Dror Bar-Natan: Talks: Banff-2504: http://drorbn.net/ba25

Pensieve header: Computing the Kauffman bracket in 4 lines of code.

Good mathematicians try not to use a theorem unless they know how to prove it (some exceptions allowed).
Those same mathematicians should not be using code unless they could have written it themselves (exceptions allowed, and even if not as well).

In[•]:=
```
SetAttributes[p, Orderless]
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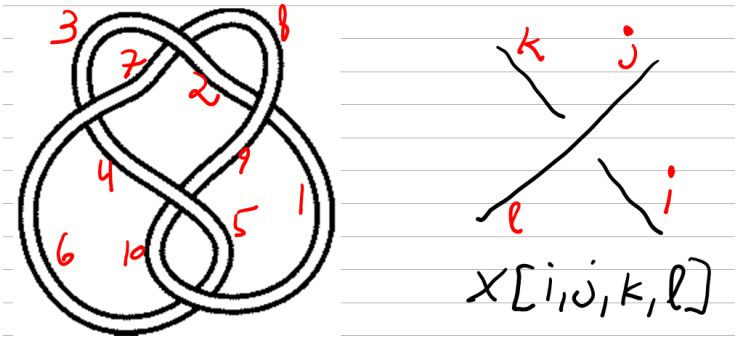
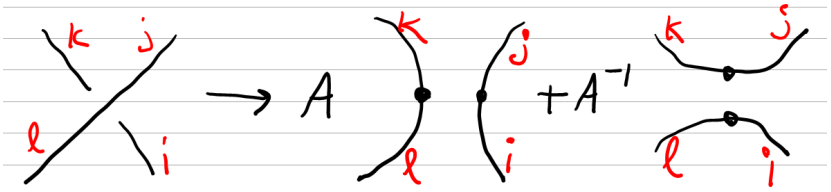


In[•]:=
```
pd = X[1, 8, 2, 9] X[3, 6, 4, 7] X[5, 10, 6, 1] X[7, 2, 8, 3] X[9, 4, 10, 5]
```

Out[•]=
```
X[1, 8, 2, 9] X[3, 6, 4, 7] X[5, 10, 6, 1] X[7, 2, 8, 3] X[9, 4, 10, 5]
```



In[•]:=
```
s1 = pd /. X[i_, j_, k_, l_] :> A p[i, j] p[k, l] + A^-1 p[i, l] p[j, k]
```

Out[•]=
$$\left(\frac{p[1,9]\,p[2,8]}{A}+A\,p[1,8]\,p[2,9]\right)\left(\frac{p[2,8]\,p[3,7]}{A}+A\,p[2,7]\,p[3,8]\right)\left(\frac{p[3,7]\,p[4,6]}{A}+A\,p[3,6]\,p[4,7]\right)\left(\frac{p[4,10]\,p[5,9]}{A}+A\,p[4,9]\,p[5,10]\right)\left(A\,p[1,6]\,p[5,10]+\frac{p[1,5]\,p[6,10]}{A}\right)$$

```
In[•]:= Short[s2 = Expand[s1], 10]
```

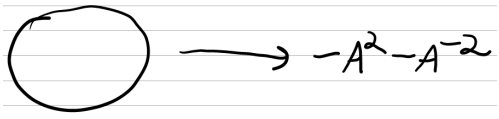
Out[•]//Short=

$$\frac{p[1,6]\,p[1,9]\,p[2,8]^2\,p[3,7]^2\,p[4,6]\,p[4,10]\,p[5,9]\,p[5,10]}{A^3} +$$
$$\frac{p[1,6]\,p[1,8]\,p[2,8]\,p[2,9]\,p[3,7]^2\,p[4,6]\,p[4,10]\,p[5,9]\,p[5,10]}{A} +$$
$$\frac{p[1,6]\,p[1,9]\,p[2,7]\,p[2,8]\,p[3,7]\,p[3,8]\,p[4,6]\,p[4,10]\,p[5,9]\,p[5,10]}{A} +$$
$$A\,p[1,6]\,p[1,8]\,p[2,7]\,p[2,9]\,p[3,7]\,p[3,8]\,p[4,6]\,p[4,10]\,p[5,9]\,p[5,10] +$$
$$\frac{p[1,6]\,p[1,9]\,p[2,8]^2\,p[3,6]\,p[3,7]\,p[4,7]\,p[4,10]\,p[5,9]\,p[5,10]}{A} + \text{<<22>>} +$$
$$A\,p[1,5]\,p[1,8]\,p[2,7]\,p[2,9]\,p[3,7]\,p[3,8]\,p[4,6]\,p[4,9]\,p[5,10]\,p[6,10] +$$
$$\frac{p[1,5]\,p[1,9]\,p[2,8]^2\,p[3,6]\,p[3,7]\,p[4,7]\,p[4,9]\,p[5,10]\,p[6,10]}{A} +$$
$$A\,p[1,5]\,p[1,8]\,p[2,8]\,p[2,9]\,p[3,6]\,p[3,7]\,p[4,7]\,p[4,9]\,p[5,10]\,p[6,10] +$$
$$A\,p[1,5]\,p[1,9]\,p[2,7]\,p[2,8]\,p[3,6]\,p[3,8]\,p[4,7]\,p[4,9]\,p[5,10]\,p[6,10] +$$
$$A^3\,p[1,5]\,p[1,8]\,p[2,7]\,p[2,9]\,p[3,6]\,p[3,8]\,p[4,7]\,p[4,9]\,p[5,10]\,p[6,10]$$


```
In[•]:= s3 = s2 //. p[i_, j_] p[j_, k_] :> p[i, k]
```

Out[•]=

$$\frac{p[2,8]^2\,p[3,7]^2\,p[5,10]^2\,p[6,9]^2}{A} + \frac{p[2,8]^2\,p[3,7]^2\,p[5,9]^2\,p[6,10]^2}{A^5} +$$
$$A\,p[5,10]^2\,p[6,9]^2\,p[7,8]^2 + \frac{p[5,9]^2\,p[6,10]^2\,p[7,8]^2}{A^3} + A\,p[2,8]^2\,p[5,10]^2\,p[7,9]^2 +$$
$$A^5\,p[5,10]^2\,p[6,8]^2\,p[7,9]^2 + \frac{p[2,8]^2\,p[5,9]^2\,p[7,10]^2}{A^3} + 3\,A^3\,p[5,10]^2\,p[8,9]^2 +$$
$$A\,p[3,7]^2\,p[5,10]^2\,p[8,9]^2 + \frac{p[6,10]^2\,p[8,9]^2}{A} + \frac{p[3,7]^2\,p[6,10]^2\,p[8,9]^2}{A^3} +$$
$$\frac{p[7,10]^2\,p[8,9]^2}{A} + \frac{p[5,9]^2\,p[8,10]^2}{A} + A^3\,p[7,9]^2\,p[8,10]^2 + 7\,A\,p[9,10]^2 + \frac{2\,p[2,8]^2\,p[9,10]^2}{A} +$$
$$\frac{2\,p[3,7]^2\,p[9,10]^2}{A} + \frac{2\,p[2,8]^2\,p[3,7]^2\,p[9,10]^2}{A^3} + A^3\,p[6,8]^2\,p[9,10]^2 + \frac{2\,p[7,8]^2\,p[9,10]^2}{A}$$




```
In[•]:= s4 = Expand[s3 /. p[_, _]^• → (-A^2 - A^-2)]
```

Out[•]=

$$\frac{1}{A^{13}} + \frac{1}{A^5} + \frac{1}{A} - A^{11}$$

When possible, make your code look like the math and include it in your papers!

A Challenge to the Masters: Can you write a readable informative 20-line program that will compute the

hyperbolic volume of any knot in the Rolfsen table?