[-β_i b_i - τ_i t_i - α_i a_i, -η_i y_i - ξ_i x_i, 1];
sϵ_{i_} := 𝔼_{{}→{i}}[0, 0, 1]; sη_{i_} := 𝔼_{{i}→{}}[0, 0, 1];
sσ_{i_→j_} := 𝔼_{{i}→{j,k}}[β_i b_j + τ_i t_j + α_i a_j, η_i y_j + ξ_i x_j, 1];
```

Booting Up QU

```
Define[aσ_{i→j} = 𝔼_{{i}→{j}}[a_j α_i, x_j ξ_i, 1],
 bσ_{i→j} = 𝔼_{{i}→{j}}[b_j β_i, y_j η_i, 1]]
```

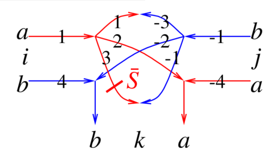
```
Define[am_{i,j→k} = 𝔼_{{i,j}→{k}}[(α_i + α_j) a_k, (𝒜_j^{-1} ξ_i + ξ_j) x_k, 1]_{$k},
 bm_{i,j→k} = 𝔼_{{i,j}→{k}}[(β_i + β_j) b_k, (η_i + η_j) y_k, e^{(e^{-ϵ β_i}-1) η_j y_k}]_{$k}]
```

```
Define[
 R_{i,j} =
  𝔼_{{}→{i,j}}[ℏ a_j b_i, ℏ x_j y_i, e^(Σ_{k=2}^{$k+1} (1 - e^{γ ϵ ℏ})^k (ℏ y_i x_j)^k / (k (1 - e^{k γ ϵ ℏ})))]_{$k},
 R̄_{i,j} = CF@𝔼_{{}→{i,j}}[-ℏ a_j b_i, -ℏ x_j y_i / B_i,
   1 + If[$k == 0, 0, (R̄_{{i,j},$k-1})_{$k}[3] -
      (((R̄_{{i,j},0})_{$k} R_{1,2} (R̄_{{3,4},$k-1})_{$k}) // (bm_{i,1→i} am_{j,2→j}) //
        (bm_{i,3→i} am_{j,4→j}))[3]]],
 P_{i,j} = 𝔼_{{i,j}→{}}[β_i α_j / ℏ, η_i ξ_j / ℏ,
   1 + If[$k == 0, 0, (P_{{i,j},$k-1})_{$k}[3] -
      (R_{1,2} // ((P_{{1,j},0})_{$k} (P_{{i,2},$k-1})_{$k}))[3]]]]
```

```
Define[aS_j = R̄_{i,j} ~ B_i ~ P_{i,j},
 aS̄_i = 𝔼_{{i}→{i}}[-a_i α_i, -x_i 𝒜_i ξ_i,
   1 + If[$k == 0, 0, (aS̄_{{i},$k-1})_{$k}[3] -
      ((aS̄_{{i},0})_{$k} ~ B_i ~ aS_i ~ B_i ~ (aS̄_{{i},$k-1})_{$k})[3]]]]
```

```
Define[bS_i = R_{i,1} ~ B_1 ~ aS_1 ~ B_1 ~ P_{i,1},
 bS̄_i = R_{i,1} ~ B_1 ~ aS̄_1 ~ B_1 ~ P_{i,1},
 aΔ_{i→j,k} = (R_{1,j} R_{2,k}) // bm_{1,2→3} // P_{3,i},
 bΔ_{i→j,k} = (R_{j,1} R_{k,2}) // am_{1,2→3} // P_{i,3}]
```

The Drinfel’d double:




```
Define[
 dm_{i,j→k} =
  ((sϒ_{i→4,4,1,1} // aΔ_{1→1,2} // aΔ_{2→2,3} // aS̄_3)
    (sϒ_{j→-1,-1,-4,-4} // bΔ_{-1→-1,-2} // bΔ_{-2→-2,-3})) //
   (P_{-1,3} P_{-3,1} am_{2,-4→k} bm_{4,-2→k})]
```

```
Define[dσ_{i→j} = aσ_{i→j} bσ_{i→j},
 dϵ_i = sϵ_i, dη_i = sη_i,
 dS_i = sϒ_{i→1,1,2,2} // (bS̄_1 aS_2) // dm_{2,1→i},
 dS̄_i = sϒ_{i→1,1,2,2} // (bS_1 aS̄_2) // dm_{2,1→i},
 dΔ_{i→j,k} = (bΔ_{i→3,1} aΔ_{i→2,4}) // (dm_{3,4→k} dm_{1,2→j})]
```

```
Define[C_i = 𝔼_{{}→{i}}[0, 0, B_i^{1/2} e^{-ℏ ϵ a_i/2}]_{$k},
 C̄_i = 𝔼_{{}→{i}}[0, 0, B_i^{-1/2} e^{ℏ ϵ a_i/2}]_{$k},
 Kink_i = (R_{1,3} C̄_2) // dm_{1,2→1} // dm_{1,3→i},
 K̄ink_i = (R̄_{1,3} C_2) // dm_{1,2→1} // dm_{1,3→i}]
```

Note. $t == \epsilon a - \gamma b$ and $b == -t/\gamma + \epsilon a/\gamma$.

```
Define[
 b2t_i = 𝔼_{{i}→{i}}[α_i a_i - β_i t_i / γ, ξ_i x_i + η_i y_i, e^{ϵ β_i a_i/γ}]_{$k},
 t2b_i = 𝔼_{{i}→{i}}[α_i a_i - τ_i γ b_i, ξ_i x_i + η_i y_i, e^{ϵ τ_i a_i}]_{$k}]
```

The CU Definitions

```
Define[cm_{i,j→k} = CF@𝔼_{{i,j}→{k}}[
    a_k (α_i + α_j) + b_k (β_i + β_j),
    y_k (η_i + η_j/𝒜_i) + γ b_k η_j ξ_i + x_k (ξ_i/𝒜_j + ξ_j),
    e^{y_k η_j (e^{-ϵ β_i}/(𝒜_i + γ ϵ 𝒜_i η_j ξ_i) - 1/𝒜_i) + ξ_i (x_k (e^{-ϵ β_j}/(𝒜_j + γ ϵ 𝒜_j η_j ξ_i) - 1/𝒜_j) - γ b_k η_j)}
     (1 + γ ϵ η_j ξ_i)^{a_k/γ + b_k/ϵ}]_{$k}]
```

```
Define[cσ_{i→j} = sσ_{i,j} /. τ_i → 0,
  cϵ_i = sϵ_i, cη_i = sη_i,
  cΔ_{i→j,k} = sΔ_{i→j,k},
  cS_i = sS_i // sϒ_{i→1,2,3,4} // cm_{4,3→i} // cm_{i,2→i} // cm_{i,1→i}];
```

The Knot Tensors

```
Define[kR_{i,j} = R_{i,j} // (b2t_i b2t_j) /. t_{i|j} → t,
 k̄R_{i,j} = R̄_{i,j} // (b2t_i b2t_j) /. {t_{i|j} → t, T_{i|j} → T},
 km_{i,j→k} = (t2b_i t2b_j) // dm_{i,j→k} //
   b2t_k /. {t_k → t, T_k → T, τ_{i|j} → 0},
 kC_i = C_i // b2t_i /. T_i → T,
 k̄C_i = C̄_i // b2t_i /. T_i → T,
 kKink_i = Kink_i // b2t_i /. {t_i → t, T_i → T},
 k̄Kink_i = K̄ink_i // b2t_i /. {t_i → t, T_i → T}]
```

A Quantum Algebra Example. ωεβ/qa

Proto-Theorem†0 (with Jesse Frohlich and Roland van der Veen). Let H be a finite dimensional Hopf algebra and let $U = H^{*cop} \otimes H$ be its Drinfel'd double, with R-matrix $R \in H^* \otimes H \subset U \otimes U$. Write $R^{\dagger 1} = \sum \rho_a \otimes r_a$, and let $\langle \cdot \mid \cdot \rangle \colon H^* \otimes H \to \mathbb{F}$ be the duality pairing. Then the functional $\int \in U^*$ defined by

$$\int \phi \otimes x := \sum \langle \phi \rho_a^{\dagger 2} \mid x r_a^{\dagger 3} \rangle$$

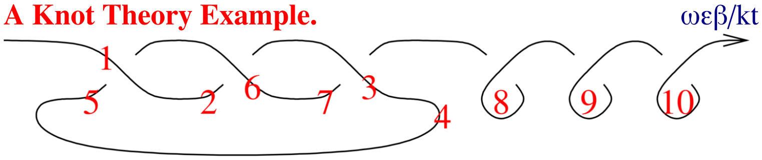
is a right†4 integral in U^*. (Meaning $\Delta^i_{jk} /\!\!/ \int_j = \int_i /\!\!/ \epsilon_k$ in $\mathrm{Hom}(U^{\otimes\{i\}} \to U^{\otimes\{k\}})$).

†0 A "proto-theorem" is something that will become a theorem once you figure out the correct statement. †1 Or did we want it to be $R /\!\!/ S_1^2$? Or $R /\!\!/ S_2^2$? †2 Or is it $\rho_a \phi$? †3 Or is it $r_a x$? †4 Or maybe "left"?

```
PP_ := Identity; $k = 1; ℏ = γ = 1;
inp = 𝔼_{{}→{1}}[3 a_1 b_1, 5 x_1 y_1, 1] // dm_{i,1→i};
Table[
  HL@TrueQ[
    (inp // (sϒ_{i→1,1,2,2} RR) // BM // AM // P_{1,2}) dϵ_j ≡
     (inp // ΔΔ // (sϒ_{i→1,1,2,2} RR) // BM // AM // P_{1,2})],
  {ΔΔ, {dΔ_{i→i,j}, dΔ_{i→j,i}}}, {AM, {dm_{2,4→2}, dm_{4,2→2}}},
  {BM, {dm_{1,3→1}, dm_{3,1→1}}},
  {RR, {R_{3,4}, R_{3,4} // dS_3 // dS_3, R_{3,4} // dS_4 // dS_4}}
 ] // MatrixForm
```

```
( ( False False False )  ( False False True  ) )
( ( False False False )  ( False False False ) )
( ( False False False )  ( False False False ) )
( ( False False True  )  ( False False False ) )
```

A Knot Theory Example. ωεβ/kt




```
$k = 2;
Simplify[
  R_{1,5} R_{6,2} R_{3,7} C̄_4 K̄ink_8 K̄ink_9 K̄ink_10 // dm_{1,2→1} // dm_{1,3→1} //
      dm_{1,4→1} // dm_{1,5→1} // dm_{1,6→1} // dm_{1,7→1} // dm_{1,8→1} //
    dm_{1,9→1} // dm_{1,10→1}] /. v_1 :→ v
```

$$\mathbb{E}_{\{\}\to\{1\}}\Big[0,\,0,\,\frac{B}{1-B+B^2}+\frac{B\left(-B+2B^2+2B^4+a\left(-1+B-B^3+B^4\right)-2xy-B^3(3+2xy)\right)\epsilon}{\left(1-B+B^2\right)^3}+$$
$$\frac{1}{2\left(1-B+B^2\right)^5}\Big(B\Big(4B^8+a^2\left(1-B+B^2\right)^2\left(1+B-6B^2+B^3+B^4\right)+6B^5x^2y^2+$$
$$2xy(-2+3xy)-B^7(11+4xy)-2B^2\left(1+6x^2y^2\right)-$$
$$2B^4\left(1-2xy+6x^2y^2\right)+B\left(1+8xy+6x^2y^2\right)+$$
$$B^6\left(6+8xy+6x^2y^2\right)+B^3\left(4+4xy+30x^2y^2\right)+$$
$$2a\left(1-B+B^2\right)\left(2B^6+2xy+8B^3(1+xy)-5B^2(1+2xy)-\right.$$
$$\left.2B^5(1+2xy)-B^4(7+2xy)+B(2+4xy)\right)\Big)\Big)\epsilon^2+O[\epsilon]^3\Big]$$

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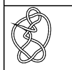
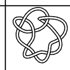
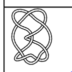
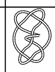
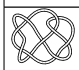
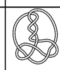
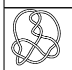
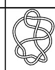
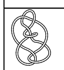
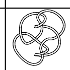
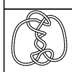
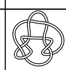
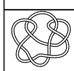
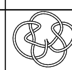
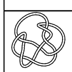
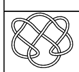
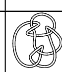
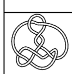
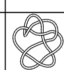
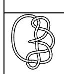
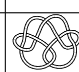
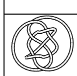
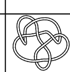
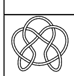
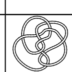
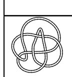
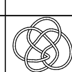
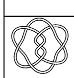
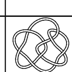
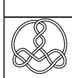
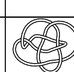
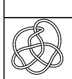
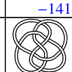
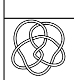
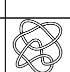
[Za] D. Zagier, *The Dilogarithm Function,* in Cartier, Moussa, Julia, and Vanhove (eds) *Frontiers in Number Theory, Physics, and Geometry II.* Springer, Berlin, Heidelberg, and ωεβ/Za.

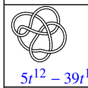
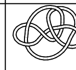
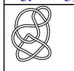
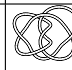
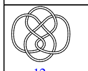
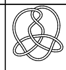
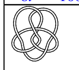
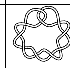
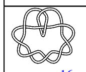
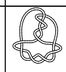
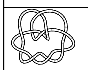
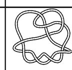
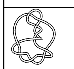
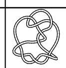
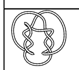
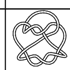
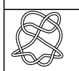
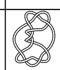
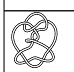
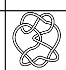
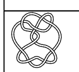
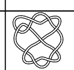
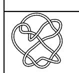
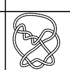
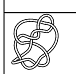
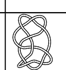
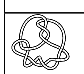
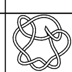
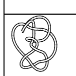
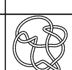
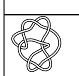
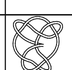
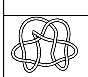
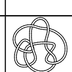
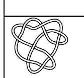
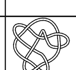
KiW 43 Abstract (ωεβ/kiw). Whether or not you like the formulas on this page, they describe the strongest truly computable knot invariant we know.

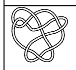
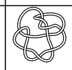
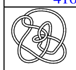
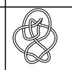
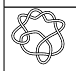
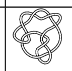
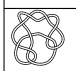
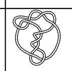
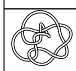
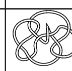
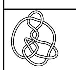
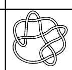
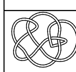
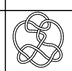
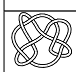
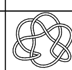
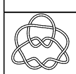
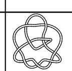
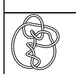
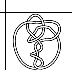
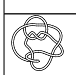
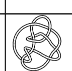
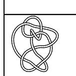
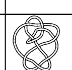
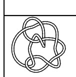
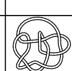
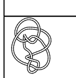
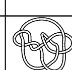
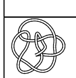
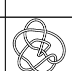
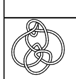
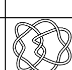
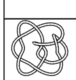
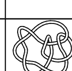
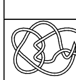
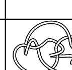
Observations. • Separates the Rolfsen table; does better than Khovanov plus HOMFLY-PT on knots with up to 12 crossings (not tested beyond). • The degrees are bounded by the genus! • ρ_1 vanishes for amphichiral knots. • Has a chance of detecting non-ribbonness (ωεβ/ind)!

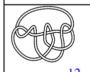
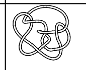
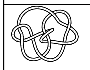
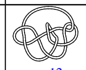
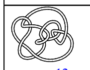
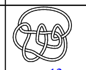
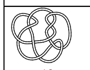
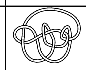
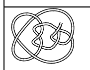
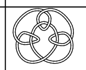
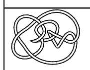
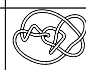
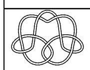
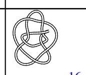
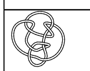
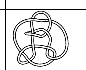
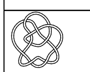
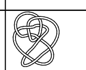
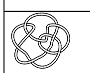
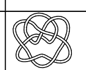
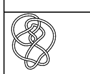
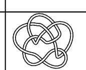
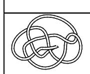
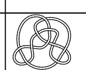
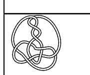
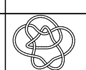
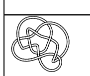
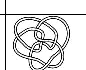
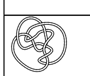
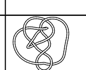
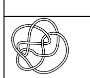
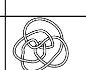
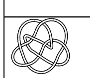
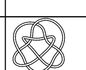
knot n_k^t	Alexander's ω^+	genus / ribbon	$(\rho_1')^+$	unknotting # / amphi?	$(\rho_2')^+$
0_1^a	1	0 / ✓	0	0 / ✓	0
3_1^a	$t-1$	1 / ✗	t	1 / ✗	$3t^3-12t^2+26t-38$
4_1^a	$3-t$	1 / ✗	0	1 / ✓	$t^4-3t^3-15t^2+74t-110$
5_1^a	t^2-t+1	2 / ✗	$2t^3+3t$	2 / ✗	$5t^7-20t^6+55t^5-120t^4+217t^3-338t^2+450t-510$
5_2^a	$2t-3$	1 / ✗	$5t-4$	1 / ✗	$-10t^4+120t^3-487t^2+1054t-1362$
6_1^a	$5-2t$	1 / ✓	$t-4$	1 / ✗	$14t^4-16t^3-293t^2+1098t-1598$
6_2^a	$-t^2+3t-3$	2 / ✗	t^3-4t^2+4t-4	1 / ✗	$3t^8-21t^7+49t^6+15t^5-433t^4+1543t^3-3431t^2+5482t-6410$
6_3^a	t^2-3t+5	2 / ✗	0	1 / ✓	$4t^8-33t^7+121t^6-203t^5-111t^4+1499t^3-4210t^2+7186t-8510$
7_1^a	t^3-t^2+t-1	3 / ✗	$3t^5+5t^3+6t$	3 / ✗	$7t^{11}-28t^{10}+77t^9-168t^8+322t^7-560t^6+891t^5-1310t^4+1777t^3-2238t^2+2604t-2772$
7_2^a	$3t-5$	1 / ✗	$14t-16$	1 / ✗	$-129t^4+1177t^3-4421t^2+9226t-11718$
7_3^a	$2t^2-3t+3$	2 / ✗	$-9t^3+8t^2-16t+12$	2 / ✗	$-18t^8+208t^7-917t^6+2666t^5-6049t^4+11283t^3-17671t^2+23356t-25736$
7_4^a	$4t-7$	1 / ✗	$32-24t$	2 / ✗	$-352t^4+3616t^3-14378t^2+30700t-39188$
7_5^a	$2t^2-4t+5$	2 / ✗	$9t^3-16t^2+29t-28$	2 / ✗	$-18t^8+264t^7-1548t^6+5680t^5-15107t^4+31152t^3-51476t^2+69252t-76414$
7_6^a	$-t^2+5t-7$	2 / ✗	$t^3-8t^2+19t-20$	1 / ✗	$3t^8-35t^7+128t^6+105t^5-2610t^4+11225t^3-28031t^2+47186t-55946$
7_7^a	t^2-5t+9	2 / ✗	$8-3t$	1 / ✗	$4t^8-55t^7+310t^6-805t^5+86t^4+6349t^3-22686t^2+43610t-53622$
8_1^a	$7-3t$	1 / ✗	$5t-16$	1 / ✗	$42t^4+215t^3-2542t^2+7562t-10542$
8_2^a	$-t^3+3t^2-3t+3$	3 / ✗	$2t^5-8t^4+10t^3-12t^2+13t-12$	2 / ✗	$5t^{12}-39t^{11}+119t^{10}-139t^9-249t^8+1660t^7-4959t^6+11131t^5-20813t^4+33595t^3-47521t^2+58988t-63556$
8_3^a	$9-4t$	1 / ✗	0	2 / ✓	$224t^4-224t^3-3910t^2+14100t-20364$
8_4^a	$-2t^2+5t-5$	2 / ✗	$3t^3-8t^2+6t-4$	2 / ✗	$54t^8-344t^7+865t^6-650t^5-2723t^4+12243t^3-28461t^2+45792t-53540$
8_5^a	$-t^3+3t^2-4t+5$	3 / ✗	$-2t^5+8t^4-13t^3+20t^2-22t+24$	2 / ✗	$5t^{12}-39t^{11}+128t^{10}-182t^9-274t^8+2476t^7-8642t^6+21517t^5-42924t^4+71719t^3-102448t^2+126480t-135628$
8_6^a	$-2t^2+6t-7$	2 / ✗	$5t^3-20t^2+28t-32$	2 / ✗	$38t^8-216t^7+112t^6+2880t^5-14787t^4+42444t^3-85415t^2+128406t-146916$
8_7^a	t^3-3t^2+5t-5	3 / ✗	$-t^5+4t^4-10t^3+12t^2-13t+12$	1 / ✗	$8t^{12}-75t^{11}+343t^{10}-979t^9+1821t^8-1782t^7-1623t^6+12083t^5-33001t^4+64599t^3-101194t^2+131404t-143216$
8_8^a	$2t^2-6t+9$	2 / ✓	$-t^3+4t^2-12t+16$	2 / ✗	$62t^8-504t^7+1736t^6-2408t^5-3717t^4+26492t^3-68493t^2+113418t-133180$
8_9^a	$-t^3+3t^2-5t+7$	3 / ✓	0	1 / ✓	$9t^{12}-87t^{11}+417t^{10}-1305t^9+2858t^8-4134t^7+2114t^6+8285t^5-31925t^4+69235t^3-112773t^2+148508t-162396$
8_{10}^a	t^3-3t^2+6t-7	3 / ✗	$-t^5+4t^4-11t^3+16t^2-21t+20$	2 / ✗	$8t^{12}-75t^{11}+362t^{10}-1122t^9+2306t^8-2540t^7-2198t^6+18817t^5-54380t^4+110103t^3-175694t^2+230080t-251346$
8_{11}^a	$-2t^2+7t-9$	2 / ✗	$5t^3-24t^2+39t-44$	1 / ✗	$38t^8-264t^7+301t^6+3514t^5-21716t^4+68785t^3-146898t^2+227828t-263172$
8_{12}^a	$t^2-7t+13$	2 / ✗	0	2 / ✓	$4t^8-77t^7+583t^6-1991t^5+987t^4+17311t^3-71802t^2+147914t-185846$
8_{13}^a	$2t^2-7t+11$	2 / ✗	$-t^3+4t^2-14t+20$	1 / ✗	$62t^8-592t^7+2351t^6-3918t^5-4235t^4+40079t^3-111533t^2+191500t-227432$
8_{14}^a	$-2t^2+8t-11$	2 / ✗	$5t^3-28t^2+57t-68$	1 / ✗	$38t^8-312t^7+444t^6+5096t^5-34777t^4+116368t^3-255750t^2+401632t-465478$
8_{15}^a	$3t^2-8t+11$	2 / ✗	$21t^3-64t^2+120t-140$	2 / ✗	$-123t^8+2128t^7-15241t^6+66120t^5-199999t^4+451912t^3-792414t^2+1101720t-1228222$
8_{16}^a	t^3-4t^2+8t-9	3 / ✗	$t^5-6t^4+17t^3-28t^2+35t-36$	2 / ✗	$8t^{12}-100t^{11}+598t^{10}-2205t^9+5292t^8-7164t^7-2380t^6+43100t^5-137314t^4+291750t^3-478742t^2+636488t-698666$
8_{17}^a	$-t^3+4t^2-8t+11$	3 / ✗	0	1 / ✓	$9t^{12}-116t^{11}+722t^{10}-2843t^9+7656t^8-13668t^7+11117t^6+21968t^5-113086t^4+273778t^3-475622t^2+649064t-717954$
8_{18}^a	$-t^3+5t^2-10t+13$	3 / ✗	0	2 / ✓	$9t^{12}-145t^{11}+1075t^{10}-4842t^9+14504t^8-28560t^7+27957t^6+35195t^5-225204t^4+573797t^3-1021641t^2+1411484t-1567262$
8_{19}^n	t^3-t^2+1	3 / ✗	$-3t^5-4t^2-3t$	3 / ✗	$7t^{11}-19t^{10}+6t^9+48t^8-52t^7-91t^6+211t^5+16t^4-431t^3+289t^2+536t-1060$
8_{20}^n	t^2-2t+3	2 / ✓	$4t-4$	1 / ✗	$4t^8-22t^7+66t^6-124t^5+52t^4+478t^3-1652t^2+3014t-3640$
8_{21}^n	$-t^2+4t-5$	2 / ✗	$t^3-8t^2+16t-20$	1 / ✗	$3t^8-28t^7+49t^6+352t^5-2489t^4+8164t^3-17530t^2+27092t-31226$

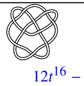
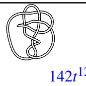
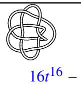
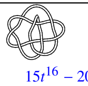
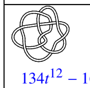
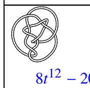
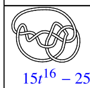
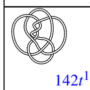
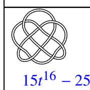
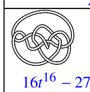
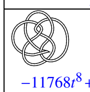
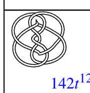
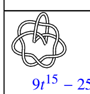
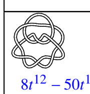
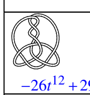
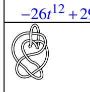
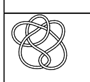
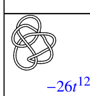
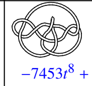
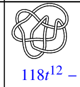
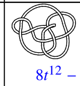
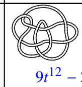
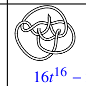
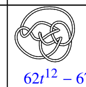
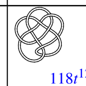
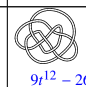
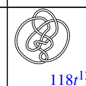
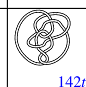
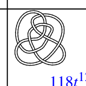
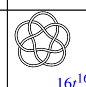
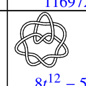
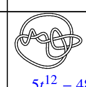
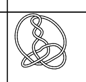
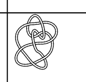
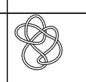
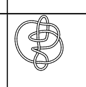
knot n_k^t	Alexander's ω^+	genus / ribbon	$(\rho_1')^+$	unknotting # / amphi?	$(\rho_2')^+$
9_1^a	$t^4-t^3+t^2-t+1$	4 / ✗	$4t^7+7t^5+9t^3+10t$	4 / ✗	$9t^{15}-36t^{14}+99t^{13}-216t^{12}+414t^{11}-720t^{10}+1170t^9-1800t^8+2630t^7-3662t^6+4853t^5-6142t^4+7423t^3-8572t^2+9420t-9780$
9_2^a	$4t-7$	1 / ✗	$30t-40$	1 / ✗	$-728t^4+6088t^3-21946t^2+44788t-56420$
9_3^a	$2t^3-3t^2+3t-3$	3 / ✗	$-13t^5+12t^4-25t^3+20t^2-32t+24$	3 / ✗	$-26t^{12}+296t^{11}-1311t^{10}+3838t^9-8867t^8+17613t^7-31407t^6+51061t^5-76085t^4+104297t^3-131779t^2+152840t-160976$
9_4^a	$3t^2-5t+5$	2 / ✗	$23t^3-28t^2+46t-44$	2 / ✗	$-219t^8+1999t^7-8389t^6+23799t^5-52835t^4+96723t^3-149121t^2+194698t-213338$
9_5^a	$6t-11$	1 / ✗	$100-65t$	2 / ✗	$-3234t^4+29792t^3-113241t^2+236818t-300294$
9_6^a	$2t^3-4t^2+5t-5$	3 / ✗	$13t^5-24t^4+45t^3-52t^2+68t-64$	3 / ✗	$-26t^{12}+376t^{11}-2212t^{10}+8280t^9-23249t^8+53488t^7-106013t^6+185990t^5-292853t^4+416673t^3-537062t^2+626488t-659788$

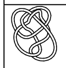
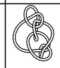
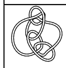
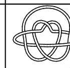
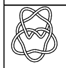
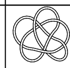
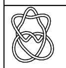
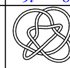
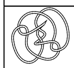
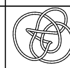
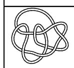
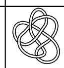
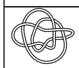
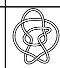
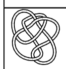
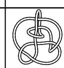
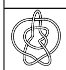
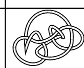
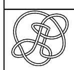
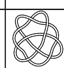
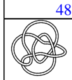
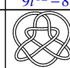
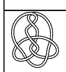
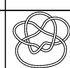
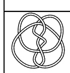
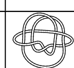
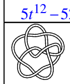
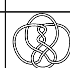
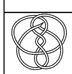
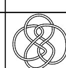
knot diag	n_k^t	Alexander's ω^+ / $(\rho_1')^+$ / $(\rho_2')^+$	genus / ribbon	unknotting # / amphi?
	9^a_7	$3t^2-7t+9$ $23t^3-56t^2+99t-108$ $-219t^8+2717t^7-15720t^6+58389t^5-157698t^4+329265t^3-548657t^2+741610t-819394$	2 / ✘	2 / ✘
	9^a_8	$-2t^2+8t-11$ $3t^3-16t^2+29t-28$ $54t^8-552t^7+2124t^6-2216t^5-12641t^4+67112t^3-172118t^2+289304t-342134$	2 / ✘	2 / ✘
	9^a_9	$2t^3-4t^2+6t-7$ $13t^5-24t^4+55t^3-72t^2+98t-96$ $-26t^{12}+376t^{11}-2296t^{10}+9328t^9-28988t^8+73584t^7-158399t^6+295928t^5-486916t^4+712094t^3-930993t^2+1092074t-1151564$	3 / ✘	3 / ✘
	9^a_{10}	$4t^2-8t+9$ $-40t^3+72t^2-114t+120$ $-608t^8+6720t^7-33776t^6+110928t^5-273462t^4+537040t^3-862768t^2+1145784t-1259748$	2 / ✘	2, 3 / ✘
	9^a_{11}	$-t^3+5t^2-7t+7$ $-2t^5+16t^4-41t^3+52t^2-66t+64$ $5t^{12}-65t^{11}+312t^{10}-463t^9-2042t^8+14588t^7-50444t^6+126967t^5-258750t^4+444545t^3-654213t^2+827220t-895336$	3 / ✘	2 / ✘
	9^a_{12}	$-2t^2+9t-13$ $5t^3-36t^2+84t-100$ $38t^8-312t^7+45t^6+9790t^5-60473t^4+202775t^3-453255t^2+722176t-841572$	2 / ✘	1 / ✘
	9^a_{13}	$4t^2-9t+11$ $-40t^3+92t^2-154t+168$ $-608t^8+7680t^7-43650t^6+158004t^5-417129t^4+856533t^3-1412461t^2+1899222t-2095210$	2 / ✘	2, 3 / ✘
	9^a_{14}	$2t^2-9t+15$ $-t^3+8t^2-35t+60$ $62t^8-752t^7+3655t^6-7178t^5-9502t^4+97737t^3-294656t^2+531720t-642168$	2 / ✘	1 / ✘
	9^a_{15}	$-2t^2+10t-15$ $-5t^3+40t^2-108t+136$ $38t^8-360t^7+208t^6+12328t^5-84103t^4+298764t^3-691161t^2+1121034t-1313504$	2 / ✘	2 / ✘
	9^a_{16}	$2t^3-5t^2+8t-9$ $-13t^5+36t^4-80t^3+120t^2-161t+168$ $-26t^{12}+456t^{11}-3331t^{10}+15554t^9-53941t^8+149494t^7-345106t^6+680900t^5-1167591t^4+1759576t^3-2347749t^2+2786466t-2949428$	3 / ✘	3 / ✘
	9^a_{17}	t^3-5t^2+9t-9 $t^5-8t^4+23t^3-32t^2+28t-24$ $8t^{12}-125t^{11}+874t^{10}-3595t^9+9462t^8-15166t^7+6162t^6+47027t^5-181220t^4+415509t^3-716070t^2+982036t-1089796$	3 / ✘	2 / ✘
	9^a_{18}	$4t^2-10t+13$ $40t^3-108t^2+193t-220$ $-608t^8+8224t^7-51208t^6+201904t^5-570516t^4+1228920t^3-2087725t^2+2850858t-3159722$	2 / ✘	2 / ✘
	9^a_{19}	$2t^2-10t+17$ $t^3-8t^2+20t-24$ $62t^8-840t^7+4536t^6-10352t^5-7041t^4+116428t^3-372683t^2+688198t-836608$	2 / ✘	1 / ✘
	9^a_{20}	$-t^3+5t^2-9t+11$ $2t^5-16t^4+47t^3-84t^2+117t-124$ $5t^{12}-65t^{11}+330t^{10}-577t^9-2439t^8+21482t^7-86959t^6+247237t^5-548658t^4+993841t^3-1502637t^2+1918532t-2080192$	3 / ✘	2 / ✘
	9^a_{21}	$-2t^2+11t-17$ $-5t^3+44t^2-127t+164$ $38t^8-408t^7+493t^6+13802t^5-105014t^4+396685t^3-954552t^2+1583140t-1868380$	2 / ✘	1 / ✘
	9^a_{22}	$t^3-5t^2+10t-11$ $-t^5+8t^4-24t^3+38t^2-40t+36$ $8t^{12}-125t^{11}+893t^{10}-3824t^9+10605t^8-17902t^7+6990t^6+64299t^5-251573t^4+584313t^3-1012133t^2+1388650t-1540398$	3 / ✘	1 / ✘
	9^a_{23}	$4t^2-11t+15$ $40t^3-128t^2+243t-288$ $-608t^8+9184t^7-62698t^6+265980t^5-794496t^4+1781111t^3-3107204t^2+4307350t-4797258$	2 / ✘	2 / ✘
	9^a_{24}	$-t^3+5t^2-10t+13$ $-4t^2+16t-20$ $9t^{12}-145t^{11}+1075t^{10}-4850t^9+14600t^8-29112t^7+29921t^6+30667t^5-218916t^4+570933t^3-1029833t^2+1433476t-1595654$	3 / ✘	1 / ✘
	9^a_{25}	$-3t^2+12t-17$ $12t^3-70t^2+153t-188$ $174t^8-1200t^7-1027t^6+42696t^5-235512t^4+740956t^3-1585864t^2+2460360t-2841166$	2 / ✘	2 / ✘
	9^a_{26}	$t^3-5t^2+11t-13$ $-t^5+8t^4-31t^3+64t^2-85t+92$ $8t^{12}-125t^{11}+900t^{10}-3861t^9+10351t^8-14356t^7-12391t^6+132473t^5-427732t^4+939309t^3-1588046t^2+2154028t-2381116$	3 / ✘	1 / ✘
	9^a_{27}	$-t^3+5t^2-11t+15$ $t^3-8t^2+24t-32$ $9t^{12}-145t^{11}+1096t^{10}-5115t^9+16088t^8-33784t^7+37362t^6+34075t^5-273854t^4+743153t^3-1374545t^2+1941332t-2171344$	3 / ✔	1 / ✘
	9^a_{28}	$t^3-5t^2+12t-15$ $t^5-8t^4+30t^3-68t^2+105t-120$ $8t^{12}-125t^{11}+923t^{10}-4138t^9+11800t^8-18092t^7-11101t^6+159415t^5-543916t^4+1228781t^3-2107809t^2+2877256t-3186008$	3 / ✘	1 / ✘
	9^a_{29}	$t^3-5t^2+12t-15$ $t^5-8t^4+26t^3-48t^2+59t-56$ $8t^{12}-125t^{11}+931t^{10}-4290t^9+13096t^8-24848t^7+13335t^6+94047t^5-409576t^4+1010237t^3-1816557t^2+2543836t-2840192$	3 / ✘	2 / ✘
	9^a_{30}	$-t^3+5t^2-12t+17$ $2t^3-10t^2+25t-32$ $9t^{12}-145t^{11}+1117t^{10}-5376t^9+17533t^8-38170t^7+43292t^6+43619t^5-347397t^4+957881t^3-1794189t^2+2553442t-2863228$	3 / ✘	1 / ✘
	9^a_{31}	$t^3-5t^2+13t-17$ $t^5-8t^4+33t^3-80t^2+132t-152$ $8t^{12}-125t^{11}+938t^{10}-4303t^9+12544t^8-19138t^7-17200t^6+204143t^5-703180t^4+1617365t^3-2818190t^2+3886636t-4319004$	3 / ✘	2 / ✘
	9^a_{32}	$t^3-6t^2+14t-17$ $-t^5+10t^4-42t^3+94t^2-133t+148$ $8t^{12}-150t^{11}+1269t^{10}-6297t^9+19455t^8-32720t^7-11156t^6+260282t^5-930836t^4+2153618t^3-3750358t^2+5165114t-5736454$	3 / ✘	2 / ✘
	9^a_{33}	$-t^3+6t^2-14t+19$ $t^3-10t^2+30t-40$ $9t^{12}-174t^{11}+1539t^{10}-8207t^9+28913t^8-67184t^7+84077t^6+55866t^5-581640t^4+1664798t^3-3166838t^2+4539202t-5100726$	3 / ✘	1 / ✘
	9^a_{34}	$-t^3+6t^2-16t+23$ $3t^3-18t^2+43t-56$ $9t^{12}-174t^{11}+1581t^{10}-8831t^9+32988t^8-81774t^7+109631t^6+73248t^5-829341t^4+2480938t^3-4869197t^2+7112552t-8043256$	3 / ✘	1 / ✘
	9^a_{35}	$7t-13$ $90t-144$ $-6355t^4+58861t^3-224539t^2+470386t-596734$	1 / ✘	2, 3 / ✘
	9^a_{36}	$-t^3+5t^2-8t+9$ $-2t^5+16t^4-44t^3+66t^2-87t+88$ $5t^{12}-65t^{11}+321t^{10}-532t^9-2081t^8+17066t^7-64846t^6+175611t^5-376739t^4+668001t^3-998037t^2+1267342t-1372104$	3 / ✘	2 / ✘
	9^a_{37}	$2t^2-11t+19$ $t^3-8t^2+22t-28$ $62t^8-928t^7+5487t^6-13814t^5-6681t^4+154867t^3-520239t^2+983348t-1204192$	2 / ✘	2 / ✘
	9^a_{38}	$5t^2-14t+19$ $62t^3-204t^2+382t-452$ $-1414t^8+22122t^7-153560t^6+657340t^5-1976110t^4+4454362t^3-7806448t^2+10855582t-12103772$	2 / ✘	2, 3 / ✘
	9^a_{39}	$-3t^2+14t-21$ $-12t^3+84t^2-210t+268$ $174t^8-1442t^7-690t^6+59068t^5-366222t^4+1247214t^3-2815796t^2+4505578t-5255776$	2 / ✘	1 / ✘
	9^a_{40}	$t^3-7t^2+18t-23$ $t^5-12t^4+57t^3-144t^2+229t-264$ $8t^{12}-175t^{11}+1712t^{10}-9738t^9+34250t^8-66108t^7-11148t^6+553509t^5-2149560t^4+5230963t^3-9406248t^2+13187800t-14730526$	3 / ✘	2 / ✘
	9^a_{41}	$3t^2-12t+19$ $3t^3-20t^2+70t-108$ $309t^8-3288t^7+13885t^6-20928t^5-55179t^4+378100t^3-1035810t^2+1787808t-2129794$	2 / ✔	2 / ✘
	9^n_{42}	$-t^2+2t-1$ $-t^3+2t^2+t-4$ $3t^8-14t^7+32t^6-96t^5+265t^4-294t^3-498t^2+2170t-3128$	2 / ✘	1 / ✘

knot diag	n_k^t	Alexander's ω^+	$(\rho'_1)^+$	$(\rho'_2)^+$	genus / ribbon	unknotting # / amphi?
	9^n_{43}	$-t^3+3t^2-2t+1$	$-2t^5+8t^4-7t^3+2t^2-5t+4$	$5t^{12}-39t^{11}+110t^{10}-108t^9-115t^8+570t^7-1477t^6+3453t^5-6651t^4+10951t^3-17188t^2+24718t-28462$	3 / ✗	2 / ✗
	9^n_{44}	t^2-4t+7	$-2t^2+9t-12$	$4t^8-48t^7+237t^6-496t^5-346t^4+4988t^3-15044t^2+26768t-32126$	2 / ✗	1 / ✗
	9^n_{45}	$-t^2+6t-9$	$t^3-14t^2+47t-60$	$3t^8-42t^7+78t^6+1376t^5-11135t^4+42574t^3-102522t^2+169806t-200284$	2 / ✗	1 / ✗
	9^n_{46}	$5-2t$	$3t-12$	$-2t^4+160t^3-1125t^2+3082t-4222$	1 / ✔	2 / ✗
	9^n_{47}	t^3-4t^2+6t-5	$-t^5+6t^4-15t^3+16t^2-10t+12$	$8t^{12}-100t^{11}+560t^{10}-1841t^9+3847t^8-4710t^7-42t^6+17494t^5-55447t^4+117058t^3-193749t^2+261386t-288924$	3 / ✗	2 / ✗
	9^n_{48}	$-t^2+7t-11$	$-t^3+12t^2-42t+52$	$3t^8-49t^7+243t^6+267t^5-8051t^4+40499t^3-112167t^2+199850t-241202$	2 / ✗	2 / ✗
	9^n_{49}	$3t^2-6t+7$	$-21t^3+38t^2-61t+60$	$-123t^8+1614t^7-8744t^6+29928t^5-75873t^4+152714t^3-250794t^2+338238t-373944$	2 / ✗	3 / ✗
	10^a_1	$9-4t$	$14t-40$	$-24t^4+2136t^3-13430t^2+34860t-47068$	1 / ✗	1 / ✗
	10^a_2	$-t^4+3t^3-3t^2+3t-3$	$3t^7-12t^6+16t^5-20t^4+24t^3-24t^2+27t-24$	$7t^{16}-57t^{15}+189t^{14}-293t^{13}-55t^{12}+1628t^{11}-5543t^{10}+13266t^9-26589t^8+47468t^7-77415t^6+116549t^5-162911t^4+212325t^3-258413t^2+292580t-305480$	4 / ✗	3 / ✗
	10^a_3	$13-6t$	$11t-28$	$870t^4+1288t^3-27795t^2+85718t-120138$	1 / ✔	2 / ✗
	10^a_4	$-3t^2+7t-7$	$4t^3-8t^2+t+8$	$294t^8-1807t^7+4570t^6-4305t^5-9550t^4+49581t^3-117456t^2+189330t-221294$	2 / ✗	2 / ✗
	10^a_5	$t^4-3t^3+5t^2-5t+5$	$-2t^7+8t^6-20t^5+28t^4-36t^3+36t^2-39t+36$	$12t^{16}-117t^{15}+565t^{14}-1757t^{13}+3847t^{12}-5960t^{11}+5381t^{10}+2968t^9-26625t^8+75008t^7-157415t^6+279173t^5-436999t^4+615297t^3-785328t^2+909916t-955948$	4 / ✗	2 / ✗
	10^a_6	$-2t^3+6t^2-7t+7$	$9t^5-36t^4+56t^3-72t^2+81t-84$	$62t^{12}-408t^{11}+712t^{10}+2280t^9-17493t^8+60652t^7-153492t^6+319048t^5-569584t^4+890397t^3-1228657t^2+1496150t-1599330$	3 / ✗	3 / ✗
	10^a_7	$-3t^2+11t-15$	$14t^3-72t^2+135t-160$	$114t^8-275t^7-5840t^6+51739t^5-222492t^4+626425t^3-1267348t^2+1914410t-2193462$	2 / ✗	1 / ✗
	10^a_8	$-2t^3+5t^2-5t+5$	$7t^5-20t^4+23t^3-28t^2+26t-24$	$94t^{12}-672t^{11}+2115t^{10}-3678t^9+2535t^8+6453t^7-30645t^6+78385t^5-154895t^4+256601t^3-367525t^2+458500t-494524$	3 / ✗	2 / ✗
	10^a_9	$-t^4+3t^3-5t^2+7t-7$	$-t^7+4t^6-10t^5+20t^4-25t^3+28t^2-28t+28$	$15t^{16}-153t^{15}+787t^{14}-2727t^{13}+7084t^{12}-14404t^{11}+22886t^{10}-26134t^9+11540t^8+39332t^7-146866t^6+325115t^5-571077t^4+856941t^3-1131013t^2+1330668t-1403980$	4 / ✗	1 / ✗
	10^a_{10}	$3t^2-11t+17$	$-5t^3+24t^2-71t+100$	$285t^8-2735t^7+10078t^6-9479t^5-64000t^4+327253t^3-827377t^2+1378130t-1624314$	2 / ✗	1 / ✗
	10^a_{11}	$-4t^2+11t-13$	$16t^3-52t^2+68t-72$	$736t^8-4672t^7+9634t^6+11132t^5-125367t^4+413121t^3-873095t^2+1336974t-1536906$	2 / ✗	2, 3 / ✗
	10^a_{12}	$2t^3-6t^2+10t-11$	$-5t^5+20t^4-50t^3+72t^2-89t+92$	$118t^{12}-1080t^{11}+4748t^{10}-12624t^9+19414t^8-2072t^7-88507t^6+320836t^5-750453t^4+1366922t^3-2053481t^2+2604638t-2816934$	3 / ✗	2 / ✗
	10^a_{13}	$2t^2-13t+23$	$t^3-12t^2+51t-84$	$62t^8-1088t^7+7367t^6-20586t^5-13356t^4+286509t^3-1005098t^2+1954280t-2416160$	2 / ✗	2 / ✗
	10^a_{14}	$-2t^3+8t^2-12t+13$	$9t^5-52t^4+119t^3-180t^2+225t-236$	$62t^{12}-584t^{11}+1720t^{10}+2816t^9-42848t^8+195040t^7-594177t^6+1407688t^5-2753604t^4+4575154t^3-6545078t^2+8106820t-8706026$	3 / ✗	2 / ✗
	10^a_{15}	$2t^3-6t^2+9t-9$	$-3t^5+12t^4-24t^3+24t^2-17t+12$	$134t^{12}-1272t^{11}+5792t^{10}-16520t^9+31765t^8-37636t^7+2396t^6+120176t^5-371368t^4+752873t^3-1195043t^2+1560190t-1702986$	3 / ✗	2 / ✗
	10^a_{16}	$-4t^2+12t-15$	$-16t^3+56t^2-76t+80$	$736t^8-5248t^7+12944t^6+6528t^5-144162t^4+522200t^3-1155370t^2+1809228t-2093696$	2 / ✗	2 / ✗
	10^a_{17}	$t^4-3t^3+5t^2-7t+9$	0	$16t^{16}-165t^{15}+861t^{14}-3043t^{13}+8173t^{12}-17514t^{11}+30162t^{10}-39958t^9+32666t^8+13998t^7-125081t^6+317743t^5-588481t^4+904569t^3-1207020t^2+1426556t-1506972$	4 / ✗	1 / ✔
	10^a_{18}	$-4t^2+14t-19$	$16t^3-68t^2+121t-140$	$736t^8-6240t^7+17736t^6+11088t^5-245648t^4+930168t^3-2109201t^2+3338706t-3874682$	2 / ✗	1 / ✗
	10^a_{19}	$2t^3-7t^2+11t-11$	$3t^5-16t^4+35t^3-40t^2+30t-24$	$134t^{12}-1480t^{11}+7641t^{10}-24194t^9+50855t^8-66007t^7+12323t^6+201357t^5-665287t^4+1397797t^3-2271085t^2+3006128t-3296368$	3 / ✗	2 / ✗
	10^a_{20}	$-3t^2+9t-11$	$14t^3-56t^2+88t-104$	$114t^8-153t^7-4783t^6+34425t^5-128711t^4+327435t^3-618704t^2+899066t-1017366$	2 / ✗	2 / ✗
	10^a_{21}	$-2t^3+7t^2-9t+9$	$9t^5-44t^4+80t^3-104t^2+121t-124$	$62t^{12}-496t^{11}+1203t^{10}+2078t^9-24456t^8+97163t^7-267878t^6+592041t^5-1106738t^4+1789591t^3-2525732t^2+3113752t-3341184$	3 / ✗	2 / ✗
	10^a_{22}	$-2t^3+6t^2-10t+13$	$-t^5+4t^4-10t^3+24t^2-37t+44$	$142t^{12}-1368t^{11}+6524t^{10}-20120t^9+42790t^8-57928t^7+16919t^6+158700t^5-540707t^4+1130294t^3-1809643t^2+2363114t-2577418$	3 / ✔	2 / ✗
	10^a_{23}	$2t^3-7t^2+13t-15$	$-5t^5+24t^4-67t^3+108t^2-137t+144$	$118t^{12}-1272t^{11}+6541t^{10}-20402t^9+38443t^8-21945t^7-132442t^6+594335t^5-1530420t^4+2960363t^3-4622193t^2+5992048t-6526360$	3 / ✗	1 / ✗
	10^a_{24}	$-4t^2+14t-19$	$24t^3-116t^2+221t-268$	$416t^8-1568t^7-13224t^6+136928t^5-604124t^4+1701008t^3-3414673t^2+5118714t-5846946$	2 / ✗	2 / ✗
	10^a_{25}	$-2t^3+8t^2-14t+17$	$9t^5-52t^4+131t^3-232t^2+314t-344$	$62t^{12}-584t^{11}+1856t^{10}+2264t^9-47052t^8+241288t^7-809541t^6+2068016t^5-4270010t^4+7347930t^3-10723331t^2+13406206t-14434208$	3 / ✗	2 / ✗
	10^a_{26}	$-2t^3+7t^2-13t+17$	$-t^5+4t^4-10t^3+28t^2-49t+60$	$142t^{12}-1600t^{11}+8823t^{10}-31058t^9+74964t^8-117897t^7+67064t^6+255997t^5-1047600t^4+2360395t^3-3947888t^2+5281288t-5805248$	3 / ✗	1 / ✗
	10^a_{27}	$2t^3-8t^2+16t-19$	$5t^5-28t^4+87t^3-164t^2+229t-252$	$118t^{12}-1464t^{11}+8536t^{10}-29792t^9+62096t^8-39696t^7-242195t^6+1151848t^5-3078140t^4+6098910t^3-9661940t^2+12621240t-13779050$	3 / ✗	1 / ✗
	10^a_{28}	$4t^2-13t+19$	$-8t^3+36t^2-100t+136$	$928t^8-7872t^7+26174t^6-22588t^5-142295t^4+689113t^3-1676391t^2+2728998t-3192146$	2 / ✗	2 / ✗
	10^a_{29}	$t^3-7t^2+15t-17$	$t^5-12t^4+52t^3-104t^2+124t-128$	$8t^{12}-175t^{11}+1659t^{10}-8913t^9+29252t^8-54292t^7+10686t^6+290989t^5-1126663t^4+2673211t^3-4723498t^2+6566572t-7317656$	3 / ✗	2 / ✗

knot diag	n_k^t	Alexander's ω^+ / $(\rho_1')^+$ / $(\rho_2')^+$	genus / ribbon / unknotting # / amphi?
	10^a_{30}	$-4t^2+17t-25$ $24t^3-148t^2+345t-440$ $416t^8-2048t^7-17490t^6+219996t^5-1101894t^4+3396907t^3-7245510t^2+11243734t-12988226$	2 / ✘ 1 / ✘
	10^a_{31}	$4t^2-14t+21$ $-4t^2+9t-12$ $992t^8-9440t^7+36936t^6-59136t^5-72624t^4+623304t^3-1691899t^2+2867550t-3391374$	2 / ✘ 1 / ✘
	10^a_{32}	$-2t^3+8t^2-15t+19$ $t^5-4t^4+13t^3-40t^2+78t-96$ $142t^{12}-1832t^{11}+11204t^{10}-42688t^9+109909t^8-184384t^7+124831t^6+360782t^5-1615391t^4+3759585t^3-6404890t^2+8655360t-9545252$	3 / ✘ 1 / ✘
	10^a_{33}	$4t^2-16t+25$ 0 $992t^8-10816t^7+47856t^6-88336t^5-84402t^4+920320t^3-2655340t^2+4640912t-5542372$	2 / ✘ 1 / ✔
	10^a_{34}	$3t^2-9t+13$ $-5t^3+20t^2-52t+68$ $285t^8-2205t^7+6601t^6-3429t^5-43369t^4+185703t^3-431857t^2+687874t-799218$	2 / ✘ 2 / ✘
	10^a_{35}	$2t^2-12t+21$ $-t^3+12t^2-47t+76$ $62t^8-1000t^7+6244t^6-15744t^5-15707t^4+232680t^3-775840t^2+1474372t-1810118$	2 / ✔ 2 / ✘
	10^a_{36}	$-3t^2+13t-19$ $14t^3-88t^2+208t-264$ $114t^8-397t^7-7597t^6+81141t^5-393441t^4+1198967t^3-2544952t^2+3941362t-4550398$	2 / ✘ 2 / ✘
	10^a_{37}	$4t^2-13t+19$ 0 $992t^8-8736t^7+31914t^6-47212t^5-64499t^4+497921t^3-1308755t^2+2181630t-2566522$	2 / ✘ 2 / ✔
	10^a_{38}	$-4t^2+15t-21$ $24t^3-128t^2+270t-336$ $416t^8-1632t^7-16122t^6+172460t^5-788845t^4+2280037t^3-4653713t^2+7038342t-8061882$	2 / ✘ 2 / ✘
	10^a_{39}	$-2t^3+8t^2-13t+15$ $9t^5-52t^4+125t^3-204t^2+263t-280$ $62t^{12}-584t^{11}+1788t^{10}+2480t^9-44191t^8+213488t^7-683173t^6+1684054t^5-3393468t^4+5753447t^3-8330571t^2+10379080t-11164828$	3 / ✘ 2 / ✘
	10^a_{40}	$2t^3-8t^2+17t-21$ $-5t^5+28t^4-89t^3+176t^2-258t+288$ $118t^{12}-1464t^{11}+8692t^{10}-31256t^9+67987t^8-49624t^7-257955t^6+1301482t^5-3582545t^4+7240253t^3-11620382t^2+15292356t-16735336$	3 / ✘ 2 / ✘
	10^a_{41}	$t^3-7t^2+17t-21$ $t^5-12t^4+54t^3-120t^2+157t-164$ $8t^{12}-175t^{11}+1697t^{10}-9543t^9+33561t^8-69114t^7+29117t^6+354127t^5-1527139t^4+3836499t^3-7019042t^2+9942516t-11145016$	3 / ✘ 2 / ✘
	10^a_{42}	$-t^3+7t^2-19t+27$ $2t^3-8t^2+11t-12$ $9t^{12}-203t^{11}+2093t^{10}-12971t^9+52885t^8-142268t^7+214987t^6+60931t^5-1368859t^4+4365895t^3-8815357t^2+13058404t-14831092$	3 / ✔ 1 / ✘
	10^a_{43}	$-t^3+7t^2-17t+23$ 0 $9t^{12}-203t^{11}+2051t^{10}-12253t^9+47594t^8-120962t^7+170450t^6+61017t^5-1045911t^4+3175271t^3-6209661t^2+9025932t-10186676$	3 / ✘ 2 / ✔
	10^a_{44}	$t^3-7t^2+19t-25$ $t^5-12t^4+56t^3-140t^2+220t-248$ $8t^{12}-175t^{11}+1735t^{10}-10157t^9+37586t^8-81160t^7+29232t^6+500937t^5-2197451t^4+5635115t^3-10448058t^2+14900236t-16735696$	3 / ✘ 1 / ✘
	10^a_{45}	$-t^3+7t^2-21t+31$ 0 $9t^{12}-203t^{11}+2135t^{10}-13689t^9+58324t^8-165246t^7+266640t^6+52413t^5-1738539t^4+5821367t^3-12123077t^2+18290148t-20900556$	3 / ✘ 2 / ✔
	10^a_{46}	$-t^4+3t^3-4t^2+5t-5$ $-3t^7+12t^6-21t^5+34t^4-43t^3+52t^2-55t+56$ $7t^{16}-57t^{15}+204t^{14}-382t^{13}+69t^{12}+2247t^{11}-9674t^{10}+27287t^9-61957t^8+121378t^7-211961t^6+335438t^5-485235t^4+644818t^3-789365t^2+891215t-928064$	4 / ✘ 3 / ✘
	10^a_{47}	$t^4-3t^3+6t^2-7t+7$ $-2t^7+8t^6-23t^5+38t^4-56t^3+60t^2-68t+64$ $12t^{16}-117t^{15}+598t^{14}-2030t^{13}+4959t^{12}-8715t^{11}+9312t^{10}+2921t^9-44823t^8+139602t^7-312112t^6+579182t^5-936546t^4+1347538t^3-1741633t^2+2029805t-2135930$	4 / ✘ 2, 3 / ✘
	10^a_{48}	$t^4-3t^3+6t^2-9t+11$ $t^5-2t^4+2t^3-3t+4$ $16t^{16}-165t^{15}+906t^{14}-3452t^{13}+10069t^{12}-23423t^{11}+43765t^{10}-63343t^9+59588t^8+8232t^7-192505t^6+537134t^5-1048176t^4+1669528t^3-2281994t^2+2735109t-2902594$	4 / ✔ 2 / ✘
	10^a_{49}	$3t^3-8t^2+12t-13$ $30t^5-94t^4+196t^3-292t^2+372t-392$ $-177t^{12}+3028t^{11}-22080t^{10}+101361t^9-341354t^8+914348t^7-2044469t^6+3931812t^5-6622778t^4+9874270t^3-13105110t^2+15522532t-16422794$	3 / ✘ 3 / ✘
	10^a_{50}	$-2t^3+7t^2-11t+13$ $-9t^5+44t^4-94t^3+150t^2-186t+200$ $62t^{12}-496t^{11}+1283t^{10}+2094t^9-29732t^8+134301t^7-412809t^6+990903t^5-1959941t^4+3278621t^3-4702408t^2+5824956t-6253664$	3 / ✘ 2 / ✘
	10^a_{51}	$2t^3-7t^2+15t-19$ $-5t^5+24t^4-73t^3+134t^2-194t+212$ $118t^{12}-1272t^{11}+6813t^{10}-22602t^9+45771t^8-28275t^7-180411t^6+857569t^5-2306697t^4+4602641t^3-7332665t^2+9612128t-10506256$	3 / ✘ 2, 3 / ✘
	10^a_{52}	$2t^3-7t^2+13t-15$ $-3t^5+16t^4-37t^3+50t^2-49t+44$ $134t^{12}-1480t^{11}+7961t^{10}-27058t^9+62159t^8-88993t^7+22042t^6+296843t^5-1040240t^4+2254967t^3-3720017t^2+4952400t-5437448$	3 / ✘ 2 / ✘
	10^a_{53}	$6t^2-18t+25$ $93t^3-346t^2+680t-828$ $-3642t^8+58248t^7-417976t^6+1846212t^5-5694639t^4+13084936t^3-23231163t^2+32545278t-36374532$	2 / ✘ 2, 3 / ✘
	10^a_{54}	$2t^3-6t^2+10t-11$ $-3t^5+12t^4-24t^3+26t^2-21t+16$ $134t^{12}-1272t^{11}+5964t^{10}-17880t^9+36606t^8-46740t^7+6565t^6+150576t^5-487825t^4+1010638t^3-1619593t^2+2120978t-2316318$	3 / ✘ 2, 3 / ✘
	10^a_{55}	$5t^2-15t+21$ $66t^3-246t^2+488t-596$ $-1966t^8+30491t^7-215627t^6+945597t^5-2905831t^4+6662951t^3-11814712t^2+16540014t-18481854$	2 / ✘ 2 / ✘
	10^a_{56}	$-2t^3+8t^2-14t+17$ $-9t^5+52t^4-133t^3+234t^2-312t+340$ $62t^{12}-584t^{11}+1800t^{10}+2840t^9-49588t^8+247616t^7-819257t^6+2077408t^5-4277830t^4+7364010t^3-10765639t^2+13481990t-14525656$	3 / ✘ 2 / ✘
	10^a_{57}	$2t^3-8t^2+18t-23$ $-5t^5+28t^4-93t^3+194t^2-300t+340$ $118t^{12}-1464t^{11}+8808t^{10}-32264t^9+71276t^8-49320t^7-305843t^6+1537376t^5-4286854t^4+8774390t^3-14221383t^2+18829374t-20648444$	3 / ✘ 2 / ✘
	10^a_{58}	$3t^2-16t+27$ $3t^3-28t^2+94t-140$ $309t^8-4384t^7+24039t^6-49896t^5-90763t^4+864784t^3-2647834t^2+4837480t-5867454$	2 / ✘ 2 / ✘
	10^a_{59}	$t^3-7t^2+18t-23$ $-t^5+12t^4-55t^3+128t^2-181t+196$ $8t^{12}-175t^{11}+1716t^{10}-9858t^9+35706t^8-76124t^7+33704t^6+412653t^5-1824096t^4+4655939t^3-8596644t^2+12230816t-13727286$	3 / ✘ 1 / ✘
	10^a_{60}	$-t^3+7t^2-20t+29$ $5t^3-40t^2+122t-176$ $9t^{12}-203t^{11}+2114t^{10}-13338t^9+55732t^8-154496t^7+241898t^6+66137t^5-1621594t^4+5326603t^3-10989858t^2+16499428t-18824860$	3 / ✘ 1 / ✘
	10^a_{61}	$-2t^3+5t^2-6t+7$ $-7t^5+20t^4-27t^3+36t^2-35t+36$ $94t^{12}-672t^{11}+2231t^{10}-4382t^9+4108t^8+6320t^7-40187t^6+113296t^5-235714t^4+400470t^3-576529t^2+714816t-767686$	3 / ✘ 2, 3 / ✘
	10^a_{62}	$t^4-3t^3+6t^2-8t+9$ $-2t^7+8t^6-23t^5+40t^4-63t^3+76t^2-89t+88$ $12t^{16}-117t^{15}+598t^{14}-2057t^{13}+5172t^{12}-9509t^{11}+10856t^{10}+2734t^9-54502t^8+178917t^7-414312t^6+786766t^5-1289208t^4+1865866t^3-2414454t^2+2812025t-2957594$	4 / ✘ 2 / ✘
	10^a_{63}	$5t^2-14t+19$ $66t^3-220t^2+416t-496$ $-1966t^8+28318t^7-188080t^6+783388t^5-2311570t^4+5141906t^3-8929148t^2+12349082t-13743884$	2 / ✘ 2 / ✘
	10^a_{64}	$-t^4+3t^3-6t^2+10t-11$ $-t^7+4t^6-11t^5+24t^4-37t^3+52t^2-60t+64$ $15t^{16}-153t^{15}+830t^{14}-3147t^{13}+9133t^{12}-20983t^{11}+37963t^{10}-50164t^9+30642t^8+68741t^7-310036t^6+745430t^5-1381735t^4+2150560t^3-2906317t^2+3464829t-3671204$	4 / ✘ 2 / ✘
	10^a_{65}	$2t^3-7t^2+14t-17$ $-5t^5+24t^4-71t^3+124t^2-169t+180$ $118t^{12}-1272t^{11}+6657t^{10}-21282t^9+40874t^8-20768t^7-166691t^6+742216t^5-1933704t^4+3781794t^3-5950947t^2+7749120t-8452246$	3 / ✘ 2 / ✘

knot diag	n^t_k	Alexander's ω^+	$(\rho'_1)^+$	$(\rho'_2)^+$	genus / ribbon	unknotting # / amphi?
	10^a_{66}	$3t^3-9t^2+16t-19$	$30t^5-112t^4+279t^3-480t^2+662t-724$	$-177t^{12}+3321t^{11}-27536t^{10}+145346t^9-561614t^8+1706788t^7-4256134t^6+8946173t^5-16135424t^4+25271935t^3-34647456t^2+41790680t-44471832$	3 / ✘	3 / ✘
	10^a_{67}	$-4t^2+16t-23$	$24t^3-140t^2+312t-392$	$416t^8-1696t^7-18592t^6+205384t^5-971474t^4+2884880t^3-6004484t^2+9188872t-10566612$	2 / ✘	2 / ✘
	10^a_{68}	$4t^2-14t+21$	$8t^3-40t^2+117t-164$	$928t^8-8448t^7+29784t^6-26736t^5-178984t^4+891736t^3-2217147t^2+3657390t-4297054$	2 / ✘	2 / ✘
	10^a_{69}	$t^3-7t^2+21t-29$	$-t^5+12t^4-68t^3+212t^2-397t+476$	$8t^{12}-175t^{11}+1753t^{10}-10339t^9+37435t^8-68174t^7-78997t^6+1015635t^5-3880779t^4+9697491t^3-17937826t^2+25646300t-28844672$	3 / ✘	2 / ✘
	10^a_{70}	$t^3-7t^2+16t-19$	$-t^5+12t^4-53t^3+114t^2-146t+152$	$8t^{12}-175t^{11}+1678t^{10}-9220t^9+31251t^8-60450t^7+14335t^6+337593t^5-1351773t^4+3275803t^3-5864336t^2+8208654t-9166724$	3 / ✘	2 / ✘
	10^a_{71}	$-t^3+7t^2-18t+25$	t^3-2t^2-t+4	$9t^{12}-203t^{11}+2072t^{10}-12608t^9+50167t^8-131082t^7+190655t^6+64937t^5-1206917t^4+3745659t^3-7436102t^2+10906778t-12346734$	3 / ✘	1 / ✘
	10^a_{72}	$-2t^3+9t^2-16t+19$	$-9t^5+60t^4-167t^3+298t^2-410t+448$	$62t^{12}-672t^{11}+2407t^{10}+2846t^9-67046t^8+358714t^7-1237440t^6+3225136t^5-6760702t^4+11767984t^3-17315777t^2+21757146t-23465324$	3 / ✘	2 / ✘
	10^a_{73}	$t^3-7t^2+20t-27$	$t^5-12t^4+65t^3-194t^2+350t-416$	$8t^{12}-175t^{11}+1738t^{10}-10112t^9+36117t^8-66038t^7-61235t^6+869449t^5-3296603t^4+8133803t^3-14880880t^2+21122890t-23697928$	3 / ✘	1 / ✘
	10^a_{74}	$-4t^2+16t-23$	$24t^3-136t^2+290t-360$	$416t^8-1984t^7-14448t^6+178832t^5-870542t^4+2626104t^3-5521764t^2+8500760t-9794748$	2 / ✘	2 / ✘
	10^a_{75}	$-t^3+7t^2-19t+27$	$-4t^3+36t^2-117t+172$	$9t^{12}-203t^{11}+2093t^{10}-12979t^9+53085t^8-144060t^7+222795t^6+45939t^5-1382507t^4+4528919t^3-9302365t^2+13926940t-15875332$	3 / ✔	2 / ✘
	10^a_{76}	$-2t^3+7t^2-12t+15$	$-9t^5+44t^4-104t^3+184t^2-245t+272$	$62t^{12}-496t^{11}+1263t^{10}+2926t^9-37611t^8+174774t^7-553794t^6+1359740t^5-2727505t^4+4595668t^3-6610039t^2+8193314t-8796596$	3 / ✘	2, 3 / ✘
	10^a_{77}	$2t^3-7t^2+14t-17$	$-5t^5+24t^4-71t^3+132t^2-189t+208$	$118t^{12}-1272t^{11}+6657t^{10}-21170t^9+39602t^8-13480t^7-193563t^6+812568t^5-2072452t^4+3997538t^3-6227879t^2+8058912t-8771174$	3 / ✘	2, 3 / ✘
	10^a_{78}	$-t^3+7t^2-16t+21$	$2t^5-24t^4+105t^3-244t^2+390t-448$	$5t^{12}-91t^{11}+626t^{10}-1310t^9-9682t^8+98268t^7-472808t^6+1558897t^5-3892200t^4+7699107t^3-12365278t^2+16351352t-17933784$	3 / ✘	2 / ✘
	10^a_{79}	$t^4-3t^3+7t^2-12t+15$	0	$16t^{16}-165t^{15}+951t^{14}-3892t^{13}+12327t^{12}-31301t^{11}+64047t^{10}-102088t^9+108942t^8-5172t^7-328635t^6+1013644t^5-2099318t^4+3486798t^3-4904824t^2+5979109t-6380898$	4 / ✘	2, 3 / ✔
	10^a_{80}	$3t^3-9t^2+15t-17$	$30t^5-112t^4+260t^3-426t^2+568t-616$	$-177t^{12}+3321t^{11}-26919t^{10}+137419t^9-511788t^8+1500906t^7-3625608t^6+7420093t^5-13101785t^4+20196767t^3-27388655t^2+32826444t-34860060$	3 / ✘	3 / ✘
	10^a_{81}	$-t^3+8t^2-20t+27$	0	$9t^{12}-232t^{11}+2632t^{10}-17347t^9+73146t^8-199476t^7+303717t^6+63516t^5-1783222t^4+5636674t^3-11239918t^2+16501092t-18681194$	3 / ✘	2 / ✔
	10^a_{82}	$-t^4+4t^3-8t^2+12t-13$	$t^7-6t^6+19t^5-42t^4+64t^3-78t^2+84t-84$	$15t^{16}-204t^{15}+1362t^{14}-5956t^{13}+19067t^{12}-46940t^{11}+89646t^{10}-125984t^9+94379t^8+118488t^7-663600t^6+1675944t^5-3187626t^4+5046508t^3-6899632t^2+8282752t-8796438$	4 / ✘	1 / ✘
	10^a_{83}	$2t^3-9t^2+19t-23$	$-5t^5+34t^4-110t^3+214t^2-301t+332$	$118t^{12}-1632t^{11}+10501t^{10}-40166t^9+92154t^8-74661t^7-344938t^6+1829049t^5-5155786t^4+10589003t^3-17184002t^2+22763416t-24966116$	3 / ✘	2 / ✘
	10^a_{84}	$2t^3-9t^2+20t-25$	$-5t^5+34t^4-116t^3+246t^2-373t+424$	$118t^{12}-1632t^{11}+10601t^{10}-40970t^9+93361t^8-60130t^7-457712t^6+2276184t^5-6379977t^4+13131088t^3-21370125t^2+28363542t-31128704$	3 / ✘	1 / ✘
	10^a_{85}	$t^4-4t^3+8t^2-10t+11$	$2t^7-12t^6+36t^5-68t^4+101t^3-124t^2+138t-140$	$12t^{16}-156t^{15}+986t^{14}-3982t^{13}+11319t^{12}-23042t^{11}+29987t^{10}-3098t^9-116460t^8+418314t^7-1005425t^6+1953048t^5-3252398t^4+4764776t^3-6220611t^2+7285042t-7676632$	4 / ✘	2 / ✘
	10^a_{86}	$-2t^3+9t^2-19t+25$	$-t^5+6t^4-21t^3+58t^2-105t+128$	$142t^{12}-2056t^{11}+14135t^{10}-60346t^9+173073t^8-322457t^7+256132t^6+640839t^5-3192178t^4+7806511t^3-13712731t^2+18852080t-20906284$	3 / ✘	2 / ✘
	10^a_{87}	$-2t^3+9t^2-18t+23$	$-t^5+6t^4-23t^3+66t^2-125t+152$	$142t^{12}-2056t^{11}+13955t^{10}-58318t^9+162798t^8-293228t^7+214867t^6+612960t^5-2882460t^4+6902570t^3-11979669t^2+16361444t-18106010$	3 / ✔	2 / ✘
	10^a_{88}	$-t^3+8t^2-24t+35$	0	$9t^{12}-232t^{11}+2716t^{10}-18955t^9+86300t^8-257664t^7+436281t^6+55760t^5-2823656t^4+9657962t^3-20306480t^2+30775472t-35215022$	3 / ✘	1 / ✔
	10^a_{89}	$t^3-8t^2+24t-33$	$t^5-14t^4+83t^3-264t^2+495t-596$	$8t^{12}-200t^{11}+2236t^{10}-14461t^9+56992t^8-117072t^7-76152t^6+1508604t^5-6093936t^4+15620030t^3-29286604t^2+42155400t-47509694$	3 / ✘	2 / ✘
	10^a_{90}	$-2t^3+8t^2-17t+23$	$-t^5+6t^4-21t^3+54t^2-93t+112$	$142t^{12}-1824t^{11}+11452t^{10}-45568t^9+123153t^8-214976t^7+138515t^6+523918t^5-2309034t^4+5458443t^3-9432309t^2+12861496t-14226804$	3 / ✘	2 / ✘
	10^a_{91}	$t^4-4t^3+9t^2-14t+17$	$t^5-2t^4+2t^3-3t+4$	$16t^{16}-220t^{15}+1535t^{14}-7166t^{13}+24885t^{12}-67476t^{11}+145070t^{10}-242014t^9+278753t^8-78212t^7-624329t^6+2091910t^5-4424108t^4+7397630t^3-10425418t^2+12711814t-13565548$	4 / ✘	1 / ✘
	10^a_{92}	$-2t^3+10t^2-20t+25$	$-9t^5+68t^4-216t^3+428t^2-622t+696$	$62t^{12}-760t^{11}+3228t^{10}+1776t^9-90686t^8+555772t^7-2114169t^6+5951964t^5-13251159t^4+24127850t^3-36624016t^2+46862460t-50844652$	3 / ✘	2 / ✘
	10^a_{93}	$2t^3-8t^2+15t-17$	$3t^5-18t^4+43t^3-58t^2+55t-48$	$134t^{12}-1696t^{11}+10180t^{10}-37880t^9+94183t^8-147272t^7+62729t^6+424866t^5-1618596t^4+3616743t^3-6059793t^2+8130868t-8948936$	3 / ✘	2 / ✘
	10^a_{94}	$-t^4+4t^3-9t^2+14t-15$	$-t^7+6t^6-20t^5+46t^4-76t^3+102t^2-115t+120$	$15t^{16}-204t^{15}+1405t^{14}-6454t^{13}+21907t^{12}-57432t^{11}+117080t^{10}-176754t^9+150405t^8+135972t^7-928717t^6+2460642t^5-4804019t^4+7729462t^3-10672990t^2+12881566t-13703760$	4 / ✘	2 / ✘
	10^a_{95}	$2t^3-9t^2+21t-27$	$-5t^5+32t^4-114t^3+248t^2-384t+436$	$118t^{12}-1656t^{11}+11045t^{10}-44462t^9+109118t^8-104035t^7-391583t^6+2298083t^5-6804711t^4+14456709t^3-24008082t^2+32236696t-35514492$	3 / ✘	1 / ✘
	10^a_{96}	$-t^3+7t^2-22t+33$	$-7t^3+50t^2-147t+212$	$9t^{12}-203t^{11}+2156t^{10}-14060t^9+61189t^8-177034t^7+287437t^6+96689t^5-2149699t^4+7231587t^3-15228082t^2+23163354t-26546674$	3 / ✘	2 / ✘
	10^a_{97}	$-5t^2+22t-33$	$-37t^3+242t^2-603t+788$	$1061t^8-5486t^7-47090t^6+615064t^5-3157165t^4+9904926t^3-21376446t^2+33395786t-38661308$	2 / ✘	2 / ✘
	10^a_{98}	$-2t^3+9t^2-18t+23$	$9t^5-60t^4+177t^3-348t^2+501t-564$	$62t^{12}-672t^{11}+2575t^{10}+1666t^9-67602t^8+398948t^7-1483813t^6+4115776t^5-9069800t^4+16396378t^3-24767965t^2+31602148t-34255402$	3 / ✘	2 / ✘
	10^a_{99}	$t^4-4t^3+10t^2-16t+19$	0	$16t^{16}-220t^{15}+1580t^{14}-7688t^{13}+27976t^{12}-79612t^{11}+179656t^{10}-315060t^9+386272t^8-148160t^7-792172t^6+2854748t^5-6237824t^4+10649644t^3-15214156t^2+18696608t-20003232$	4 / ✔	2 / ✔

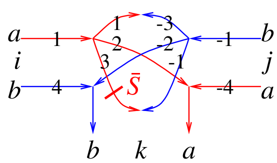
knot diag	n^t_k	Alexander's ω^+	genus / ribbon	$(\rho'_1)^+$	unknotting # / amphi?	$(\rho'_2)^+$
	10^a_{100}	$t^4-4t^3+9t^2-12t+13$	4 / ✘	$2t^7-12t^6+39t^5-80t^4+128t^3-164t^2+192t-196$	2, 3 / ✘	$12t^{16}-156t^{15}+1019t^{14}-4340t^{13}+13189t^{12}-29012t^{11}+41715t^{10}-11232t^9-153611t^8+603116t^7-1520513t^6+3049452t^5-5190414t^4+7715304t^3-10164234t^2+11961684t-12623974$
	10^a_{102}	$-2t^3+8t^2-16t+21$	3 / ✘	$-t^5+6t^4-19t^3+50t^2-89t+108$	1 / ✘	$142t^{12}-1824t^{11}+11296t^{10}-44000t^9+115984t^8-197200t^7+123203t^6+462512t^5-1996064t^4+4649298t^3-7951840t^2+10777160t-11897326$
	10^a_{104}	$t^4-4t^3+9t^2-15t+19$	4 / ✘	$t^5-2t^4+2t^3-3t+4$	1 / ✘	$16t^{16}-220t^{15}+1535t^{14}-7197t^{13}+25227t^{12}-69332t^{11}+151513t^{10}-257279t^9+301366t^8-83393t^7-710402t^6+2409469t^5-5162297t^4+8726478t^3-12397663t^2+15191203t-16238052$
	10^a_{106}	$-t^4+4t^3-9t^2+15t-17$	4 / ✘	$-t^7+6t^6-20t^5+48t^4-82t^3+114t^2-134t+140$	2 / ✘	$15t^{16}-204t^{15}+1405t^{14}-6481t^{13}+22197t^{12}-58948t^{11}+122017t^{10}-186937t^9+159252t^8+161653t^7-1073190t^6+2872671t^5-5674479t^4+9221494t^3-12827310t^2+15551003t-16568312$
	10^a_{108}	$2t^3-8t^2+14t-15$	3 / ✘	$-3t^5+18t^4-41t^3+50t^2-40t+32$	2 / ✘	$134t^{12}-1696t^{11}+10032t^{10}-36416t^9+87916t^8-133860t^7+58617t^6+353392t^5-1337642t^4+2961006t^3-4930449t^2+6594854t-7251776$
	10^a_{110}	$t^3-8t^2+20t-25$	3 / ✘	$t^5-14t^4+69t^3-160t^2+219t-236$	2 / ✘	$8t^{12}-200t^{11}+2180t^{10}-13569t^9+52114t^8-116472t^7+61616t^6+604668t^5-2747906t^4+7072274t^3-13103918t^2+18672836t-20967250$
	10^a_{112}	$-t^4+5t^3-11t^2+17t-19$	4 / ✘	$t^7-8t^6+29t^5-68t^4+115t^3-152t^2+175t-180$	2 / ✘	$15t^{16}-255t^{15}+2068t^{14}-10699t^{13}+39650t^{12}-111160t^{11}+239401t^{10}-381338t^9+357595t^8+215240t^7-1900590t^6+5252099t^5-10470652t^4+17062683t^3-23747257t^2+28786648t-30666904$
	10^a_{114}	$-2t^3+10t^2-21t+27$	3 / ✘	$t^5-8t^4+30t^3-78t^2+140t-168$	1 / ✘	$142t^{12}-2280t^{11}+16976t^{10}-76976t^9+230999t^8-445876t^7+369450t^6+890044t^5-4554487t^4+11256519t^3-19890736t^2+27431686t-30450926$
	10^a_{116}	$-t^4+5t^3-12t^2+19t-21$	4 / ✘	$t^7-8t^6+30t^5-74t^4+132t^3-184t^2+217t-228$	2 / ✘	$15t^{16}-255t^{15}+2111t^{14}-11302t^{13}+43668t^{12}-128023t^{11}+288575t^{10}-482307t^9+485985t^8+215018t^7-2416711t^6+6942030t^5-14142246t^4+23374622t^3-32832655t^2+40008697t-42694444$
	10^a_{118}	$t^4-5t^3+12t^2-19t+23$	4 / ✘	0	1 / ✔	$16t^{16}-275t^{15}+2305t^{14}-12526t^{13}+49379t^{12}-149077t^{11}+352067t^{10}-641987t^9+825146t^8-399494t^7-1458086t^6+5641784t^5-12589879t^4+21712756t^3-31187934t^2+38432195t-41152780$
	10^a_{120}	$8t^2-26t+37$	2 / ✘	$166t^3-692t^2+1433t-1788$	2, 3 / ✘	$-11768t^8+201320t^7-1541132t^6+7193960t^5-23193562t^4+55098408t^3-100101157t^2+142136186t-159564534$
	10^a_{122}	$-2t^3+11t^2-24t+31$	3 / ✘	$-t^5+8t^4-34t^3+104t^2-211t+264$	2 / ✘	$142t^{12}-2512t^{11}+20355t^{10}-99362t^9+318535t^8-657014t^7+617040t^6+1199636t^5-6869579t^4+17663208t^3-31953091t^2+44656222t-49787168$
	10^n_{124}	t^4-t^3+t-1	4 / ✘	$-4t^7-6t^4-4t^2-6t$	4 / ✘	$9t^{15}-25t^{14}+10t^{13}+75t^{12}-177t^{11}+155t^{10}+113t^9-570t^8+850t^7-428t^6-824t^5+2167t^4-2340t^3+510t^2+2375t-3832$
	10^n_{126}	t^3-2t^2+4t-5	3 / ✘	$t^5-2t^4+10t^3-12t^2+22t-20$	2 / ✘	$8t^{12}-50t^{11}+185t^{10}-457t^9+666t^8-18t^7-3074t^6+10724t^5-24495t^4+43738t^3-64631t^2+81072t-87356$
	10^n_{128}	$2t^3-3t^2+t+1$	3 / ✘	$-13t^5+12t^4-3t^3-10t^2-9t+12$	3 / ✘	$-26t^{12}+296t^{11}-1071t^{10}+1750t^9-1107t^8+287t^7-2938t^6+7959t^5-7820t^4+3175t^3-8727t^2+28392t-40368$
	10^n_{130}	$2t^2-4t+5$	2 / ✘	$t^3-2t^2+19t-24$	2 / ✘	$62t^8-336t^7+924t^6-1568t^5+253t^4+8384t^3-28668t^2+53628t-65374$
	10^n_{132}	t^2-t+1	2 / ✘	$2t^2+5t-4$	1 / ✘	$4t^8-7t^7+12t^6-145t^5+508t^4-631t^3-322t^2+2150t-3150$
	10^n_{134}	$2t^3-4t^2+4t-3$	3 / ✘	$-13t^5+24t^4-33t^3+30t^2-41t+40$	3 / ✘	$-26t^{12}+376t^{11}-2056t^{10}+6760t^9-16248t^8+32568t^7-58951t^6+98316t^5-150194t^4+210738t^3-273246t^2+324124t-344346$
	10^a_{101}	$7t^2-21t+29$	2 / ✘	$-129t^3+480t^2-942t+1148$	2, 3 / ✘	$-7453t^8+115979t^7-819947t^6+3586847t^5-10987573t^4+25120359t^3-44443695t^2+62133778t-69396618$
	10^a_{103}	$2t^3-8t^2+17t-21$	3 / ✘	$5t^5-30t^4+93t^3-178t^2+254t-280$	3 / ✘	$118t^{12}-1440t^{11}+8404t^{10}-29584t^9+61863t^8-33736t^7-289763t^6+1355186t^5-3666373t^4+7367413t^3-11802974t^2+15525908t-16990056$
	10^a_{105}	$t^3-8t^2+22t-29$	3 / ✘	$-t^5+14t^4-71t^3+184t^2-292t+332$	2 / ✘	$8t^{12}-200t^{11}+2218t^{10}-14261t^9+57123t^8-132986t^7+65302t^6+805306t^5-3722841t^4+9784430t^3-18400587t^2+26441286t-29769592$
	10^a_{107}	$-t^3+8t^2-22t+31$	3 / ✘	$2t^3-8t^2+13t-16$	1 / ✘	$9t^{12}-232t^{11}+2674t^{10}-18155t^9+79705t^8-227986t^7+366663t^6+65430t^5-2285283t^4+7518398t^3-15408513t^2+22997470t-26180364$
	10^a_{109}	$t^4-4t^3+10t^2-17t+21$	4 / ✘	0	2 / ✔	$16t^{16}-220t^{15}+1580t^{14}-7719t^{13}+28318t^{12}-81525t^{11}+186591t^{10}-332351t^9+413696t^8-158284t^7-889129t^6+3239371t^5-7165411t^4+12361738t^3-17799197t^2+21979657t-23554274$
	10^a_{111}	$-2t^3+9t^2-17t+21$	3 / ✘	$-9t^5+60t^4-171t^3+316t^2-436t+480$	2 / ✘	$62t^{12}-672t^{11}+2507t^{10}+1894t^9-64067t^8+361705t^7-1299145t^6+3506889t^5-7575591t^4+13510069t^3-20234835t^2+25700228t-27818092$
	10^a_{113}	$2t^3-11t^2+26t-33$	3 / ✘	$-5t^5+42t^4-167t^3+394t^2-623t+720$	1 / ✘	$118t^{12}-2016t^{11}+15681t^{10}-71126t^9+190712t^8-187416t^7-827053t^6+4935892t^5-14986146t^4+32456282t^3-54606535t^2+73872380t-81581546$
	10^a_{115}	$-t^3+9t^2-26t+37$	3 / ✘	0	2 / ✔	$9t^{12}-261t^{11}+3345t^{10}-24942t^9+118870t^8-365932t^7+636497t^6+31527t^5-3907730t^4+13472649t^3-28298039t^2+42798944t-48929878$
	10^a_{117}	$2t^3-10t^2+24t-31$	3 / ✘	$-5t^5+38t^4-144t^3+330t^2-522t+600$	2 / ✘	$118t^{12}-1824t^{11}+13156t^{10}-56312t^9+143746t^8-128212t^7-648731t^6+3701012t^5-11080717t^4+23844230t^3-39994730t^2+54033352t-59650184$
	10^a_{119}	$-2t^3+10t^2-23t+31$	3 / ✘	$-t^5+6t^4-26t^3+86t^2-175t+220$	1 / ✘	$142t^{12}-2288t^{11}+17392t^{10}-81560t^9+255719t^8-521820t^7+483354t^6+990524t^5-5618050t^4+14499405t^3-26339835t^2+36916418t-41198798$
	10^a_{121}	$2t^3-11t^2+27t-35$	3 / ✘	$5t^5-42t^4+167t^3-396t^2+634t-732$	2 / ✘	$118t^{12}-2016t^{11}+15853t^{10}-73450t^9+204605t^8-232351t^7-764251t^6+5054205t^5-15890853t^4+35160633t^3-59996079t^2+81831748t-90616328$
	10^a_{123}	$t^4-6t^3+15t^2-24t+29$	4 / ✔	0	2 / ✔	$16t^{16}-330t^{15}+3216t^{14}-19770t^{13}+86170t^{12}-282500t^{11}+715162t^{10}-1388790t^9+1917350t^8-1169720t^7-2832520t^6+12363784t^5-28689660t^4+50560110t^3-73579700t^2+91325158t-98015944$
	10^n_{125}	t^3-2t^2+2t-1	3 / ✘	$-t^5+2t^4-2t^3+3t-4$	2 / ✘	$8t^{12}-50t^{11}+151t^{10}-289t^9+417t^8-524t^7+536t^6-150t^5-1168t^4+3942t^3-8130t^2+12314t-14126$
	10^n_{127}	$-t^3+4t^2-6t+7$	3 / ✘	$2t^5-14t^4+32t^3-52t^2+67t-72$	2 / ✘	$5t^{12}-48t^{11}+128t^{10}+289t^9-3551t^8+15554t^7-46589t^6+109206t^5-211625t^4+348370t^3-494107t^2+608154t-651576$
	10^n_{129}	$2t^2-6t+9$	2 / ✔	$-t^3-2t^2+14t-20$	1 / ✘	$62t^8-568t^7+2280t^6-4308t^5-553t^4+25616t^3-76125t^2+132258t-157332$
	10^n_{131}	$-2t^2+8t-11$	2 / ✘	$5t^3-38t^2+87t-112$	1 / ✘	$38t^8-272t^7-580t^6+12792t^5-66417t^4+202096t^3-422662t^2+646440t-742870$
	10^n_{133}	$-t^2+5t-7$	2 / ✘	$t^3-14t^2+37t-48$	1 / ✘	$3t^8-43t^7+16t^6+1489t^5-9322t^4+30945t^3-68047t^2+106954t-123994$
	10^n_{135}	$3t^2-9t+13$	2 / ✘	$t^3-6t^2+18t-24$	2 / ✘	$321t^8-2613t^7+8905t^6-12033t^5-19329t^4+132451t^3-337025t^2+553002t-647370$

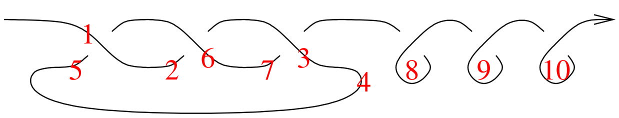
diag	knot n^t_k	Alexander's ω^+	$(\rho'_1)^+$	genus / ribbon	unknotting # / amphi?	$(\rho'_2)^+$
	10^n_{136}	$-t^2+4t-5$	$-t^3+4t^2-2t-4$	2 / ✗	1 / ✗	$3t^8-36t^7+189t^6-512t^5+347t^4+2660t^3-11142t^2+22668t-28354$
	10^n_{137}	$t^2-6t+11$	$-4t^2+24t-44$	2 / ✔	1 / ✗	$4t^8-74t^7+512t^6-1420t^5-1160t^4+21074t^3-72904t^2+140922t-173900$
	10^n_{138}	t^3-5t^2+8t-7	$-t^5+8t^4-22t^3+24t^2-11t+8$	3 / ✗	2 / ✗	$8t^{12}-125t^{11}+855t^{10}-3374t^9+8458t^8-13328t^7+8173t^6+25863t^5-114602t^4+277037t^3-497313t^2+702260t-787812$
	10^n_{139}	t^4-t^3+2t-3	$-4t^7-12t^4+5t^3-4t^2-16t+12$	4 / ✗	4 / ✗	$9t^{15}-25t^{14}-3t^{13}+172t^{12}-425t^{11}+290t^{10}+924t^9-3099t^8+4327t^7-1756t^6-5200t^5+12117t^4-11846t^3+1547t^2+12451t-19002$
	10^n_{140}	t^2-2t+3	$8t-8$	2 / ✔	2 / ✗	$4t^8-22t^7+90t^6-292t^5+424t^4+430t^3-3056t^2+6470t-8104$
	10^n_{141}	$-t^3+3t^2-4t+5$	$t^3-8t^2+16t-20$	3 / ✗	1 / ✗	$9t^{12}-87t^{11}+396t^{10}-1150t^9+2382t^8-3516t^7+2746t^6+3397t^5-19148t^4+46359t^3-80476t^2+109936t-121692$
	10^n_{142}	$2t^3-3t^2+2t-1$	$-13t^5+12t^4-13t^3+4t^2-17t+12$	3 / ✗	3 / ✗	$-26t^{12}+296t^{11}-1155t^{10}+2582t^9-4276t^8+6812t^7-11749t^6+19392t^5-27878t^4+36798t^3-48891t^2+62932t-69706$
	10^n_{143}	t^3-3t^2+6t-7	$t^5-4t^4+15t^3-28t^2+45t-48$	3 / ✗	1 / ✗	$8t^{12}-75t^{11}+362t^{10}-1106t^9+2070t^8-1092t^7-7698t^6+33841t^5-86216t^4+164927t^3-254838t^2+327896t-356170$
	10^n_{144}	$-3t^2+10t-13$	$10t^3-44t^2+80t-96$	2 / ✗	2 / ✗	$222t^8-1642t^7+3140t^6+12252t^5-94326t^4+307146t^3-651636t^2+998418t-1147140$
	10^n_{145}	t^2+t-3	$2t^3+8t^2+6t-8$	2 / ✗	2 / ✗	$-5t^7+7t^6+113t^5-141t^4-465t^3+730t^2+850t-2198$
	10^n_{146}	$2t^2-8t+13$	$t^3-8t^2+21t-28$	2 / ✗	1 / ✗	$62t^8-664t^7+2844t^6-4544t^5-9663t^4+71376t^3-197106t^2+340392t-405394$
	10^n_{147}	$-2t^2+7t-9$	$-3t^3+12t^2-15t+12$	2 / ✗	1 / ✗	$54t^8-488t^7+1697t^6-1694t^5-8312t^4+42905t^3-107222t^2+177492t-208860$
	10^n_{148}	t^3-3t^2+7t-9	$t^5-4t^4+18t^3-36t^2+62t-68$	3 / ✗	2 / ✗	$8t^{12}-75t^{11}+377t^{10}-1209t^9+2330t^8-864t^7-11900t^6+51677t^5-135261t^4+266207t^3-420746t^2+549160t-599424$
	10^n_{149}	$-t^3+5t^2-9t+11$	$2t^5-18t^4+55t^3-104t^2+149t-164$	3 / ✗	2 / ✗	$5t^{12}-61t^{11}+226t^{10}+339t^9-7195t^8+38874t^7-135727t^6+357173t^5-753890t^4+1318245t^3-1945105t^2+2447584t-2640944$
	10^n_{150}	$-t^3+4t^2-6t+7$	$-2t^5+12t^4-26t^3+38t^2-45t+44$	3 / ✗	2 / ✗	$5t^{12}-52t^{11}+216t^{10}-355t^9-719t^8+6578t^7-24361t^6+64526t^5-137117t^4+243126t^3-364723t^2+464942t-504136$
	10^n_{151}	$t^3-4t^2+10t-13$	$-t^5+6t^4-21t^3+42t^2-66t+72$	3 / ✗	2 / ✗	$8t^{12}-100t^{11}+632t^{10}-2529t^9+6645t^8-9606t^7-5854t^6+80466t^5-270269t^4+605378t^3-1033839t^2+1408362t-1558600$
	10^n_{152}	$t^4-t^3-t^2+4t-5$	$4t^7-7t^5+18t^4-7t^3-12t^2+45t-52$	4 / ✗	4 / ✗	$9t^{15}-14t^{14}-92t^{13}+396t^{12}-419t^{11}-1212t^{10}+5444t^9-9692t^8+6412t^7+11488t^6-39344t^5+55244t^4-33234t^3-30168t^2+102115t-133894$
	10^n_{153}	t^3-t^2-t+3	$t^5-2t^4+t^3+2t^2-t$	3 / ✔	2 / ✗	$8t^{12}-17t^{11}-46t^{10}+231t^9-381t^8+364t^7-367t^6+157t^5+1142t^4-2815t^3+1874t^2+2128t-4572$
	10^n_{154}	t^3-4t+7	$-3t^5-6t^4+13t^3-47t+68$	3 / ✗	3 / ✗	$48t^{10}-93t^9-546t^8+2396t^7-1956t^6-8376t^5+25906t^4-23802t^3-25690t^2+102540t-140874$
	10^n_{155}	$-t^3+3t^2-5t+7$	$-2t^3+12t^2-22t+28$	3 / ✔	2 / ✗	$9t^{12}-87t^{11}+417t^{10}-1321t^9+3014t^8-4806t^7+3646t^6+6917t^5-34773t^4+82963t^3-142781t^2+193836t-214060$
	10^n_{156}	t^3-4t^2+8t-9	$t^5-6t^4+19t^3-30t^2+33t-32$	3 / ✗	1 / ✗	$8t^{12}-100t^{11}+594t^{10}-2165t^9+5120t^8-6852t^7-2208t^6+41208t^5-134214t^4+293026t^3-493422t^2+668112t-738218$
	10^n_{157}	$-t^3+6t^2-11t+13$	$-2t^5+22t^4-78t^3+148t^2-218t+240$	3 / ✗	2 / ✗	$5t^{12}-74t^{11}+340t^{10}+321t^9-11314t^8+67637t^7-250977t^6+688036t^5-1493487t^4+2661131t^3-3974091t^2+5034465t-5444000$
	10^n_{158}	$-t^3+4t^2-10t+15$	$2t^2-7t+12$	3 / ✗	2 / ✗	$9t^{12}-116t^{11}+764t^{10}-3275t^9+9743t^8-19422t^7+18439t^6+32898t^5-196271t^4+513374t^3-940025t^2+1323614t-1479452$
	10^n_{159}	$t^3-4t^2+9t-11$	$t^5-6t^4+26t^3-60t^2+98t-112$	3 / ✗	1 / ✗	$8t^{12}-100t^{11}+609t^{10}-2267t^9+5047t^8-3237t^7-23513t^6+115362t^5-318739t^4+648093t^3-1045247t^2+1379659t-1511358$
	10^n_{160}	$-t^3+4t^2-4t+3$	$-2t^5+12t^4-20t^3+14t^2-16t+12$	3 / ✗	2 / ✗	$5t^{12}-52t^{11}+198t^{10}-255t^9-522t^8+3092t^7-8443t^6+18756t^5-37588t^4+67858t^3-108568t^2+148444t-165862$
	10^n_{161}	t^3-2t+3	$3t^5+6t^4-3t^3+4t^2+14t-12$	3 / ✗	3 / ✗	$30t^{10}-53t^9-145t^8+630t^7-674t^6-870t^5+3591t^4-4450t^3+581t^2+6166t-9640$
	10^n_{162}	$-3t^2+9t-11$	$10t^3-38t^2+58t-68$	2 / ✗	2 / ✗	$222t^8-1473t^7+2609t^6+8829t^5-65543t^4+206079t^3-427536t^2+647498t-741358$
	10^n_{163}	$t^3-5t^2+12t-15$	$-t^5+8t^4-30t^3+62t^2-89t+96$	3 / ✗	1, 2 / ✗	$8t^{12}-125t^{11}+923t^{10}-4154t^9+12040t^8-19732t^7-4345t^6+140575t^5-506052t^4+1171653t^3-2040193t^2+2809224t-3119648$
	10^n_{164}	$3t^2-11t+17$	$t^3-10t^2+29t-40$	2 / ✗	1 / ✗	$321t^8-3179t^7+12782t^6-20103t^5-32876t^4+254013t^3-688337t^2+1170838t-1386922$
	10^n_{165}	$-2t^2+10t-15$	$-5t^3+50t^2-146t+196$	2 / ✗	2 / ✗	$38t^8-344t^7-848t^6+23020t^5-137555t^4+465256t^3-1047705t^2+1673914t-1951560$

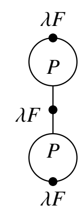
Hidden, Scaffolding, Recycling

Talk Plan.

1. Headers and abstract.
2. Ops grid.
3. sl^{ϵ}_{2+} and 4D Lie algebras, then the details of CU and QU.
4. The **DoPeGDO** box, "one abstraction level up".
5. A glance through the **DoPeGDO** footnotes.
6. Naive **DoPeGDO** compositions.
7. The "debts" box, and then go through the debts as follows.
8. A quantum algebra example.
9. A knot theory example, followed by the knot table noting affinities with topology.



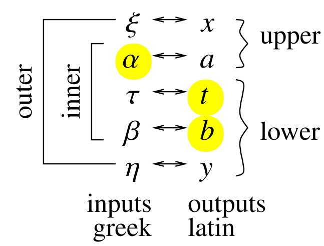




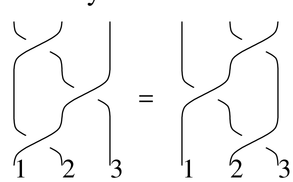
Variable Taxonomy

light

$t = \epsilon a - \gamma b$ is "central"



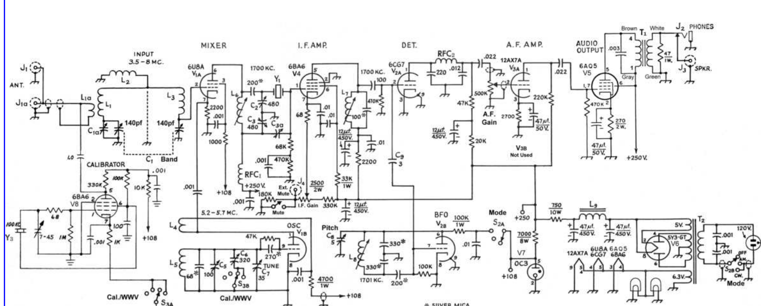
Categories are overrated (1)! • Tangles are artificially made to have a "top" and a "bottom". • Tangles are accessed by their ends and not by their strands; crossings are named by their position and not by the strands involved:



Is this $\sigma_1\sigma_2\sigma_1 = \sigma_2\sigma_1\sigma_2$
or $\sigma_{12}\sigma_{13}\sigma_{23} = \sigma_{23}\sigma_{13}\sigma_{12}$?

• Easier to talk about "skein theory".
• Harder to talk about "universal quantum invariants".

Series always converge!



Circuit diagram of the 6X2 Super-heterodyne. Unless indicated otherwise, decimal capacitances are in μf., other capacitances in μμf., resistors are ½ watt.

Representation theory is overrated!