



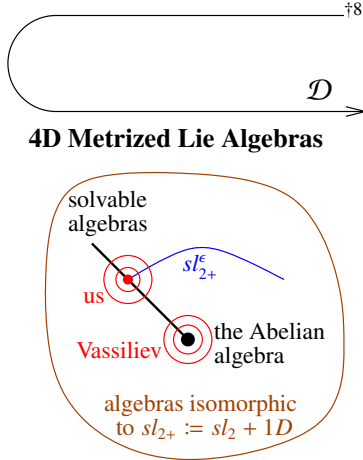
Everything around sl_{2+}^ϵ is DoPeGDO. So what?

Abstract. I'll explain what "everything around" means: classical and quantum $m, \Delta, S, tr, R, C,$ and $\theta,$ as well as $P, \Phi, J, \mathbb{D},$ and more, and all of their compositions. What **DoPeGDO** means: the category of **Docile Perturbed Gaussian Differential Operators**. And what sl_{2+}^ϵ means: a solvable approximation of the semi-simple Lie algebra sl_2 .

Knot theorists should rejoice because all this leads to very powerful and well-behaved poly-time-computable knot invariants. Quantum algebraists should rejoice because it's a realistic playground for testing complicated equations and theories.

Conventions. 1. For a set $A,$ let $z_A := \{z_i\}_{i \in A}$ and let $\zeta_A := \{z_i^* = \zeta_i\}_{i \in A}.$ †1. Everything converges!

Less Abstract



DoPeGDO := The category with objects finite sets^{†2} and $\text{mor}(A \rightarrow B):$

$$\{\mathcal{F} = \omega \exp(Q + P)\} \subset \mathbb{Q}[[\zeta_A, z_B]]$$

Where: • ω is a scalar.^{†3} • Q is a "small" quadratic in $\zeta_A \cup z_B.$ ^{†4} • P is a "docile perturbation": $P = \sum_{k \geq 1} \epsilon^k P^{(k)},$ where $\text{deg } P^{(k)} \leq 2k + 2.$ ^{†5} • Compositions:^{†6}

$$\mathcal{F} // \mathcal{G} = \mathcal{G} \circ \mathcal{F} := (\mathcal{G}|_{\zeta_i \rightarrow \partial_{z_i}} \mathcal{F})_{z_i=0} = (\mathcal{F}|_{z_i \rightarrow \partial_{\zeta_i}} \mathcal{G})_{\zeta_i=0}.$$

Cool! $(V^*)^{\otimes \infty} \otimes V^{\otimes \infty}$ explodes; the ranks of quadratics and bounded-degree polynomials grow slowly!^{†7} **Representation theory is over-rated!**

Cool! How often do you see a computational toolbox so successful?

Our Algebras. Let $sl_{2+}^\epsilon := L\langle y, b, a, x \rangle$ subject to $[a, x] = x, [b, y] = -\epsilon y, [a, b] = 0, [a, y] = -y, [b, x] = \epsilon x,$ and $[x, y] = \epsilon a + b.$ So $t := \epsilon a - b$ is central and if $\exists \epsilon^{-1}, sl_{2+}^\epsilon / \langle t \rangle \cong sl_2.$

U is either $CU = \hat{U}(sl_{2+}^\epsilon)$ or $QU = \mathcal{U}_\hbar(sl_{2+}^\epsilon) = A\langle y, b, a, x \rangle$ with $[a, x] = x, [b, y] = -\epsilon y, [a, b] = 0, [a, y] = -y, [b, x] = \epsilon x,$ and $xy - qyx = (1 - AB)/\hbar,$ where $q = e^{\hbar \epsilon}, A = e^{-\hbar \epsilon a},$ and $B = e^{-\hbar b}.$ Set also $T = A^{-1}B = e^{\hbar t}.$

The Quantum Leap. Also decree that in $QU,$

$$\Delta(y, b, a, x) = (y_1 + B_1 y_2, b_1 + b_2, a_1 + a_2, x_1 + A_1 x_2),$$
$$S(y, b, a, x) = (-B^{-1}y, -b, -a, -A^{-1}x),$$

and $R = \sum \hbar^{j+k} y^k b^j \otimes a^j x^k / j! [k]_q!$

Mid-Talk Debts. • What is this good for in quantum algebra?

- In knot theory?
- How does the "inclusion" $\mathcal{D}: \text{Hom}(U^{\otimes \infty} \rightarrow U^{\otimes S}) \rightsquigarrow$ **DoPeGDO** work?
- Proofs that everything around sl_{2+}^ϵ really is **DoPeGDO**.
- Relations with prior art.
- The rest of the "compositions" story.

Theorem ([BG], conjectured [MM], elucidated [Ro1]). Let $J_d(K)$ be the coloured Jones polynomial of $K,$ in the d -dimensional representation of $sl_2.$ Writing

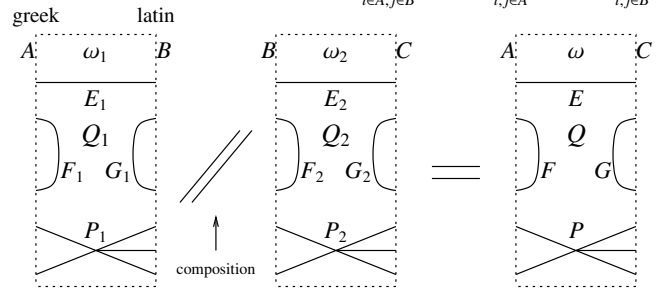
$$\left. \frac{(q^{1/2} - q^{-1/2}) J_d(K)}{q^{d/2} - q^{-d/2}} \right|_{q=e^\hbar} = \sum_{j,m \geq 0} a_{jm}(K) d^j \hbar^m,$$

"below diagonal" coefficients vanish, $a_{jm}(K) = 0$ if $j > m,$ and "on diagonal" coefficients give the inverse of the Alexander polynomial: $(\sum_{m=0}^\infty a_{mm}(K) \hbar^m) \cdot \omega(K)(e^\hbar) = 1.$

"Above diagonal" we have **Rozansky's Theorem** [Ro3, (1.2)]:

$$J_d(K)(q) = \frac{q^d - q^{-d}}{(q - q^{-1}) \omega(K)(q^d)} \left(1 + \sum_{k=1}^\infty \frac{(q-1)^k \rho_k(K)(q^d)}{\omega^{2k}(K)(q^d)} \right).$$

Compositions (1). In $\text{mor}(A \rightarrow B), Q = \sum_{i \in A, j \in B} E_{ij} \zeta_i z_j + \frac{1}{2} \sum_{i, j \in A} F_{ij} \zeta_i \zeta_j + \frac{1}{2} \sum_{i, j \in B} G_{ij} z_i z_j$



Where • $E = E_1(I - F_2 G_1)^{-1} E_2.$
 • $F = F_1 + E_1 F_2 (I - G_1 F_2)^{-1} E_1^T.$
 • $G = G_2 + E_2^T G_1 (I - F_2 G_1)^{-1} E_2.$
 • $\omega = \omega_1 \omega_2 \det(I - F_2 G_1)^{-1}.$
 • P is computed using "connected Feynman diagrams" or as the solution of a messy PDE (yet we're still in algebra!).



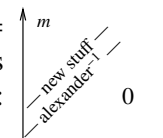
One abstraction level up from tangles!
 {tangles} → { }
 with compositions:

DoPeGDO Footnotes. †1. Each variable has a "weight" $\in \{0, 1, 2\},$ and always $\text{wt } z_i + \text{wt } \zeta_i = 2.$

- †2. Really, "weight-graded finite sets" $A = A_0 \sqcup A_1 \sqcup A_2.$
- †3. Really, a power series in the weight-0 variables^{†9}.
- †4. The weight of Q must be 2, so it decomposes as $Q = Q_{20} + Q_{11}.$ The coefficients of Q_{20} are rational numbers while the coefficients of Q_{11} may be weight-0 power series^{†9}.
- †5. Setting $\text{wt } \epsilon = -2,$ the weight of P is ≤ 2 (so the powers of the weight-0 variables are not constrained^{†9}).
- †6. There's also an obvious product $\text{mor}(A_1 \rightarrow B_1) \times \text{mor}(A_2 \rightarrow B_2) \rightarrow \text{mor}(A_1 \sqcup A_2 \rightarrow B_1 \sqcup B_2).$
- †7. That is, if the weight-0 variables are ignored. Otherwise more care is needed yet the conclusion remains.
- †8. $\text{Hom}(U^{\otimes \infty} \rightarrow U^{\otimes S}) \rightsquigarrow \text{mor}(\{\eta_i, \beta_i, \tau_i, \alpha_i, \xi_i\}_{i \in S} \rightarrow \{y_i, b_i, t_i, a_i, x_i\}_{i \in S}),$ where $\text{wt}(\eta_i, \xi_i, y_i, x_i) = 1$ and $\text{wt}(\beta_i, \tau_i, \alpha_i; b_i, t_i, a_i) = (2, 2, 0; 0, 0, 2).$
- †9. For tangle invariants the weight-0 power series are always rational functions in the exponentials of the weight-0 variables (for knots: just one variable).



Melvin, Morton, Garoufalidis



$\mathcal{D}: \text{Hom}(U^{\otimes \Sigma} \rightarrow U^{\otimes S}) \rightarrow \mathbb{Q}[[\eta_\Sigma, \beta_\Sigma, \alpha_\Sigma, \xi_\Sigma, y_S, b_S, a_S, x_S]]$. The PBW theorem for CU (always in the $ybax$ order), or its quantum analog for QU , say that if $U = CU$ or QU then $U^{\otimes S}$ is isomorphic as a vector space to $\mathbb{Q}[[y_i, b_i, a_i, x_i]]_{i \in S}$; so it is enough to understand $\text{Hom}(\mathbb{Q}[[z_A]] \rightarrow \mathbb{Q}[[z_B]])$ for finite sets A and B . Using the pairing

$$\langle z_i^m, \zeta_j^n \rangle = \partial_{\zeta_i}^m z_j^n \Big|_{\zeta_A \rightarrow 0} = \delta_{ij} \delta_{mn} n!,$$

we get a map

$$\begin{aligned} \mathcal{D}: \text{Hom}(\mathbb{Q}[[z_A]] \rightarrow \mathbb{Q}[[z_B]]) &\cong \mathbb{Q}[[z_A]]^* \otimes \mathbb{Q}[[z_B]] \\ &\cong \mathbb{Q}[[\zeta_A]] \otimes \mathbb{Q}[[z_B]] \cong \mathbb{Q}[[\zeta_A, z_B]] \end{aligned}$$

Example. $\mathcal{D}(id: \mathbb{Q}[[z]] \rightarrow \mathbb{Q}[[z]]) = e^{\zeta z}$. Indeed,

$$\langle z^n, e^{\zeta z} \rangle = \left\langle z^n, \sum_m \frac{(\zeta z)^m}{m!} \right\rangle = \sum_m \frac{z^m}{m!} \delta_{mn} n! = z^n.$$

Example. $\mathcal{D}(id: U \rightarrow U) = e^{\eta y + \beta b + \alpha a + \xi x}$.

Claim. Assuming convergence, if $F \in \text{Hom}(\mathbb{Q}[[z_A]] \rightarrow \mathbb{Q}[[z_B]])$, $G \in \text{Hom}(\mathbb{Q}[[z_B]] \rightarrow \mathbb{Q}[[z_C]])$, $\mathcal{F} = \mathcal{D}(F)$, and $\mathcal{G} = \mathcal{D}(G)$, then

$$\mathcal{D}(F \circ G) = (\mathcal{F}|_{z_i \rightarrow \partial_{\zeta_i} \mathcal{G}})_{\zeta_i=0}.$$

And so the title of the talk finally makes sense!

Other GDOs. Claim. If $L: \mathbb{Q}[[z_A]] \rightarrow \mathbb{Q}[[z_B]]$ is linear, then $\mathcal{D}(L) = L(e^{\sum_{i \in A} \zeta_i z_i})$. **Proof.** Exercise.

Example. Let $c\Delta_{jk}^i: CU^{\otimes \{i\}} \rightarrow CU^{\otimes \{j,k\}}$ be the standard coproduct, given by $c\Delta_{jk}^i(y_i, b_i, a_i, x_i) = (y_j + y_k, b_j + b_k, a_j + a_k, x_j + x_k)$. Then

$$\begin{aligned} \mathcal{D}(c\Delta_{jk}^i) &= c\Delta_{jk}^i(e^{\eta y_i + \beta b_i + \alpha a_i + \xi x_i}) \\ &= e^{\eta_i(y_j + y_k) + \beta_i(b_j + b_k) + \alpha_i(a_j + a_k) + \xi_i(x_j + x_k)}. \end{aligned}$$

Example. The standard commutative product m_k^{ij} of polynomials is given by $z_i, z_j \rightarrow z_k$. Hence $\mathcal{D}(m_k^{ij}) = m_k^{ij}(e^{\zeta_i z_i + \zeta_j z_j}) = e^{(\zeta_i + \zeta_j) z_k}$.

$$\begin{array}{ccc} \mathbb{Q}[[z]]_i \otimes \mathbb{Q}[[z]]_j & \xrightarrow{m_k^{ij}} & \mathbb{Q}[[z]]_k \\ \parallel & & \parallel \\ \mathbb{Q}[[z_i, z_j]] & \xrightarrow{m_k^{ij}} & \mathbb{Q}[[z_k]] \end{array}$$

A real DoPeGDO Example. Let $cm_k^{ij}: CU_i \otimes CU_j \rightarrow CU_k$ be ‘‘classical multiplication’’ for sl_{2+}^ϵ , and let $\mathcal{O}_i: \mathbb{Q}[[y_i, b_i, a_i, x_i]] \rightarrow CU_i$ be the PBW ordering map.

$$\begin{array}{ccc} CU_i \otimes CU_j & \xrightarrow{cm_k^{ij}} & CU_k \\ \uparrow \mathcal{O}_{i,j} & & \uparrow \mathcal{O}_k \\ \mathbb{Q}[[y_i, b_i, a_i, x_i, y_j, b_j, a_j, x_j]] & & \mathbb{Q}[[y_k, b_k, a_k, x_k]] \end{array}$$

Claim. Let

$$\begin{aligned} \Lambda &= \left(\eta_i + \frac{e^{-\alpha_i - \epsilon \beta_i} \eta_j}{1 + \epsilon \eta_j \xi_i} \right) y_k + \left(\beta_i + \beta_j + \frac{\log(1 + \epsilon \eta_j \xi_i)}{\epsilon} \right) b_k + \\ &\quad \left(\alpha_i + \alpha_j + \log(1 + \epsilon \eta_j \xi_i) \right) a_k + \left(\frac{e^{-\alpha_j - \epsilon \beta_j} \xi_i}{1 + \epsilon \eta_j \xi_i} + \xi_j \right) x_k \end{aligned}$$

Then $e^{\eta_i y_i + \beta_i b_i + \alpha_i a_i + \xi_i x_i + \eta_j y_j + \beta_j b_j + \alpha_j a_j + \xi_j x_j} \Big|_{\mathcal{O}_{i,j}} \Big|_{cm_k^{ij}} = e^\Lambda \Big|_{\mathcal{O}_k}$, and hence $\mathcal{D}(cm_k^{ij}) = e^\Lambda$ and cm_k^{ij} is DoPeGDO.

Proof. We compute in a faithful 2D representation ρ of CU :

($\omega \epsilon \beta / \text{cm}$)

$\text{HL}[\mathcal{E}_-] := \text{Style}[\mathcal{E}, \text{Background} \rightarrow \text{If}[\text{TrueQ}[\mathcal{E}], \blacksquare, \blacksquare]];$

$$\left\{ \rho y = \begin{pmatrix} \theta & \theta \\ \theta & \theta \end{pmatrix}, \rho b = \begin{pmatrix} \theta & \theta \\ \theta & -\epsilon \end{pmatrix}, \rho a = \begin{pmatrix} 1 & \theta \\ \theta & \theta \end{pmatrix}, \rho x = \begin{pmatrix} \theta & 1 \\ \theta & \theta \end{pmatrix} \right\};$$

$$\begin{aligned} \text{HL} / @ \{ \rho a \cdot \rho x - \rho x \cdot \rho a &= \rho x, \rho a \cdot \rho y - \rho y \cdot \rho a = -\rho y, \\ \rho b \cdot \rho y - \rho y \cdot \rho b &= -\epsilon \rho y, \rho b \cdot \rho x - \rho x \cdot \rho b = \epsilon \rho x, \\ \rho x \cdot \rho y - \rho y \cdot \rho x &= \rho b + \epsilon \rho a \} \end{aligned}$$

{True, True, True, True, True}

HL@Simplify@With[{E = MatrixExp},

$$\begin{aligned} &E[\eta_i \rho y] \cdot E[\beta_i \rho b] \cdot E[\alpha_i \rho a] \cdot E[\xi_i \rho x] \cdot E[\eta_j \rho y] \cdot E[\beta_j \rho b] \cdot \\ &E[\alpha_j \rho a] \cdot E[\xi_j \rho x] = \\ &E[\partial_{y_k} \Lambda \rho y] \cdot E[\partial_{b_k} \Lambda \rho b] \cdot E[\partial_{a_k} \Lambda \rho a] \cdot E[\partial_{x_k} \Lambda \rho x] \end{aligned}$$

True

Series[\Lambda, {\epsilon, \theta, 1}]

$$\begin{aligned} &(\mathfrak{a}_k (\alpha_i + \alpha_j) + \mathfrak{y}_k (\eta_i + e^{-\alpha_i} \eta_j) + \\ &\mathfrak{b}_k (\beta_i + \beta_j + \eta_j \xi_i) + \mathfrak{x}_k (e^{-\alpha_j} \xi_i + \xi_j)) + \\ &\left(\mathfrak{a}_k \eta_j \xi_i - \frac{1}{2} \mathfrak{b}_k \eta_j^2 \xi_i^2 - e^{-\alpha_i} \mathfrak{y}_k \eta_j (\beta_i + \eta_j \xi_i) - \right. \\ &\left. e^{-\alpha_j} \mathfrak{x}_k \xi_i (\beta_j + \eta_j \xi_i) \right) \epsilon + \mathcal{O}[\epsilon]^2 \end{aligned}$$

(Shame, but this technique fails for QU).

Claim. In QU , R is DoPeGDO.

Proof. Recall that with $q = e^{\hbar \epsilon}$,

$$R = \sum \hbar^{j+k} y^k b^j \otimes a^j x^k / j! [k]_q! = \mathcal{O} \left(e^{\hbar b_1 a_2} e_q^{\hbar y_1 x_2} \right).$$

Now expand $e_q^{\hbar y_1 x_2}$ in powers of ϵ using:

Faddeev’s Formula (In as much as we can tell, first appeared without proof in Faddeev [Fa], rediscovered and proven in Quesne [Qu], and again with easier proof, in Zagier [Za]). With $[n]_q := \frac{q^n - 1}{q - 1}$, with $[n]_q! := [1]_q [2]_q \cdots [n]_q$ and with $e_q^x := \sum_{n \geq 0} \frac{x^n}{[n]_q!}$, we have

$$\log e_q^x = \sum_{k \geq 1} \frac{(1-q)^k x^k}{k(1-q^k)} = x + \frac{(1-q)^2 x^2}{2(1-q^2)} + \dots$$

Proof. We have that $e_q^x = \frac{e^{qx} - e^x}{qx - x}$ (‘‘the q -derivative of e_q^x is itself’’), and hence $e_q^{qx} = (1 + (1-q)x)e_q^x$, and

$$\log e_q^{qx} = \log(1 + (1-q)x) + \log e_q^x.$$

Writing $\log e_q^x = \sum_{k \geq 1} a_k x^k$ and comparing powers of x , we get $q^k a_k = -(1-q)^k / k + a_k$, or $a_k = \frac{(1-q)^k}{k(1-q^k)}$. \square

Compositions (2). Recall that with all indices i running in some set B ,

$$\mathcal{F} \circ \mathcal{G} = \left(\mathcal{F}|_{z_i \rightarrow \partial_{\zeta_i} \mathcal{G}} \right)_{\zeta_i=0} = e^{\sum \partial_{\zeta_i} \mathcal{G}} \left(\mathcal{F} \mathcal{G} \right) \Big|_{z_i = \zeta_i = 0},$$

so in general we wish to understand

$[F: \mathcal{E}]_B := e^{\frac{1}{2} \sum_{i,j \in B} F_{ij} \partial_{z_i} \partial_{z_j} \mathcal{E}}$ and $\langle F: \mathcal{E} \rangle_B := [F: \mathcal{E}]_B|_{z_B \rightarrow 0}$, where \mathcal{E} is a docile perturbed Gaussian. The following lemma allows us to restrict to the case where \mathcal{E} has no B - B quadratic part:

Lemma 1. With convergences left to the reader,

$$\left\langle F: \mathcal{E} \Big|_{e^{\frac{1}{2} \sum_{i,j \in B} G_{ij} z_i z_j}} \right\rangle_B = \det(1 - GF)^{-1} \left\langle F(1 - GF)^{-1}: \mathcal{E} \right\rangle_B.$$

The next lemma dispatches the case where \mathcal{E} has a B -linear part:

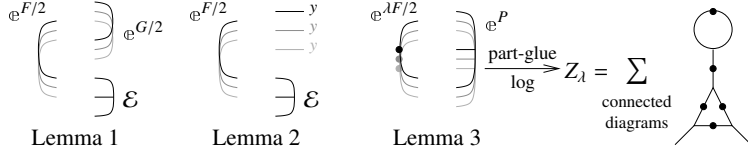
Lemma 2. $\left\langle F: \mathcal{E} \Big|_{e^{\sum_{i \in B} y_i z_i}} \right\rangle_B = \left\langle F: \mathcal{E}|_{z_B \rightarrow z_B + F y_B} \right\rangle_B$.

(1) strictly speaking, true only if $B \cap \text{Anc} = \emptyset$

Finally, we deal with the docile perturbation case:

Lemma 3. With an extra variable λ , $Z_\lambda := \log[\lambda F : \mathbb{C}^P]_B$ satisfies and is determined by the following PDE / IVP:

$$Z_0 = P \quad \text{and} \quad \partial_\lambda Z_\lambda = \sum_{i,j \in B} F_{ij} (\partial_{z_i} \partial_{z_j} Z_\lambda + (\partial_{z_i} Z_\lambda)(\partial_{z_j} Z_\lambda)).$$



Warning. Some implementation details match earlier versions of the theory.

The “Speedy” Engine

$\omega\epsilon\beta$ /engine

Internal Utilities

Canonical Form:

```
CCF [E_] :=
  PPCF@ExpandDenominator@
  ExpandNumerator@PPTogether@Together [PPExp [
    Expand [E] /. {e^x - e^y -> e^{x+y} /. e^x -> e^{CCF[x]}}];
CF [E_List] := CF /@ E;
CF [sd_SeriesData] := MapAt [CF, sd, 3];
CF [E_] := PPCF@Module [
  {vs = Cases [E, (y | b | t | a | x | η | β | τ | α | ξ)_, ∞] U
  {y, b, t, a, x, η, β, τ, α, ξ}},
  Total [CoefficientRules [Expand [E], vs] /.
  (ps_ -> c_) -> CCF [c] (Times @@ vs^{ps})
];
CF [E_E] := CF /@ E;
CF [IE_sp__ [ES___]] := CF /@ IE_sp [ES];
```

The Kronecker δ :

$K\delta$ /: $K\delta_{i,j} := \text{If}[i == j, 1, 0]$;

Equality, multiplication, and degree-adjustment of perturbed Gaussians; $\mathbb{E}[L, Q, P]$ stands for $e^{L+Q}P$:

```
E /: E [L1_, Q1_, P1_] == E [L2_, Q2_, P2_] :=
  CF [L1 == L2] ^ CF [Q1 == Q2] ^ CF [Normal [P1 - P2] == 0];
E /: E [L1_, Q1_, P1_] * E [L2_, Q2_, P2_] :=
  E [L1 + L2, Q1 + Q2, P1 * P2];
E [L_, Q_, P_]_{k} := E [L, Q, Series [Normal@P, {e, 0, $k}]];
```

Zip and Bind

Variables and their duals:

```
{t*, b*, y*, a*, x*, z*} = {τ, β, η, α, ξ, ζ};
{τ*, β*, η*, α*, ξ*, ζ*} = {t, b, y, a, x, z};
(u_i_)* := (u*)_i;
```

Upper to lower and lower to Upper:

```
U21 = {B_i^{p-} -> e^{-p h y b_i}, B_i^{p-} -> e^{-p h y b}, T_i^{p-} -> e^{p h t_i},
  T_i^{p-} -> e^{p h t}, A_i^{p-} -> e^{p y a_i}, A_i^{p-} -> e^{p y a}};
L2U = {e^{c- b_i + d-} -> B_i^{-c/(h y)} e^d, e^{c- b + d-} -> B^{-c/(h y)} e^d,
  e^{c- t_i + d-} -> T_i^{c/h} e^d, e^{c- t + d-} -> T^{c/h} e^d,
  e^{c- a_i + d-} -> A_i^{c/y} e^d, e^{c- a + d-} -> A^{c/y} e^d,
  e^E -> Expand@E};
```

Derivatives in the presence of exponentiated variables:

```
D_b [f_-] := ∂_b f - h y B ∂_b f; D_{b_i} [f_-] := ∂_{b_i} f - h y B_i ∂_{b_i} f;
D_t [f_-] := ∂_t f + h T ∂_t f; D_{t_i} [f_-] := ∂_{t_i} f + h T_i ∂_{t_i} f;
D_α [f_-] := ∂_α f + y A ∂_α f; D_{α_i} [f_-] := ∂_{α_i} f + y A_i ∂_{α_i} f;
D_{v_-} [f_-] := ∂_v f; D_{(v,0)} [f_-] := f; D_{(v)} [f_-] := f;
D_{(v,n_Integer)} [f_-] := D_v [D_{(v,n-1)} [f]];
D_{(L_List, Ls___)} [f_-] := D_{(Ls)} [D_L [f]];
```

Finite Zips:

```
collect [sd_SeriesData, E_] :=
  MapAt [collect [#, E] &, sd, 3];
collect [E_, E_] := PPCollect@Collect [E, E];
Zip {} [P_] := P;
Zip_{E_S} [Ps_List] := Zip_{E_S} /@ Ps;
Zip_{(E_S, E_S___)} [P_] := PPCollect [
  (collect [P // Zip_{E_S}, E] /. f_- . E^{d-} -> (D_{(E_S, d)} [f])) /.
  E^* -> 0 /. ((E^* /. {b -> B, t -> T, α -> A}) -> 1) ]
```

QZip implements the “Q-level zips” on $\mathbb{E}(L, Q, P) = e^{L+Q}P(\epsilon)$.

Such zips regard the L variables as scalars.

```
QZip_{E_S_List}@E [L_, Q_, P_] :=
  PPQZip@Module [{ξ, z, zs, c, ys, ηs, qt, zrule, grule, out},
  zs = Table [ξ*, {ξ, ζs}];
  c = CF [Q /. Alternatives @@ (ζs U zs) -> 0];
  ys = CF@Table [∂_ξ (Q /. Alternatives @@ zs -> 0),
  {ξ, ζs}];
  ηs = CF@Table [∂_z (Q /. Alternatives @@ ζs -> 0), {z, zs}];
  qt = CF@Inverse@Table [Kδ_{z,ξ*} - ∂_{z,ξ} Q, {ξ, ζs}, {z, zs}];
  zrule = Thread [zs -> CF [qt. (zs + ys)]];
  grule = Thread [ζs -> ζs + ηs.qt];
  CF /@ E [L, c + ηs.qt.y,
  Det [qt] Zip_{E_S} [P /. (zrule U grule)]]];
```

LZip implements the “L-level zips” on $\mathbb{E}(L, Q, P) = P e^{L+Q}$. Such zips regard all of $P e^Q$ as a single “P”. Here the z ’s are b and α and the ζ ’s are β and a .

```
LZip_{E_S_List}@E [L_, Q_, P_] :=
  PPLZip@Module [{ξ, z, zs, Zs, c, ys, ηs, lt, zrule,
  Zrule, grule, Q1, EEQ, EQ},
  zs = Table [ξ*, {ξ, ζs}];
  Zs = zs /. {b -> B, t -> T, α -> A};
  c = L /. Alternatives @@ (ζs U zs) -> 0 /.
  Alternatives @@ Zs -> 1;
  ys = Table [∂_ξ (L /. Alternatives @@ zs -> 0), {ξ, ζs}];
  ηs = Table [∂_z (L /. Alternatives @@ ζs -> 0), {z, zs}];
  lt = Inverse@Table [Kδ_{z,ξ*} - ∂_{z,ξ} L, {ξ, ζs}, {z, zs}];
  zrule = Thread [zs -> lt. (zs + ys)];
  Zrule = Join [zrule,
  zrule /.
  r_Rule -> ((U = r[[1]] /. {b -> B, t -> T, α -> A}) ->
  (U /. U21 /. r // L2U))];
  grule = Thread [ζs -> ζs + ηs.lt];
  Q1 = Q /. (Zrule U grule);
  EEQ [ps___] :=
  EEQ [ps] =
  PP“EEQ”@ (CF [e^{-Q1} DThread [{zs, {ps}}] [e^{Q1}]] /.
  {Alternatives @@ zs -> 0, Alternatives @@ Zs -> 1});
  CF@E [c + ηs.lt.y,
  Q1 /. {Alternatives @@ zs -> 0, Alternatives @@ Zs -> 1},
  Det [lt]
  (Zip_{E_S} [(EQ @@ zs) (P /. (Zrule U grule))] /.
  Derivative [ps___] [EQ] [___] -> EEQ [ps] /.
  _EQ -> 1) ]];
```

```

B{}[L_, R_] := LR;
B{is_}[L_E, R_E] := PPB@Module[{n},
  Times[
    L /. Table[(v : b | B | t | T | a | x | y)_i -> v_{nei},
      {i, {is}}],
    R /. Table[(v : beta | tau | alpha | A | xi | eta)_i -> v_{nei}, {i, {is}}]
  ] // LZipJoin@@Table[{beta_{nei}, tau_{nei}, alpha_{nei}}, {i, {is}}] //
  QZipJoin@@Table[{xi_{nei}, eta_{nei}}, {i, {is}}];
B{is_}[L_, R_] := B{is}[L, R];

```

E morphisms with domain and range.

```

B{is_List}[E_{d1 -> r1}[L1_, Q1_, P1_], E_{d2 -> r2}[L2_, Q2_, P2_]] :=
  E_{(d1 U Complement[d2, is]) -> (r2 U Complement[r1, is])} @@
  B{is}[E[L1, Q1, P1], E[L2, Q2, P2]];
E_{d1 -> r1}[L1_, Q1_, P1_] // E_{d2 -> r2}[L2_, Q2_, P2_] :=
  B_{r1} E_{d1 -> r1}[L1, Q1, P1], E_{d2 -> r2}[L2, Q2, P2];
E_{d1 -> r1}[L1_, Q1_, P1_] == E_{d2 -> r2}[L2_, Q2_, P2_] ^:=
  (d1 == d2) ^ (r1 == r2) ^ (E[L1, Q1, P1] == E[L2, Q2, P2]);
E_{d1 -> r1}[L1_, Q1_, P1_] E_{d2 -> r2}[L2_, Q2_, P2_] ^:=
  E_{(d1 U d2) -> (r1 U r2)} @@ (E[L1, Q1, P1] x E[L2, Q2, P2]);
E_{dr_}[L_, Q_, P_]_{$k_} := E_{dr} @@ E[L, Q, P]_{$k};
E_{[E_...]}[i_] := {E}[i];

```

E[^]

```

E_{dr_}[A_] :=
  CF@Module[{L, A0 = Limit[A, e -> 0]},
    E_{dr}[L = A0 /. (eta | y | xi | x)_ -> 0, A0 - L, e^{A-A0}]_{$k} /. 12U]

```

Exponentials as needed.

Task. Define $\text{Exp}_{m,i,k}[P]$ to compute $e^{Q(P)}$ to ϵ^k in the using the $m_{i,j \rightarrow i}$ multiplication, where P is an ϵ -dependent near-docile element, giving the answer in \mathbb{E} -form.

Methodology. If $P_0 := P_{\epsilon=0}$ and $e^{\lambda Q(P)} = O(e^{\lambda P_0} F(\lambda))$, then

$F(\lambda = 0) = 1$ and we have:

$$O(e^{\lambda P_0} (P_0 F(\lambda) + \partial_\lambda F)) = O(\partial_\lambda e^{\lambda P_0} F(\lambda)) =$$

$$\partial_\lambda O(e^{\lambda P_0} F(\lambda)) = \partial_\lambda e^{\lambda Q(P)} = e^{\lambda Q(P)} O(P) = O(e^{\lambda P_0} F(\lambda)) O(P)$$

This is a linear ODE for F . Setting inductively $F_k = F_{k-1} + \epsilon^k \varphi$ we find that $F_0 = 1$ and solve for φ .

(* Bug: The first line is valid only if $O(e^{P_0}) = e^{O(P_0)}$. *)

```

Exp_{m,i,k}[P_] := Module[{LQ = Normal@P /. e -> 0},
  E[LQ /. (x | y)_i -> 0, LQ /. (b | a | t)_i -> 0, 1]];

```

```

Exp_{m,i,k}[P_] := Block[{$k = k},
  Module[{P0, lambda, phi, phiS, F, j, rhs, eqn, pows, at0, atlambda},
    P0 = Normal@P /. e -> 0;
    F = Normal@Last@Exp_{m,i,k-1}[lambda P];
    While[
      rhs =
        m_{i,j -> i}[
          E_{i -> {i}}[lambda P0 /. (x | y)_i -> 0, lambda P0 /. (b | a | t)_i -> 0,
            F]_k s_{i -> j} @ E_{i -> {i}}[0, 0, P]_k // Last // Normal;
          eqn = CF[(partial lambda F) + P0 F - rhs];
          eqn != 0, (*do*)
          pows = First/@CoefficientRules[eqn, {y_i, b_i, a_i, x_i}];
          F += Sum[e^k phi_{js}[lambda] Times@@{y_i, b_i, a_i, x_i}^{js},
            {js, pows}];
          rhs =
            m_{i,j -> i}[
              E_{i -> {i}}[lambda P0 /. (x | y)_i -> 0, lambda P0 /. (b | a | t)_i -> 0,
                F]_k s_{i -> j} @ E_{i -> {i}}[0, 0, P]_k // Last // Normal;
              eqn = CF[(partial lambda F) + P0 F - rhs];
              phiS = Table[phi_{js}[lambda], {js, pows}];
              at0 = Table[phi_{js}[0] == 0, {js, pows}];
              atlambda = (# == 0) & /@
                (pows /. CoefficientRules[eqn, {y_i, b_i, a_i, x_i}]);
              F = F /. DSolve[And@@(at0 U atlambda), phiS, lambda][[1]]
            ];
          E_{i -> {i}}[P0 /. (x | y)_i -> 0, P0 /. (b | a | t)_i -> 0,
            F + O[epsilon]^{k+1} /. lambda -> 1] ] ]

```

“Define” Code

Define[lhs = rhs, ...] defines the lhs to be rhs, except that rhs is computed only once for each value of \$k. Fancy Mathematica not for the faint of heart. Most readers should ignore.

```

SetAttributes[Define, HoldAll];
Define[def_, defs_] := (Define[def]; Define[defs]);
Define[op_is_ = E_] :=
  Module[{SD, ii, jj, kk, isp, nis, nisp, sis},
    Block[{i, j, k},
      ReleaseHold[Hold[
        SD[op_nisp, $k_Integer, PPBoot@Block[{i, j, k}, op_isp, $k = E;
          op_nis, $k]];
        SD[op_isp, op_{is}, $k]; SD[op_sis_, op_{sis}];
      ] /. {SD -> SetDelayed,
        isp -> {is} /. {i -> i_, j -> j_, k -> k_},
        nis -> {is} /. {i -> ii, j -> jj, k -> kk},
        nisp -> {is} /. {i -> ii_, j -> jj_, k -> kk_}
      } ] ]

```

The Objects

Symmetric Algebra Objects


```

sm_{i,j} \to k :=
  E_{\{i,j\} \to \{k\}} [b_k (\beta_i + \beta_j) + t_k (\tau_i + \tau_j) + a_k (\alpha_i + \alpha_j) +
    y_k (\eta_i + \eta_j) + x_k (\xi_i + \xi_j)];
s\Delta_{i,j} \to k :=
  E_{\{i\} \to \{j,k\}} [\beta_i (b_j + b_k) + \tau_i (t_j + t_k) + \alpha_i (a_j + a_k) +
    \eta_i (y_j + y_k) + \xi_i (x_j + x_k)];
ss_{i,j} := E_{\{i\} \to \{i\}} [-\beta_i b_i - \tau_i t_i - \alpha_i a_i - \eta_i y_i - \xi_i x_i];
se_{i,j} := E_{\{i\} \to \{i\}} [0];
s\eta_{i,j} := E_{\{i\} \to \{i\}} [0];
s\sigma_{i,j} := E_{\{i\} \to \{j\}} [\beta_i b_j + \tau_i t_j + \alpha_i a_j + \eta_i y_j + \xi_i x_j];
sY_{i,j,k,l,m} := E_{\{i\} \to \{j,k,l,m\}} [\beta_i b_k + \tau_i t_k + \alpha_i a_l + \eta_i y_j + \xi_i x_m];

```

The CU Definitions

$$c\Delta = \left(\eta_i + \frac{e^{-\gamma \alpha_i - \epsilon \beta_i} \eta_j}{1 + \gamma \epsilon \eta_j \xi_i} \right) y_k + \left(\beta_i + \beta_j + \frac{\text{Log}[1 + \gamma \epsilon \eta_j \xi_i]}{\epsilon} \right) b_k + \left(\alpha_i + \alpha_j + \frac{\text{Log}[1 + \gamma \epsilon \eta_j \xi_i]}{\gamma} \right) a_k + \left(\frac{e^{-\gamma \alpha_j - \epsilon \beta_j} \xi_i}{1 + \gamma \epsilon \eta_j \xi_i} + \xi_j \right) x_k;$$

```

Define [cm_{i,j} \to k = E_{\{i,j\} \to \{k\}} [c\Delta]];
Define [c\sigma_{i,j} = s\sigma_{i,j} / \tau_i \to \theta, c\epsilon_i = se_i, c\eta_i = s\eta_i,
  c\Delta_{i,j,k} = s\Delta_{i,j,k},
  cS_i = sS_i // sY_{i-1,2,3,4} // cm_{4,3-i} // cm_{i,2-i} // cm_{i,1-i}];

```

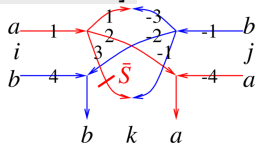
Booting Up QU

```

Define [a\sigma_{i,j} = E_{\{i\} \to \{j\}} [a_j \alpha_i + x_j \xi_i],
  b\sigma_{i,j} = E_{\{i\} \to \{j\}} [b_j \beta_i + y_j \eta_i]];
Define [am_{i,j,k} = E_{\{i,j\} \to \{k\}} [(\alpha_i + \alpha_j) a_k + (\mathcal{A}_j^{-1} \xi_i + \xi_j) x_k],
  bm_{i,j,k} = E_{\{i,j\} \to \{k\}} [(\beta_i + \beta_j) b_k + (\eta_i + e^{-\epsilon \beta_i} \eta_j) y_k]];
Define [R_{i,j} = E_{\{i\} \to \{i,j\}} [\hbar a_j b_i + \sum_{k=1}^{j+1} \frac{(1 - e^{\gamma \epsilon \hbar})^k (\hbar y_i x_j)^k}{k (1 - e^{k \gamma \epsilon \hbar})}],
  \bar{R}_{i,j} = CF @ E_{\{i\} \to \{i,j\}} [-\hbar a_j b_i, -\hbar x_j y_i / B_i,
  1 + If[$k == \theta, \theta, (\bar{R}_{\{i,j\}, \$k-1} \$k [3] -
  ((\bar{R}_{\{i,j\}, \theta} \$k R_{\{3,4\}, \$k-1} \$k) // (bm_{i,1-i} am_{j,2-j}) //
  (bm_{i,3-i} am_{j,4-j})) [3] ]],
  P_{i,j} = E_{\{i,j\} \to \{i\}} [\beta_i \alpha_j / \hbar, \eta_i \xi_j / \hbar,
  1 + If[$k == \theta, \theta, (P_{\{i,j\}, \$k-1} \$k [3] -
  (R_{\{1,2\} // ((P_{\{1,j\}, \theta} \$k (P_{\{i,2\}, \$k-1} \$k)) [3] ])]];
Define [aS_i = (a\sigma_{i-2} \bar{R}_{1,i}) // P_{1,2},
  \bar{aS}_i = E_{\{i\} \to \{i\}} [-a_i \alpha_i, -x_i \xi_i,
  1 + If[$k == \theta, \theta, (\bar{aS}_{\{i\}, \$k-1} \$k [3] -
  ((\bar{aS}_{\{i\}, \theta} \$k // aS_i // (\bar{aS}_{\{i\}, \$k-1} \$k) [3] )]];
Define [bS_i = b\sigma_{i-1} R_{i,2} // aS_2 // P_{1,2},
  \bar{bS}_i = b\sigma_{i-1} R_{i,2} // \bar{aS}_2 // P_{1,2},
  a\Delta_{i,j,k} = (R_{1,j} R_{k,2}) // bm_{1,2-3} // P_{3,i},
  b\Delta_{i,j,k} = (R_{j,1} R_{k,2}) // am_{1,2-3} // P_{i,3}];

```

The Drinfel'd double:



```

Define [
  dm_{i,j} \to k =
    ((sY_{i-4,4,1,1} // a\Delta_{1-1,2} // a\Delta_{2-2,3} // \bar{aS}_3)
    (sY_{j \to -1, -1, -4, -4} // b\Delta_{-1-1, -2} // b\Delta_{-2-2, -3})) //
    (P_{-1,3} P_{-3,1} am_{2,-4-k} bm_{4,-2-k}]);
Define [d\sigma_{i,j} = a\sigma_{i,j} b\sigma_{i,j},
  d\epsilon_i = se_i, d\eta_i = s\eta_i,
  dS_i = sY_{i-1,1,2,2} // (\bar{bS}_1 aS_2) // dm_{2,1-i},
  \bar{dS}_i = sY_{i-1,1,2,2} // (bS_1 \bar{aS}_2) // dm_{2,1-i},
  d\Delta_{i,j,k} = (b\Delta_{i-3,1} a\Delta_{i-2,4}) // (dm_{3,4-k} dm_{1,2-j}]);

```

```

Define [C_i = E_{\{i\} \to \{i\}} [\theta, \theta, B_i^{1/2} e^{-\hbar \epsilon a_i / 2}]_{\$k},
  \bar{C}_i = E_{\{i\} \to \{i\}} [\theta, \theta, B_i^{-1/2} e^{\hbar \epsilon a_i / 2}]_{\$k},
  Kink_i = (R_{1,3} \bar{C}_2) // dm_{1,2-1} // dm_{1,3-i},
  \bar{Kink}_i = (\bar{R}_{1,3} C_2) // dm_{1,2-1} // dm_{1,3-i}];

```

Note. $t = \epsilon a - \gamma b$ and $b = -t / \gamma + \epsilon a / \gamma$.

```

Define [b2t_i = E_{\{i\} \to \{i\}} [\alpha_i a_i + \beta_i (\epsilon a_i - t_i) / \gamma + \xi_i x_i + \eta_i y_i],
  t2b_i = E_{\{i\} \to \{i\}} [\alpha_i a_i + \tau_i (\epsilon a_i - \gamma b_i) + \xi_i x_i + \eta_i y_i]];

```

The Knot Tensors

```

Define [kR_{i,j} = R_{i,j} // (b2t_i b2t_j) / \tau_i | j \to t,
  \bar{kR}_{i,j} = \bar{R}_{i,j} // (b2t_i b2t_j) / \tau_i | j \to t, T_i | j \to T},
  km_{i,j} \to k = (t2b_i t2b_j) // dm_{i,j-k} //
  b2t_k / \{t_k \to t, T_k \to T, \tau_i | j \to \theta\},
  kC_i = C_i // b2t_i / T_i \to T,
  \bar{kC}_i = \bar{C}_i // b2t_i / T_i \to T,
  kKink_i = Kink_i // b2t_i / \{t_i \to t, T_i \to T\},
  \bar{kKink}_i = \bar{Kink}_i // b2t_i / \{t_i \to t, T_i \to T\}];

```

A Quantum Algebra Example.

$\omega\epsilon\beta/\text{qa}$

Proto-Proposition^{†0} (with Jesse Frohlich and Roland van der Veen, near [Ma, Proposition 1.7.3]). Let H be a finite dimensional Hopf algebra and let $U = H^{*cop} \otimes H$ be its Drinfel'd double, with R -matrix $R \in H^* \otimes H \subset U \otimes U$. Write $R^{\dagger 1} = \sum \rho_a \otimes r_a$, and let $\langle \cdot | \cdot \rangle: H^* \otimes H \rightarrow \mathbb{F}$ be the duality pairing. Then the functional $\int \in U^*$ defined by

$$\int \phi \otimes x := \sum \langle \phi \rho_a^{\dagger 2} | x r_a^{\dagger 3} \rangle$$

is a right^{†4} integral in U^* . (Meaning $\Delta_{jk}^i // \int_j = \int_i // \epsilon_k$ in $\text{Hom}(U^{\otimes \{i\}} \rightarrow U^{\otimes \{k\}})$).

†0 A “proto-proposition” is something that will become a proposition once you figure out the correct statement. †1 Or did we want it to be $R // S_1^2$? Or $R // S_2^2$? †2 Or is it $\rho_a \phi$? †3 Or is it $r_a x$? †4 Or maybe “left”?

PP_ := Identity; \$k = 1; \hbar = \gamma = 1;

inp = E_{\{i\} \to \{1\}} [3 a_1 b_1, 5 x_1 y_1, 1] // dm_{1,1-i};

Table[

```

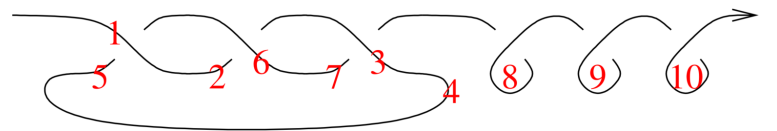
HL@TrueQ[
  (inp // (sY_{i-1,1,2,2} RR) // BM // AM // P_{1,2}) de_j \equiv
  (inp // \Delta\Delta // (sY_{i-1,1,2,2} RR) // BM // AM // P_{1,2}),
  {\Delta\Delta, {d\Delta_{i-1,j}, d\Delta_{i,j,i}}}, {AM, {dm_{2,4-2}, dm_{4,2-2}}},
  {BM, {dm_{1,3-1}, dm_{3,1-1}}},
  {RR, {R_{3,4}, R_{3,4} // dS_3 // dS_3, R_{3,4} // dS_4 // dS_4}}];
] // MatrixForm

```

(False False False)	(False False True)
(False False False)	(False False False)
(False False False)	(False False False)
(False False True)	(False False False)

A Knot Theory Example.

$\omega\epsilon\beta/\text{kt}$



\$k = 2;

Simplify[

```

R_{1,5} R_{6,2} R_{3,7} \bar{C}_4 \bar{Kink}_8 \bar{Kink}_9 \bar{Kink}_{10} // dm_{1,2-1} // dm_{1,3-1} //
  dm_{1,4-1} // dm_{1,5-1} // dm_{1,6-1} // dm_{1,7-1} // dm_{1,8-1} //
  dm_{1,9-1} // dm_{1,10-1}]] / \cdot v_{-1} \to v

```

$$E_{\{1\} \rightarrow \{1\}} \left[\theta, \theta, \frac{B}{1 - B + B^2} + \frac{B(-B + 2B^2 + 2B^4 + a(-1 + B - B^3 + B^4) - 2xy - B^3(3 + 2xy)) \in}{(1 - B + B^2)^3} + \frac{1}{2(1 - B + B^2)^5} B(4B^8 + a^2(1 - B + B^2)^2(1 + B - 6B^2 + B^3 + B^4) + 6B^5x^2y^2 + 2xy(-2 + 3xy) - B^7(11 + 4xy) - 2B^2(1 + 6x^2y^2) - 2B^4(1 - 2xy + 6x^2y^2) + B(1 + 8xy + 6x^2y^2) + B^6(6 + 8xy + 6x^2y^2) + B^3(4 + 4xy + 30x^2y^2) + 2a(1 - B + B^2)(2B^6 + 2xy + 8B^3(1 + xy) - 5B^2(1 + 2xy) - 2B^5(1 + 2xy) - B^4(7 + 2xy) + B(2 + 4xy)) \in^2 + 0[\in] \right]$$

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KiW 43 Abstract (ωεβ/kiw). Whether or not you like the formulas on this page, they describe the strongest truly computable knot invariant we know.







































Observations. • Separates the Rolfsen table; does better than

Khovanov plus HOMFLY-PT on knots with up to 12 crossings (not tested beyond). • The degrees are bounded by the genus!
 • ρ₁ vanishes for amphichiral knots. • Has a chance of detecting non-ribbonness (ωεβ/ind)!



































knot diag	n'_k (ρ'_1) ⁺	Alexander’s ω^+ (ρ'_2) ⁺	genus / ribbon unknotting # / amphi?	knot diag	n'_k (ρ'_1) ⁺	Alexander’s ω^+ (ρ'_2) ⁺	genus / ribbon unknotting # / amphi?	knot diag	n'_k (ρ'_1) ⁺	Alexander’s ω^+ (ρ'_2) ⁺	genus / ribbon unknotting # / amphi?
	0_1^a	1	0 / ✓ 0 / ✓		3_1^a	$t - 1$	1 / ✗ 1 / ✗		4_1^a	$3 - t$	1 / ✗ 1 / ✓
	5_1^a	$t^2 - t + 1$	2 / ✗ 2 / ✗		5_2^a	$2t - 3$	1 / ✗ 1 / ✗		6_1^a	$5 - 2t$	1 / ✓ 1 / ✗
	6_2^a	$-t^2 + 3t - 3$	2 / ✗ 1 / ✗		6_3^a	$t^2 - 3t + 5$	2 / ✗ 1 / ✓		7_1^a	$t^3 - t^2 + t - 1$	3 / ✗ 3 / ✗
	7_2^a	$3t - 5$	1 / ✗ 1 / ✗		7_3^a	$2t^2 - 3t + 3$	2 / ✗ 2 / ✗		7_4^a	$4t - 7$	1 / ✗ 2 / ✗
	7_5^a	$2t^2 - 4t + 5$	2 / ✗ 2 / ✗		7_6^a	$-t^2 + 5t - 7$	2 / ✗ 1 / ✗		7_7^a	$t^2 - 5t + 9$	2 / ✗ 1 / ✗
	8_1^a	$7 - 3t$	1 / ✗ 1 / ✗		8_2^a	$-t^3 + 3t^2 - 3t + 3$	3 / ✗ 2 / ✗		8_3^a	$9 - 4t$	1 / ✗ 2 / ✓
	8_4^a	$-2t^2 + 5t - 5$	2 / ✗ 2 / ✗		8_5^a	$-t^3 + 3t^2 - 4t + 5$	3 / ✗ 2 / ✗		8_6^a	$-2t^2 + 6t - 7$	2 / ✗ 2 / ✗
	8_7^a	$t^3 - 3t^2 + 5t - 5$	3 / ✗ 1 / ✗		8_8^a	$2t^2 - 6t + 9$	2 / ✓ 2 / ✗		8_9^a	$-t^3 + 3t^2 - 5t + 7$	3 / ✓ 1 / ✓

knot diag	n'_k $(\rho'_1)^+$	Alexander's ω^+	genus / ribbon unknotting # / amphi?	knot diag	n'_k $(\rho'_1)^+$	Alexander's ω^+	genus / ribbon unknotting # / amphi?	knot diag	n'_k $(\rho'_1)^+$	Alexander's ω^+	genus / ribbon unknotting # / amphi?
	8_{10}^a	$t^3 - 3t^2 + 6t - 7$ $-t^3 + 4t^4 - 11t^3 + 16t^2 - 21t + 20$	3 / ✗ 2 / ✗		8_{11}^a	$-2t^2 + 7t - 9$ $5t^3 - 24t^2 + 39t - 44$	2 / ✗ 1 / ✗		8_{12}^a	$t^2 - 7t + 13$ 0	2 / ✗ 2 / ✓
		$8t^{12} - 75t^{11} + 362t^{10} - 1122t^9 + 2306t^8 - 2540t^7 - 2198t^6 + 18817t^5 - 54380t^4 + 110103t^3 - 175694t^2 + 230080t - 251346$				$38t^8 - 264t^7 + 301t^6 + 3514t^5 - 21716t^4 + 68785t^3 - 146898t^2 + 227828t - 263172$				$4t^8 - 77t^7 + 583t^6 - 1991t^5 + 987t^4 + 17311t^3 - 71802t^2 + 147914t - 185846$	
	8_{13}^a	$2t^2 - 7t + 11$ $-t^3 + 4t^2 - 14t + 20$	2 / ✗ 1 / ✗		8_{14}^a	$-2t^2 + 8t - 11$ $5t^3 - 28t^2 + 57t - 68$	2 / ✗ 1 / ✗		8_{15}^a	$3t^2 - 8t + 11$ $21t^3 - 64t^2 + 120t - 140$	2 / ✗ 2 / ✗
		$62t^8 - 592t^7 + 2351t^6 - 3918t^5 - 4235t^4 + 40079t^3 - 111533t^2 + 191500t - 227432$				$38t^8 - 312t^7 + 444t^6 + 5096t^5 - 34777t^4 + 116368t^3 - 255750t^2 + 401632t - 465478$				$-123t^8 + 2128t^7 - 15241t^6 + 66120t^5 - 199999t^4 + 451912t^3 - 792414t^2 + 1101720t - 1228222$	
	8_{16}^a	$t^3 - 4t^2 + 8t - 9$ $t^3 - 6t^4 + 17t^3 - 28t^2 + 35t - 36$	3 / ✗ 2 / ✗		8_{17}^a	$-t^3 + 4t^2 - 8t + 11$ 0	3 / ✗ 1 / ✓		8_{18}^a	$-t^3 + 5t^2 - 10t + 13$ 0	3 / ✗ 2 / ✓
		$8t^{12} - 100t^{11} + 598t^{10} - 2205t^9 + 5292t^8 - 7164t^7 - 2380t^6 + 43100t^5 - 137314t^4 + 291750t^3 - 478742t^2 + 636488t - 698666$				$9t^{12} - 116t^{11} + 722t^{10} - 2843t^9 + 7656t^8 - 13668t^7 + 11117t^6 + 21968t^5 - 113086t^4 + 273778t^3 - 475622t^2 + 649064t - 717954$				$9t^{12} - 145t^{11} + 1075t^{10} - 4842t^9 + 14504t^8 - 28560t^7 + 27957t^6 + 35195t^5 - 225204t^4 + 573797t^3 - 1021641t^2 + 1411484t - 1567262$	
	8_{19}^a	$t^3 - t^2 + 1$ $-3t^5 - 4t^2 - 3t$	3 / ✗ 3 / ✗		8_{20}^a	$t^2 - 2t + 3$ $4t - 4$	2 / ✓ 1 / ✗		8_{21}^a	$-t^2 + 4t - 5$ $t^3 - 8t^2 + 16t - 20$	2 / ✗ 1 / ✗
		$7t^{11} - 19t^{10} + 6t^9 + 48t^8 - 52t^7 - 91t^6 + 211t^5 + 16t^4 - 431t^3 + 289t^2 + 536t - 1060$				$4t^8 - 22t^7 + 66t^6 - 124t^5 + 52t^4 + 478t^3 - 1652t^2 + 3014t - 3640$				$3t^8 - 28t^7 + 49t^6 + 352t^5 - 2489t^4 + 8164t^3 - 17530t^2 + 27092t - 31226$	

knot diag	n'_k $(\rho'_1)^+$	Alexander's ω^+	genus / ribbon unknotting # / amphi?	knot diag	n'_k $(\rho'_1)^+$	Alexander's ω^+	genus / ribbon unknotting # / amphi?
	9_1^a	$t^4 - t^3 + t^2 - t + 1$ $4t^7 + 7t^5 + 9t^3 + 10t$	4 / ✗ 4 / ✗		9_2^a	$4t - 7$ $30t - 40$	1 / ✗ 1 / ✗
		$9t^{15} - 36t^{14} + 99t^{13} - 216t^{12} + 414t^{11} - 720t^{10} + 1170t^9 - 1800t^8 + 2630t^7 - 3662t^6 + 4853t^5 - 6142t^4 + 7423t^3 - 8572t^2 + 9420t - 9780$				$-728t^4 + 6088t^3 - 21946t^2 + 44788t - 56420$	
	9_3^a	$2t^3 - 3t^2 + 3t - 3$ $-13t^5 + 12t^4 - 25t^3 + 20t^2 - 32t + 24$	3 / ✗ 3 / ✗		9_4^a	$3t^2 - 5t + 5$ $23t^3 - 28t^2 + 46t - 44$	2 / ✗ 2 / ✗
		$-26t^{12} + 296t^{11} - 1311t^{10} + 3838t^9 - 8867t^8 + 17613t^7 - 31407t^6 + 51061t^5 - 76085t^4 + 104297t^3 - 131779t^2 + 152840t - 160976$				$-219t^8 + 1999t^7 - 8389t^6 + 23799t^5 - 52835t^4 + 96723t^3 - 149121t^2 + 194698t - 213338$	
	9_5^a	$6t - 11$ $100 - 65t$	1 / ✗ 2 / ✗		9_6^a	$2t^3 - 4t^2 + 5t - 5$ $13t^5 - 24t^4 + 45t^3 - 52t^2 + 68t - 64$	3 / ✗ 3 / ✗
		$-3234t^4 + 29792t^3 - 113241t^2 + 236818t - 300294$				$-26t^{12} + 376t^{11} - 2212t^{10} + 8280t^9 - 23249t^8 + 53488t^7 - 106013t^6 + 185990t^5 - 292853t^4 + 416673t^3 - 537062t^2 + 626488t - 659788$	
	9_7^a	$3t^2 - 7t + 9$ $23t^3 - 56t^2 + 99t - 108$	2 / ✗ 2 / ✗		9_8^a	$-2t^2 + 8t - 11$ $3t^3 - 16t^2 + 29t - 28$	2 / ✗ 2 / ✗
		$-219t^8 + 2717t^7 - 15720t^6 + 58389t^5 - 157698t^4 + 329265t^3 - 548657t^2 + 741610t - 819394$				$54t^8 - 552t^7 + 2124t^6 - 2216t^5 - 12641t^4 + 67112t^3 - 172118t^2 + 289304t - 342134$	
	9_9^a	$2t^3 - 4t^2 + 6t - 7$ $13t^5 - 24t^4 + 55t^3 - 72t^2 + 98t - 96$	3 / ✗ 3 / ✗		9_{10}^a	$4t^2 - 8t + 9$ $-40t^3 + 72t^2 - 114t + 120$	2 / ✗ 2, 3 / ✗
		$-26t^{12} + 376t^{11} - 2296t^{10} + 9328t^9 - 28988t^8 + 73584t^7 - 158399t^6 + 295928t^5 - 486916t^4 + 712094t^3 - 930993t^2 + 1092074t - 1151564$				$-608t^8 + 6720t^7 - 33776t^6 + 110928t^5 - 273462t^4 + 537040t^3 - 862768t^2 + 1145784t - 1259748$	
	9_{11}^a	$-t^3 + 5t^2 - 7t + 7$ $-2t^5 + 16t^4 - 41t^3 + 52t^2 - 66t + 64$	3 / ✗ 2 / ✗		9_{12}^a	$-2t^2 + 9t - 13$ $5t^3 - 36t^2 + 84t - 100$	2 / ✗ 1 / ✗
		$5t^{12} - 65t^{11} + 312t^{10} - 463t^9 - 2042t^8 + 14588t^7 - 50444t^6 + 126967t^5 - 258750t^4 + 444545t^3 - 654213t^2 + 827220t - 895336$				$38t^8 - 312t^7 + 45t^6 + 9790t^5 - 60473t^4 + 202775t^3 - 453255t^2 + 722176t - 841572$	
	9_{13}^a	$4t^2 - 9t + 11$ $-40t^3 + 92t^2 - 154t + 168$	2 / ✗ 2, 3 / ✗		9_{14}^a	$2t^2 - 9t + 15$ $-t^3 + 8t^2 - 35t + 60$	2 / ✗ 1 / ✗
		$-608t^8 + 7680t^7 - 43650t^6 + 158004t^5 - 417129t^4 + 856533t^3 - 1412461t^2 + 1899222t - 2095210$				$62t^8 - 752t^7 + 3655t^6 - 7178t^5 - 9502t^4 + 97737t^3 - 294656t^2 + 531720t - 642168$	
	9_{15}^a	$-2t^2 + 10t - 15$ $-5t^3 + 40t^2 - 108t + 136$	2 / ✗ 2 / ✗		9_{16}^a	$2t^3 - 5t^2 + 8t - 9$ $-13t^5 + 36t^4 - 80t^3 + 120t^2 - 161t + 168$	3 / ✗ 3 / ✗
		$38t^8 - 360t^7 + 208t^6 + 12328t^5 - 84103t^4 + 298764t^3 - 691161t^2 + 1121034t - 1313504$				$-26t^{12} + 456t^{11} - 3331t^{10} + 15554t^9 - 53941t^8 + 149494t^7 - 345106t^6 + 680900t^5 - 1167591t^4 + 1759576t^3 - 2347749t^2 + 2786466t - 2949428$	
	9_{17}^a	$t^3 - 5t^2 + 9t - 9$ $t^5 - 8t^4 + 23t^3 - 32t^2 + 28t - 24$	3 / ✗ 2 / ✗		9_{18}^a	$4t^2 - 10t + 13$ $40t^3 - 108t^2 + 193t - 220$	2 / ✗ 2 / ✗
		$8t^{12} - 125t^{11} + 874t^{10} - 3595t^9 + 9462t^8 - 15166t^7 + 6162t^6 + 47027t^5 - 181220t^4 + 415509t^3 - 716070t^2 + 982036t - 1089796$				$-608t^8 + 8224t^7 - 51208t^6 + 201904t^5 - 570516t^4 + 1228920t^3 - 2087725t^2 + 2850858t - 3159722$	
	9_{19}^a	$2t^2 - 10t + 17$ $t^3 - 8t^2 + 20t - 24$	2 / ✗ 1 / ✗		9_{20}^a	$-t^3 + 5t^2 - 9t + 11$ $2t^5 - 16t^4 + 47t^3 - 84t^2 + 117t - 124$	3 / ✗ 2 / ✗
		$62t^8 - 840t^7 + 4536t^6 - 10352t^5 - 7041t^4 + 116428t^3 - 372683t^2 + 688198t - 836608$				$5t^{12} - 65t^{11} + 330t^{10} - 577t^9 - 2439t^8 + 21482t^7 - 86959t^6 + 247237t^5 - 548658t^4 + 993841t^3 - 1502637t^2 + 1918532t - 2080192$	
	9_{21}^a	$-2t^2 + 11t - 17$ $-5t^3 + 44t^2 - 127t + 164$	2 / ✗ 1 / ✗		9_{22}^a	$t^3 - 5t^2 + 10t - 11$ $-t^5 + 8t^4 - 24t^3 + 38t^2 - 40t + 36$	3 / ✗ 1 / ✗
		$38t^8 - 408t^7 + 493t^6 + 13802t^5 - 105014t^4 + 396685t^3 - 954552t^2 + 1583140t - 1868380$				$8t^{12} - 125t^{11} + 893t^{10} - 3824t^9 + 10605t^8 - 17902t^7 + 6990t^6 + 64299t^5 - 251573t^4 + 584313t^3 - 1012133t^2 + 1388650t - 1540398$	
	9_{23}^a	$4t^2 - 11t + 15$ $40t^3 - 128t^2 + 243t - 288$	2 / ✗ 2 / ✗		9_{24}^a	$-t^3 + 5t^2 - 10t + 13$ $-4t^2 + 16t - 20$	3 / ✗ 1 / ✗
		$-608t^8 + 9184t^7 - 62698t^6 + 265980t^5 - 794496t^4 + 1781111t^3 - 3107204t^2 + 4307350t - 4792758$				$9t^{12} - 145t^{11} + 1075t^{10} - 4850t^9 + 14600t^8 - 29112t^7 + 29921t^6 + 30667t^5 - 218916t^4 + 570933t^3 - 1029833t^2 + 1433476t - 1595654$	











knot diag	n_k^a Alexander's ω^+ $(\rho_1)^+$	genus / ribbon unknotting # / amphi?	knot diag	n_k^a Alexander's ω^+ $(\rho_1)^+$	genus / ribbon unknotting # / amphi?
	9_{25}^a $-3t^2 + 12t - 17$ $12t^3 - 70t^2 + 153t - 188$ $174t^8 - 1200t^7 - 1027t^6 + 42696t^5 - 235512t^4 + 740956t^3 - 1585864t^2 + 2460360t - 2841166$	2 / ✗ 2 / ✗		9_{26}^a $t^3 - 5t^2 + 11t - 13$ $-t^5 + 8t^4 - 31t^3 + 64t^2 - 85t + 92$ $8t^{12} - 125t^{11} + 900t^{10} - 3861t^9 + 10351t^8 - 14356t^7 - 12391t^6 + 132473t^5 - 427732t^4 + 939309t^3 - 1588046t^2 + 2154028t - 2381116$	3 / ✗ 1 / ✗
	9_{27}^a $-t^3 + 5t^2 - 11t + 15$ $t^3 - 8t^2 + 24t - 32$ $9t^{12} - 145t^{11} + 1096t^{10} - 5115t^9 + 16088t^8 - 33784t^7 + 37362t^6 + 34075t^5 - 273854t^4 + 743153t^3 - 1374545t^2 + 1941332t - 2171344$	3 / ✓ 1 / ✗		9_{28}^a $t^3 - 5t^2 + 12t - 15$ $t^5 - 8t^4 + 30t^3 - 68t^2 + 105t - 120$ $8t^{12} - 125t^{11} + 923t^{10} - 4138t^9 + 11800t^8 - 18092t^7 - 11101t^6 + 159415t^5 - 543916t^4 + 1228781t^3 - 2107809t^2 + 2877256t - 3186008$	3 / ✗ 1 / ✗
	9_{29}^a $t^3 - 5t^2 + 12t - 15$ $t^5 - 8t^4 + 26t^3 - 48t^2 + 59t - 56$ $8t^{12} - 125t^{11} + 931t^{10} - 4290t^9 + 13096t^8 - 24848t^7 + 13335t^6 + 94047t^5 - 409576t^4 + 1010237t^3 - 1816557t^2 + 2543836t - 2840192$	3 / ✗ 2 / ✗		9_{30}^a $-t^3 + 5t^2 - 12t + 17$ $2t^3 - 10t^2 + 25t - 32$ $9t^{12} - 145t^{11} + 1117t^{10} - 5376t^9 + 17533t^8 - 38170t^7 + 43292t^6 + 43619t^5 - 347397t^4 + 957881t^3 - 1794189t^2 + 2553442t - 2863228$	3 / ✗ 1 / ✗
	9_{31}^a $t^3 - 5t^2 + 13t - 17$ $t^5 - 8t^4 + 33t^3 - 80t^2 + 132t - 152$ $8t^{12} - 125t^{11} + 938t^{10} - 4303t^9 + 12544t^8 - 19138t^7 - 17200t^6 + 204143t^5 - 703180t^4 + 1617365t^3 - 2818190t^2 + 3886636t - 4319004$	3 / ✗ 2 / ✗		9_{32}^a $t^3 - 6t^2 + 14t - 17$ $-t^5 + 10t^4 - 42t^3 + 94t^2 - 133t + 148$ $8t^{12} - 150t^{11} + 1269t^{10} - 6297t^9 + 19455t^8 - 32720t^7 - 11156t^6 + 260282t^5 - 930836t^4 + 2153618t^3 - 3750358t^2 + 5165114t - 5736454$	3 / ✗ 2 / ✗
	9_{33}^a $-t^3 + 6t^2 - 14t + 19$ $t^3 - 10t^2 + 30t - 40$ $9t^{12} - 174t^{11} + 1539t^{10} - 8207t^9 + 28913t^8 - 67184t^7 + 84077t^6 + 55866t^5 - 581640t^4 + 1664798t^3 - 3166838t^2 + 4539202t - 5100726$	3 / ✗ 1 / ✗		9_{34}^a $-t^3 + 6t^2 - 16t + 23$ $3t^3 - 18t^2 + 43t - 56$ $9t^{12} - 174t^{11} + 1581t^{10} - 8831t^9 + 32988t^8 - 81774t^7 + 109631t^6 + 73248t^5 - 829341t^4 + 2480938t^3 - 4869197t^2 + 7112552t - 8043256$	3 / ✗ 1 / ✗
	9_{35}^a $7t - 13$ $90t - 144$ $-6355t^4 + 58861t^3 - 224539t^2 + 470386t - 596734$	1 / ✗ 2, 3 / ✗		9_{36}^a $-t^3 + 5t^2 - 8t + 9$ $-2t^5 + 16t^4 - 44t^3 + 66t^2 - 87t + 88$ $5t^{12} - 65t^{11} + 321t^{10} - 532t^9 - 2081t^8 + 17066t^7 - 64846t^6 + 175611t^5 - 376739t^4 + 668001t^3 - 998037t^2 + 1267342t - 1372104$	3 / ✗ 2 / ✗
	9_{37}^a $2t^2 - 11t + 19$ $t^3 - 8t^2 + 22t - 28$ $62t^8 - 928t^7 + 5487t^6 - 13814t^5 - 6681t^4 + 154867t^3 - 520239t^2 + 983348t - 1204192$	2 / ✗ 2 / ✗		9_{38}^a $5t^2 - 14t + 19$ $62t^3 - 204t^2 + 382t - 452$ $-1414t^8 + 22122t^7 - 153560t^6 + 657340t^5 - 1976110t^4 + 4454362t^3 - 7806448t^2 + 10855582t - 12103772$	2 / ✗ 2, 3 / ✗
	9_{39}^a $-3t^2 + 14t - 21$ $-12t^3 + 84t^2 - 210t + 268$ $174t^8 - 1442t^7 - 690t^6 + 59068t^5 - 366222t^4 + 1247214t^3 - 2815796t^2 + 4505578t - 5255776$	2 / ✗ 1 / ✗		9_{40}^a $t^3 - 7t^2 + 18t - 23$ $t^5 - 12t^4 + 57t^3 - 144t^2 + 229t - 264$ $8t^{12} - 175t^{11} + 1712t^{10} - 9738t^9 + 34250t^8 - 66108t^7 - 11148t^6 + 553509t^5 - 2149560t^4 + 5230963t^3 - 9406248t^2 + 13187800t - 14730526$	3 / ✗ 2 / ✗
	9_{41}^a $3t^2 - 12t + 19$ $3t^3 - 20t^2 + 70t - 108$ $309t^8 - 3288t^7 + 13885t^6 - 20928t^5 - 55179t^4 + 378100t^3 - 1035810t^2 + 1787808t - 2129794$	2 / ✓ 2 / ✗		9_{42}^a $-t^2 + 2t - 1$ $-t^3 + 2t^2 + t - 4$ $3t^8 - 14t^7 + 32t^6 - 96t^5 + 265t^4 - 294t^3 - 498t^2 + 2170t - 3128$	2 / ✗ 1 / ✗
	9_{43}^a $-t^3 + 3t^2 - 2t + 1$ $-2t^5 + 8t^4 - 7t^3 + 2t^2 - 5t + 4$ $5t^{12} - 39t^{11} + 110t^{10} - 108t^9 - 115t^8 + 570t^7 - 1477t^6 + 3453t^5 - 6651t^4 + 10951t^3 - 17188t^2 + 24718t - 28462$	3 / ✗ 2 / ✗		9_{44}^a $t^2 - 4t + 7$ $-2t^2 + 9t - 12$ $4t^8 - 48t^7 + 237t^6 - 496t^5 - 346t^4 + 4988t^3 - 15044t^2 + 26768t - 32126$	2 / ✗ 1 / ✗
	9_{45}^a $-t^2 + 6t - 9$ $t^3 - 14t^2 + 47t - 60$ $3t^8 - 42t^7 + 78t^6 + 1376t^5 - 11135t^4 + 42574t^3 - 102522t^2 + 169806t - 200284$	2 / ✗ 1 / ✗		9_{46}^a $5 - 2t$ $3t - 12$ $-2t^4 + 160t^3 - 1125t^2 + 3082t - 4222$	1 / ✓ 2 / ✗
	9_{47}^a $t^3 - 4t^2 + 6t - 5$ $-t^5 + 6t^4 - 15t^3 + 16t^2 - 10t + 12$ $8t^{12} - 100t^{11} + 560t^{10} - 1841t^9 + 3847t^8 - 4710t^7 - 42t^6 + 17494t^5 - 55447t^4 + 117058t^3 - 193749t^2 + 261386t - 288924$	3 / ✗ 2 / ✗		9_{48}^a $-t^2 + 7t - 11$ $-t^3 + 12t^2 - 42t + 52$ $3t^8 - 49t^7 + 243t^6 + 267t^5 - 8051t^4 + 40499t^3 - 112167t^2 + 199850t - 241202$	2 / ✗ 2 / ✗
	9_{49}^a $3t^2 - 6t + 7$ $-21t^3 + 38t^2 - 61t + 60$ $-123t^8 + 1614t^7 - 8744t^6 + 29928t^5 - 75873t^4 + 152714t^3 - 250794t^2 + 338238t - 373944$	2 / ✗ 3 / ✗		10_1^a $9 - 4t$ $14t - 40$ $-24t^4 + 2136t^3 - 13430t^2 + 34860t - 47068$	1 / ✗ 1 / ✗
	10_2^a $-t^4 + 3t^3 - 3t^2 + 3t - 3$ $3t^7 - 12t^6 + 16t^5 - 20t^4 + 24t^3 - 24t^2 + 27t - 24$ $7t^{16} - 57t^{15} + 189t^{14} - 293t^{13} - 55t^{12} + 1628t^{11} - 5543t^{10} + 13266t^9 - 26589t^8 + 47468t^7 - 77415t^6 + 116549t^5 - 162911t^4 + 212325t^3 - 258413t^2 + 292580t - 305480$	4 / ✗ 3 / ✗		10_3^a $13 - 6t$ $11t - 28$ $870t^4 + 1288t^3 - 27795t^2 + 85718t - 120138$	1 / ✓ 2 / ✗
	10_4^a $-3t^2 + 7t - 7$ $4t^3 - 8t^2 + t + 8$ $294t^8 - 1807t^7 + 4570t^6 - 4305t^5 - 9550t^4 + 49581t^3 - 117456t^2 + 189330t - 221294$	2 / ✗ 2 / ✗		10_5^a $t^4 - 3t^3 + 5t^2 - 5t + 5$ $-2t^7 + 8t^6 - 20t^5 + 28t^4 - 36t^3 + 36t^2 - 39t + 36$ $12t^{16} - 117t^{15} + 565t^{14} - 1757t^{13} + 3847t^{12} - 5960t^{11} + 5381t^{10} + 2968t^9 - 26625t^8 + 75008t^7 - 157415t^6 + 279173t^5 - 436999t^4 + 615297t^3 - 785328t^2 + 909916t - 955948$	4 / ✗ 2 / ✗
	10_6^a $-2t^3 + 6t^2 - 7t + 7$ $9t^5 - 36t^4 + 56t^3 - 72t^2 + 81t - 84$ $62t^{12} - 408t^{11} + 712t^{10} + 2280t^9 - 17493t^8 + 60652t^7 - 153492t^6 + 319048t^5 - 569584t^4 + 890397t^3 - 1228657t^2 + 1496150t - 1599330$	3 / ✗ 3 / ✗		10_7^a $-3t^2 + 11t - 15$ $14t^3 - 72t^2 + 135t - 160$ $114t^8 - 275t^7 - 5840t^6 + 51739t^5 - 222492t^4 + 626425t^3 - 1267348t^2 + 1914410t - 2193462$	2 / ✗ 1 / ✗
	10_8^a $-2t^3 + 5t^2 - 5t + 5$ $7t^5 - 20t^4 + 23t^3 - 28t^2 + 26t - 24$ $94t^{12} - 672t^{11} + 2115t^{10} - 3678t^9 + 2535t^8 + 6453t^7 - 30645t^6 + 78385t^5 - 154895t^4 + 256601t^3 - 367525t^2 + 458500t - 494524$	3 / ✗ 2 / ✗		10_9^a $-t^4 + 3t^3 - 5t^2 + 7t - 7$ $-t^7 + 4t^6 - 10t^5 + 20t^4 - 25t^3 + 28t^2 - 28t + 28$ $15t^{16} - 153t^{15} + 787t^{14} - 2727t^{13} + 7084t^{12} - 14404t^{11} + 22886t^{10} - 26134t^9 + 11540t^8 + 39332t^7 - 146866t^6 + 325115t^5 - 571077t^4 + 856941t^3 - 1131013t^2 + 1330668t - 1403980$	4 / ✗ 1 / ✗
	10_{10}^a $3t^2 - 11t + 17$ $-5t^3 + 24t^2 - 71t + 100$ $285t^8 - 2735t^7 + 10078t^6 - 9479t^5 - 64000t^4 + 327253t^3 - 827377t^2 + 1378130t - 1624314$	2 / ✗ 1 / ✗		10_{11}^a $-4t^2 + 11t - 13$ $16t^3 - 52t^2 + 68t - 72$ $736t^8 - 4672t^7 + 9634t^6 + 11132t^5 - 125367t^4 + 413121t^3 - 873095t^2 + 1336974t - 1536906$	2 / ✗ 2, 3 / ✗
	10_{12}^a $2t^3 - 6t^2 + 10t - 11$ $-5t^5 + 20t^4 - 50t^3 + 72t^2 - 89t + 92$ $118t^{12} - 1080t^{11} + 4748t^{10} - 12624t^9 + 19414t^8 - 2072t^7 - 88507t^6 + 320836t^5 - 750453t^4 + 1366922t^3 - 2053481t^2 + 2604638t - 2816934$	3 / ✗ 2 / ✗		10_{13}^a $2t^2 - 13t + 23$ $t^3 - 12t^2 + 51t - 84$ $62t^8 - 1088t^7 + 7367t^6 - 20586t^5 - 13356t^4 + 286509t^3 - 1005098t^2 + 1954280t - 2416160$	2 / ✗ 2 / ✗

knot diag	n_k^t Alexander's ω^+ $(\rho_1^t)^+$	genus / ribbon unknotting # / amphi?	knot diag	n_k^t Alexander's ω^+ $(\rho_2^t)^+$	genus / ribbon unknotting # / amphi?
	$10^a_{14} \quad -2t^3 + 8t^2 - 12t + 13$ $9t^5 - 52t^4 + 119t^3 - 180t^2 + 225t - 236$ $62t^{12} - 584t^{11} + 1720t^{10} + 2816t^9 - 42848t^8 + 195040t^7 - 594177t^6 + 1407688t^5 - 2753604t^4 + 4575154t^3 - 6545078t^2 + 8106820t - 8706026$	3 / ✗ 2 / ✗		$10^a_{15} \quad 2t^3 - 6t^2 + 9t - 9$ $-3t^5 + 12t^4 - 24t^3 + 24t^2 - 17t + 12$ $134t^{12} - 1272t^{11} + 5792t^{10} - 16520t^9 + 31765t^8 - 37636t^7 + 2396t^6 + 120176t^5 - 371368t^4 + 752873t^3 - 1195043t^2 + 1560190t - 1702986$	3 / ✗ 2 / ✗
	$10^a_{16} \quad -4t^2 + 12t - 15$ $-16t^3 + 56t^2 - 76t + 80$ $736t^8 - 5248t^7 + 12944t^6 + 6528t^5 - 14416t^4 + 522200t^3 - 1155370t^2 + 1809228t - 2093696$	2 / ✗ 2 / ✗		$10^a_{17} \quad t^4 - 3t^3 + 5t^2 - 7t + 9$ 0 $16t^{16} - 165t^{15} + 861t^{14} - 3043t^{13} + 8173t^{12} - 17514t^{11} + 30162t^{10} - 39958t^9 + 32666t^8 + 13998t^7 - 125081t^6 + 317743t^5 - 588481t^4 + 904569t^3 - 1207020t^2 + 1426556t - 1506972$	4 / ✗ 1 / ✓
	$10^a_{18} \quad -4t^2 + 14t - 19$ $16t^3 - 68t^2 + 121t - 140$ $736t^8 - 6240t^7 + 17736t^6 + 11088t^5 - 245648t^4 + 930168t^3 - 2109201t^2 + 3338706t - 3874682$	2 / ✗ 1 / ✗		$10^a_{19} \quad 2t^3 - 7t^2 + 11t - 11$ $3t^5 - 16t^4 + 35t^3 - 40t^2 + 30t - 24$ $134t^{12} - 1480t^{11} + 7641t^{10} - 24194t^9 + 50855t^8 - 66007t^7 + 12323t^6 + 201357t^5 - 665287t^4 + 1397797t^3 - 2271085t^2 + 3006128t - 3296368$	3 / ✗ 2 / ✗
	$10^a_{20} \quad -3t^2 + 9t - 11$ $14t^3 - 56t^2 + 88t - 104$ $114t^8 - 153t^7 - 4783t^6 + 34425t^5 - 128711t^4 + 327435t^3 - 618704t^2 + 899066t - 1017366$	2 / ✗ 2 / ✗		$10^a_{21} \quad -2t^3 + 7t^2 - 9t + 9$ $9t^5 - 44t^4 + 80t^3 - 104t^2 + 121t - 124$ $62t^{12} - 496t^{11} + 1203t^{10} + 2078t^9 - 24456t^8 + 97163t^7 - 267878t^6 + 592041t^5 - 1106738t^4 + 1789591t^3 - 2525732t^2 + 3113752t - 3341184$	3 / ✗ 2 / ✗
	$10^a_{22} \quad -2t^3 + 6t^2 - 10t + 13$ $-t^5 + 4t^4 - 10t^3 + 24t^2 - 37t + 44$ $142t^{12} - 1368t^{11} + 6524t^{10} - 20120t^9 + 42790t^8 - 57928t^7 + 16919t^6 + 158700t^5 - 540707t^4 + 1130294t^3 - 1809643t^2 + 2363114t - 2577418$	3 / ✓ 2 / ✗		$10^a_{23} \quad 2t^3 - 7t^2 + 13t - 15$ $-5t^5 + 24t^4 - 67t^3 + 108t^2 - 137t + 144$ $118t^{12} - 1272t^{11} + 6541t^{10} - 20402t^9 + 38443t^8 - 21945t^7 - 132442t^6 + 594335t^5 - 1530420t^4 + 2960363t^3 - 4622193t^2 + 5992048t - 6526360$	3 / ✗ 1 / ✗
	$10^a_{24} \quad -4t^2 + 14t - 19$ $24t^3 - 116t^2 + 221t - 268$ $416t^8 - 1568t^7 - 13224t^6 + 136928t^5 - 604124t^4 + 1701008t^3 - 3414673t^2 + 5118714t - 5846946$	2 / ✗ 2 / ✗		$10^a_{25} \quad -2t^3 + 8t^2 - 14t + 17$ $9t^5 - 52t^4 + 131t^3 - 232t^2 + 314t - 344$ $62t^{12} - 584t^{11} + 1856t^{10} + 2264t^9 - 47052t^8 + 241288t^7 - 809541t^6 + 2068016t^5 - 4270010t^4 + 7347930t^3 - 10723331t^2 + 13406206t - 14434208$	3 / ✗ 2 / ✗
	$10^a_{26} \quad -2t^3 + 7t^2 - 13t + 17$ $-t^5 + 4t^4 - 10t^3 + 28t^2 - 49t + 60$ $142t^{12} - 1600t^{11} + 8823t^{10} - 31058t^9 + 74964t^8 - 117897t^7 + 67064t^6 + 255997t^5 - 1047600t^4 + 2360395t^3 - 3947888t^2 + 5281288t - 5805248$	3 / ✗ 1 / ✗		$10^a_{27} \quad 2t^3 - 8t^2 + 16t - 19$ $5t^5 - 28t^4 + 87t^3 - 164t^2 + 229t - 252$ $118t^{12} - 1464t^{11} + 8536t^{10} - 29792t^9 + 62096t^8 - 39696t^7 - 242195t^6 + 1151848t^5 - 3078140t^4 + 6098910t^3 - 9661940t^2 + 12621240t - 13779050$	3 / ✗ 1 / ✗
	$10^a_{28} \quad 4t^2 - 13t + 19$ $-8t^3 + 36t^2 - 100t + 136$ $928t^8 - 7872t^7 + 26174t^6 - 22588t^5 - 142295t^4 + 689113t^3 - 1676391t^2 + 2728998t - 3192146$	2 / ✗ 2 / ✗		$10^a_{29} \quad t^3 - 7t^2 + 15t - 17$ $t^5 - 12t^4 + 52t^3 - 104t^2 + 124t - 128$ $8t^{12} - 175t^{11} + 1659t^{10} - 8913t^9 + 29252t^8 - 54292t^7 + 10680t^6 + 290989t^5 - 1126663t^4 + 2673211t^3 - 4723498t^2 + 6566572t - 7317656$	3 / ✗ 2 / ✗
	$10^a_{30} \quad -4t^2 + 17t - 25$ $24t^3 - 148t^2 + 345t - 440$ $416t^8 - 2048t^7 - 17490t^6 + 219996t^5 - 1101894t^4 + 3396907t^3 - 7245510t^2 + 11243734t - 12988226$	2 / ✗ 1 / ✗		$10^a_{31} \quad 4t^2 - 14t + 21$ $-4t^2 + 9t - 12$ $992t^8 - 9440t^7 + 36936t^6 - 59136t^5 - 72624t^4 + 623304t^3 - 1691899t^2 + 2867550t - 3391374$	2 / ✗ 1 / ✗
	$10^a_{32} \quad -2t^3 + 8t^2 - 15t + 19$ $t^5 - 4t^4 + 13t^3 - 40t^2 + 78t - 96$ $142t^{12} - 1832t^{11} + 11204t^{10} - 42688t^9 + 109909t^8 - 184384t^7 + 124831t^6 + 360782t^5 - 1615391t^4 + 3759585t^3 - 6404890t^2 + 8655360t - 9545252$	3 / ✗ 1 / ✗		$10^a_{33} \quad 4t^2 - 16t + 25$ 0 $992t^8 - 10816t^7 + 47856t^6 - 88336t^5 - 84402t^4 + 920320t^3 - 2655340t^2 + 4640912t - 5542372$	2 / ✗ 1 / ✓
	$10^a_{34} \quad 3t^2 - 9t + 13$ $-5t^3 + 20t^2 - 52t + 68$ $285t^8 - 2205t^7 + 6601t^6 - 3429t^5 - 43369t^4 + 185703t^3 - 431857t^2 + 687874t - 799218$	2 / ✗ 2 / ✗		$10^a_{35} \quad 2t^2 - 12t + 21$ $-t^3 + 12t^2 - 47t + 76$ $62t^8 - 1000t^7 + 6244t^6 - 15744t^5 - 15707t^4 + 232680t^3 - 775840t^2 + 1474372t - 1810118$	2 / ✓ 2 / ✗
	$10^a_{36} \quad -3t^2 + 13t - 19$ $14t^3 - 88t^2 + 208t - 264$ $114t^8 - 397t^7 - 7597t^6 + 81141t^5 - 393441t^4 + 1198967t^3 - 2544952t^2 + 3941362t - 4550398$	2 / ✗ 2 / ✗		$10^a_{37} \quad 4t^2 - 13t + 19$ 0 $992t^8 - 8736t^7 + 31914t^6 - 47212t^5 - 64499t^4 + 497921t^3 - 1308755t^2 + 2181630t - 2566522$	2 / ✗ 2 / ✓
	$10^a_{38} \quad -4t^2 + 15t - 21$ $24t^3 - 128t^2 + 270t - 336$ $416t^8 - 1632t^7 - 16122t^6 + 172460t^5 - 788845t^4 + 2280037t^3 - 4653713t^2 + 7038342t - 8061882$	2 / ✗ 2 / ✗		$10^a_{39} \quad -2t^3 + 8t^2 - 13t + 15$ $9t^5 - 52t^4 + 125t^3 - 204t^2 + 263t - 280$ $62t^{12} - 584t^{11} + 1788t^{10} + 2480t^9 - 44191t^8 + 213488t^7 - 683173t^6 + 1684054t^5 - 3393468t^4 + 5753447t^3 - 8330571t^2 + 10379080t - 11164828$	3 / ✗ 2 / ✗
	$10^a_{40} \quad 2t^3 - 8t^2 + 17t - 21$ $-5t^5 + 28t^4 - 89t^3 + 176t^2 - 258t + 288$ $118t^{12} - 1464t^{11} + 8692t^{10} - 31256t^9 + 67987t^8 - 49624t^7 - 257955t^6 + 1301482t^5 - 3582545t^4 + 7240253t^3 - 11620382t^2 + 15292356t - 16735336$	3 / ✗ 2 / ✗		$10^a_{41} \quad t^3 - 7t^2 + 17t - 21$ $t^5 - 12t^4 + 54t^3 - 120t^2 + 157t - 164$ $8t^{12} - 175t^{11} + 1697t^{10} - 9543t^9 + 33561t^8 - 69114t^7 + 29117t^6 + 354127t^5 - 1527139t^4 + 3836499t^3 - 7019042t^2 + 9942516t - 11145016$	3 / ✗ 2 / ✗
	$10^a_{42} \quad -t^3 + 7t^2 - 19t + 27$ $2t^3 - 8t^2 + 11t - 12$ $9t^{12} - 203t^{11} + 2093t^{10} - 12971t^9 + 52885t^8 - 142268t^7 + 214987t^6 + 60931t^5 - 1368859t^4 + 4365895t^3 - 8815357t^2 + 13058404t - 14831092$	3 / ✓ 1 / ✗		$10^a_{43} \quad -t^3 + 7t^2 - 17t + 23$ 0 $9t^{12} - 203t^{11} + 2051t^{10} - 12253t^9 + 47594t^8 - 120962t^7 + 170450t^6 + 61017t^5 - 1045911t^4 + 3175271t^3 - 6209661t^2 + 9025932t - 10186676$	3 / ✗ 2 / ✓
	$10^a_{44} \quad t^3 - 7t^2 + 19t - 25$ $t^5 - 12t^4 + 56t^3 - 140t^2 + 220t - 248$ $8t^{12} - 175t^{11} + 1735t^{10} - 10157t^9 + 37586t^8 - 81160t^7 + 29232t^6 + 500937t^5 - 2197451t^4 + 5635115t^3 - 10448058t^2 + 14900236t - 16735696$	3 / ✗ 1 / ✗		$10^a_{45} \quad -t^3 + 7t^2 - 21t + 31$ 0 $9t^{12} - 203t^{11} + 2135t^{10} - 13689t^9 + 58324t^8 - 165246t^7 + 266640t^6 + 52413t^5 - 1738539t^4 + 5821367t^3 - 12123077t^2 + 18290148t - 20900556$	3 / ✗ 2 / ✓
	$10^a_{46} \quad -t^4 + 3t^3 - 4t^2 + 5t - 5$ $-3t^7 + 12t^6 - 21t^5 + 34t^4 - 43t^3 + 52t^2 - 55t + 56$ $7t^{16} - 57t^{15} + 204t^{14} - 382t^{13} + 69t^{12} + 2247t^{11} - 9674t^{10} + 27287t^9 - 61957t^8 + 121378t^7 - 211961t^6 + 335438t^5 - 485235t^4 + 644818t^3 - 789365t^2 + 891215t - 928064$	4 / ✗ 3 / ✗		$10^a_{47} \quad t^4 - 3t^3 + 6t^2 - 7t + 7$ $-2t^7 + 8t^6 - 23t^5 + 38t^4 - 56t^3 + 60t^2 - 68t + 64$ $12t^{16} - 117t^{15} + 598t^{14} - 2030t^{13} + 4959t^{12} - 8715t^{11} + 9312t^{10} + 2921t^9 - 44823t^8 + 139602t^7 - 312112t^6 + 579182t^5 - 936546t^4 + 1347538t^3 - 1741633t^2 + 2029805t - 2135930$	4 / ✗ 2, 3 / ✗
	$10^a_{48} \quad t^4 - 3t^3 + 6t^2 - 9t + 11$ $t^5 - 2t^4 + 2t^3 - 3t + 4$ $16t^{16} - 165t^{15} + 906t^{14} - 3452t^{13} + 10069t^{12} - 23423t^{11} + 43765t^{10} - 63343t^9 + 59588t^8 + 8232t^7 - 192505t^6 + 537134t^5 - 1048176t^4 + 1669528t^3 - 2281994t^2 + 2735109t - 2902594$	4 / ✓ 2 / ✗		$10^a_{49} \quad 3t^3 - 8t^2 + 12t - 13$ $30t^5 - 94t^4 + 196t^3 - 292t^2 + 372t - 392$ $-177t^{12} + 3028t^{11} - 22080t^{10} + 101361t^9 - 341354t^8 + 914348t^7 - 2044469t^6 + 3931812t^5 - 6622778t^4 + 9874270t^3 - 13105110t^2 + 15522532t - 16422794$	3 / ✗ 3 / ✗

knot diag	n_k^+ Alexander's ω^+ $(\rho_1^+)^+$	genus / ribbon unknotting # / amphi?	knot diag	n_k^+ Alexander's ω^+ $(\rho_1^+)^+$	genus / ribbon unknotting # / amphi?
	10_{50}^a $-2t^3 + 7t^2 - 11t + 13$ $-9r^5 + 44r^4 - 94r^3 + 150r^2 - 186t + 200$ $62r^{12} - 496r^{11} + 1283r^{10} + 2094r^9 - 29732r^8 + 134301r^7 - 412809r^6 + 990903r^5 - 1959941r^4 + 3278621r^3 - 4702408r^2 + 5824956r - 6253664$	3 / ✗ 2 / ✗		10_{51}^a $2t^3 - 7t^2 + 15t - 19$ $-5r^5 + 24r^4 - 73r^3 + 134r^2 - 194t + 212$ $118r^{12} - 1272r^{11} + 6813r^{10} - 22602r^9 + 45771r^8 - 28275r^7 - 180411r^6 + 857569r^5 - 2306697r^4 + 4602641r^3 - 7332665r^2 + 9612128r - 10506256$	3 / ✗ 2, 3 / ✗
	10_{52}^a $2t^3 - 7t^2 + 13t - 15$ $-3r^5 + 16r^4 - 37r^3 + 50r^2 - 49t + 44$ $134r^{12} - 1480r^{11} + 7961r^{10} - 27058r^9 + 62159r^8 - 88993r^7 + 22042r^6 + 296843r^5 - 1040240r^4 + 2254967r^3 - 3720017r^2 + 4952400r - 5437448$	3 / ✗ 2 / ✗		10_{53}^a $6t^2 - 18t + 25$ $93r^3 - 346t^2 + 680t - 828$ $-3642r^8 + 58248r^7 - 417976r^6 + 1846212r^5 - 5694639r^4 + 13084936r^3 - 23231163r^2 + 32545278r - 36374532$	2 / ✗ 2, 3 / ✗
	10_{54}^a $2t^3 - 6t^2 + 10t - 11$ $-3r^5 + 12r^4 - 24r^3 + 26t^2 - 21t + 16$ $134r^{12} - 1272r^{11} + 5964r^{10} - 17880r^9 + 36606r^8 - 46740r^7 + 6565r^6 + 150576r^5 - 487825r^4 + 1010638r^3 - 1619593r^2 + 2120978r - 2316318$	3 / ✗ 2, 3 / ✗		10_{55}^a $5t^2 - 15t + 21$ $66r^3 - 246t^2 + 488t - 596$ $-1966r^8 + 30491r^7 - 215627r^6 + 945597r^5 - 2905831r^4 + 6662951r^3 - 11814712r^2 + 16540014r - 18481854$	2 / ✗ 2 / ✗
	10_{56}^a $-2t^3 + 8t^2 - 14t + 17$ $-9r^5 + 52r^4 - 133r^3 + 234t^2 - 312t + 340$ $62r^{12} - 584r^{11} + 1800r^{10} + 2840r^9 - 49588r^8 + 247616r^7 - 819257r^6 + 2077408r^5 - 4277830r^4 + 7364010r^3 - 10765639r^2 + 13481990r - 14525656$	3 / ✗ 2 / ✗		10_{57}^a $2t^3 - 8t^2 + 18t - 23$ $-5r^5 + 28r^4 - 93r^3 + 194t^2 - 300t + 340$ $118r^{12} - 1464r^{11} + 8808r^{10} - 32264r^9 + 71276r^8 - 49320r^7 - 305843r^6 + 1537376r^5 - 4286854r^4 + 8774390r^3 - 14221383r^2 + 18829374r - 20648444$	3 / ✗ 2 / ✗
	10_{58}^a $3t^2 - 16t + 27$ $3r^3 - 28t^2 + 94t - 140$ $309r^8 - 4384r^7 + 24039r^6 - 49896r^5 - 90763r^4 + 864784r^3 - 2647834r^2 + 4837480r - 5867454$	2 / ✗ 2 / ✗		10_{59}^a $t^3 - 7t^2 + 18t - 23$ $-r^5 + 12r^4 - 55r^3 + 128t^2 - 181t + 196$ $8r^{12} - 175r^{11} + 1716r^{10} - 9858r^9 + 35706r^8 - 76124r^7 + 33704r^6 + 412653r^5 - 1824096r^4 + 4655939r^3 - 8596644r^2 + 12230816r - 13727286$	3 / ✗ 1 / ✗
	10_{60}^a $-t^3 + 7t^2 - 20t + 29$ $5r^3 - 40r^2 + 122t - 176$ $9r^{12} - 203r^{11} + 2114r^{10} - 13338r^9 + 55732r^8 - 154496r^7 + 241898r^6 + 66137r^5 - 1621594r^4 + 5326603r^3 - 10989858r^2 + 16499428r - 18824860$	3 / ✗ 1 / ✗		10_{61}^a $-2t^3 + 5t^2 - 6t + 7$ $-7r^5 + 20r^4 - 27r^3 + 36t^2 - 35t + 36$ $94r^{12} - 672r^{11} + 2231r^{10} - 4382r^9 + 4108r^8 + 6320r^7 - 40187r^6 + 113296r^5 - 235714r^4 + 400470r^3 - 576529r^2 + 714816r - 767686$	3 / ✗ 2, 3 / ✗
	10_{62}^a $t^4 - 3r^3 + 6t^2 - 8t + 9$ $-2t^7 + 8t^6 - 23r^5 + 40r^4 - 63r^3 + 76t^2 - 89t + 88$ $12r^{16} - 117r^{15} + 598r^{14} - 2057r^{13} + 5172r^{12} - 9509r^{11} + 10856r^{10} + 2734r^9 - 54502r^8 + 178917r^7 - 414312r^6 + 786766r^5 - 1289208r^4 + 1865866r^3 - 2414454r^2 + 2812025r - 2957594$	4 / ✗ 2 / ✗		10_{63}^a $5t^2 - 14t + 19$ $66r^3 - 220t^2 + 416t - 496$ $-1966r^8 + 28318r^7 - 188800r^6 + 783388r^5 - 2311570r^4 + 5141906r^3 - 8929148r^2 + 12349082r - 13743884$	2 / ✗ 2 / ✗
	10_{64}^a $-t^4 + 3r^2 - 6t^2 + 10t - 11$ $-t^7 + 4t^6 - 11r^5 + 24r^4 - 37r^3 + 52t^2 - 60t + 64$ $15r^{16} - 153r^{15} + 830r^{14} - 3147r^{13} + 9133r^{12} - 20983r^{11} + 37963r^{10} - 50164r^9 + 30642r^8 + 68741r^7 - 310036r^6 + 745430r^5 - 1381735r^4 + 2150560r^3 - 2906317r^2 + 3464829r - 3671204$	4 / ✗ 2 / ✗		10_{65}^a $2t^3 - 7t^2 + 14t - 17$ $-5r^5 + 24r^4 - 71r^3 + 124t^2 - 169t + 180$ $118r^{12} - 1272r^{11} + 6657r^{10} - 21282r^9 + 40874r^8 - 20768r^7 - 166691r^6 + 742216r^5 - 1933704r^4 + 3781794r^3 - 5950947r^2 + 7749120r - 8452246$	3 / ✗ 2 / ✗
	10_{66}^a $3r^3 - 9t^2 + 16t - 19$ $30r^5 - 112r^4 + 279r^3 - 480r^2 + 662t - 724$ $-177r^{12} + 3321r^{11} - 27536r^{10} + 145346r^9 - 561614r^8 + 1706788r^7 - 4256134r^6 + 8946173r^5 - 16135424r^4 + 25271935r^3 - 34647456r^2 + 41790680r - 44471832$	3 / ✗ 3 / ✗		10_{67}^a $-4t^2 + 16t - 23$ $24r^3 - 140t^2 + 312t - 392$ $416r^8 - 1696r^7 - 18592r^6 + 205384r^5 - 971474r^4 + 2884880r^3 - 6004484r^2 + 9188872r - 10566612$	2 / ✗ 2 / ✗
	10_{68}^a $4t^2 - 14t + 21$ $8r^3 - 40r^2 + 117t - 164$ $928r^8 - 8448r^7 + 29784r^6 - 26736r^5 - 178984r^4 + 891736r^3 - 2217147r^2 + 3657390r - 4297054$	2 / ✗ 2 / ✗		10_{69}^a $t^3 - 7t^2 + 21t - 29$ $-r^5 + 12r^4 - 68r^3 + 212t^2 - 397t + 476$ $8r^{12} - 175r^{11} + 1753r^{10} - 10339r^9 + 37435r^8 - 68174r^7 - 78997r^6 + 1015635r^5 - 3880779r^4 + 9697491r^3 - 17937826r^2 + 25646300r - 28844672$	3 / ✗ 2 / ✗
	10_{70}^a $t^3 - 7t^2 + 16t - 19$ $-r^5 + 12r^4 - 53r^3 + 114t^2 - 146t + 152$ $8r^{12} - 175r^{11} + 1678r^{10} - 9220r^9 + 31251r^8 - 60450r^7 + 14335r^6 + 337593r^5 - 1351773r^4 + 3275803r^3 - 5864336r^2 + 8208654r - 9166724$	3 / ✗ 2 / ✗		10_{71}^a $-t^3 + 7t^2 - 18t + 25$ $t^3 - 2t^2 - t + 4$ $9r^{12} - 203r^{11} + 2072r^{10} - 12608r^9 + 50167r^8 - 131082r^7 + 190655r^6 + 64937r^5 - 1206917r^4 + 3745659r^3 - 7436102r^2 + 10906778r - 12346734$	3 / ✗ 1 / ✗
	10_{72}^a $-2t^3 + 9t^2 - 16t + 19$ $-9r^5 + 60r^4 - 167r^3 + 298t^2 - 410t + 448$ $62r^{12} - 672r^{11} + 2407r^{10} + 2846r^9 - 67046r^8 + 358714r^7 - 1237440r^6 + 3225136r^5 - 6760702r^4 + 11767984r^3 - 17315777r^2 + 21757146r - 23465324$	3 / ✗ 2 / ✗		10_{73}^a $t^3 - 7t^2 + 20t - 27$ $t^5 - 12r^4 + 65r^3 - 194t^2 + 350t - 416$ $8r^{12} - 175r^{11} + 1738r^{10} - 10112r^9 + 36117r^8 - 66038r^7 - 61235r^6 + 869449r^5 - 3296603r^4 + 8133803r^3 - 14880880r^2 + 21122890r - 23697928$	3 / ✗ 1 / ✗
	10_{74}^a $-4t^2 + 16t - 23$ $24r^3 - 136t^2 + 290t - 360$ $416r^8 - 1984r^7 - 14448r^6 + 178832r^5 - 870542r^4 + 2626104r^3 - 5521764r^2 + 8500760r - 9794748$	2 / ✗ 2 / ✗		10_{75}^a $-t^3 + 7t^2 - 19t + 27$ $-4r^3 + 36t^2 - 117t + 172$ $9r^{12} - 203r^{11} + 2093r^{10} - 12979r^9 + 53085r^8 - 144060r^7 + 222795r^6 + 45939r^5 - 1382507r^4 + 4528919r^3 - 9302365r^2 + 13926940r - 15875332$	3 / ✗ 2 / ✗
	10_{76}^a $-2t^3 + 7t^2 - 12t + 15$ $-9r^5 + 44r^4 - 104r^3 + 184t^2 - 245t + 272$ $62r^{12} - 496r^{11} + 1263r^{10} + 2926r^9 - 37611r^8 + 174774r^7 - 553794r^6 + 1359740r^5 - 2727505r^4 + 4595668r^3 - 6610039r^2 + 8193314r - 8796596$	3 / ✗ 2, 3 / ✗		10_{77}^a $2t^3 - 7t^2 + 14t - 17$ $-5r^5 + 24r^4 - 71r^3 + 132t^2 - 189t + 208$ $118r^{12} - 1272r^{11} + 6657r^{10} - 21170r^9 + 39602r^8 - 13480r^7 - 193563r^6 + 812568r^5 - 2072452r^4 + 3997538r^3 - 6227879r^2 + 8058912r - 8771174$	3 / ✗ 2, 3 / ✗
	10_{78}^a $-t^3 + 7t^2 - 16t + 21$ $2r^5 - 24r^4 + 105r^3 - 244t^2 + 390t - 448$ $5r^{12} - 91r^{11} + 626r^{10} - 1310r^9 - 9682r^8 + 98268r^7 - 472808r^6 + 1558897r^5 - 3892200r^4 + 7699107r^3 - 12365278r^2 + 16351352r - 17933784$	3 / ✗ 2 / ✗		10_{79}^a $t^4 - 3r^3 + 7t^2 - 12t + 15$ 0 $16r^{16} - 165r^{15} + 951r^{14} - 3892r^{13} + 12327r^{12} - 31301r^{11} + 64047r^{10} - 102088r^9 + 108942r^8 - 5172r^7 - 328635r^6 + 1013644r^5 - 2099318r^4 + 3486798r^3 - 4904824r^2 + 5979109r - 6380898$	4 / ✗ 2, 3 / ✓
	10_{80}^a $3r^3 - 9t^2 + 15t - 17$ $30r^5 - 112r^4 + 260r^3 - 426t^2 + 568t - 616$ $-177r^{12} + 3321r^{11} - 26919r^{10} + 137419r^9 - 511788r^8 + 1500906r^7 - 3625608r^6 + 7420093r^5 - 13101785r^4 + 20196767r^3 - 27388655r^2 + 32826444r - 34860060$	3 / ✗ 3 / ✗		10_{81}^a $-t^3 + 8t^2 - 20t + 27$ 0 $9r^{12} - 232r^{11} + 2632r^{10} - 17347r^9 + 73146r^8 - 199476r^7 + 303717r^6 + 63516r^5 - 1783222r^4 + 5636674r^3 - 11239918r^2 + 16501092r - 18681194$	3 / ✗ 2 / ✓
	10_{82}^a $-t^4 + 4r^3 - 8t^2 + 12t - 13$ $t^7 - 6t^6 + 19r^5 - 42r^4 + 64r^3 - 78t^2 + 84t - 84$ $15r^{16} - 204r^{15} + 1362r^{14} - 5956r^{13} + 19067r^{12} - 46940r^{11} + 89646r^{10} - 125984r^9 + 94379r^8 + 118488r^7 - 663600r^6 + 1675944r^5 - 3187626r^4 + 5046508r^3 - 6899632r^2 + 8282752r - 8796438$	4 / ✗ 1 / ✗		10_{83}^a $2t^3 - 9t^2 + 19t - 23$ $-5r^5 + 34r^4 - 110r^3 + 214t^2 - 301t + 332$ $118r^{12} - 1632r^{11} + 10501r^{10} - 40166r^9 + 92154r^8 - 74661r^7 - 344938r^6 + 1829049r^5 - 5155786r^4 + 10589003r^3 - 17184002r^2 + 22763416r - 24966116$	3 / ✗ 2 / ✗

knot diag	n_k^+ Alexander's ω^+ $(\rho_1)^+$	genus / ribbon unknotting # / amphi?	knot diag	n_k^+ Alexander's ω^+ $(\rho_1)^+$	genus / ribbon unknotting # / amphi?
	10_{84}^a $2t^3 - 9t^2 + 20t - 25$ $-5t^5 + 34t^4 - 116t^3 + 246t^2 - 373t + 424$ $118t^{12} - 1632t^{11} + 10601t^{10} - 40970t^9 + 93361t^8 - 60130t^7 - 457712t^6 + 2276184t^5 - 6379977t^4 + 13131088t^3 - 21370125t^2 + 28363542t - 31128704$	3 / ✗ 1 / ✗		10_{85}^a $t^4 - 4t^3 + 8t^2 - 10t + 11$ $2t^7 - 12t^6 + 36t^5 - 68t^4 + 101t^3 - 124t^2 + 138t - 140$ $12t^{16} - 156t^{15} + 986t^{14} - 3982t^{13} + 11319t^{12} - 23042t^{11} + 29987t^{10} - 3098t^9 - 116460t^8 + 418314t^7 - 1005425t^6 + 1953048t^5 - 3252398t^4 + 4764776t^3 - 6220611t^2 + 7285042t - 7676632$	4 / ✗ 2 / ✗
	10_{86}^a $-2t^3 + 9t^2 - 19t + 25$ $-t^5 + 6t^4 - 21t^3 + 58t^2 - 105t + 128$ $142t^{12} - 2056t^{11} + 14135t^{10} - 60346t^9 + 173073t^8 - 322457t^7 + 256132t^6 + 640839t^5 - 3192178t^4 + 7806511t^3 - 13712731t^2 + 18852080t - 20906284$	3 / ✗ 2 / ✗		10_{87}^a $-2t^3 + 9t^2 - 18t + 23$ $-t^5 + 6t^4 - 23t^3 + 66t^2 - 125t + 152$ $142t^{12} - 2056t^{11} + 13955t^{10} - 58318t^9 + 162798t^8 - 293228t^7 + 214867t^6 + 612960t^5 - 2882460t^4 + 6902570t^3 - 11979669t^2 + 16361444t - 18106010$	3 / ✓ 2 / ✗
	10_{88}^a $-t^3 + 8t^2 - 24t + 35$ 0	3 / ✗ 1 / ✓		10_{89}^a $t^3 - 8t^2 + 24t - 33$ $t^5 - 14t^4 + 83t^3 - 264t^2 + 495t - 596$ $8t^{12} - 200t^{11} + 2236t^{10} - 14461t^9 + 56992t^8 - 117072t^7 - 76152t^6 + 1508604t^5 - 6093936t^4 + 15620030t^3 - 29286604t^2 + 42155400t - 47509694$	3 / ✗ 2 / ✗
	10_{90}^a $-2t^3 + 8t^2 - 17t + 23$ $-t^5 + 6t^4 - 21t^3 + 54t^2 - 93t + 112$ $142t^{12} - 1824t^{11} + 11452t^{10} - 45568t^9 + 123153t^8 - 214976t^7 + 138515t^6 + 523918t^5 - 2309034t^4 + 5458443t^3 - 9432309t^2 + 12861496t - 14226804$	3 / ✗ 2 / ✗		10_{91}^a $t^4 - 4t^3 + 9t^2 - 14t + 17$ $t^5 - 2t^4 + 2t^3 - 3t + 4$ $16t^{16} - 220t^{15} + 1535t^{14} - 7166t^{13} + 24885t^{12} - 67476t^{11} + 145070t^{10} - 242014t^9 + 278753t^8 - 78212t^7 - 624329t^6 + 2091910t^5 - 4424108t^4 + 7397630t^3 - 10425418t^2 + 12711814t - 13565348$	4 / ✗ 1 / ✗
	10_{92}^a $-2t^3 + 10t^2 - 20t + 25$ $-9t^5 + 68t^4 - 216t^3 + 428t^2 - 622t + 696$ $62t^{12} - 760t^{11} + 3228t^{10} + 1776t^9 - 90686t^8 + 555772t^7 - 2114169t^6 + 5951964t^5 - 13251159t^4 + 24127850t^3 - 36624016t^2 + 46862460t - 50844652$	3 / ✗ 2 / ✗		10_{93}^a $2t^3 - 8t^2 + 15t - 17$ $3t^5 - 18t^4 + 43t^3 - 58t^2 + 55t - 48$ $134t^{12} - 1696t^{11} + 10180t^{10} - 37880t^9 + 94183t^8 - 147272t^7 + 62729t^6 + 424866t^5 - 1618596t^4 + 3616743t^3 - 6059793t^2 + 8130868t - 8948936$	3 / ✗ 2 / ✗
	10_{94}^a $-t^4 + 4t^3 - 9t^2 + 14t - 15$ $-t^7 + 6t^6 - 20t^5 + 46t^4 - 76t^3 + 102t^2 - 115t + 120$ $15t^{16} - 204t^{15} + 1405t^{14} - 6454t^{13} + 21907t^{12} - 57432t^{11} + 117080t^{10} - 176754t^9 + 150405t^8 + 135972t^7 - 928717t^6 + 2460642t^5 - 4804019t^4 + 7729462t^3 - 10672990t^2 + 12881566t - 13703760$	4 / ✗ 2 / ✗		10_{95}^a $2t^3 - 9t^2 + 21t - 27$ $-5t^5 + 32t^4 - 114t^3 + 248t^2 - 384t + 436$ $118t^{12} - 1656t^{11} + 11045t^{10} - 44462t^9 + 109118t^8 - 104035t^7 - 391583t^6 + 2298083t^5 - 6804711t^4 + 14456709t^3 - 24008082t^2 + 32236696t - 35514492$	3 / ✗ 1 / ✗
	10_{96}^a $-t^3 + 7t^2 - 22t + 33$ $-7t^3 + 50t^2 - 147t + 212$ $9t^{12} - 203t^{11} + 2156t^{10} - 14060t^9 + 61189t^8 - 177034t^7 + 287437t^6 + 96689t^5 - 2149699t^4 + 7231587t^3 - 15228082t^2 + 23163354t - 26546674$	3 / ✗ 2 / ✗		10_{97}^a $-5t^2 + 22t - 33$ $-37t^3 + 242t^2 - 603t + 788$ $106t^8 - 5486t^7 - 47090t^6 + 615064t^5 - 3157165t^4 + 9904926t^3 - 21376446t^2 + 33395786t - 38661308$	2 / ✗ 2 / ✗
	10_{98}^a $-2t^3 + 9t^2 - 18t + 23$ $9t^5 - 60t^4 + 177t^3 - 348t^2 + 501t - 564$ $62t^{12} - 672t^{11} + 2575t^{10} + 1666t^9 - 67602t^8 + 398948t^7 - 1483813t^6 + 4115776t^5 - 9069800t^4 + 16396378t^3 - 24767965t^2 + 31602148t - 34255402$	3 / ✗ 2 / ✗		10_{99}^a $t^4 - 4t^3 + 10t^2 - 16t + 19$ 0	4 / ✓ 2 / ✓
	10_{100}^a $t^4 - 4t^3 + 9t^2 - 12t + 13$ $2t^7 - 12t^6 + 39t^5 - 80t^4 + 128t^3 - 164t^2 + 192t - 196$ $12t^{16} - 156t^{15} + 1019t^{14} - 4340t^{13} + 13189t^{12} - 29012t^{11} + 41715t^{10} - 11232t^9 - 153611t^8 + 603116t^7 - 1520513t^6 + 3049452t^5 - 5190414t^4 + 7715304t^3 - 10164234t^2 + 11961684t - 12623974$	4 / ✗ 2, 3 / ✗		10_{101}^a $7t^2 - 21t + 29$ $-129t^3 + 480t^2 - 942t + 1148$ $-7453t^8 + 115979t^7 - 819947t^6 + 3586847t^5 - 10987573t^4 + 25120359t^3 - 44443695t^2 + 62133778t - 69396618$	2 / ✗ 2, 3 / ✗
	10_{102}^a $-2t^3 + 8t^2 - 16t + 21$ $-t^5 + 6t^4 - 19t^3 + 50t^2 - 89t + 108$ $142t^{12} - 1824t^{11} + 11296t^{10} - 44000t^9 + 115984t^8 - 197200t^7 + 123203t^6 + 462512t^5 - 1996064t^4 + 4649298t^3 - 7951840t^2 + 10777160t - 11897326$	3 / ✗ 1 / ✗		10_{103}^a $2t^3 - 8t^2 + 17t - 21$ $5t^5 - 30t^4 + 93t^3 - 178t^2 + 254t - 280$ $118t^{12} - 1440t^{11} + 8404t^{10} - 29584t^9 + 61863t^8 - 33736t^7 - 289763t^6 + 1355186t^5 - 3666373t^4 + 7367413t^3 - 11802974t^2 + 15525908t - 16990056$	3 / ✗ 3 / ✗
	10_{104}^a $t^4 - 4t^3 + 9t^2 - 15t + 19$ $t^5 - 2t^4 + 2t^3 + 3t + 4$ $16t^{16} - 220t^{15} + 1535t^{14} - 7197t^{13} + 25227t^{12} - 69332t^{11} + 151513t^{10} - 257279t^9 + 301366t^8 - 83393t^7 - 710402t^6 + 2409469t^5 - 5162297t^4 + 8726478t^3 - 12397663t^2 + 15191203t - 16238052$	4 / ✗ 1 / ✗		10_{105}^a $t^3 - 8t^2 + 22t - 29$ $-t^5 + 14t^4 - 71t^3 + 184t^2 - 292t + 332$ $8t^{12} - 200t^{11} + 2218t^{10} - 14261t^9 + 57123t^8 - 132986t^7 + 65302t^6 + 805306t^5 - 3722841t^4 + 9784430t^3 - 18400587t^2 + 26441286t - 29769592$	3 / ✗ 2 / ✗
	10_{106}^a $-t^4 + 4t^3 - 9t^2 + 15t - 17$ $-t^7 + 6t^6 - 20t^5 + 48t^4 - 82t^3 + 114t^2 - 134t + 140$ $15t^{16} - 204t^{15} + 1405t^{14} - 6481t^{13} + 22197t^{12} - 58948t^{11} + 122017t^{10} - 186937t^9 + 159252t^8 + 161653t^7 - 1073190t^6 + 2872671t^5 - 5674479t^4 + 9221494t^3 - 12827310t^2 + 15551003t - 16568312$	4 / ✗ 2 / ✗		10_{107}^a $-t^3 + 8t^2 - 22t + 31$ $2t^3 - 8t^2 + 13t - 16$ $9t^{12} - 232t^{11} + 2674t^{10} - 18155t^9 + 79705t^8 - 227986t^7 + 366663t^6 + 65430t^5 - 2285283t^4 + 7518398t^3 - 15408513t^2 + 22997470t - 26180364$	3 / ✗ 1 / ✗
	10_{108}^a $2t^3 - 8t^2 + 14t - 15$ $-3t^5 + 18t^4 - 41t^3 + 50t^2 - 40t + 32$ $134t^{12} - 1696t^{11} + 10032t^{10} - 36416t^9 + 87916t^8 - 133860t^7 + 58617t^6 + 353392t^5 - 1337642t^4 + 2961006t^3 - 4930449t^2 + 6594854t - 7251776$	3 / ✗ 2 / ✗		10_{109}^a $t^4 - 4t^3 + 10t^2 - 17t + 21$ 0	4 / ✗ 2 / ✓
	10_{110}^a $t^3 - 8t^2 + 20t - 25$ $t^5 - 14t^4 + 69t^3 - 160t^2 + 219t - 236$ $8t^{12} - 200t^{11} + 2180t^{10} - 13569t^9 + 52114t^8 - 116472t^7 + 61616t^6 + 604668t^5 - 2747906t^4 + 7072274t^3 - 13103918t^2 + 18672836t - 20967250$	3 / ✗ 2 / ✗		10_{111}^a $-2t^3 + 9t^2 - 17t + 21$ $-9t^5 + 60t^4 - 171t^3 + 316t^2 - 436t + 480$ $62t^{12} - 672t^{11} + 2507t^{10} + 1894t^9 - 64067t^8 + 361705t^7 - 1299145t^6 + 3506889t^5 - 7575591t^4 + 13510069t^3 - 20234835t^2 + 25700228t - 27818092$	3 / ✗ 2 / ✗
	10_{112}^a $-t^4 + 5t^3 - 11t^2 + 17t - 19$ $t^7 - 8t^6 + 29t^5 - 68t^4 + 115t^3 - 152t^2 + 175t - 180$ $15t^{16} - 255t^{15} + 2068t^{14} - 10699t^{13} + 39650t^{12} - 111160t^{11} + 239401t^{10} - 381338t^9 + 357595t^8 + 215240t^7 - 1900590t^6 + 5252099t^5 - 10470652t^4 + 17062683t^3 - 23742257t^2 + 28786648t - 30666904$	4 / ✗ 2 / ✗		10_{113}^a $2t^3 - 11t^2 + 26t - 33$ $-5t^5 + 42t^4 - 167t^3 + 394t^2 - 623t + 720$ $118t^{12} - 2016t^{11} + 15681t^{10} - 71126t^9 + 190712t^8 - 187416t^7 - 827053t^6 + 4935892t^5 - 14986146t^4 + 32456282t^3 - 54606535t^2 + 73872380t - 81581546$	3 / ✗ 1 / ✗
	10_{114}^a $-2t^3 + 10t^2 - 21t + 27$ $t^5 - 8t^4 + 30t^3 - 78t^2 + 140t - 168$ $142t^{12} - 2280t^{11} + 16976t^{10} - 76976t^9 + 230999t^8 - 445876t^7 + 369450t^6 + 890044t^5 - 4554487t^4 + 11256519t^3 - 19890736t^2 + 27431686t - 30450926$	3 / ✗ 1 / ✗		10_{115}^a $-t^3 + 9t^2 - 26t + 37$ 0	3 / ✗ 2 / ✓
	10_{116}^a $-t^4 + 5t^3 - 12t^2 + 19t - 21$ $t^7 - 8t^6 + 30t^5 - 74t^4 + 132t^3 - 184t^2 + 217t - 228$ $15t^{16} - 255t^{15} + 2111t^{14} - 11302t^{13} + 43668t^{12} - 128023t^{11} + 288575t^{10} - 482307t^9 + 485985t^8 + 215018t^7 - 2416711t^6 + 6942030t^5 - 14142246t^4 + 23374622t^3 - 32832655t^2 + 40008697t - 42694444$	4 / ✗ 2 / ✗		10_{117}^a $2t^3 - 10t^2 + 24t - 31$ $-5t^5 + 38t^4 - 144t^3 + 330t^2 - 522t + 600$ $118t^{12} - 1824t^{11} + 13156t^{10} - 56312t^9 + 143746t^8 - 128212t^7 - 648731t^6 + 3701012t^5 - 11080717t^4 + 23844230t^3 - 39994730t^2 + 54033352t - 59650184$	3 / ✗ 2 / ✗

knot diag	n_k^t Alexander's ω^+ $(\rho_1^t)^+$	genus / ribbon unknotting # / amphi?	knot diag	n_k^t Alexander's ω^+ $(\rho_1^t)^+$	genus / ribbon unknotting # / amphi?
	10_{118}^a 0	$t^4 - 5t^3 + 12t^2 - 19t + 23$ 4 / ✗ 1 / ✓		10_{119}^a $-t^5 + 6t^4 - 26t^3 + 86t^2 - 175t + 220$	$-2t^3 + 10t^2 - 23t + 31$ 3 / ✗ 1 / ✗
	10_{120}^a $166t^3 - 692t^2 + 1433t - 1788$	$8t^2 - 26t + 37$ 2 / ✗ 2, 3 / ✗		10_{121}^a $5t^5 - 42t^4 + 167t^3 - 396t^2 + 634t - 732$	$2t^3 - 11t^2 + 27t - 35$ 3 / ✗ 2 / ✗
	10_{122}^a $-t^5 + 8t^4 - 34t^3 + 104t^2 - 211t + 264$	$-2t^3 + 11t^2 - 24t + 31$ 3 / ✗ 2 / ✗		10_{123}^a 0	$t^4 - 6t^3 + 15t^2 - 24t + 29$ 4 / ✓ 2 / ✓
	10_{124}^a $-4t^7 - 6t^4 - 4t^2 - 6t$	$t^4 - t^3 + t - 1$ 4 / ✗ 4 / ✗		10_{125}^a $-t^5 + 2t^4 - 2t^3 + 3t - 4$	$t^5 - 2t^2 + 2t - 1$ 3 / ✗ 2 / ✗
	10_{126}^a $t^5 - 2t^4 + 10t^3 - 12t^2 + 22t - 20$	$t^3 - 2t^2 + 4t - 5$ 3 / ✗ 2 / ✗		10_{127}^a $2t^5 - 14t^4 + 32t^3 - 52t^2 + 67t - 72$	$-t^3 + 4t^2 - 6t + 7$ 3 / ✗ 2 / ✗
	10_{128}^a $-13t^5 + 12t^4 - 3t^3 - 10t^2 - 9t + 12$	$2t^3 - 3t^2 + t + 1$ 3 / ✗ 3 / ✗		10_{129}^a $-t^3 - 2t^2 + 14t - 20$	$2t^2 - 6t + 9$ 2 / ✓ 1 / ✗
	10_{130}^a $t^3 - 2t^2 + 19t - 24$	$2t^2 - 4t + 5$ 2 / ✗ 2 / ✗		10_{131}^a $5t^3 - 38t^2 + 87t - 112$	$-2t^2 + 8t - 11$ 2 / ✗ 1 / ✗
	10_{132}^a $2t^2 + 5t - 4$	$t^2 - t + 1$ 2 / ✗ 1 / ✗		10_{133}^a $t^3 - 14t^2 + 37t - 48$	$-t^2 + 5t - 7$ 2 / ✗ 1 / ✗
	10_{134}^a $-13t^5 + 24t^4 - 33t^3 + 30t^2 - 41t + 40$	$2t^3 - 4t^2 + 4t - 3$ 3 / ✗ 3 / ✗		10_{135}^a $t^3 - 6t^2 + 18t - 24$	$3t^2 - 9t + 13$ 2 / ✗ 2 / ✗
	10_{136}^a $-t^3 + 4t^2 - 2t - 4$	$-t^2 + 4t - 5$ 2 / ✗ 1 / ✗		10_{137}^a $-4t^2 + 24t - 44$	$t^2 - 6t + 11$ 2 / ✓ 1 / ✗
	10_{138}^a $-t^5 + 8t^4 - 22t^3 + 24t^2 - 11t + 8$	$t^3 - 5t^2 + 8t - 7$ 3 / ✗ 2 / ✗		10_{139}^a $-4t^7 - 12t^4 + 5t^3 - 4t^2 - 16t + 12$	$t^4 - t^3 + 2t - 3$ 4 / ✗ 4 / ✗
	10_{140}^a 8t - 8	$t^2 - 2t + 3$ 2 / ✓ 2 / ✗		10_{141}^a $t^3 - 8t^2 + 16t - 20$	$-t^3 + 3t^2 - 4t + 5$ 3 / ✗ 1 / ✗
	10_{142}^a $-13t^5 + 12t^4 - 13t^3 + 4t^2 - 17t + 12$	$2t^3 - 3t^2 + 2t - 1$ 3 / ✗ 3 / ✗		10_{143}^a $t^5 - 4t^4 + 15t^3 - 28t^2 + 45t - 48$	$t^3 - 3t^2 + 6t - 7$ 3 / ✗ 1 / ✗
	10_{144}^a $10t^3 - 44t^2 + 80t - 96$	$-3t^2 + 10t - 13$ 2 / ✗ 2 / ✗		10_{145}^a $2t^3 + 8t^2 + 6t - 8$	$t^2 + t - 3$ 2 / ✗ 2 / ✗
	10_{146}^a $t^3 - 8t^2 + 21t - 28$	$2t^2 - 8t + 13$ 2 / ✗ 1 / ✗		10_{147}^a $-3t^3 + 12t^2 - 15t + 12$	$-2t^2 + 7t - 9$ 2 / ✗ 1 / ✗
	10_{148}^a $t^5 - 4t^4 + 18t^3 - 36t^2 + 62t - 68$	$t^3 - 3t^2 + 7t - 9$ 3 / ✗ 2 / ✗		10_{149}^a $2t^5 - 18t^4 + 55t^3 - 104t^2 + 149t - 164$	$-t^3 + 5t^2 - 9t + 11$ 3 / ✗ 2 / ✗
	10_{150}^a $-2t^5 + 12t^4 - 26t^3 + 38t^2 - 45t + 44$	$-t^3 + 4t^2 - 6t + 7$ 3 / ✗ 2 / ✗		10_{151}^a $-t^5 + 6t^4 - 21t^3 + 42t^2 - 66t + 72$	$t^3 - 4t^2 + 10t - 13$ 3 / ✗ 2 / ✗
	10_{152}^a $4t^7 - 7t^5 + 18t^4 - 7t^3 - 12t^2 + 45t - 52$	$t^4 - t^3 - t^2 + 4t - 5$ 4 / ✗ 4 / ✗		10_{153}^a $t^5 - 2t^4 + t^3 + 2t^2 - t$	$t^3 - t^2 - t + 3$ 3 / ✓ 2 / ✗
	10_{154}^a $-3t^5 - 6t^4 + 13t^3 - 47t + 68$	$t^3 - 4t + 7$ 3 / ✗ 3 / ✗		10_{155}^a $-2t^3 + 12t^2 - 22t + 28$	$-t^3 + 3t^2 - 5t + 7$ 3 / ✓ 2 / ✗

knot diag	n_k^t Alexander's ω^+ $(\rho_1^t)^+$	genus / ribbon unknotting # / amphi?	knot diag	n_k^t Alexander's ω^+ $(\rho_1^t)^+$	genus / ribbon unknotting # / amphi?
	10_{156}^n $t^3 - 4t^2 + 8t - 9$ $t^5 - 6t^4 + 19t^3 - 30t^2 + 33t - 32$ $8t^{12} - 100t^{11} + 594t^{10} - 2165t^9 + 5120t^8 - 6852t^7 - 2208t^6 + 41208t^5 - 134214t^4 + 293026t^3 - 493422t^2 + 668112t - 738218$	3 / ✗ 1 / ✗		10_{157}^n $-t^3 + 6t^2 - 11t + 13$ $-2t^5 + 22t^4 - 78t^3 + 148t^2 - 218t + 240$ $5t^{12} - 74t^{11} + 340t^{10} + 321t^9 - 11314t^8 + 67637t^7 - 250977t^6 + 688036t^5 - 1493487t^4 + 2661131t^3 - 3974091t^2 + 5034465t - 5444000$	3 / ✗ 2 / ✗
	10_{158}^n $-t^3 + 4t^2 - 10t + 15$ $2t^2 - 7t + 12$ $9t^{12} - 116t^{11} + 764t^{10} - 3275t^9 + 9743t^8 - 19422t^7 + 18439t^6 + 32898t^5 - 196271t^4 + 513374t^3 - 940025t^2 + 1323614t - 1479452$	3 / ✗ 2 / ✗		10_{159}^n $t^3 - 4t^2 + 9t - 11$ $t^5 - 6t^4 + 26t^3 - 60t^2 + 98t - 112$ $8t^{12} - 100t^{11} + 609t^{10} - 2267t^9 + 5047t^8 - 3237t^7 - 23513t^6 + 115362t^5 - 318739t^4 + 648093t^3 - 1045247t^2 + 1379659t - 1511358$	3 / ✗ 1 / ✗
	10_{160}^n $-t^3 + 4t^2 - 4t + 3$ $-2t^5 + 12t^4 - 20t^3 + 14t^2 - 16t + 12$ $5t^{12} - 52t^{11} + 198t^{10} - 255t^9 - 522t^8 + 3092t^7 - 8443t^6 + 18756t^5 - 37588t^4 + 67858t^3 - 108568t^2 + 148444t - 165862$	3 / ✗ 2 / ✗		10_{161}^n $t^3 - 2t + 3$ $3t^5 + 6t^4 - 3t^3 + 4t^2 + 14t - 12$ $30t^{10} - 53t^9 - 145t^8 + 630t^7 - 674t^6 - 870t^5 + 3591t^4 - 4450t^3 + 581t^2 + 6166t - 9640$	3 / ✗ 3 / ✗
	10_{162}^n $-3t^2 + 9t - 11$ $10t^3 - 38t^2 + 58t - 68$ $222t^8 - 1473t^7 + 2609t^6 + 8829t^5 - 65543t^4 + 206079t^3 - 427536t^2 + 647498t - 741358$	2 / ✗ 2 / ✗		10_{163}^n $t^3 - 5t^2 + 12t - 15$ $-t^5 + 8t^4 - 30t^3 + 62t^2 - 89t + 96$ $8t^{12} - 125t^{11} + 923t^{10} - 4154t^9 + 12040t^8 - 19732t^7 - 4345t^6 + 140575t^5 - 506052t^4 + 1171653t^3 - 2040193t^2 + 2809224t - 3119648$	3 / ✗ 1, 2 / ✗
	10_{164}^n $3t^2 - 11t + 17$ $t^3 - 10t^2 + 29t - 40$ $321t^8 - 3179t^7 + 12782t^6 - 20103t^5 - 32876t^4 + 254013t^3 - 688337t^2 + 1170838t - 1386922$	2 / ✗ 1 / ✗		10_{165}^n $-2t^2 + 10t - 15$ $-5t^3 + 50t^2 - 146t + 196$ $38t^8 - 344t^7 - 848t^6 + 23020t^5 - 137555t^4 + 465256t^3 - 1047705t^2 + 1673914t - 1951560$	2 / ✗ 2 / ✗