Pensieve header: A full implementation of gl_n^ϵ.

In[•]:=
```
SetDirectory["C:/drorbn/AcademicPensieve/Projects/glneps"];
Once[
 << "KnotTheory`";
 << "Rot.m";
 << "PolyPlot.m";
]
```

```
Loading KnotTheory` version of October 29, 2024, 10:29:52.1301.
Read more at http://katlas.org/wiki/KnotTheory.

Loading Rot.m to compute rotation numbers.

Loading PolyPlot.m to plot 2-variable polynomials.
```

Canonical Forms:

In[•]:=
```
CF[ℰ_] := Expand@Collect[ℰ, Cases[ℰ, (y | a | x | η | β | τ | ξ)__[_], ∞] ∪
      Cases[ℰ, (y | a | x | η | β | τ | ξ)[_], ∞], F] /. F → Factor;
CF[ℰ : 𝔼[_][___]] := CF /@ ℰ;
```

“Define” Code

In[•]:=
```
SetAttributes[Define, HoldAll];
Define[def_, defs__] := (Define[def]; Define[defs];);
Define[op_[is__] = ℰ_] := Module[{SD, ii, jj, kk, isp, nis, nisp, sis}, Block[{i, j, k},
   ReleaseHold[Hold[
      SD[op[nisp, $k_Integer], Block[{i, j, k}, op[isp, $k] = ℰ;
        op[nis, $k]]];
      SD[op[isp], op[{is}, $k]];
      SD[op[sis__], op[{sis}]];
     ] /. {SD → SetDelayed,
      isp → {is} /. {i → i_, j → j_, k → k_},
      nis → {is} /. {i → ii, j → jj, k → kk},
      nisp → {is} /. {i → ii_, j → jj_, k → kk_}
     }] ]]
```

```
In[•]:= utm_n[i_, j_ → k_] := (*utm_n[i,j→k]=*)𝔼[{i, j} → {k}][
          B = IdentityMatrix[n];
          Do[B = B.MatrixExp[SparseArray[{α, β} → ξ_{α,β}, {n, n}]], {β, 1, n}, {α, 1, β}];
          (* This specifies the PBW ordering *)
          eqns =
           And @@ Thread[Flatten[CF[(Inverse[B /. ξ_{αβ__} :↦ ξ_{αβ}[k]] /. ξ_{α_,α_}[k] :↦ ξ_{α,α}[i] + ξ_{α,α}[j]).
                  (B /. ξ_{αβ__} :↦ ξ_{αβ}[i]).(B /. ξ_{αβ__} :↦ ξ_{αβ}[j])]] == Flatten[IdentityMatrix[n]]];
          vars = Union@Cases[eqns, ξ__[k], ∞];
          {sol} = Solve[eqns, vars];
          CF[(vars /. ξ → x).(vars /. sol) + Sum[x_{α,α}[k] (ξ_{α,α}[i] + ξ_{α,α}[j]), {α, n}]]
         ];

In[•]:= utm_2[i, j → k]

Out[•]=
𝔼[{i, j} → {k}][x_{1,1}[k] ξ_{1,1}[i] + x_{1,1}[k] ξ_{1,1}[j] +
  e^{-ξ_{1,1}[j]} x_{1,2}[k] ξ_{1,2}[i] + e^{-ξ_{2,2}[i]} x_{1,2}[k] ξ_{1,2}[j] + x_{2,2}[k] ξ_{2,2}[i] + x_{2,2}[k] ξ_{2,2}[j]]

In[•]:= Table[First@Timing[n → utm_n[1, 2 → 3]], {n, 10}]

Out[•]=
{0., 0., 0.015625, 0.015625, 0.078125, 0.25, 0.78125, 2.20313, 5.92188, 15.9219}
```