

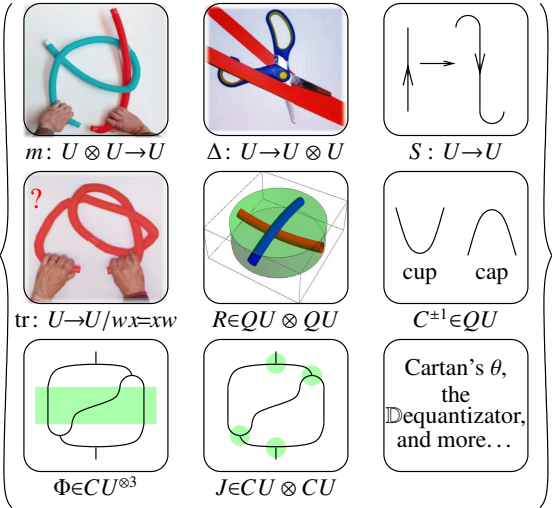


Everything around sl_{2+}^ϵ is DoPeGDO. So what?

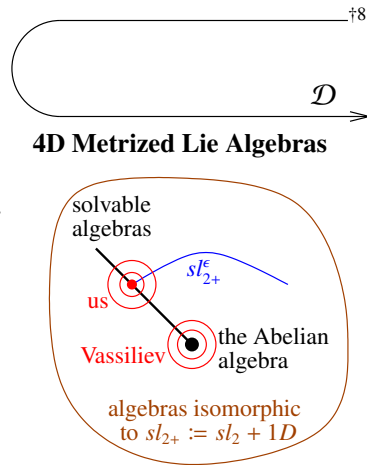
Abstract. I'll explain what "everything around" means: classical and quantum $m, \Delta, S, tr, R, C,$ and $\theta,$ as well as $P, \Phi, J, \mathbb{D},$ and more, and all of their compositions. What **DoPeGDO** means: the category of **Docile Perturbed Gaussian Differential Operators**. And what sl_{2+}^ϵ means: a solvable approximation of the semi-simple Lie algebra sl_2 .

Knot theorists should rejoice because all this leads to very powerful and well-behaved poly-time-computable knot invariants. Quantum algebraists should rejoice because it's a realistic playground for testing complicated equations and theories.

Conventions. 1. For a set $A,$ let $z_A := \{z_i\}_{i \in A}$ and let $\zeta_A := \{z_i^* = \zeta_i\}_{i \in A}.$ †1. Everything converges!



Less Abstract



DoPeGDO := The category with objects finite sets †2 and $\text{mor}(A \rightarrow B):$

$$\{\mathcal{F} = \omega \exp(Q + P)\} \subset \mathbb{Q}[[\zeta_A, z_B]]$$

Where: • ω is a scalar. †3 • Q is a "small" quadratic in $\zeta_A \cup z_B.$ †4 • P is a "docile perturbation": $P = \sum_{k \geq 1} \epsilon^k P^{(k)},$ where $\text{deg } P^{(k)} \leq 2k + 2.$ †5 • Compositions: †6

$$\mathcal{F} // \mathcal{G} = \mathcal{G} \circ \mathcal{F} := (\mathcal{G}|_{\zeta_i \rightarrow \partial_{z_i} \mathcal{F}})_{z_i=0} = (\mathcal{F}|_{z_i \rightarrow \partial_{\zeta_i} \mathcal{G}})_{\zeta_i=0}.$$

Cool! $(V^*)^{\otimes \infty} \otimes V^{\otimes \infty}$ explodes; the ranks of quadratics and bounded-degree polynomials grow slowly! †7 **Representation theory is over-rated!**

Cool! How often do you see a computational toolbox so successful?

Our Algebras. Let $sl_{2+}^\epsilon := L\langle y, b, a, x \rangle$ subject to $[a, x] = x, [b, y] = -\epsilon y, [a, b] = 0, [a, y] = -y, [b, x] = \epsilon x,$ and $[x, y] = \epsilon a + b.$ So $t := \epsilon a - b$ is central and if $\exists \epsilon^{-1}, sl_{2+}^\epsilon / \langle t \rangle \cong sl_2.$

U is either $CU = \hat{U}(sl_{2+}^\epsilon)$ or $QU = \mathcal{U}_\hbar(sl_{2+}^\epsilon) = A\langle y, b, a, x \rangle$ with $[a, x] = x, [b, y] = -\epsilon y, [a, b] = 0, [a, y] = -y, [b, x] = \epsilon x,$ and $xy - qyx = (1 - AB)/\hbar,$ where $q = e^{\hbar \epsilon}, A = e^{-\hbar \epsilon a},$ and $B = e^{-\hbar b}.$ Set also $T = A^{-1}B = e^{\hbar t}.$

The Quantum Leap. Also decree that in $QU,$

$$\Delta(y, b, a, x) = (y_1 + B_1 y_2, b_1 + b_2, a_1 + a_2, x_1 + A_1 x_2),$$
$$S(y, b, a, x) = (-B^{-1}y, -b, -a, -A^{-1}x),$$

and $R = \sum \hbar^{j+k} y^k b^j \otimes a^j x^k / j! [k]_q!$

Mid-Talk Debts. • What is this good for in quantum algebra?

- In knot theory?
- How does the "inclusion" $\mathcal{D}: \text{Hom}(U^{\otimes \Sigma} \rightarrow U^{\otimes S}) \rightsquigarrow$ **DoPeGDO** work?
- Proofs that everything around sl_{2+}^ϵ really is **DoPeGDO**.
- Relations with prior art.
- The rest of the "compositions" story.

Theorem ([BG], conjectured [MM], elucidated [Ro1]). Let $J_d(K)$ be the coloured Jones polynomial of $K,$ in the d -dimensional representation of $sl_2.$ Writing

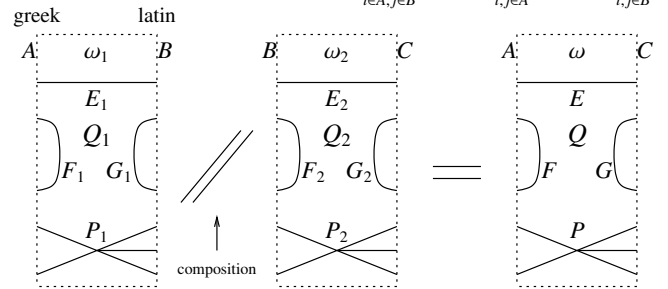
$$\left. \frac{(q^{1/2} - q^{-1/2}) J_d(K)}{q^{d/2} - q^{-d/2}} \right|_{q=e^\hbar} = \sum_{j,m \geq 0} a_{jm}(K) d^j \hbar^m,$$

"below diagonal" coefficients vanish, $a_{jm}(K) = 0$ if $j > m,$ and "on diagonal" coefficients give the inverse of the Alexander polynomial: $(\sum_{m=0}^\infty a_{mm}(K) \hbar^m) \cdot \omega(K)(e^\hbar) = 1.$

"Above diagonal" we have **Rozansky's Theorem** [Ro3, (1.2)]:

$$J_d(K)(q) = \frac{q^d - q^{-d}}{(q - q^{-1}) \omega(K)(q^d)} \left(1 + \sum_{k=1}^\infty \frac{(q-1)^k \rho_k(K)(q^d)}{\omega^{2k}(K)(q^d)} \right).$$

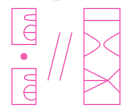
Compositions (1). In $\text{mor}(A \rightarrow B), Q = \sum_{i \in A, j \in B} E_{ij} \zeta_i z_j + \frac{1}{2} \sum_{i, j \in A} F_{ij} \zeta_i \zeta_j + \frac{1}{2} \sum_{i, j \in B} G_{ij} z_i z_j$



Where • $E = E_1(I - F_2 G_1)^{-1} E_2.$
 • $F = F_1 + E_1 F_2 (I - G_1 F_2)^{-1} E_1^T.$
 • $G = G_2 + E_2^T G_1 (I - F_2 G_1)^{-1} E_2.$
 • $\omega = \omega_1 \omega_2 \det(I - F_2 G_1)^{-1}.$
 • P is computed using "connected Feynman diagrams" or as the solution of a messy PDE (yet we're still in algebra!).



One abstraction level up from tangles! {tangles} → { } with compositions:

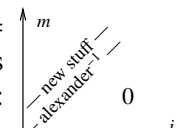


DoPeGDO Footnotes. †1. Each variable has a "weight" $\in \{0, 1, 2\},$ and always $\text{wt } z_i + \text{wt } \zeta_i = 2.$

- †2. Really, "weight-graded finite sets" $A = A_0 \sqcup A_1 \sqcup A_2.$
- †3. Really, a power series in the weight-0 variables †9.
- †4. The weight of Q must be 2, so it decomposes as $Q = Q_{20} + Q_{11}.$ The coefficients of Q_{20} are rational numbers while the coefficients of Q_{11} may be weight-0 power series †9.
- †5. Setting $\text{wt } \epsilon = -2,$ the weight of P is ≤ 2 (so the powers of the weight-0 variables are not constrained †9).
- †6. There's also an obvious product $\text{mor}(A_1 \rightarrow B_1) \times \text{mor}(A_2 \rightarrow B_2) \rightarrow \text{mor}(A_1 \sqcup A_2 \rightarrow B_1 \sqcup B_2).$
- †7. That is, if the weight-0 variables are ignored. Otherwise more care is needed yet the conclusion remains.
- †8. $\text{Hom}(U^{\otimes \Sigma} \rightarrow U^{\otimes S}) \rightsquigarrow \text{mor}(\{\eta_i, \beta_i, \tau_i, \alpha_i, \xi_i\}_{i \in \Sigma} \rightarrow \{y_i, b_i, t_i, a_i, x_i\}_{i \in S}),$ where $\text{wt}(\eta_i, \xi_i, y_i, x_i) = 1$ and $\text{wt}(\beta_i, \tau_i, \alpha_i; b_i, t_i, a_i) = (2, 2, 0; 0, 0, 2).$
- †9. For tangle invariants the weight-0 power series are always rational functions in the exponentials of the weight-0 variables (for knots: just one variable).



Melvin, Morton, Garoufalidis



$\mathcal{D}: \text{Hom}(U^{\otimes \Sigma} \rightarrow U^{\otimes S}) \rightarrow \mathbb{Q}[[\eta_\Sigma, \beta_\Sigma, \alpha_\Sigma, \xi_\Sigma, y_S, b_S, a_S, x_S]]$. The PBW theorem for CU (always in the $ybax$ order), or its quantum analog for QU , say that if $U = CU$ or QU then $U^{\otimes S}$ is isomorphic as a vector space to $\mathbb{Q}[[y_i, b_i, a_i, x_i]]_{i \in S}$; so it is enough to understand $\text{Hom}(\mathbb{Q}[[z_A]] \rightarrow \mathbb{Q}[[z_B]])$ for finite sets A and B . Using the pairing

$$\langle z_i^m, \zeta_j^n \rangle = \partial_{\zeta_i}^m z_j^n \Big|_{\zeta_i \rightarrow 0} = \delta_{ij} \delta_{mn} n!,$$

we get a map

$$\begin{aligned} \mathcal{D}: \text{Hom}(\mathbb{Q}[[z_A]] \rightarrow \mathbb{Q}[[z_B]]) &\cong \mathbb{Q}[[z_A]]^* \otimes \mathbb{Q}[[z_B]] \\ &\cong \mathbb{Q}[[\zeta_A]] \otimes \mathbb{Q}[[z_B]] \cong \mathbb{Q}[[\zeta_A, z_B]] \end{aligned}$$

Example. $\mathcal{D}(id: \mathbb{Q}[[z]] \rightarrow \mathbb{Q}[[z]]) = e^{\zeta z}$. Indeed,

$$\langle z^n, e^{\zeta z} \rangle = \left\langle z^n, \sum_m \frac{(\zeta z)^m}{m!} \right\rangle = \sum_m \frac{z^m}{m!} \delta_{mn} n! = z^n.$$

Example. $\mathcal{D}(id: U \rightarrow U) = e^{\eta y + \beta b + \alpha a + \xi x}$.

Claim. Assuming convergence, if $F \in \text{Hom}(\mathbb{Q}[[z_A]] \rightarrow \mathbb{Q}[[z_B]])$, $G \in \text{Hom}(\mathbb{Q}[[z_B]] \rightarrow \mathbb{Q}[[z_C]])$, $\mathcal{F} = \mathcal{D}(F)$, and $\mathcal{G} = \mathcal{D}(G)$, then

$$\mathcal{D}(F \circ G) = (\mathcal{F}|_{z_i \rightarrow \partial_{\zeta_i} \mathcal{G}})_{\zeta_i=0}.$$

And so the title of the talk finally makes sense!

Other GDOs. Claim. If $L: \mathbb{Q}[[z_A]] \rightarrow \mathbb{Q}[[z_B]]$ is linear, then $\mathcal{D}(L) = L(e^{\sum_{i \in A} \zeta_i z_i})$. **Proof.** Exercise.

Example. Let $c\Delta_{jk}^i: CU^{\otimes \{i\}} \rightarrow CU^{\otimes \{j,k\}}$ be the standard coproduct, given by $c\Delta_{jk}^i(y_i, b_i, a_i, x_i) = (y_j + y_k, b_j + b_k, a_j + a_k, x_j + x_k)$. Then

$$\begin{aligned} \mathcal{D}(c\Delta_{jk}^i) &= c\Delta_{jk}^i(e^{\eta y_i + \beta b_i + \alpha a_i + \xi x_i}) \\ &= e^{\eta_i(y_j + y_k) + \beta_i(b_j + b_k) + \alpha_i(a_j + a_k) + \xi_i(x_j + x_k)}. \end{aligned}$$

Example. The standard commutative product m_k^{ij} of polynomials is given by $z_i, z_j \rightarrow z_k$. Hence $\mathcal{D}(m_k^{ij}) = m_k^{ij}(e^{\zeta_i z_i + \zeta_j z_j}) = e^{(\zeta_i + \zeta_j) z_k}$.

$$\begin{array}{ccc} \mathbb{Q}[[z]]_i \otimes \mathbb{Q}[[z]]_j & \xrightarrow{m_k^{ij}} & \mathbb{Q}[[z]]_k \\ \parallel & & \parallel \\ \mathbb{Q}[[z_i, z_j]] & \xrightarrow{m_k^{ij}} & \mathbb{Q}[[z_k]] \end{array}$$

A real DoPeGDO Example. Let $cm_k^{ij}: CU_i \otimes CU_j \rightarrow CU_k$ be ‘‘classical multiplication’’ for sl_{2+}^ϵ , and let $\mathcal{O}_i: \mathbb{Q}[[y_i, b_i, a_i, x_i]] \rightarrow CU_i$ be the PBW ordering map.

$$\begin{array}{ccc} CU_i \otimes CU_j & \xrightarrow{cm_k^{ij}} & CU_k \\ \uparrow \mathcal{O}_{i,j} & & \uparrow \mathcal{O}_k \\ \mathbb{Q}[[y_i, b_i, a_i, x_i, y_j, b_j, a_j, x_j]] & & \mathbb{Q}[[y_k, b_k, a_k, x_k]] \end{array}$$

Claim. Let

$$\begin{aligned} \Lambda &= \left(\eta_i + \frac{e^{-\alpha_i - \epsilon \beta_i} \eta_j}{1 + \epsilon \eta_j \xi_i} \right) y_k + \left(\beta_i + \beta_j + \frac{\log(1 + \epsilon \eta_j \xi_i)}{\epsilon} \right) b_k + \\ &\quad \left(\alpha_i + \alpha_j + \log(1 + \epsilon \eta_j \xi_i) \right) a_k + \left(\frac{e^{-\alpha_j - \epsilon \beta_j} \xi_i}{1 + \epsilon \eta_j \xi_i} + \xi_j \right) x_k \end{aligned}$$

Then $e^{\eta_i y_i + \beta_i b_i + \alpha_i a_i + \xi_i x_i + \eta_j y_j + \beta_j b_j + \alpha_j a_j + \xi_j x_j} \Big|_{\mathcal{O}_{i,j}} \Big|_{cm_k^{ij}} = e^\Lambda \Big|_{\mathcal{O}_k}$, and hence $\mathcal{D}(cm_k^{ij}) = e^\Lambda$ and cm_k^{ij} is DoPeGDO.

Proof. We compute in a faithful 2D representation ρ of CU : (weβ/cm)

$$\begin{aligned} \text{HL}[\mathcal{E}_-] &:= \text{Style}[\mathcal{E}, \text{Background} \rightarrow \text{If}[\text{TrueQ}@\mathcal{E}, \blacksquare, \blacksquare]]; \\ \{\rho y = \begin{pmatrix} \theta & \theta \\ \theta & \theta \end{pmatrix}, \rho b = \begin{pmatrix} \theta & \theta \\ \theta & -\epsilon \end{pmatrix}, \rho a = \begin{pmatrix} 1 & \theta \\ \theta & \theta \end{pmatrix}, \rho x = \begin{pmatrix} \theta & 1 \\ \theta & \theta \end{pmatrix}\}; \\ \text{HL} / @ \{ &\rho a . \rho x - \rho x . \rho a = \rho x, \rho a . \rho y - \rho y . \rho a = -\rho y, \\ &\rho b . \rho y - \rho y . \rho b = -\epsilon \rho y, \rho b . \rho x - \rho x . \rho b = \epsilon \rho x, \\ &\rho x . \rho y - \rho y . \rho x = \rho b + \epsilon \rho a \} \\ \{\text{True}, \text{True}, \text{True}, \text{True}, \text{True}\} \\ \text{HL} @ \text{Simplify} @ \text{With} [& \{\mathbb{E} = \text{MatrixExp}\}, \\ & \mathbb{E}[\eta_i \rho y] . \mathbb{E}[\beta_j \rho b] . \mathbb{E}[\alpha_i \rho a] . \mathbb{E}[\xi_i \rho x] . \mathbb{E}[\eta_j \rho y] . \mathbb{E}[\beta_j \rho b] . \\ & \mathbb{E}[\alpha_j \rho a] . \mathbb{E}[\xi_j \rho x] = \\ & \mathbb{E}[\partial_{y_k} \Lambda \rho y] . \mathbb{E}[\partial_{b_k} \Lambda \rho b] . \mathbb{E}[\partial_{a_k} \Lambda \rho a] . \mathbb{E}[\partial_{x_k} \Lambda \rho x]] \end{aligned}$$

True

Series $[\Lambda, \{\epsilon, \theta, 1\}]$

$$\begin{aligned} &(\mathfrak{a}_k (\alpha_i + \alpha_j) + y_k (\eta_i + e^{-\alpha_i} \eta_j) + \\ & \mathfrak{b}_k (\beta_i + \beta_j + \eta_j \xi_i) + x_k (e^{-\alpha_j} \xi_i + \xi_j)) + \\ & \left(\mathfrak{a}_k \eta_j \xi_i - \frac{1}{2} \mathfrak{b}_k \eta_j^2 \xi_i^2 - e^{-\alpha_i} y_k \eta_j (\beta_i + \eta_j \xi_i) - \right. \\ & \left. e^{-\alpha_j} x_k \xi_i (\beta_j + \eta_j \xi_i) \right) \epsilon + \mathcal{O}[\epsilon]^2 \end{aligned}$$

(Shame, but this technique fails for QU).

Claim. In QU , R is DoPeGDO.

Proof. Recall that with $q = e^{\hbar \epsilon}$,

$$R = \sum \hbar^{j+k} y^k b^j \otimes a^j x^k / j! [k]_q! = \mathcal{O} \left(e^{\hbar b_1 a_2} e_q^{\hbar y_1 x_2} \right).$$

Now expand $e_q^{\hbar y_1 x_2}$ in powers of ϵ using:

Faddeev’s Formula (In as much as we can tell, first appeared without proof in Faddeev [Fa], rediscovered and proven in Quesne [Qu], and again with easier proof, in Zagier [Za]). With $[n]_q := \frac{q^n - 1}{q - 1}$, with $[n]_q! := [1]_q [2]_q \cdots [n]_q$ and with $e_q^x := \sum_{n \geq 0} \frac{x^n}{[n]_q!}$, we have

$$\log e_q^x = \sum_{k \geq 1} \frac{(1-q)^k x^k}{k(1-q^k)} = x + \frac{(1-q)^2 x^2}{2(1-q^2)} + \dots$$

Proof. We have that $e_q^x = \frac{e^{qx} - e^x}{qx - x}$ (‘‘the q -derivative of e_q^x is itself’’), and hence $e_q^{qx} = (1 + (1-q)x)e_q^x$, and

$$\log e_q^{qx} = \log(1 + (1-q)x) + \log e_q^x.$$

Writing $\log e_q^x = \sum_{k \geq 1} a_k x^k$ and comparing powers of x , we get $q^k a_k = -(1-q)^k / k + a_k$, or $a_k = \frac{(1-q)^k}{k(1-q^k)}$. \square

Compositions (2). Recall that with all indices i running in some set B ,

$$\mathcal{F} \circ \mathcal{G} = \left(\mathcal{F}|_{z_i \rightarrow \partial_{\zeta_i} \mathcal{G}} \right)_{\zeta_i=0} = e^{\sum \partial_{z_i} \partial_{\zeta_i} (\mathcal{F} \mathcal{G})} \Big|_{z_i = \zeta_i = 0},$$

so in general we wish to understand

$[F: \mathcal{E}]_B := e^{\frac{1}{2} \sum_{i,j \in B} F_{ij} \partial_{z_i} \partial_{z_j} \mathcal{E}}$ and $\langle F: \mathcal{E} \rangle_B := [F: \mathcal{E}]_B|_{z_B \rightarrow 0}$, where \mathcal{E} is a docile perturbed Gaussian. The following lemma allows us to restrict to the case where \mathcal{E} has no B - B quadratic part:

Lemma 1. With convergences left to the reader,

$$\left\langle F: \mathcal{E} e^{\frac{1}{2} \sum_{i,j \in B} G_{ij} z_i z_j} \right\rangle_B = \det(1 - GF)^{-1/2} \left\langle F(1 - GF)^{-1}: \mathcal{E} \right\rangle_B.$$

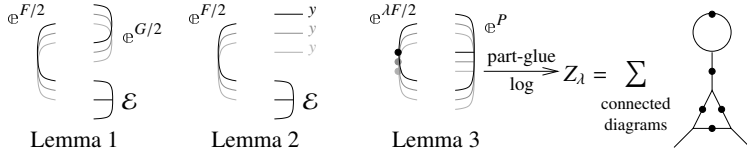
The next lemma dispatches the case where \mathcal{E} has a B -linear part:

Lemma 2. $\left\langle F: \mathcal{E} e^{\sum_{i \in B} y_i z_i} \right\rangle_B = e^{\frac{1}{2} \sum_{i,j \in B} F_{ij} y_i y_j} \left\langle F: \mathcal{E}|_{z_B \rightarrow z_B + F y_B} \right\rangle_B.$

Finally, we deal with the docile perturbation case:

Lemma 3. With an extra variable λ , $Z_\lambda := \log[\lambda F : \mathbb{e}^P]_B$ satisfies and is determined by the following PDE / IVP:

$$Z_0 = P \quad \text{and} \quad \partial_\lambda Z_\lambda = \frac{1}{2} \sum_{i,j \in B} F_{ij} (\partial_{z_i} \partial_{z_j} Z_\lambda + (\partial_{z_i} Z_\lambda) (\partial_{z_j} Z_\lambda)).$$



Warning. Some implementation details match earlier versions of the theory.

The “Speedy” Engine

$\omega\epsilon\beta/\text{engine}$

Internal Utilities

Canonical Form:

```
CCF[ε_] :=
  PPCF@ExpandDenominator@
  ExpandNumerator@PPTogether@Together[PPExp[
    Expand[ε] //. e^x e^y -> e^{x+y} /. e^x -> e^{CCF[x]}];
CF[ε_List] := CF/@ε;
CF[sd_SeriesData] := MapAt[CF, sd, 3];
CF[ε_] := PPCF@Module[
  {vs = Cases[ε, (y | b | t | a | x | η | β | τ | α | ξ), ∞] U
    {y, b, t, a, x, η, β, τ, α, ξ}},
  Total[CoefficientRules[Expand[ε], vs] /.
    (ps_ -> c_) -> CCF[c] (Times@@vs^{ps})
];
CF[ε_E] := CF/@ε;
CF[IE_sp__][εS___] := CF/@IE_sp[εS];
```

The Kronecker δ :

$K\delta /: K\delta_{i,j} := \text{If}[i === j, 1, 0];$

Equality, multiplication, and degree-adjustment of perturbed Gaussians; $\mathbb{E}[L, Q, P]$ stands for $\mathbb{e}^{L+Q} P$:

```
IE /: IE[L1_, Q1_, P1_] == IE[L2_, Q2_, P2_] :=
  CF[L1 == L2] ^ CF[Q1 == Q2] ^ CF[Normal[P1 - P2] == 0];
IE /: IE[L1_, Q1_, P1_] * IE[L2_, Q2_, P2_] :=
  IE[L1 + L2, Q1 + Q2, P1 * P2];
IE[L_, Q_, P_]_{k} := IE[L, Q, Series[Normal@P, {ε, 0, $k}]];
```

Zip and Bind

Variables and their duals:

```
{t*, b*, y*, a*, x*, z*} = {τ, β, η, α, ξ, ζ};
{τ*, β*, η*, α*, ξ*, ζ*} = {t, b, y, a, x, z};
(u_{-i})^* := (u^*)_i;
```

Upper to lower and lower to Upper:

```
U21 = {B_{-i}^{p_{-}} -> e^{-p h y b_i}, B_{-i}^{p_{-}} -> e^{-p h y b}, T_{-i}^{p_{-}} -> e^{p h t_i},
  T_{-i}^{p_{-}} -> e^{p h t}, A_{-i}^{p_{-}} -> e^{p y a_i}, A_{-i}^{p_{-}} -> e^{p y a}};
l2U = {e^{c_{-} b_i + d_{-}} -> B_{-i}^{-c/(h y)} e^d, e^{c_{-} b + d_{-}} -> B^{-c/(h y)} e^d,
  e^{c_{-} t_i + d_{-}} -> T_{-i}^{c/h} e^d, e^{c_{-} t + d_{-}} -> T^{c/h} e^d,
  e^{c_{-} a_i + d_{-}} -> A_{-i}^{c/y} e^d, e^{c_{-} a + d_{-}} -> A^{c/y} e^d,
  e^{ε_{-}} -> e^{Expand[ε]};
```

Derivatives in the presence of exponentiated variables:

```
D_b[f_-] := ∂_b f - h y B ∂_b f; D_{b_i}[f_-] := ∂_{b_i} f - h y B_i ∂_{b_i} f;
D_t[f_-] := ∂_t f + h T ∂_t f; D_{t_i}[f_-] := ∂_{t_i} f + h T_i ∂_{t_i} f;
D_α[f_-] := ∂_α f + y A ∂_α f; D_{α_i}[f_-] := ∂_{α_i} f + y A_i ∂_{α_i} f;
D_{v_-}[f_-] := ∂_v f; D_{(v,0)}[f_-] := f; D_{( )}[f_-] := f;
D_{(v, n_Integer)}[f_-] := D_v[D_{(v, n-1)}[f]];
D_{(L_List, Ls___)}[f_-] := D_{(Ls)}[D_L[f]];
```

Finite Zips:

```
collect[sd_SeriesData, ε_] :=
  MapAt[collect[#, ε] &, sd, 3];
collect[ε_, ε_] := PPCollect@Collect[ε, ε];
Zip_{( )}[P_] := P;
Zip_{εS_}[Ps_List] := Zip_{εS}/@Ps;
Zip_{(ε, εS___)}[P_] := PPZip[
  (collect[P // Zip_{εS}, ε] /. f_- . ε^{d_{-}} -> (D_{(ε*, d)}[f])) /.
  ε* -> 0 /. ((ε* /. {b -> B, t -> T, α -> A}) -> 1)];
```

QZip implements the “Q-level zips” on $\mathbb{E}(L, Q, P) = \mathbb{e}^{L+Q} P(\epsilon)$.

Such zips regard the L variables as scalars.

```
QZip_{εS_List}@E[L_, Q_, P_] :=
  PPQZip@Module[{ξ, z, zs, c, ys, ηs, qt, zrule, grule, out},
  zs = Table[ξ*, {ξ, εS}];
  c = CF[Q /. Alternatives@@(εS U zs) -> 0];
  ys = CF@Table[∂_ε(Q /. Alternatives@@zs -> 0),
    {ξ, εS}];
  ηs = CF@Table[∂_z(Q /. Alternatives@@εS -> 0), {z, zs}];
  qt = CF@Inverse@Table[Kδ_{z, ε*} - ∂_{z, ε} Q, {ξ, εS}, {z, zs}];
  zrule = Thread[zs -> CF[qt . (zs + ys)]];
  grule = Thread[εS -> εS + ηs . qt];
  CF /@ E[L, c + ηs . qt . ys,
    Det[qt] Zip_{εS}[P /. (zrule U grule)]];]
```

LZip implements the “L-level zips” on $\mathbb{E}(L, Q, P) = \mathbb{P}\mathbb{e}^{L+Q}$. Such zips regard all of $\mathbb{P}\mathbb{e}^Q$ as a single “P”. Here the z ’s are b and α and the ζ ’s are β and a .

```
LZip_{εS_List}@E[L_, Q_, P_] :=
  PPLZip@Module[{ξ, z, zs, Zs, c, ys, ηs, lt, zrule,
  Zrule, grule, Q1, EEQ, EQ},
  zs = Table[ξ*, {ξ, εS}];
  Zs = zs /. {b -> B, t -> T, α -> A};
  c = L /. Alternatives@@(εS U zs) -> 0 /.
  Alternatives@@Zs -> 1;
  ys = Table[∂_ε(L /. Alternatives@@zs -> 0), {ξ, εS}];
  ηs = Table[∂_z(L /. Alternatives@@εS -> 0), {z, zs}];
  lt = Inverse@Table[Kδ_{z, ε*} - ∂_{z, ε} L, {ξ, εS}, {z, zs}];
  zrule = Thread[zs -> lt . (zs + ys)];
  Zrule = Join[zrule,
  zrule /.
  r_Rule -> ((U = r[[1]] /. {b -> B, t -> T, α -> A}) ->
    (U /. U21 /. r // l2U))];
  grule = Thread[εS -> εS + ηs . lt];
  Q1 = Q /. (Zrule U grule);
  EEQ[ps___] :=
  EEQ[ps] =
  PP^EEQ"@ (CF[e^{-Q1} DThread[{zs, {ps}}][e^{Q1}]] /.
    {Alternatives@@zs -> 0, Alternatives@@Zs -> 1});
  CF@E[c + ηs . lt . ys,
  Q1 /. {Alternatives@@zs -> 0, Alternatives@@Zs -> 1},
  Det[lt]
  (Zip_{εS}[(EQ@@zs) (P /. (Zrule U grule))] /.
    Derivative[ps___][EQ][___] -> EEQ[ps] /.
    _EQ -> 1) ]];]
```

```

B{}[L_, R_] := LR;
B{is_}[L_E, R_E] := PPB@Module[{n},
  Times[
    L /. Table[(v : b | B | t | T | a | x | y)_i → v_{nei},
      {i, {is}}],
    R /. Table[(v : β | τ | α | ℳ | ξ | η)_i → v_{nei}, {i, {is}}]
  ] // LZJoin@Table[{β_{nei}, τ_{nei}, α_{nei}}, {i, {is}}] //
  QZipJoin@Table[{ξ_{nei}, η_{nei}}, {i, {is}}];
B{is_}[L_, R_] := B{is}[L, R];

```

E morphisms with domain and range.

```

B{is_List}[E_{d1→r1}[L1_, Q1_, P1_], E_{d2→r2}[L2_, Q2_, P2_]] :=
  E_{(d1 ∪ Complement[d2, is]) → (r2 ∪ Complement[r1, is])} @@
  B{is}[E[L1, Q1, P1], E[L2, Q2, P2]];
E_{d1→r1}[L1_, Q1_, P1_] // E_{d2→r2}[L2_, Q2_, P2_] :=
  B_{r1 ∩ d2}[E_{d1→r1}[L1, Q1, P1], E_{d2→r2}[L2, Q2, P2]];
E_{d1→r1}[L1_, Q1_, P1_] ≡ E_{d2→r2}[L2_, Q2_, P2_] ^:=
  (d1 == d2) ∧ (r1 == r2) ∧ (E[L1, Q1, P1] ≡ E[L2, Q2, P2]);
E_{d1→r1}[L1_, Q1_, P1_] E_{d2→r2}[L2_, Q2_, P2_] ^:=
  E_{(d1 ∪ d2) → (r1 ∪ r2)} @@ (E[L1, Q1, P1] × E[L2, Q2, P2]);
E_{dr_}[L_, Q_, P_]_{$k_} := E_{dr} @@ E[L, Q, P]_{$k};
E_{[E_...]}[i_] := {E}[i];

```

E[Λ]

```

E_{dr_}[A_] :=
  CF@Module[{L, Δ0 = Limit[A, ε → 0]},
    E_{dr}[L = Δ0 /. (η | y | ξ | x)_ → 0, Δ0 - L, e^{A-Δ0}]_{$k} /. 12U]

```

Exponentials as needed.

Task. Define $\text{Exp}_{m,i,k}[P]$ to compute $e^{Q(P)}$ to ϵ^k in the using the $m_{i,j \rightarrow i}$ multiplication, where P is an ϵ -dependent near-docile element, giving the answer in \mathbf{E} -form.

Methodology. If $P_0 := P_{\epsilon=0}$ and $e^{\lambda Q(P)} = \mathbf{O}(e^{\lambda P_0} F(\lambda))$, then

$F(\lambda = 0) = 1$ and we have:

$$\mathbf{O}(e^{\lambda P_0} (P_0 F(\lambda) + \partial_\lambda F)) = \mathbf{O}(\partial_\lambda e^{\lambda P_0} F(\lambda)) = \partial_\lambda \mathbf{O}(e^{\lambda P_0} F(\lambda)) = \partial_\lambda e^{\lambda P_0} F(\lambda) = e^{\lambda P_0} \mathbf{O}(P) = \mathbf{O}(e^{\lambda P_0} F(\lambda)) \mathbf{O}(P)$$

This is a linear ODE for F . Setting inductively $F_k = F_{k-1} + \epsilon^k \varphi$ we find that $F_0 = 1$ and solve for φ .

(* Bug: The first line is valid only if $\mathbf{O}(e^{P_0}) = e^{\mathbf{O}(P_0)}$.) *

```

Exp_{m,i,k}[P_] := Module[{LQ = Normal@P /. ε → 0},
  E[LQ /. (x | y)_i → 0, LQ /. (b | a | t)_i → 0, 1]];

```

```

Exp_{m,i,k}[P_] := Block[{$k = k},
  Module[{P0, λ, φ, φs, F, j, rhs, eqn, pows, at0, atλ},
    P0 = Normal@P /. ε → 0;
    F = Normal@Last@Exp_{m,i,k-1}[λ P];
    While[
      rhs =
        m_{i,j→i}[
          E_{i→{i}}[λ P0 /. (x | y)_i → 0, λ P0 /. (b | a | t)_i → 0,
            F]_k s_{σ_{i→j}} @ E_{i→{i}}[0, 0, P]_k // Last // Normal;
          eqn = CF[(∂_λ F) + P0 F - rhs];
          eqn != 0, (*do*)
          pows = First/@CoefficientRules[eqn, {y_i, b_i, a_i, x_i}];
          F += Sum[ε^k φ_{js}[λ] Times@@{y_i, b_i, a_i, x_i}^{js},
            {js, pows}];
          rhs =
            m_{i,j→i}[
              E_{i→{i}}[λ P0 /. (x | y)_i → 0, λ P0 /. (b | a | t)_i → 0,
                F]_k s_{σ_{i→j}} @ E_{i→{i}}[0, 0, P]_k // Last // Normal;
              eqn = CF[(∂_λ F) + P0 F - rhs];
              φs = Table[φ_{js}[λ], {js, pows}];
              at0 = Table[φ_{js}[0] == 0, {js, pows}];
              atλ = (# == 0) & /@
                (pows /. CoefficientRules[eqn, {y_i, b_i, a_i, x_i}]);
              F = F /. DSolve[And@@(at0 ∪ atλ), φs, λ][[1]]
            ];
          E_{i→{i}}[P0 /. (x | y)_i → 0, P0 /. (b | a | t)_i → 0,
            F + 0[ε]^{k+1} /. λ → 1] ] ]

```

“Define” Code

Define[lhs = rhs, ...] defines the lhs to be rhs, except that rhs is computed only once for each value of \$k. Fancy Mathematica not for the faint of heart. Most readers should ignore.

```

SetAttributes[Define, HoldAll];
Define[def_, defs_] := (Define[def]; Define[defs]);
Define[op_is_ = ε_] :=
  Module[{SD, ii, jj, kk, isp, nis, nisp, sis},
    Block[{i, j, k},
      ReleaseHold[Hold[
        SD[op_nisp, $k_Integer, PPBoot@Block[{i, j, k}, op_isp, $k = ε;
          op_nis, $k]];
        SD[op_isp, op_{is}, $k]; SD[op_sis_, op_{sis}];
      ] /. {SD → SetDelayed,
        isp → {is} /. {i → i_, j → j_, k → k_},
        nis → {is} /. {i → ii, j → jj, k → kk},
        nisp → {is} /. {i → ii_, j → jj_, k → kk_}
      } ] ]

```

The Objects

Symmetric Algebra Objects

```

sm_{i,j} \to k :=
  E_{\{i,j\} \to \{k\}} [b_k (\beta_i + \beta_j) + t_k (\tau_i + \tau_j) + a_k (\alpha_i + \alpha_j) +
    y_k (\eta_i + \eta_j) + x_k (\xi_i + \xi_j)];
s\Delta_{i,j} \to k :=
  E_{\{i\} \to \{j,k\}} [\beta_i (b_j + b_k) + \tau_i (t_j + t_k) + \alpha_i (a_j + a_k) +
    \eta_i (y_j + y_k) + \xi_i (x_j + x_k)];
ss_{i,j} := E_{\{i\} \to \{i\}} [-\beta_i b_i - \tau_i t_i - \alpha_i a_i - \eta_i y_i - \xi_i x_i];
se_{i,j} := E_{\{i\} \to \{i\}} [0];
s\eta_{i,j} := E_{\{i\} \to \{i\}} [0];
s\sigma_{i,j} := E_{\{i\} \to \{j\}} [\beta_i b_j + \tau_i t_j + \alpha_i a_j + \eta_i y_j + \xi_i x_j];
sY_{i,j,k,l,m} := E_{\{i\} \to \{j,k,l,m\}} [\beta_i b_k + \tau_i t_k + \alpha_i a_l + \eta_i y_j + \xi_i x_m];

```

The CU Definitions

```

c\Lambda = \left( \eta_i + \frac{e^{-\gamma \alpha_i - \epsilon \beta_i} \eta_j}{1 + \gamma \epsilon \eta_j \xi_i} \right) y_k + \left( \beta_i + \beta_j + \frac{\text{Log}[1 + \gamma \epsilon \eta_j \xi_i]}{\epsilon} \right) b_k +
  \left( \alpha_i + \alpha_j + \frac{\text{Log}[1 + \gamma \epsilon \eta_j \xi_i]}{\gamma} \right) a_k + \left( \frac{e^{-\gamma \alpha_j - \epsilon \beta_j} \xi_i}{1 + \gamma \epsilon \eta_j \xi_i} + \xi_j \right) x_k;

```

```

Define [cm_{i,j} \to k = E_{\{i,j\} \to \{k\}} [c\Lambda]];
Define [c\sigma_{i,j} = s\sigma_{i,j} / \tau_i \to 0, c\epsilon_i = se_i, c\eta_i = s\eta_i,
  c\Delta_{i,j,k} = s\Delta_{i,j,k},
  cS_i = ss_i // sY_{i-1,2,3,4} // cm_{4,3-i} // cm_{i,2-i} // cm_{i,1-i}];

```

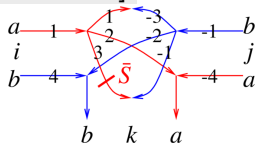
Booting Up QU

```

Define [a\sigma_{i,j} = E_{\{i\} \to \{j\}} [a_j \alpha_i + x_j \xi_i],
  b\sigma_{i,j} = E_{\{i\} \to \{j\}} [b_j \beta_i + y_j \eta_i]];
Define [am_{i,j,k} = E_{\{i,j\} \to \{k\}} [(\alpha_i + \alpha_j) a_k + (\eta_j^{-1} \xi_i + \xi_j) x_k],
  bm_{i,j,k} = E_{\{i,j\} \to \{k\}} [(\beta_i + \beta_j) b_k + (\eta_i + e^{-\epsilon \beta_i} \eta_j) y_k]];
Define [R_{i,j} = E_{\{i\} \to \{i,j\}} [\hbar a_j b_i + \sum_{k=1}^{j+1} \frac{(1 - e^{\gamma \epsilon \hbar})^k (\hbar y_i x_j)^k}{k (1 - e^{k \gamma \epsilon \hbar})}],
  \bar{R}_{i,j} = CF @ E_{\{i\} \to \{i,j\}} [-\hbar a_j b_i, -\hbar x_j y_i / B_i,
  1 + If[$k == 0, 0, (\bar{R}_{\{i,j\}, $k-1} $k [3] -
  ((\bar{R}_{\{i,j\}, 0} $k R_{\{3,4\}, $k-1} $k) // (bm_{i,1-i} am_{j,2-j} //
  (bm_{i,3-i} am_{j,4-j}))) [3]]],
  P_{i,j} = E_{\{i,j\} \to \{i\}} [\beta_i \alpha_j / \hbar, \eta_i \xi_j / \hbar,
  1 + If[$k == 0, 0, (P_{\{i,j\}, $k-1} $k [3] -
  (R_{\{1,2\} // ((P_{\{1,j\}, 0} $k (P_{\{i,2\}, $k-1} $k))) [3]])]];
Define [aS_i = (a\sigma_{i-2} \bar{R}_{1,i}) // P_{1,2},
  \bar{aS}_i = E_{\{i\} \to \{i\}} [-a_i \alpha_i, -x_i \xi_i,
  1 + If[$k == 0, 0, (\bar{aS}_{\{i\}, $k-1} $k [3] -
  ((\bar{aS}_{\{i\}, 0} $k // aS_i // (\bar{aS}_{\{i\}, $k-1} $k) [3]])]];
Define [bS_i = b\sigma_{i-1} R_{i,2} // aS_2 // P_{1,2},
  \bar{bS}_i = b\sigma_{i-1} R_{i,2} // \bar{aS}_2 // P_{1,2},
  a\Delta_{i,j,k} = (R_{1,j} R_{k,2}) // bm_{1,2-3} // P_{3,i},
  b\Delta_{i,j,k} = (R_{j,1} R_{k,2}) // am_{1,2-3} // P_{i,3}];

```

The Drinfel'd double:



```

Define [
  dm_{i,j} \to k =
    ((sY_{i-4,4,1,1} // a\Delta_{1-1,2} // a\Delta_{2-2,3} // \bar{aS}_3)
    (sY_{j \to -1, -1, -4, -4} // b\Delta_{-1-1, -2} // b\Delta_{-2-2, -3})) //
    (P_{-1,3} P_{-3,1} am_{2,-4-k} bm_{4,-2-k}]);
Define [d\sigma_{i,j} = a\sigma_{i,j} b\sigma_{i,j},
  d\epsilon_i = se_i, d\eta_i = s\eta_i,
  dS_i = sY_{i-1,1,2,2} // (\bar{bS}_1 aS_2) // dm_{2,1-i},
  \bar{dS}_i = sY_{i-1,1,2,2} // (bS_1 \bar{aS}_2) // dm_{2,1-i},
  d\Delta_{i,j,k} = (b\Delta_{i-3,1} a\Delta_{i-2,4}) // (dm_{3,4-k} dm_{1,2-j}]);

```

```

Define [C_i = E_{\{i\} \to \{i\}} [0, 0, B_i^{1/2} e^{-\hbar \epsilon a_i / 2}]_{\$k},
  \bar{C}_i = E_{\{i\} \to \{i\}} [0, 0, B_i^{-1/2} e^{\hbar \epsilon a_i / 2}]_{\$k},
  Kink_i = (R_{1,3} \bar{C}_2) // dm_{1,2-1} // dm_{1,3-i},
  \bar{Kink}_i = (\bar{R}_{1,3} C_2) // dm_{1,2-1} // dm_{1,3-i}];

```

Note. $t = \epsilon a - \gamma b$ and $b = -t / \gamma + \epsilon a / \gamma$.

```

Define [b2t_i = E_{\{i\} \to \{i\}} [\alpha_i a_i + \beta_i (\epsilon a_i - t_i) / \gamma + \xi_i x_i + \eta_i y_i],
  t2b_i = E_{\{i\} \to \{i\}} [\alpha_i a_i + \tau_i (\epsilon a_i - \gamma b_i) + \xi_i x_i + \eta_i y_i]];

```

The Knot Tensors

```

Define [kR_{i,j} = R_{i,j} // (b2t_i b2t_j) / \tau_i | j \to t,
  \bar{kR}_{i,j} = \bar{R}_{i,j} // (b2t_i b2t_j) / \tau_i | j \to t, T_i | j \to T},
  km_{i,j} \to k = (t2b_i t2b_j) // dm_{i,j-k} //
  b2t_k / \{t_k \to t, T_k \to T, \tau_i | j \to 0\},
  kC_i = C_i // b2t_i / T_i \to T,
  \bar{kC}_i = \bar{C}_i // b2t_i / T_i \to T,
  kKink_i = Kink_i // b2t_i / \{t_i \to t, T_i \to T\},
  \bar{kKink}_i = \bar{Kink}_i // b2t_i / \{t_i \to t, T_i \to T\}];

```

A Quantum Algebra Example.

$\omega\epsilon\beta/\text{qa}$

Proto-Proposition^{†0} (with Jesse Frohlich and Roland van der Veen, near [Ma, Proposition 1.7.3]). Let H be a finite dimensional Hopf algebra and let $U = H^{*cop} \otimes H$ be its Drinfel'd double, with R -matrix $R \in H^* \otimes H \subset U \otimes U$. Write $R^{\dagger 1} = \sum \rho_a \otimes r_a$, and let $\langle \cdot | \cdot \rangle: H^* \otimes H \rightarrow \mathbb{F}$ be the duality pairing. Then the functional $\int \in U^*$ defined by

$$\int \phi \otimes x := \sum \langle \phi \rho_a^{\dagger 2} | x r_a^{\dagger 3} \rangle$$

is a right^{†4} integral in U^* . (Meaning $\Delta_{jk}^i // \int_j = \int_i // \epsilon_k$ in $\text{Hom}(U^{\otimes \{i\}} \rightarrow U^{\otimes \{k\}})$).

†0 A “proto-proposition” is something that will become a proposition once you figure out the correct statement. †1 Or did we want it to be $R // S_1^2$? Or $R // S_2^2$? †2 Or is it $\rho_a \phi$? †3 Or is it $r_a x$? †4 Or maybe “left”?

PP_ := Identity; \$k = 1; \hbar = \gamma = 1;

inp = E_{\{i\} \to \{1\}} [3 a_1 b_1, 5 x_1 y_1, 1] // dm_{1,1-i};

Table[

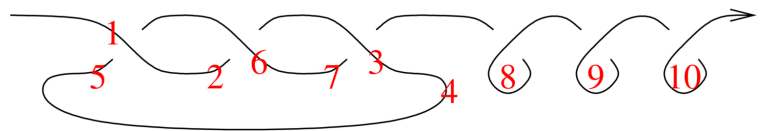
```

  HL@TrueQ[
    (inp // (sY_{i-1,1,2,2} RR) // BM // AM // P_{1,2}) de_j \equiv
    (inp // \Delta \Delta // (sY_{i-1,1,2,2} RR) // BM // AM // P_{1,2}),
    {\Delta \Delta, {d\Delta_{i-1,j}, d\Delta_{j,i}}}, {AM, {dm_{2,4-2}, dm_{4,2-2}}},
    {BM, {dm_{1,3-1}, dm_{3,1-1}}},
    {RR, {R_{3,4}, R_{3,4} // dS_3 // dS_3, R_{3,4} // dS_4 // dS_4}}
  ] // MatrixForm
  ( (False False False) (False False True)
  (False False False) (False False False)
  (False False False) (False False False)
  (False False True) (False False False) )

```

A Knot Theory Example.

$\omega\epsilon\beta/\text{kt}$



\$k = 2;

Simplify[

```

  R_{1,5} R_{6,2} R_{3,7} \bar{C}_4 \bar{Kink}_8 \bar{Kink}_9 \bar{Kink}_{10} // dm_{1,2-1} // dm_{1,3-1} //
  dm_{1,4-1} // dm_{1,5-1} // dm_{1,6-1} // dm_{1,7-1} // dm_{1,8-1} //
  dm_{1,9-1} // dm_{1,10-1}] / \cdot v_{-1} \to v

```

$$E_{\{\} \rightarrow \{1\}} \left[\theta, \theta, \frac{B}{1-B+B^2} + \frac{B(-B+2B^2+2B^4+a(-1+B-B^3+B^4)-2xy-B^3(3+2xy))}{(1-B+B^2)^3} \in + \frac{1}{2(1-B+B^2)^5} B(4B^8+a^2(1-B+B^2)^2(1+B-6B^2+B^3+B^4)+6B^5x^2y^2+2xy(-2+3xy)-B^7(11+4xy)-2B^2(1+6x^2y^2)-2B^4(1-2xy+6x^2y^2)+B(1+8xy+6x^2y^2)+B^6(6+8xy+6x^2y^2)+B^3(4+4xy+30x^2y^2)+2a(1-B+B^2)(2B^6+2xy+8B^3(1+xy)-5B^2(1+2xy)-2B^5(1+2xy)-B^4(7+2xy)+B(2+4xy)) \in^2 + 0[\in] \right]$$

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KiW 43 Abstract (ωεβ/kiw). Whether or not you like the formulas on this page, they describe the strongest truly computable knot invariant we know.

Observations. • Separates the Rolfsen table; does better than

Khovanov plus HOMFLY-PT on knots with up to 12 crossings (not tested beyond). • The degrees are bounded by the genus!
 • ρ₁ vanishes for amphichiral knots. • Has a chance of detecting non-ribbonness (ωεβ/ind)!

knot diag	n'_k (ρ'_1) ⁺	Alexander’s ω^+ (ρ'_2) ⁺	genus / ribbon unknotting # / amphi?	knot diag	n'_k (ρ'_1) ⁺	Alexander’s ω^+ (ρ'_2) ⁺	genus / ribbon unknotting # / amphi?	knot diag	n'_k (ρ'_1) ⁺	Alexander’s ω^+ (ρ'_2) ⁺	genus / ribbon unknotting # / amphi?
	0_1^a	1	0 / ✓ 0 / ✓		3_1^a	$t-1$	1 / ✗ 1 / ✗		4_1^a	$3-t$	1 / ✗ 1 / ✓
	5_1^a	t^2-t+1	2 / ✗ 2 / ✗		5_2^a	$2t-3$	1 / ✗ 1 / ✗		6_1^a	$5-2t$	1 / ✓ 1 / ✗
	6_2^a	$-t^2+3t-3$	2 / ✗ 1 / ✗		6_3^a	t^2-3t+5	2 / ✗ 1 / ✓		7_1^a	t^3-t^2+t-1	3 / ✗ 3 / ✗
	7_2^a	$3t-5$	1 / ✗ 1 / ✗		7_3^a	$2t^2-3t+3$	2 / ✗ 2 / ✗		7_4^a	$4t-7$	1 / ✗ 2 / ✗
	7_5^a	$2t^2-4t+5$	2 / ✗ 2 / ✗		7_6^a	$-t^2+5t-7$	2 / ✗ 1 / ✗		7_7^a	t^2-5t+9	2 / ✗ 1 / ✗
	8_1^a	$7-3t$	1 / ✗ 1 / ✗		8_2^a	$-t^3+3t^2-3t+3$	3 / ✗ 2 / ✗		8_3^a	$9-4t$	1 / ✗ 2 / ✓
	8_4^a	$-2t^2+5t-5$	2 / ✗ 2 / ✗		8_5^a	$-t^3+3t^2-4t+5$	3 / ✗ 2 / ✗		8_6^a	$-2t^2+6t-7$	2 / ✗ 2 / ✗
	8_7^a	t^3-3t^2+5t-5	3 / ✗ 1 / ✗		8_8^a	$2t^2-6t+9$	2 / ✓ 2 / ✗		8_9^a	$-t^3+3t^2-5t+7$	3 / ✓ 1 / ✓











knot diag	n'_k $(\rho'_1)^+$	Alexander's ω^+	genus / ribbon unknotting # / amphi?	knot diag	n'_k $(\rho'_1)^+$	Alexander's ω^+	genus / ribbon unknotting # / amphi?	knot diag	n'_k $(\rho'_1)^+$	Alexander's ω^+	genus / ribbon unknotting # / amphi?
	8_{10}^a	$t^3 - 3t^2 + 6t - 7$ $-t^3 + 4t^4 - 11t^3 + 16t^2 - 21t + 20$	3 / ✗ 2 / ✗		8_{11}^a	$-2t^2 + 7t - 9$ $5t^3 - 24t^2 + 39t - 44$	2 / ✗ 1 / ✗		8_{12}^a	$t^2 - 7t + 13$ 0	2 / ✗ 2 / ✓
$8t^{12} - 75t^{11} + 362t^{10} - 1122t^9 + 2306t^8 - 2540t^7 - 2198t^6 + 18817t^5 - 54380t^4 + 110103t^3 - 175694t^2 + 230080t - 251346$	$27t^2 - 7t + 11$ $-t^3 + 4t^2 - 14t + 20$	2 / ✗ 1 / ✗	$38t^8 - 264t^7 + 301t^6 + 3514t^5 - 21716t^4 + 68785t^3 - 146898t^2 + 227828t - 263172$	$4t^8 - 77t^7 + 583t^6 - 1991t^5 + 987t^4 + 17311t^3 - 71802t^2 + 147914t - 185846$	8_{13}^a	$2t^2 - 7t + 11$ $-t^3 + 4t^2 - 14t + 20$	2 / ✗ 1 / ✗	8_{14}^a	$3t^2 - 8t + 11$ $21t^3 - 64t^2 + 120t - 140$	2 / ✗ 2 / ✗	
$62t^8 - 592t^7 + 2351t^6 - 3918t^5 - 4235t^4 + 40079t^3 - 111533t^2 + 191500t - 227432$	$t^3 - 4t^2 + 8t - 9$ $t^3 - 6t^4 + 17t^3 - 28t^2 + 35t - 36$	3 / ✗ 2 / ✗	$38t^8 - 312t^7 + 444t^6 + 5096t^5 - 34777t^4 + 116368t^3 - 255750t^2 + 401632t - 465478$	$9t^{12} - 116t^{11} + 722t^{10} - 2843t^9 + 7656t^8 - 13668t^7 + 11117t^6 + 21968t^5 - 113086t^4 + 273778t^3 - 475622t^2 + 649064t - 717954$	8_{15}^a	$t^3 - 4t^2 + 8t - 9$ $t^3 - 6t^4 + 17t^3 - 28t^2 + 35t - 36$	3 / ✗ 2 / ✗	8_{16}^a	$-t^3 + 5t^2 - 10t + 13$ 0	3 / ✗ 2 / ✓	
$8t^{12} - 100t^{11} + 598t^{10} - 2205t^9 + 5292t^8 - 7164t^7 - 2380t^6 + 43100t^5 - 137314t^4 + 291750t^3 - 478742t^2 + 636488t - 698666$	$t^3 - t^2 + 1$ $-3t^5 - 4t^2 - 3t$	3 / ✗ 3 / ✗	$9t^{12} - 116t^{11} + 722t^{10} - 2843t^9 + 7656t^8 - 13668t^7 + 11117t^6 + 21968t^5 - 113086t^4 + 273778t^3 - 475622t^2 + 649064t - 717954$	$9t^{12} - 145t^{11} + 1075t^{10} - 4842t^9 + 14504t^8 - 28560t^7 + 27957t^6 + 35195t^5 - 225204t^4 + 573797t^3 - 1021641t^2 + 1411484t - 1567262$	8_{17}^a	$t^3 - t^2 + 1$ $-3t^5 - 4t^2 - 3t$	3 / ✗ 3 / ✗	8_{18}^a	$-t^3 + 4t - 5$ $t^3 - 8t^2 + 16t - 20$	2 / ✗ 1 / ✗	
$7t^{11} - 19t^{10} + 6t^9 + 48t^8 - 52t^7 - 91t^6 + 211t^5 + 16t^4 - 431t^3 + 289t^2 + 536t - 1060$	$t^2 - 2t + 3$ $4t - 4$	2 / ✓ 1 / ✗	$4t^8 - 22t^7 + 66t^6 - 124t^5 + 52t^4 + 478t^3 - 1652t^2 + 3014t - 3640$	$3t^8 - 28t^7 + 49t^6 + 352t^5 - 2489t^4 + 8164t^3 - 17530t^2 + 27092t - 31226$	8_{19}^a	$t^2 - 2t + 3$ $4t - 4$	2 / ✓ 1 / ✗	8_{20}^a	$t^3 - 8t^2 + 16t - 20$	2 / ✗ 1 / ✗	

knot diag	n'_k $(\rho'_1)^+$	Alexander's ω^+	genus / ribbon unknotting # / amphi?	knot diag	n'_k $(\rho'_1)^+$	Alexander's ω^+	genus / ribbon unknotting # / amphi?
	9_1^a	$t^4 - t^3 + t^2 - t + 1$ $4t^7 + 7t^5 + 9t^3 + 10t$	4 / ✗ 4 / ✗		9_2^a	$4t - 7$ $30t - 40$	1 / ✗ 1 / ✗
$9t^{15} - 36t^{14} + 99t^{13} - 216t^{12} + 414t^{11} - 720t^{10} + 1170t^9 - 1800t^8 + 2630t^7 - 3662t^6 + 4853t^5 - 6142t^4 + 7423t^3 - 8572t^2 + 9420t - 9780$	$2t^3 - 3t^2 + 3t - 3$ $-13t^5 + 12t^4 - 25t^3 + 20t^2 - 32t + 24$	3 / ✗ 3 / ✗	$-728t^4 + 6088t^3 - 21946t^2 + 44788t - 56420$		9_4^a	$3t^2 - 5t + 5$ $23t^3 - 28t^2 + 46t - 44$	2 / ✗ 2 / ✗
$-26t^{12} + 296t^{11} - 1311t^{10} + 3838t^9 - 8867t^8 + 17613t^7 - 31407t^6 + 51061t^5 - 76085t^4 + 104297t^3 - 131779t^2 + 152840t - 160976$	$6t - 11$ $100 - 65t$	1 / ✗ 2 / ✗	$-219t^8 + 1999t^7 - 8389t^6 + 23799t^5 - 52835t^4 + 96723t^3 - 149121t^2 + 194698t - 213338$		9_6^a	$2t^3 - 4t^2 + 5t - 5$ $13t^5 - 24t^4 + 45t^3 - 52t^2 + 68t - 64$	3 / ✗ 3 / ✗
$-3234t^4 + 29792t^3 - 113241t^2 + 236818t - 300294$	$3t^2 - 7t + 9$ $23t^3 - 56t^2 + 99t - 108$	2 / ✗ 2 / ✗	$-26t^{12} + 376t^{11} - 2296t^{10} + 9328t^9 - 28988t^8 + 73584t^7 - 158399t^6 + 295928t^5 - 486916t^4 + 712094t^3 - 930993t^2 + 1092074t - 1151564$		9_8^a	$-2t^2 + 8t - 11$ $3t^3 - 16t^2 + 29t - 28$	2 / ✗ 2 / ✗
$-219t^8 + 2717t^7 - 15720t^6 + 58389t^5 - 157698t^4 + 329265t^3 - 548657t^2 + 741610t - 819394$	$2t^3 - 4t^2 + 6t - 7$ $13t^5 - 24t^4 + 55t^3 - 72t^2 + 98t - 96$	3 / ✗ 3 / ✗	$54t^8 - 552t^7 + 2124t^6 - 2216t^5 - 12641t^4 + 67112t^3 - 172118t^2 + 289304t - 342134$		9_{10}^a	$4t^2 - 8t + 9$ $-40t^3 + 72t^2 - 114t + 120$	2 / ✗ 2, 3 / ✗
$-26t^{12} + 376t^{11} - 2296t^{10} + 9328t^9 - 28988t^8 + 73584t^7 - 158399t^6 + 295928t^5 - 486916t^4 + 712094t^3 - 930993t^2 + 1092074t - 1151564$	$-t^3 + 5t^2 - 7t + 7$ $-2t^5 + 16t^4 - 41t^3 + 52t^2 - 66t + 64$	3 / ✗ 2 / ✗	$-608t^8 + 6720t^7 - 33776t^6 + 110928t^5 - 273462t^4 + 537040t^3 - 862768t^2 + 1145784t - 1259748$		9_{12}^a	$-2t^2 + 9t - 13$ $5t^3 - 36t^2 + 84t - 100$	2 / ✗ 1 / ✗
$5t^{12} - 65t^{11} + 312t^{10} - 463t^9 - 2042t^8 + 14588t^7 - 50444t^6 + 126967t^5 - 258750t^4 + 444545t^3 - 654213t^2 + 827220t - 895336$	$4t^2 - 9t + 11$ $-40t^3 + 92t^2 - 154t + 168$	2 / ✗ 2, 3 / ✗	$38t^8 - 312t^7 + 45t^6 + 9790t^5 - 60473t^4 + 202775t^3 - 453255t^2 + 722176t - 841572$		9_{14}^a	$2t^2 - 9t + 15$ $-t^3 + 8t^2 - 35t + 60$	2 / ✗ 1 / ✗
$-608t^8 + 7680t^7 - 43650t^6 + 158004t^5 - 417129t^4 + 856533t^3 - 1412461t^2 + 1899222t - 2095210$	$-2t^2 + 10t - 15$ $-5t^3 + 40t^2 - 108t + 136$	2 / ✗ 2 / ✗	$62t^8 - 752t^7 + 3655t^6 - 7178t^5 - 9502t^4 + 97737t^3 - 294656t^2 + 531720t - 642168$		9_{16}^a	$2t^3 - 5t^2 + 8t - 9$ $-13t^5 + 36t^4 - 80t^3 + 120t^2 - 161t + 168$	3 / ✗ 3 / ✗
$38t^8 - 360t^7 + 208t^6 + 12328t^5 - 84103t^4 + 298764t^3 - 691161t^2 + 1121034t - 1313504$	$t^3 - 5t^2 + 9t - 9$ $t^5 - 8t^4 + 23t^3 - 32t^2 + 28t - 24$	3 / ✗ 2 / ✗	$-26t^{12} + 456t^{11} - 3331t^{10} + 15554t^9 - 53941t^8 + 149494t^7 - 345106t^6 + 680900t^5 - 1167591t^4 + 1759576t^3 - 2347749t^2 + 2786466t - 2949428$		9_{18}^a	$4t^2 - 10t + 13$ $40t^3 - 108t^2 + 193t - 220$	2 / ✗ 2 / ✗
$8t^{12} - 125t^{11} + 874t^{10} - 3595t^9 + 9462t^8 - 15166t^7 + 6162t^6 + 47027t^5 - 181220t^4 + 415509t^3 - 716070t^2 + 982036t - 1089796$	$2t^2 - 10t + 17$ $t^3 - 8t^2 + 20t - 24$	2 / ✗ 1 / ✗	$-608t^8 + 8224t^7 - 51208t^6 + 201904t^5 - 570516t^4 + 1228920t^3 - 2087725t^2 + 2850858t - 3159722$		9_{20}^a	$-t^3 + 5t^2 - 9t + 11$ $2t^5 - 16t^4 + 47t^3 - 84t^2 + 117t - 124$	3 / ✗ 2 / ✗
$62t^8 - 840t^7 + 4536t^6 - 10352t^5 - 7041t^4 + 116428t^3 - 372683t^2 + 688198t - 836608$	$-2t^2 + 11t - 17$ $-5t^3 + 44t^2 - 127t + 164$	2 / ✗ 1 / ✗	$5t^{12} - 65t^{11} + 330t^{10} - 577t^9 - 2439t^8 + 21482t^7 - 86959t^6 + 247237t^5 - 548658t^4 + 993841t^3 - 1502637t^2 + 1918532t - 2080192$		9_{22}^a	$t^3 - 5t^2 + 10t - 11$ $-t^5 + 8t^4 - 24t^3 + 38t^2 - 40t + 36$	3 / ✗ 1 / ✗
$38t^8 - 408t^7 + 493t^6 + 13802t^5 - 105014t^4 + 396685t^3 - 954552t^2 + 1583140t - 1868380$	$4t^2 - 11t + 15$ $40t^3 - 128t^2 + 243t - 288$	2 / ✗ 2 / ✗	$8t^{12} - 125t^{11} + 893t^{10} - 3824t^9 + 10605t^8 - 17902t^7 + 6990t^6 + 64299t^5 - 251573t^4 + 584313t^3 - 1012133t^2 + 1388650t - 1540398$		9_{24}^a	$-t^3 + 5t^2 - 10t + 13$ $-4t^2 + 16t - 20$	3 / ✗ 1 / ✗
$-608t^8 + 9184t^7 - 62698t^6 + 265980t^5 - 794496t^4 + 1781111t^3 - 3107204t^2 + 4307350t - 4792758$			$9t^{12} - 145t^{11} + 1075t^{10} - 4850t^9 + 14600t^8 - 29112t^7 + 29921t^6 + 30667t^5 - 218916t^4 + 570933t^3 - 1029833t^2 + 1433476t - 1595654$				

knot diag	n_k^l Alexander's ω^+ $(\rho_1)^+$	genus / ribbon unknotting # / amphi?	knot diag	n_k^l Alexander's ω^+ $(\rho_1)^+$	genus / ribbon unknotting # / amphi?
	$9a_{25}$ $-3t^2 + 12t - 17$ $12t^3 - 70t^2 + 153t - 188$ $174t^8 - 1200t^7 - 1027t^6 + 42696t^5 - 235512t^4 + 740956t^3 - 1585864t^2 + 2460360t - 2841166$	2 / ✗ 2 / ✗		$9a_{26}$ $t^3 - 5t^2 + 11t - 13$ $-t^5 + 8t^4 - 31t^3 + 64t^2 - 85t + 92$ $8t^{12} - 125t^{11} + 900t^{10} - 3861t^9 + 10351t^8 - 14356t^7 - 12391t^6 + 132473t^5 - 427732t^4 + 939309t^3 - 1588046t^2 + 2154028t - 2381116$	3 / ✗ 1 / ✗
	$9a_{27}$ $-t^3 + 5t^2 - 11t + 15$ $t^3 - 8t^2 + 24t - 32$ $9t^{12} - 145t^{11} + 1096t^{10} - 5115t^9 + 16088t^8 - 33784t^7 + 37362t^6 + 34075t^5 - 273854t^4 + 743153t^3 - 1374545t^2 + 1941332t - 2171344$	3 / ✓ 1 / ✗		$9a_{28}$ $t^3 - 5t^2 + 12t - 15$ $t^5 - 8t^4 + 30t^3 - 68t^2 + 105t - 120$ $8t^{12} - 125t^{11} + 923t^{10} - 4138t^9 + 11800t^8 - 18092t^7 - 11101t^6 + 159415t^5 - 543916t^4 + 1228781t^3 - 2107809t^2 + 2877256t - 3186008$	3 / ✗ 1 / ✗
	$9a_{29}$ $t^3 - 5t^2 + 12t - 15$ $t^5 - 8t^4 + 26t^3 - 48t^2 + 59t - 56$ $8t^{12} - 125t^{11} + 931t^{10} - 4290t^9 + 13096t^8 - 24848t^7 + 13335t^6 + 94047t^5 - 409576t^4 + 1010237t^3 - 1816557t^2 + 2543836t - 2840192$	3 / ✗ 2 / ✗		$9a_{30}$ $-t^3 + 5t^2 - 12t + 17$ $2t^3 - 10t^2 + 25t - 32$ $9t^{12} - 145t^{11} + 1117t^{10} - 5376t^9 + 17533t^8 - 38170t^7 + 43292t^6 + 43619t^5 - 347397t^4 + 957881t^3 - 1794189t^2 + 2553442t - 2863228$	3 / ✗ 1 / ✗
	$9a_{31}$ $t^3 - 5t^2 + 13t - 17$ $t^5 - 8t^4 + 33t^3 - 80t^2 + 132t - 152$ $8t^{12} - 125t^{11} + 938t^{10} - 4303t^9 + 12544t^8 - 19138t^7 - 17200t^6 + 204143t^5 - 703180t^4 + 1617365t^3 - 2818190t^2 + 3886636t - 4319004$	3 / ✗ 2 / ✗		$9a_{32}$ $t^3 - 6t^2 + 14t - 17$ $-t^5 + 10t^4 - 42t^3 + 94t^2 - 133t + 148$ $8t^{12} - 150t^{11} + 1269t^{10} - 6297t^9 + 19455t^8 - 32720t^7 - 11156t^6 + 260282t^5 - 930836t^4 + 2153618t^3 - 3750358t^2 + 5165114t - 5736454$	3 / ✗ 2 / ✗
	$9a_{33}$ $-t^3 + 6t^2 - 14t + 19$ $t^3 - 10t^2 + 30t - 40$ $9t^{12} - 174t^{11} + 1539t^{10} - 8207t^9 + 28913t^8 - 67184t^7 + 84077t^6 + 55866t^5 - 581640t^4 + 1664798t^3 - 3166838t^2 + 4539202t - 5100726$	3 / ✗ 1 / ✗		$9a_{34}$ $-t^3 + 6t^2 - 16t + 23$ $3t^3 - 18t^2 + 43t - 56$ $9t^{12} - 174t^{11} + 1581t^{10} - 8831t^9 + 32988t^8 - 81774t^7 + 109631t^6 + 73248t^5 - 829341t^4 + 2480938t^3 - 4869197t^2 + 7112552t - 8043256$	3 / ✗ 1 / ✗
	$9a_{35}$ $7t - 13$ $90t - 144$ $-6355t^4 + 58861t^3 - 224539t^2 + 470386t - 596734$	1 / ✗ 2, 3 / ✗		$9a_{36}$ $-t^3 + 5t^2 - 8t + 9$ $-2t^5 + 16t^4 - 44t^3 + 66t^2 - 87t + 88$ $5t^{12} - 65t^{11} + 321t^{10} - 532t^9 - 2081t^8 + 17066t^7 - 64846t^6 + 175611t^5 - 376739t^4 + 668001t^3 - 998037t^2 + 1267342t - 1372104$	3 / ✗ 2 / ✗
	$9a_{37}$ $2t^2 - 11t + 19$ $t^3 - 8t^2 + 22t - 28$ $62t^8 - 928t^7 + 5487t^6 - 13814t^5 - 6681t^4 + 154867t^3 - 520239t^2 + 983348t - 1204192$	2 / ✗ 2 / ✗		$9a_{38}$ $5t^2 - 14t + 19$ $62t^3 - 204t^2 + 382t - 452$ $-1414t^8 + 22122t^7 - 153560t^6 + 657340t^5 - 1976110t^4 + 4454362t^3 - 7806448t^2 + 10855582t - 12103772$	2 / ✗ 2, 3 / ✗
	$9a_{39}$ $-3t^2 + 14t - 21$ $-12t^3 + 84t^2 - 210t + 268$ $174t^8 - 1442t^7 - 690t^6 + 59068t^5 - 366222t^4 + 1247214t^3 - 2815796t^2 + 4505578t - 5255776$	2 / ✗ 1 / ✗		$9a_{40}$ $t^3 - 7t^2 + 18t - 23$ $t^5 - 12t^4 + 57t^3 - 144t^2 + 229t - 264$ $8t^{12} - 175t^{11} + 1712t^{10} - 9738t^9 + 34250t^8 - 66108t^7 - 11148t^6 + 553509t^5 - 2149560t^4 + 5230963t^3 - 9406248t^2 + 13187800t - 14730526$	3 / ✗ 2 / ✗
	$9a_{41}$ $3t^2 - 12t + 19$ $3t^3 - 20t^2 + 70t - 108$ $309t^8 - 3288t^7 + 13885t^6 - 20928t^5 - 55179t^4 + 378100t^3 - 1035810t^2 + 1787808t - 2129794$	2 / ✓ 2 / ✗		$9a_{42}$ $-t^2 + 2t - 1$ $-t^3 + 2t^2 + t - 4$ $3t^8 - 14t^7 + 32t^6 - 96t^5 + 265t^4 - 294t^3 - 498t^2 + 2170t - 3128$	2 / ✗ 1 / ✗
	$9a_{43}$ $-t^3 + 3t^2 - 2t + 1$ $-2t^5 + 8t^4 - 7t^3 + 2t^2 - 5t + 4$ $5t^{12} - 39t^{11} + 110t^{10} - 108t^9 - 115t^8 + 570t^7 - 1477t^6 + 3453t^5 - 6651t^4 + 10951t^3 - 17188t^2 + 24718t - 28462$	3 / ✗ 2 / ✗		$9a_{44}$ $t^2 - 4t + 7$ $-2t^2 + 9t - 12$ $4t^8 - 48t^7 + 237t^6 - 496t^5 - 346t^4 + 4988t^3 - 15044t^2 + 26768t - 32126$	2 / ✗ 1 / ✗
	$9a_{45}$ $-t^2 + 6t - 9$ $t^3 - 14t^2 + 47t - 60$ $3t^8 - 42t^7 + 78t^6 + 1376t^5 - 11135t^4 + 42574t^3 - 102522t^2 + 169806t - 200284$	2 / ✗ 1 / ✗		$9a_{46}$ $5 - 2t$ $3t - 12$ $-2t^4 + 160t^3 - 1125t^2 + 3082t - 4222$	1 / ✓ 2 / ✗
	$9a_{47}$ $t^3 - 4t^2 + 6t - 5$ $-t^5 + 6t^4 - 15t^3 + 16t^2 - 10t + 12$ $8t^{12} - 100t^{11} + 560t^{10} - 1841t^9 + 3847t^8 - 4710t^7 - 42t^6 + 17494t^5 - 55447t^4 + 117058t^3 - 193749t^2 + 261386t - 288924$	3 / ✗ 2 / ✗		$9a_{48}$ $-t^2 + 7t - 11$ $-t^3 + 12t^2 - 42t + 52$ $3t^8 - 49t^7 + 243t^6 + 267t^5 - 8051t^4 + 40499t^3 - 112167t^2 + 199850t - 241202$	2 / ✗ 2 / ✗
	$9a_{49}$ $3t^2 - 6t + 7$ $-21t^3 + 38t^2 - 61t + 60$ $-123t^8 + 1614t^7 - 8744t^6 + 29928t^5 - 75873t^4 + 152714t^3 - 250794t^2 + 338238t - 373944$	2 / ✗ 3 / ✗		$10a_1$ $9 - 4t$ $14t - 40$ $-24t^4 + 2136t^3 - 13430t^2 + 34860t - 47068$	1 / ✗ 1 / ✗
	$10a_2$ $-t^4 + 3t^3 - 3t^2 + 3t - 3$ $3t^7 - 12t^6 + 16t^5 - 20t^4 + 24t^3 - 24t^2 + 27t - 24$ $7t^{16} - 57t^{15} + 189t^{14} - 293t^{13} - 55t^{12} + 1628t^{11} - 5543t^{10} + 13266t^9 - 26589t^8 + 47468t^7 - 77415t^6 + 116549t^5 - 162911t^4 + 212325t^3 - 258413t^2 + 292580t - 305480$	4 / ✗ 3 / ✗		$10a_3$ $13 - 6t$ $11t - 28$ $870t^4 + 1288t^3 - 27795t^2 + 85718t - 120138$	1 / ✓ 2 / ✗
	$10a_4$ $-3t^2 + 7t - 7$ $4t^3 - 8t^2 + t + 8$ $294t^8 - 1807t^7 + 4570t^6 - 4305t^5 - 9550t^4 + 49581t^3 - 117456t^2 + 189330t - 221294$	2 / ✗ 2 / ✗		$10a_5$ $t^4 - 3t^3 + 5t^2 - 5t + 5$ $-2t^7 + 8t^6 - 20t^5 + 28t^4 - 36t^3 + 36t^2 - 39t + 36$ $12t^{16} - 117t^{15} + 565t^{14} - 1757t^{13} + 3847t^{12} - 5960t^{11} + 5381t^{10} + 2968t^9 - 26625t^8 + 75008t^7 - 157415t^6 + 279173t^5 - 436999t^4 + 615297t^3 - 785328t^2 + 909916t - 955948$	4 / ✗ 2 / ✗
	$10a_6$ $-2t^3 + 6t^2 - 7t + 7$ $9t^5 - 36t^4 + 56t^3 - 72t^2 + 81t - 84$ $62t^{12} - 408t^{11} + 712t^{10} + 2280t^9 - 17493t^8 + 60652t^7 - 153492t^6 + 319048t^5 - 569584t^4 + 890397t^3 - 1228657t^2 + 1496150t - 1599330$	3 / ✗ 3 / ✗		$10a_7$ $-3t^2 + 11t - 15$ $14t^3 - 72t^2 + 135t - 160$ $114t^8 - 275t^7 - 5840t^6 + 51739t^5 - 222492t^4 + 626425t^3 - 1267348t^2 + 1914410t - 2193462$	2 / ✗ 1 / ✗
	$10a_8$ $-2t^3 + 5t^2 - 5t + 5$ $7t^5 - 20t^4 + 23t^3 - 28t^2 + 26t - 24$ $94t^{12} - 672t^{11} + 2115t^{10} - 3678t^9 + 2535t^8 + 6453t^7 - 30645t^6 + 78385t^5 - 154895t^4 + 256601t^3 - 367525t^2 + 458500t - 494524$	3 / ✗ 2 / ✗		$10a_9$ $-t^4 + 3t^3 - 5t^2 + 7t - 7$ $-t^7 + 4t^6 - 10t^5 + 20t^4 - 25t^3 + 28t^2 - 28t + 28$ $15t^{16} - 153t^{15} + 787t^{14} - 2727t^{13} + 7084t^{12} - 14404t^{11} + 22886t^{10} - 26134t^9 + 11540t^8 + 39332t^7 - 146866t^6 + 325115t^5 - 571077t^4 + 856941t^3 - 1131013t^2 + 1330668t - 1403980$	4 / ✗ 1 / ✗
	$10a_{10}$ $3t^2 - 11t + 17$ $-5t^3 + 24t^2 - 71t + 100$ $285t^8 - 2735t^7 + 10078t^6 - 9479t^5 - 64000t^4 + 327253t^3 - 827377t^2 + 1378130t - 1624314$	2 / ✗ 1 / ✗		$10a_{11}$ $-4t^2 + 11t - 13$ $16t^3 - 52t^2 + 68t - 72$ $736t^8 - 4672t^7 + 9634t^6 + 11132t^5 - 125367t^4 + 413121t^3 - 873095t^2 + 1336974t - 1536906$	2 / ✗ 2, 3 / ✗
	$10a_{12}$ $2t^3 - 6t^2 + 10t - 11$ $-5t^5 + 20t^4 - 50t^3 + 72t^2 - 89t + 92$ $118t^{12} - 1080t^{11} + 4748t^{10} - 12624t^9 + 19414t^8 - 2072t^7 - 88507t^6 + 320836t^5 - 750453t^4 + 1366922t^3 - 2053481t^2 + 2604638t - 2816934$	3 / ✗ 2 / ✗		$10a_{13}$ $2t^2 - 13t + 23$ $t^3 - 12t^2 + 51t - 84$ $62t^8 - 1088t^7 + 7367t^6 - 20586t^5 - 13356t^4 + 286509t^3 - 1005098t^2 + 1954280t - 2416160$	2 / ✗ 2 / ✗

knot diag	n_k^t Alexander's ω^+ $(\rho_1^t)^+$	genus / ribbon unknotting # / amphi?	knot diag	n_k^t Alexander's ω^+ $(\rho_2^t)^+$	genus / ribbon unknotting # / amphi?
	$10^a_{14} \quad -2t^3 + 8t^2 - 12t + 13$ $9t^5 - 52t^4 + 119t^3 - 180t^2 + 225t - 236$ $62t^{12} - 584t^{11} + 1720t^{10} + 2816t^9 - 42848t^8 + 195040t^7 - 594177t^6 + 1407688t^5 - 2753604t^4 + 4575154t^3 - 6545078t^2 + 8106820t - 8706026$	3 / ✗ 2 / ✗		$10^a_{15} \quad 2t^3 - 6t^2 + 9t - 9$ $-3t^5 + 12t^4 - 24t^3 + 24t^2 - 17t + 12$ $134t^{12} - 1272t^{11} + 5792t^{10} - 16520t^9 + 31765t^8 - 37636t^7 + 2396t^6 + 120176t^5 - 371368t^4 + 752873t^3 - 1195043t^2 + 1560190t - 1702986$	3 / ✗ 2 / ✗
	$10^a_{16} \quad -4t^2 + 12t - 15$ $-16t^3 + 56t^2 - 76t + 80$ $736t^8 - 5248t^7 + 12944t^6 + 6528t^5 - 14416t^4 + 522200t^3 - 1155370t^2 + 1809228t - 2093696$	2 / ✗ 2 / ✗		$10^a_{17} \quad t^4 - 3t^3 + 5t^2 - 7t + 9$ 0 $16t^{16} - 165t^{15} + 861t^{14} - 3043t^{13} + 8173t^{12} - 17514t^{11} + 30162t^{10} - 39958t^9 + 32666t^8 + 13998t^7 - 125081t^6 + 317743t^5 - 588481t^4 + 904569t^3 - 1207020t^2 + 1426556t - 1506972$	4 / ✗ 1 / ✓
	$10^a_{18} \quad -4t^2 + 14t - 19$ $16t^3 - 68t^2 + 121t - 140$ $736t^8 - 6240t^7 + 17736t^6 + 11088t^5 - 245648t^4 + 930168t^3 - 2109201t^2 + 3338706t - 3874682$	2 / ✗ 1 / ✗		$10^a_{19} \quad 2t^3 - 7t^2 + 11t - 11$ $3t^5 - 16t^4 + 35t^3 - 40t^2 + 30t - 24$ $134t^{12} - 1480t^{11} + 7641t^{10} - 24194t^9 + 50855t^8 - 66007t^7 + 12323t^6 + 201357t^5 - 665287t^4 + 1397797t^3 - 2271085t^2 + 3006128t - 3296368$	3 / ✗ 2 / ✗
	$10^a_{20} \quad -3t^2 + 9t - 11$ $14t^3 - 56t^2 + 88t - 104$ $114t^8 - 153t^7 - 4783t^6 + 34425t^5 - 128711t^4 + 327435t^3 - 618704t^2 + 899066t - 1017366$	2 / ✗ 2 / ✗		$10^a_{21} \quad -2t^3 + 7t^2 - 9t + 9$ $9t^5 - 44t^4 + 80t^3 - 104t^2 + 121t - 124$ $62t^{12} - 496t^{11} + 1203t^{10} + 2078t^9 - 24456t^8 + 97163t^7 - 267878t^6 + 592041t^5 - 1106738t^4 + 1789591t^3 - 2525732t^2 + 3113752t - 3341184$	3 / ✗ 2 / ✗
	$10^a_{22} \quad -2t^3 + 6t^2 - 10t + 13$ $-t^5 + 4t^4 - 10t^3 + 24t^2 - 37t + 44$ $142t^{12} - 1368t^{11} + 6524t^{10} - 20120t^9 + 42790t^8 - 57928t^7 + 16919t^6 + 158700t^5 - 540707t^4 + 1130294t^3 - 1809643t^2 + 2363114t - 2577418$	3 / ✓ 2 / ✗		$10^a_{23} \quad 2t^3 - 7t^2 + 13t - 15$ $-5t^5 + 24t^4 - 67t^3 + 108t^2 - 137t + 144$ $118t^{12} - 1272t^{11} + 6541t^{10} - 20402t^9 + 38443t^8 - 21945t^7 - 132442t^6 + 594335t^5 - 1530420t^4 + 2960363t^3 - 4622193t^2 + 5992048t - 6526360$	3 / ✗ 1 / ✗
	$10^a_{24} \quad -4t^2 + 14t - 19$ $24t^3 - 116t^2 + 221t - 268$ $416t^8 - 1568t^7 - 13224t^6 + 136928t^5 - 604124t^4 + 1701008t^3 - 3414673t^2 + 5118714t - 5846946$	2 / ✗ 2 / ✗		$10^a_{25} \quad -2t^3 + 8t^2 - 14t + 17$ $9t^5 - 52t^4 + 131t^3 - 232t^2 + 314t - 344$ $62t^{12} - 584t^{11} + 1856t^{10} + 2264t^9 - 47052t^8 + 241288t^7 - 809541t^6 + 2068016t^5 - 4270010t^4 + 7347930t^3 - 10723331t^2 + 13406206t - 14434208$	3 / ✗ 2 / ✗
	$10^a_{26} \quad -2t^3 + 7t^2 - 13t + 17$ $-t^5 + 4t^4 - 10t^3 + 28t^2 - 49t + 60$ $142t^{12} - 1600t^{11} + 8823t^{10} - 31058t^9 + 74964t^8 - 117897t^7 + 67064t^6 + 255997t^5 - 1047600t^4 + 2360395t^3 - 3947888t^2 + 5281288t - 5805248$	3 / ✗ 1 / ✗		$10^a_{27} \quad 2t^3 - 8t^2 + 16t - 19$ $5t^5 - 28t^4 + 87t^3 - 164t^2 + 229t - 252$ $118t^{12} - 1464t^{11} + 8536t^{10} - 29792t^9 + 62096t^8 - 39696t^7 - 242195t^6 + 1151848t^5 - 3078140t^4 + 6098910t^3 - 9661940t^2 + 12621240t - 13779050$	3 / ✗ 1 / ✗
	$10^a_{28} \quad 4t^2 - 13t + 19$ $-8t^3 + 36t^2 - 100t + 136$ $928t^8 - 7872t^7 + 26174t^6 - 22588t^5 - 142295t^4 + 689113t^3 - 1676391t^2 + 2728998t - 3192146$	2 / ✗ 2 / ✗		$10^a_{29} \quad t^3 - 7t^2 + 15t - 17$ $t^5 - 12t^4 + 52t^3 - 104t^2 + 124t - 128$ $8t^{12} - 175t^{11} + 1659t^{10} - 8913t^9 + 29252t^8 - 54292t^7 + 10680t^6 + 290989t^5 - 1126663t^4 + 2673211t^3 - 4723498t^2 + 6566572t - 7317656$	3 / ✗ 2 / ✗
	$10^a_{30} \quad -4t^2 + 17t - 25$ $24t^3 - 148t^2 + 345t - 440$ $416t^8 - 2048t^7 - 17490t^6 + 219996t^5 - 1101894t^4 + 3396907t^3 - 7245510t^2 + 11243734t - 12988226$	2 / ✗ 1 / ✗		$10^a_{31} \quad 4t^2 - 14t + 21$ $-4t^2 + 9t - 12$ $992t^8 - 9440t^7 + 36936t^6 - 59136t^5 - 72624t^4 + 623304t^3 - 1691899t^2 + 2867550t - 3391374$	2 / ✗ 1 / ✗
	$10^a_{32} \quad -2t^3 + 8t^2 - 15t + 19$ $t^5 - 4t^4 + 13t^3 - 40t^2 + 78t - 96$ $142t^{12} - 1832t^{11} + 11204t^{10} - 42688t^9 + 109909t^8 - 184384t^7 + 124831t^6 + 360782t^5 - 1615391t^4 + 3759585t^3 - 6404890t^2 + 8655360t - 9545252$	3 / ✗ 1 / ✗		$10^a_{33} \quad 4t^2 - 16t + 25$ 0 $992t^8 - 10816t^7 + 47856t^6 - 88336t^5 - 84402t^4 + 920320t^3 - 2655340t^2 + 4640912t - 5542372$	2 / ✗ 1 / ✓
	$10^a_{34} \quad 3t^2 - 9t + 13$ $-5t^3 + 20t^2 - 52t + 68$ $285t^8 - 2205t^7 + 6601t^6 - 3429t^5 - 43369t^4 + 185703t^3 - 431857t^2 + 687874t - 799218$	2 / ✗ 2 / ✗		$10^a_{35} \quad 2t^2 - 12t + 21$ $-t^3 + 12t^2 - 47t + 76$ $62t^8 - 1000t^7 + 6244t^6 - 15744t^5 - 15707t^4 + 232680t^3 - 775840t^2 + 1474372t - 1810118$	2 / ✓ 2 / ✗
	$10^a_{36} \quad -3t^2 + 13t - 19$ $14t^3 - 88t^2 + 208t - 264$ $114t^8 - 397t^7 - 7597t^6 + 81141t^5 - 393441t^4 + 1198967t^3 - 2544952t^2 + 3941362t - 4550398$	2 / ✗ 2 / ✗		$10^a_{37} \quad 4t^2 - 13t + 19$ 0 $992t^8 - 8736t^7 + 31914t^6 - 47212t^5 - 64499t^4 + 497921t^3 - 1308755t^2 + 2181630t - 2566522$	2 / ✗ 2 / ✓
	$10^a_{38} \quad -4t^2 + 15t - 21$ $24t^3 - 128t^2 + 270t - 336$ $416t^8 - 1632t^7 - 16122t^6 + 172460t^5 - 788845t^4 + 2280037t^3 - 4653713t^2 + 7038342t - 8061882$	2 / ✗ 2 / ✗		$10^a_{39} \quad -2t^3 + 8t^2 - 13t + 15$ $9t^5 - 52t^4 + 125t^3 - 204t^2 + 263t - 280$ $62t^{12} - 584t^{11} + 1788t^{10} + 2480t^9 - 44191t^8 + 213488t^7 - 683173t^6 + 1684054t^5 - 3393468t^4 + 5753447t^3 - 8330571t^2 + 10379080t - 11164828$	3 / ✗ 2 / ✗
	$10^a_{40} \quad 2t^3 - 8t^2 + 17t - 21$ $-5t^5 + 28t^4 - 89t^3 + 176t^2 - 258t + 288$ $118t^{12} - 1464t^{11} + 8692t^{10} - 31256t^9 + 67987t^8 - 49624t^7 - 257955t^6 + 1301482t^5 - 3582545t^4 + 7240253t^3 - 11620382t^2 + 15292356t - 16735336$	3 / ✗ 2 / ✗		$10^a_{41} \quad t^3 - 7t^2 + 17t - 21$ $t^5 - 12t^4 + 54t^3 - 120t^2 + 157t - 164$ $8t^{12} - 175t^{11} + 1697t^{10} - 9543t^9 + 33561t^8 - 69114t^7 + 29117t^6 + 354127t^5 - 1527139t^4 + 3836499t^3 - 7019042t^2 + 9942516t - 11145016$	3 / ✗ 2 / ✗
	$10^a_{42} \quad -t^3 + 7t^2 - 19t + 27$ $2t^3 - 8t^2 + 11t - 12$ $9t^{12} - 203t^{11} + 2093t^{10} - 12971t^9 + 52885t^8 - 142268t^7 + 214987t^6 + 60931t^5 - 1368859t^4 + 4365895t^3 - 8815357t^2 + 13058404t - 14831092$	3 / ✓ 1 / ✗		$10^a_{43} \quad -t^3 + 7t^2 - 17t + 23$ 0 $9t^{12} - 203t^{11} + 2051t^{10} - 12253t^9 + 47594t^8 - 120962t^7 + 170450t^6 + 61017t^5 - 1045911t^4 + 3175271t^3 - 6209661t^2 + 9025932t - 10186676$	3 / ✗ 2 / ✓
	$10^a_{44} \quad t^3 - 7t^2 + 19t - 25$ $t^5 - 12t^4 + 56t^3 - 140t^2 + 220t - 248$ $8t^{12} - 175t^{11} + 1735t^{10} - 10157t^9 + 37586t^8 - 81160t^7 + 29232t^6 + 500937t^5 - 2197451t^4 + 5635115t^3 - 10448058t^2 + 14900236t - 16735696$	3 / ✗ 1 / ✗		$10^a_{45} \quad -t^3 + 7t^2 - 21t + 31$ 0 $9t^{12} - 203t^{11} + 2135t^{10} - 13689t^9 + 58324t^8 - 165246t^7 + 266640t^6 + 52413t^5 - 1738539t^4 + 5821367t^3 - 12123077t^2 + 18290148t - 20900556$	3 / ✗ 2 / ✓
	$10^a_{46} \quad -t^4 + 3t^3 - 4t^2 + 5t - 5$ $-3t^7 + 12t^6 - 21t^5 + 34t^4 - 43t^3 + 52t^2 - 55t + 56$ $7t^{16} - 57t^{15} + 204t^{14} - 382t^{13} + 69t^{12} + 2247t^{11} - 9674t^{10} + 27287t^9 - 61957t^8 + 121378t^7 - 211961t^6 + 335438t^5 - 485235t^4 + 644818t^3 - 789365t^2 + 891215t - 928064$	4 / ✗ 3 / ✗		$10^a_{47} \quad t^4 - 3t^3 + 6t^2 - 7t + 7$ $-2t^7 + 8t^6 - 23t^5 + 38t^4 - 56t^3 + 60t^2 - 68t + 64$ $12t^{16} - 117t^{15} + 598t^{14} - 2030t^{13} + 4959t^{12} - 8715t^{11} + 9312t^{10} + 2921t^9 - 44823t^8 + 139602t^7 - 312112t^6 + 579182t^5 - 936546t^4 + 1347538t^3 - 1741633t^2 + 2029805t - 2135930$	4 / ✗ 2, 3 / ✗
	$10^a_{48} \quad t^4 - 3t^3 + 6t^2 - 9t + 11$ $t^5 - 2t^4 + 2t^3 - 3t + 4$ $16t^{16} - 165t^{15} + 906t^{14} - 3452t^{13} + 10069t^{12} - 23423t^{11} + 43765t^{10} - 63343t^9 + 59588t^8 + 8232t^7 - 192505t^6 + 537134t^5 - 1048176t^4 + 1669528t^3 - 2281994t^2 + 2735109t - 2902594$	4 / ✓ 2 / ✗		$10^a_{49} \quad 3t^3 - 8t^2 + 12t - 13$ $30t^5 - 94t^4 + 196t^3 - 292t^2 + 372t - 392$ $-177t^{12} + 3028t^{11} - 22080t^{10} + 101361t^9 - 341354t^8 + 914348t^7 - 2044469t^6 + 3931812t^5 - 6622778t^4 + 9874270t^3 - 13105110t^2 + 15522532t - 16422794$	3 / ✗ 3 / ✗

knot diag	n_k^t Alexander's ω^+ $(\rho_1^t)^+$	genus / ribbon unknotting # / amphi?	knot diag	n_k^t Alexander's ω^+ $(\rho_1^t)^+$	genus / ribbon unknotting # / amphi?
	10_{118}^a 0 $t^4 - 5t^3 + 12t^2 - 19t + 23$ $16r^{16} - 275r^{15} + 2305r^{14} - 12526r^{13} + 49379r^{12} - 149077r^{11} + 352067r^{10} - 641987r^9 + 825146r^8 - 399494r^7 - 1458086r^6 + 5641784r^5 - 12589879r^4 + 21712756r^3 - 31187934r^2 + 38432195r - 41152780$	4 / ✗ 1 / ✓		10_{119}^a $-2t^3 + 10t^2 - 23t + 31$ $-t^5 + 6t^4 - 26t^3 + 86t^2 - 175t + 220$ $142r^{12} - 2288r^{11} + 17392r^{10} - 81560r^9 + 255719r^8 - 521820r^7 + 483354r^6 + 990524r^5 - 5618050r^4 + 14499405r^3 - 26339835r^2 + 36916418r - 41198798$	3 / ✗ 1 / ✗
	10_{120}^a $8t^2 - 26t + 37$ $166t^3 - 692t^2 + 1433t - 1788$ $-11768r^8 + 201320r^7 - 1541132r^6 + 7193960r^5 - 23193562r^4 + 55098408r^3 - 100101157r^2 + 142136186r - 159564534$	2 / ✗ 2, 3 / ✗		10_{121}^a $2t^3 - 11t^2 + 27t - 35$ $5t^5 - 42t^4 + 167t^3 - 396t^2 + 634t - 732$ $118r^{12} - 2016r^{11} + 15853r^{10} - 73450r^9 + 204605r^8 - 232351r^7 - 764251r^6 + 5054205r^5 - 15890853r^4 + 35160633r^3 - 59996079r^2 + 81831748r - 90616328$	3 / ✗ 2 / ✗
	10_{122}^a $-2t^3 + 11t^2 - 24t + 31$ $-t^5 + 8t^4 - 34t^3 + 104t^2 - 211t + 264$ $142r^{12} - 2512r^{11} + 20355r^{10} - 99362r^9 + 318535r^8 - 657014r^7 + 617040r^6 + 1199636r^5 - 6869579r^4 + 17663208r^3 - 31953091r^2 + 44656222r - 49787168$	3 / ✗ 2 / ✗		10_{123}^a 0 $t^4 - 6r^3 + 15r^2 - 24t + 29$ $16r^{16} - 330r^{15} + 3216r^{14} - 19770r^{13} + 86170r^{12} - 282500r^{11} + 715162r^{10} - 1388790r^9 + 1917350r^8 - 1169720r^7 - 2832520r^6 + 12363784r^5 - 28689660r^4 + 50560110r^3 - 73579700r^2 + 91325158r - 98015944$	4 / ✓ 2 / ✓
	10_{124}^n $t^4 - t^3 + t - 1$ $-4t^7 - 6t^4 - 4t^2 - 6t$ $9r^{15} - 25r^{14} + 10r^{13} + 75r^{12} - 177r^{11} + 155r^{10} + 113r^9 - 570r^8 + 850r^7 - 428r^6 - 824r^5 + 2167r^4 - 2340r^3 + 510r^2 + 2375r - 3832$	4 / ✗ 4 / ✗		10_{125}^n $t^5 - 2r^2 + 2t - 1$ $-t^5 + 2t^4 - 2t^3 + 3t - 4$ $8r^{12} - 50r^{11} + 151r^{10} - 289r^9 + 417r^8 - 524r^7 + 536r^6 - 150r^5 - 1168r^4 + 3942r^3 - 8130r^2 + 12314r - 14126$	3 / ✗ 2 / ✗
	10_{126}^n $t^3 - 2t^2 + 4t - 5$ $t^5 - 2t^4 + 10t^3 - 12t^2 + 22t - 20$ $8r^{12} - 50r^{11} + 185r^{10} - 457r^9 + 666r^8 - 18r^7 - 3074r^6 + 10724r^5 - 24495r^4 + 43738r^3 - 64631r^2 + 81072r - 87356$	3 / ✗ 2 / ✗		10_{127}^n $-t^3 + 4t^2 - 6t + 7$ $2t^5 - 14t^4 + 32t^3 - 52t^2 + 67t - 72$ $5r^{12} - 48r^{11} + 128r^{10} + 289r^9 - 3551r^8 + 15554r^7 - 46589r^6 + 109206r^5 - 211625r^4 + 348370r^3 - 494107r^2 + 608154r - 651576$	3 / ✗ 2 / ✗
	10_{128}^n $2t^3 - 3t^2 + t + 1$ $-13t^5 + 12t^4 - 3t^3 - 10t^2 - 9t + 12$ $-26r^{12} + 296r^{11} - 1071r^{10} + 1750r^9 - 1107r^8 + 287r^7 - 2938r^6 + 7959r^5 - 7820r^4 + 3175r^3 - 8727r^2 + 28392r - 40368$	3 / ✗ 3 / ✗		10_{129}^n $2t^2 - 6t + 9$ $-t^3 - 2t^2 + 14t - 20$ $62r^8 - 568r^7 + 2280r^6 - 4308r^5 - 553r^4 + 25616r^3 - 76125r^2 + 132258r - 157332$	2 / ✓ 1 / ✗
	10_{130}^n $2t^2 - 4t + 5$ $t^3 - 2t^2 + 19t - 24$ $62r^8 - 336r^7 + 924r^6 - 1568r^5 + 253r^4 + 8384r^3 - 28668r^2 + 53628r - 65374$	2 / ✗ 2 / ✗		10_{131}^n $-2t^2 + 8t - 11$ $5t^3 - 38t^2 + 87t - 112$ $38r^8 - 272r^7 - 580r^6 + 12792r^5 - 66417r^4 + 202096r^3 - 422662r^2 + 646440r - 742870$	2 / ✗ 1 / ✗
	10_{132}^n $t^2 - t + 1$ $2t^2 + 5t - 4$ $4r^8 - 7r^7 + 12r^6 - 145r^5 + 508r^4 - 631r^3 - 322t^2 + 2150r - 3150$	2 / ✗ 1 / ✗		10_{133}^n $-t^2 + 5t - 7$ $t^3 - 14t^2 + 37t - 48$ $3r^8 - 43r^7 + 16r^6 + 1489r^5 - 9322r^4 + 30945r^3 - 68047r^2 + 106954r - 123994$	2 / ✗ 1 / ✗
	10_{134}^n $2t^3 - 4t^2 + 4t - 3$ $-13t^5 + 24t^4 - 33t^3 + 30t^2 - 41t + 40$ $-26r^{12} + 376r^{11} - 2056r^{10} + 6760r^9 - 16248r^8 + 32568r^7 - 58951r^6 + 98316r^5 - 150194r^4 + 210738r^3 - 273246r^2 + 324124r - 344346$	3 / ✗ 3 / ✗		10_{135}^n $3t^2 - 9t + 13$ $t^3 - 6t^2 + 18t - 24$ $321r^8 - 2613r^7 + 8905r^6 - 12033r^5 - 19329r^4 + 132451r^3 - 337025r^2 + 553002r - 647370$	2 / ✗ 2 / ✗
	10_{136}^n $-t^2 + 4t - 5$ $-t^3 + 4t^2 - 2t - 4$ $3r^8 - 36r^7 + 189r^6 - 512r^5 + 347r^4 + 2660r^3 - 11142r^2 + 22668r - 28354$	2 / ✗ 1 / ✗		10_{137}^n $t^2 - 6t + 11$ $-4t^2 + 24t - 44$ $4r^8 - 74r^7 + 512r^6 - 1420r^5 - 1160r^4 + 21074r^3 - 72904r^2 + 140922r - 173900$	2 / ✓ 1 / ✗
	10_{138}^n $t^3 - 5t^2 + 8t - 7$ $-t^5 + 8t^4 - 22t^3 + 24t^2 - 11t + 8$ $8r^{12} - 125r^{11} + 855r^{10} - 3374r^9 + 8458r^8 - 13328r^7 + 8173r^6 + 25863r^5 - 114602r^4 + 277037r^3 - 497313r^2 + 702260r - 787812$	3 / ✗ 2 / ✗		10_{139}^n $t^4 - t^3 + 2t - 3$ $-4t^7 - 12t^4 + 5t^3 - 4t^2 - 16t + 12$ $9r^{15} - 25r^{14} - 3r^{13} + 172r^{12} - 425r^{11} + 290r^{10} + 924r^9 - 3099r^8 + 4327r^7 - 1756r^6 - 5200r^5 + 12117r^4 - 11846r^3 + 1547r^2 + 12451r - 19002$	4 / ✗ 4 / ✗
	10_{140}^n $t^2 - 2t + 3$ $8t - 8$ $4r^8 - 22r^7 + 90r^6 - 292r^5 + 424r^4 + 430r^3 - 3056r^2 + 6470r - 8104$	2 / ✓ 2 / ✗		10_{141}^n $-t^3 + 3t^2 - 4t + 5$ $t^3 - 8t^2 + 16t - 20$ $9r^{12} - 87r^{11} + 396r^{10} - 1150r^9 + 2382r^8 - 3516r^7 + 2746r^6 + 3397r^5 - 19148r^4 + 46359r^3 - 80476r^2 + 109936r - 121692$	3 / ✗ 1 / ✗
	10_{142}^n $2t^3 - 3t^2 + 2t - 1$ $-13t^5 + 12t^4 - 13t^3 + 4t^2 - 17t + 12$ $-26r^{12} + 296r^{11} - 1155r^{10} + 2582r^9 - 4276r^8 + 6812r^7 - 11749r^6 + 19392r^5 - 27878r^4 + 36798r^3 - 48891r^2 + 62932r - 69706$	3 / ✗ 3 / ✗		10_{143}^n $t^3 - 3t^2 + 6t - 7$ $t^5 - 4t^4 + 15t^3 - 28t^2 + 45t - 48$ $8r^{12} - 75r^{11} + 362r^{10} - 1106r^9 + 2070r^8 - 1092r^7 - 7698r^6 + 33841r^5 - 86216r^4 + 164927r^3 - 254838r^2 + 327896r - 356170$	3 / ✗ 1 / ✗
	10_{144}^n $-3t^2 + 10t - 13$ $10t^3 - 44t^2 + 80t - 96$ $222r^8 - 1642r^7 + 3140r^6 + 12252r^5 - 94326r^4 + 307146r^3 - 651636r^2 + 998418r - 1147140$	2 / ✗ 2 / ✗		10_{145}^n $t^2 + t - 3$ $2t^3 + 8t^2 + 6t - 8$ $-5r^7 + 7r^6 + 113r^5 - 141r^4 - 465r^3 + 730r^2 + 850r - 2198$	2 / ✗ 2 / ✗
	10_{146}^n $2t^2 - 8t + 13$ $t^3 - 8t^2 + 21t - 28$ $62r^8 - 664r^7 + 2844r^6 - 4544r^5 - 9663r^4 + 71376r^3 - 197106r^2 + 340392r - 405394$	2 / ✗ 1 / ✗		10_{147}^n $-2t^2 + 7t - 9$ $-3t^3 + 12t^2 - 15t + 12$ $54r^8 - 488r^7 + 1697r^6 - 1694r^5 - 8312r^4 + 42905r^3 - 107222r^2 + 177492r - 208860$	2 / ✗ 1 / ✗
	10_{148}^n $t^3 - 3t^2 + 7t - 9$ $t^5 - 4t^4 + 18t^3 - 36t^2 + 62t - 68$ $8r^{12} - 75r^{11} + 377r^{10} - 1209r^9 + 2330r^8 - 864r^7 - 11900r^6 + 51677r^5 - 135261r^4 + 266207r^3 - 420746r^2 + 549160r - 599424$	3 / ✗ 2 / ✗		10_{149}^n $-t^3 + 5t^2 - 9t + 11$ $2t^5 - 18t^4 + 55t^3 - 104t^2 + 149t - 164$ $5r^{12} - 61r^{11} + 226r^{10} + 339r^9 - 7195r^8 + 38874r^7 - 135727r^6 + 357173r^5 - 753890r^4 + 1318245r^3 - 1945105r^2 + 2447584r - 2640944$	3 / ✗ 2 / ✗
	10_{150}^n $-t^3 + 4t^2 - 6t + 7$ $-2t^5 + 12t^4 - 26t^3 + 38t^2 - 45t + 44$ $5r^{12} - 52r^{11} + 216r^{10} - 355r^9 - 719r^8 + 6578r^7 - 24361r^6 + 64526r^5 - 137117r^4 + 243126r^3 - 364723r^2 + 464942r - 504136$	3 / ✗ 2 / ✗		10_{151}^n $t^3 - 4t^2 + 10t - 13$ $-t^5 + 6t^4 - 21t^3 + 42t^2 - 66t + 72$ $8r^{12} - 100r^{11} + 632r^{10} - 2529r^9 + 6645r^8 - 9606r^7 - 5854r^6 + 80466r^5 - 270269r^4 + 605378r^3 - 1033839r^2 + 1408362r - 1558600$	3 / ✗ 2 / ✗
	10_{152}^n $t^4 - t^3 - t^2 + 4t - 5$ $4t^7 - 7t^5 + 18t^4 - 7t^3 - 12t^2 + 45t - 52$ $9r^{15} - 14r^{14} - 92r^{13} + 396r^{12} - 419r^{11} - 1212r^{10} + 5444r^9 - 9692r^8 + 6412r^7 + 11488r^6 - 39344r^5 + 55244r^4 - 33234r^3 - 30168r^2 + 102115r - 133894$	4 / ✗ 4 / ✗		10_{153}^n $t^3 - t^2 - t + 3$ $t^5 - 2t^4 + t^3 + 2t^2 - t$ $8r^{12} - 17r^{11} - 46r^{10} + 231r^9 - 381r^8 + 364r^7 - 367r^6 + 157r^5 + 1142r^4 - 2815r^3 + 1874r^2 + 2128r - 4572$	3 / ✓ 2 / ✗
	10_{154}^n $t^3 - 4t + 7$ $-3t^5 - 6t^4 + 13t^3 - 47t + 68$ $48r^{10} - 93r^9 - 546r^8 + 2396r^7 - 1956r^6 - 8376r^5 + 25906r^4 - 23802r^3 - 25690r^2 + 102540r - 140874$	3 / ✗ 3 / ✗		10_{155}^n $-t^3 + 3t^2 - 5t + 7$ $-2t^3 + 12t^2 - 22t + 28$ $9r^{12} - 87r^{11} + 417r^{10} - 1321r^9 + 3014r^8 - 4806r^7 + 3646r^6 + 6917r^5 - 34773r^4 + 82963r^3 - 142781r^2 + 193836r - 214060$	3 / ✓ 2 / ✗

knot diag	n_k^t Alexander's ω^+ $(\rho_1^t)^+$	genus / ribbon unknotting # / amphi?	knot diag	n_k^t Alexander's ω^+ $(\rho_1^t)^+$	genus / ribbon unknotting # / amphi?
	10_{156}^n $t^3 - 4t^2 + 8t - 9$ $t^5 - 6t^4 + 19t^3 - 30t^2 + 33t - 32$ $8t^{12} - 100t^{11} + 594t^{10} - 2165t^9 + 5120t^8 - 6852t^7 - 2208t^6 + 41208t^5 - 134214t^4 + 293026t^3 - 493422t^2 + 668112t - 738218$	3 / ✗ 1 / ✗		10_{157}^n $-t^3 + 6t^2 - 11t + 13$ $-2t^5 + 22t^4 - 78t^3 + 148t^2 - 218t + 240$ $5t^{12} - 74t^{11} + 340t^{10} + 321t^9 - 11314t^8 + 67637t^7 - 250977t^6 + 688036t^5 - 1493487t^4 + 2661131t^3 - 3974091t^2 + 5034465t - 5444000$	3 / ✗ 2 / ✗
	10_{158}^n $-t^3 + 4t^2 - 10t + 15$ $2t^2 - 7t + 12$ $9t^{12} - 116t^{11} + 764t^{10} - 3275t^9 + 9743t^8 - 19422t^7 + 18439t^6 + 32898t^5 - 196271t^4 + 513374t^3 - 940025t^2 + 1323614t - 1479452$	3 / ✗ 2 / ✗		10_{159}^n $t^3 - 4t^2 + 9t - 11$ $t^5 - 6t^4 + 26t^3 - 60t^2 + 98t - 112$ $8t^{12} - 100t^{11} + 609t^{10} - 2267t^9 + 5047t^8 - 3237t^7 - 23513t^6 + 115362t^5 - 318739t^4 + 648093t^3 - 1045247t^2 + 1379659t - 1511358$	3 / ✗ 1 / ✗
	10_{160}^n $-t^3 + 4t^2 - 4t + 3$ $-2t^5 + 12t^4 - 20t^3 + 14t^2 - 16t + 12$ $5t^{12} - 52t^{11} + 198t^{10} - 255t^9 - 522t^8 + 3092t^7 - 8443t^6 + 18756t^5 - 37588t^4 + 67858t^3 - 108568t^2 + 148444t - 165862$	3 / ✗ 2 / ✗		10_{161}^n $t^3 - 2t + 3$ $3t^5 + 6t^4 - 3t^3 + 4t^2 + 14t - 12$ $30t^{10} - 53t^9 - 145t^8 + 630t^7 - 674t^6 - 870t^5 + 3591t^4 - 4450t^3 + 581t^2 + 6166t - 9640$	3 / ✗ 3 / ✗
	10_{162}^n $-3t^2 + 9t - 11$ $10t^3 - 38t^2 + 58t - 68$ $222t^8 - 1473t^7 + 2609t^6 + 8829t^5 - 65543t^4 + 206079t^3 - 427536t^2 + 647498t - 741358$	2 / ✗ 2 / ✗		10_{163}^n $t^3 - 5t^2 + 12t - 15$ $-t^5 + 8t^4 - 30t^3 + 62t^2 - 89t + 96$ $8t^{12} - 125t^{11} + 923t^{10} - 4154t^9 + 12040t^8 - 19732t^7 - 4345t^6 + 140575t^5 - 506052t^4 + 1171653t^3 - 2040193t^2 + 2809224t - 3119648$	3 / ✗ 1, 2 / ✗
	10_{164}^n $3t^2 - 11t + 17$ $t^3 - 10t^2 + 29t - 40$ $321t^8 - 3179t^7 + 12782t^6 - 20103t^5 - 32876t^4 + 254013t^3 - 688337t^2 + 1170838t - 1386922$	2 / ✗ 1 / ✗		10_{165}^n $-2t^2 + 10t - 15$ $-5t^3 + 50t^2 - 146t + 196$ $38t^8 - 344t^7 - 848t^6 + 23020t^5 - 137555t^4 + 465256t^3 - 1047705t^2 + 1673914t - 1951560$	2 / ✗ 2 / ✗