```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\Bonn-2505"];
Once[<< IType.m];
T3 = T1 T2;
```

```
Loading KnotTheory` version of October 29, 2024, 10:29:52.1301.
Read more at http://katlas.org/wiki/KnotTheory.
Loading Rot.m from http://drorbn.net/AP/Talks/Bonn-2505 to compute rotation numbers.
```

exec

In[•]:=
```
nb2tex$PDFWidth *= 1.25;
```

The Programs

tex

{\red\bf A faster program,} in which the Feynman diagrams are ``pre-computed'' (see theta.nb at \web{ap}):

pdf

In[•]:=
```
R1[s_, i_, j_] = CF[
   s (1/2 - g3ii + T2^s g1ii g2ji - g1ii g2jj - (T2^s - 1) g2ji g3ii + 2 g2jj g3ii - (1 - T3^s) g2ji g3ji -
      g2ii g3jj - T2^s g2ji g3jj + g1ii g3jj + ((T1^s - 1) g1ji (T2^2 s g2ji - T2^s g2jj + T2^s g3jj) +
         (T3^s - 1) g3ji (1 - T2^s g1ii - (T1^s - 1) (T2^s + 1) g1ji + (T2^s - 2) g2jj + g2ij)) / (T2^s - 1))];
```

In[•]:=
```
CF[
 s (T2^s g1,i,i g2,j,i + ((-1 + T1^s) T2^2 s g1,j,i g2,j,i)/(-1 + T2^s) - g1,i,i g2,j,j -
    ((-1 + T1^s) T2^s g1,j,i g2,j,j)/(-1 + T2^s) - g3,i,i - (-1 + T2^s) g2,j,i g3,i,i + 2 g2,j,j g3,i,i +
    ((-1 + T3^s) g3,j,i)/(-1 + T2^s) - (T2^s (-1 + T3^s) g1,i,i g3,j,i)/(-1 + T2^s) - ((-1 + T1^s) (1 + T2^s) (-1 + T3^s) g1,j,i g3,j,i)/(-1 + T2^s) +
    ((-1 + T3^s) g2,i,j g3,j,i)/(-1 + T2^s) - (1 - T3^s) g2,j,i g3,j,i + ((-2 + T2^s) (-1 + T3^s) g2,j,j g3,j,i)/(-1 + T2^s) +
    g1,i,i g3,j,j + ((-1 + T1^s) T2^s g1,j,i g3,j,j)/(-1 + T2^s) - g2,i,i g3,j,j - T2^s g2,j,i g3,j,j + 1/2) - R1[s, i, j]]
```

Out[•]=
```
0
```

pdf

In[•]:=
```
Θ[{s0_, i0_, j0_}, {s1_, i1_, j1_}] := CF[
  s1 (T1^s0 - 1) (T2^s1 - 1)^-1 (T3^s1 - 1) g1,j1,i0 g3,j0,i1 ((T2^s0 g2,i1,i0 - g2,i1,j0) - (T2^s0 g2,j1,i0 - g2,j1,j0))]
```

In[•]:=
```
CF[
  1/(-1 + T2^s1) s1 (-1 + (T1 T2)^s1) ((-1 + T1^s0) g1,j1,i0 (T2^s0 g2,i1,i0 - g2,i1,j0) g3,j0,i1 -
      (-1 + T1^s0) g1,j1,i0 (T2^s0 g2,j1,i0 - g2,j1,j0) g3,j0,i1) - Θ[{s0, i0, j0}, {s1, i1, j1}]]
```

Out[•]=
```
0
```

pdf

In[•]:=
```
Γ1[φ_, k_] = -φ / 2 + φ g3kk;
```

tex

We call the invariant computed θ:

pdf

In[•]:=
```
Θ[K_] := Module[{Cs, φ, n, A, s, i, j, k, Δ, G, ν, α, β, gEval, c, z},
  {Cs, φ} = Rot[K]; n = Length[Cs];
  A = IdentityMatrix[2 n + 1];
  Cases[Cs, {s_, i_, j_} :> (A[[{i, j}, {i + 1, j + 1}]] += ({{-T^s, T^s - 1}, {0, -1}}))];
  Δ = T^((-Total[φ] - Total[Cs[[All, 1]]])/2) Det[A];
  G = Inverse[A]; gEval[ℰ_] := Factor[ℰ /. gν_,α_,β_ :> (G[[α, β]] /. T → Tν)];
  z = gEval[∑k1=1^n ∑k2=1^n Θ[Cs[[k1]], Cs[[k2]]]];
  z += gEval[∑k=1^n R1 @@ Cs[[k]]];
  z += gEval[∑k=1^(2 n) Γ1[φ[[k]], k]];
  {Δ, (Δ /. T → T1) (Δ /. T → T2) (Δ /. T → T3) z} // Factor];
```

exec
```
nb2tex$PDFWidth /= 1.25;
```

Some Knots

tex
```
\needspace{15mm}
{\bf\red Some Knots.}
```

pdf

In[•]:=
```
Expand[Θ[Knot[3, 1]]]
```

pdf

KnotTheory: Loading precomputed data in PD4Knots`.

Out[•]=
pdf

$$\left\{-1+\frac{1}{T}+T,\ -\frac{1}{T_1^2}-T_1^2-\frac{1}{T_2^2}-\frac{1}{T_1^2\,T_2^2}+\frac{1}{T_1\,T_2^2}+\frac{1}{T_1^2\,T_2}+\frac{T_1}{T_2}+\frac{T_2}{T_1}+T_1^2\,T_2-T_2^2+T_1\,T_2^2-T_1^2\,T_2^2\right\}$$

exec
```
nb2tex$PDFWidth *= 1.25;
```

pdf

In[•]:=
```
PolyPlot[0] = Graphics[{}];
PolyPlot[p_] := Module[{crs, m1, m2, maxc, minc, s, hex},
   crs = CoefficientRules[T1^(m1=-Exponent[p,T1,Min]) T2^(m2=-Exponent[p,T2,Min]) p, {T1, T2}];
   maxc = N@Log@Max@Abs[Last /@ crs];
   minc = N@Log@Min@Select[Abs[Last /@ crs], # > 0 &];
   If[minc == maxc, s[_] = 0, s[c_] := s[c] = (maxc - Log@c) / (maxc - minc)];
   hex = Table[{Cos[α], Sin[α]} / Cos[2 π / 12] / 2, {α, 2 π / 12, 2 π, 2 π / 6}];
   Graphics[crs /. ({x1_, x2_} → c_) :> {
       If[c == 0, White, Lighter[If[c > 0, Red, Blue], 0.88 s[Abs@c]]],
       Polygon[({{1, -1/2}, {0, Sqrt[3]/2}}.{x1 + m1, x2 + m2} + #) & /@ hex]}]];
PolyPlot[{Δ_, θ_}] := PolyPlot[θ]
```

exec
```
nb2tex$PDFWidth /= 1.25;
```

tex
```
\needspace{25mm}
\parpic[r]{$
 {\includegraphics[height=0.45in]{../Beijing-2407/K11n34.png}
  \atop\text{\tiny K11n34}}
 {\includegraphics[height=0.45in]{../Beijing-2407/K11n42.png}
  \atop\text{\tiny K11n42}}
$}
```

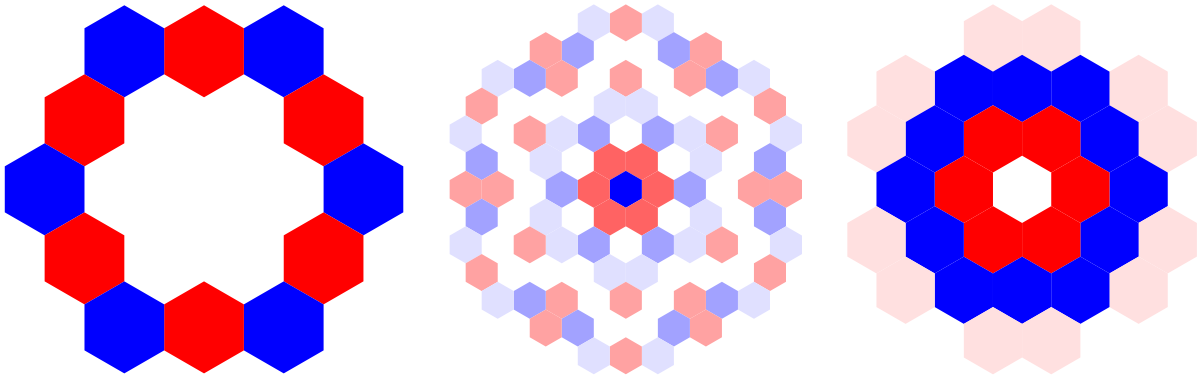
pdf

In[•]:=
```
GraphicsRow[PolyPlot[θ[Knot[#]]] &
  /@ {"3_1", "K11n34", "K11n42"}]
```

pdf
```
KnotTheory: Loading precomputed data in DTCode4KnotsTo11`.
```

pdf
```
KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of
      2005.
```

Out[•]=
pdf



tex
```
\parpic[r]{$
 {\includegraphics[height=0.6in]{../../Projects/Gallery/Conway.png}
```

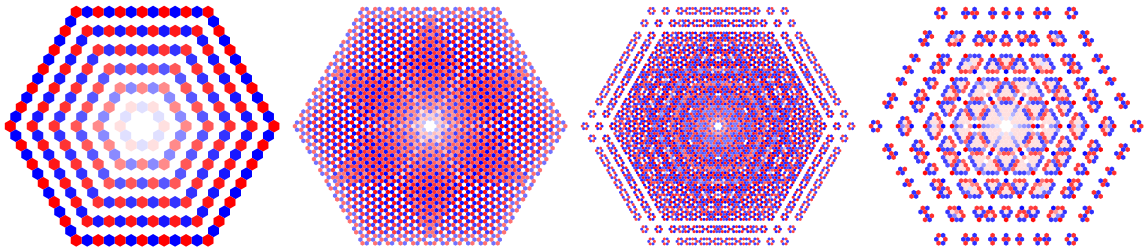
```
  \atop\text{\scriptsize Conway}}
 {\includegraphics[height=0.6in]{../../Projects/Gallery/PhotoNotAvailable.png}
  \atop\text{\scriptsize Kinoshita}}
 {\includegraphics[height=0.6in]{../../Projects/Gallery/Terasaka.jpg}
  \atop\text{\scriptsize Terasaka}}
$}
```

So θ detects knot mutation and separates the Conway knot K11n34 from the Kinoshita-Terasaka knot K11n42!

pdf

In[•]:=
```
GraphicsRow[PolyPlot[θ[TorusKnot @@ #]] &
  /@ {{13, 2}, {17, 3}, {13, 5}, {7, 6}}, Spacings → 0]
```

Out[•]=
pdf



tex

```
%\needspace{50mm}
\parpic[r]{$
 {\includegraphics[height=0.6in]{../../Projects/Gallery/Gompf.jpg}
  \atop\text{\scriptsize Gompf}}
 {\includegraphics[height=0.6in]{../../Projects/Gallery/Scharlemann.jpg}
  \atop\text{\scriptsize Scharlemann}}
 {\includegraphics[height=0.6in]{../../Projects/Gallery/Thompson.jpg}
  \atop\text{\scriptsize Thompson}}
$}
```

The 48-crossing Gompf-Scharlemann-Thompson knot \cite{GompfScharlemannThompson:Counterexample} is significant because it may be a counterexample to the slice-ribbon conjecture:

\[\resizebox{\linewidth}{!}{\import{../Waco-2203/}{GST48-Marked.pdf_t}} \]

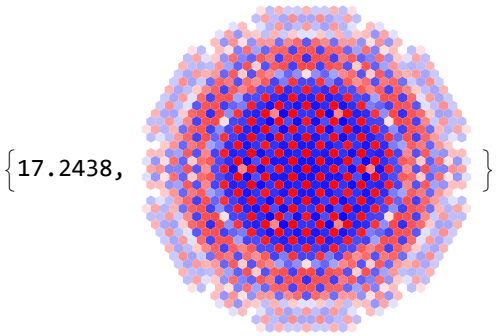
exec

```
nb2tex$PDFWidth *= 1.25;
```

pdf

In[•]:=
```
AbsoluteTiming@
 PolyPlot[θ[EPD[X14,1, X̄2,29, X3,40, X43,4, X̄26,5, X6,95, X96,7, X13,8, X̄9,28, X10,41, X42,11, X̄27,12,
    X30,15, X̄16,61, X̄17,72, X̄18,83, X19,34, X̄89,20, X̄21,92, X̄79,22, X̄68,23, X̄57,24, X̄25,56, X62,31,
    X73,32, X84,33, X̄50,35, X36,81, X37,70, X38,59, X̄39,54, X44,55, X58,45, X69,46, X80,47, X48,91,
    X90,49, X51,82, X52,71, X53,60, X̄63,74, X̄64,85, X̄76,65, X̄87,66, X̄67,94, X̄75,86, X̄88,77, X̄78,93]]]
```

Out[•]=
pdf



```
{17.2438, }
```

exec

In[•]:=
```
nb2tex$PDFWidth /= 1.25;
```

In[•]:=
```
tab250 = {0} ~ Join ~ Table[θ[K], {K, AllKnots[{3, 10}]}];
```

In[•]:=
```
g250 = GraphicsGrid[Partition[PolyPlot /@ tab250, 25], Spacings → 0]
```

Out[•]=