

$$\begin{aligned}
\text{lhs} &= \int (\mathbb{E} [\dot{\pi}_{\mathbf{i}} \mathbf{p}_{\mathbf{i}} + \dot{\pi}_{\mathbf{j}} \mathbf{p}_{\mathbf{j}} + \dot{\pi}_{\mathbf{k}} \mathbf{p}_{\mathbf{k}}] \times \mathcal{L} / @ (X_{\mathbf{i},\mathbf{j}}[\mathbf{1}] X_{\mathbf{i}+1,\mathbf{k}}[\mathbf{1}] X_{\mathbf{j}+1,\mathbf{k}+1}[\mathbf{1}])) \\
&\quad \mathbb{d}\{\mathbf{p}_{\mathbf{i}}, \mathbf{p}_{\mathbf{j}}, \mathbf{p}_{\mathbf{k}}, \mathbf{x}_{\mathbf{i}}, \mathbf{x}_{\mathbf{j}}, \mathbf{x}_{\mathbf{k}}, \mathbf{p}_{\mathbf{i}+1}, \mathbf{p}_{\mathbf{j}+1}, \mathbf{p}_{\mathbf{k}+1}, \mathbf{x}_{\mathbf{i}+1}, \mathbf{x}_{\mathbf{j}+1}, \mathbf{x}_{\mathbf{k}+1}\}; \\
\text{rhs} &= \int (\mathbb{E} [\dot{\pi}_{\mathbf{i}} \mathbf{p}_{\mathbf{i}} + \dot{\pi}_{\mathbf{j}} \mathbf{p}_{\mathbf{j}} + \dot{\pi}_{\mathbf{k}} \mathbf{p}_{\mathbf{k}}] \times \mathcal{L} / @ (X_{\mathbf{j},\mathbf{k}}[\mathbf{1}] X_{\mathbf{i},\mathbf{k}+1}[\mathbf{1}] X_{\mathbf{i}+1,\mathbf{j}+1}[\mathbf{1}])) \\
&\quad \mathbb{d}\{\mathbf{p}_{\mathbf{i}}, \mathbf{p}_{\mathbf{j}}, \mathbf{p}_{\mathbf{k}}, \mathbf{x}_{\mathbf{i}}, \mathbf{x}_{\mathbf{j}}, \mathbf{x}_{\mathbf{k}}, \mathbf{p}_{\mathbf{i}+1}, \mathbf{p}_{\mathbf{j}+1}, \mathbf{p}_{\mathbf{k}+1}, \mathbf{x}_{\mathbf{i}+1}, \mathbf{x}_{\mathbf{j}+1}, \mathbf{x}_{\mathbf{k}+1}\}; \\
\text{lhs} &= \text{rhs}
\end{aligned}$$