

Bed[K_, ω_] :=

Module[{t, r, XingsByArmpits, bends, faces, p, A, is},

t = 1 - ω; r = t + t*;

XingsByArmpits =

List@@PD[K] /. x : X[i_, j_, k_, L_] =>

If[PositiveQ[x], X+[-i, j, k, -L], X-[-j, k, L, -i]];

bends = Times@@XingsByArmpits /.

_ [X][a_, b_, c_, d_] => p_{a,-d} p_{b,-a} p_{c,-b} p_{d,-c};

faces = bends //. p_{x_,y_} p_{y_,z_} => p_{x,y,z};

A = Table[0, Length@faces, Length@faces];

Do[is = Position[faces, #][[1, 1]] & /@ List@@x;

A[[is, is]] += If[Head[x] === X+,

$$\begin{pmatrix} -r & -t & 2t & t^* \\ -t^* & 0 & t^* & 0 \\ 2t^* & t & -r & -t^* \\ t & 0 & -t & 0 \end{pmatrix}, \begin{pmatrix} r & -t & -2t^* & t^* \\ -t^* & 0 & t^* & 0 \\ -2t & t & r & -t^* \\ t & 0 & -t & 0 \end{pmatrix},$$

{x, XingsByArmpits}];

MatrixSignature[A];