In[ ]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\Theta"]
Once[<< KnotTheory`]
```

Out[ ]=
```
C:\drorbn\AcademicPensieve\Projects\Theta
```

A version of Dror’s Rot program that computes the rotation numbers from PD code of a link (the component containing edge 1 is assumed to be opened)
The output is a list of crossings each written as {sign, i, j, i^+, j^+} and a list of rotation numbers for the edges. There are 2cr+1 edges where cr is the number of crossings.

In[∘]:=
```
NextHead[fr_] := Min[Select[fr, (Abs[#] > 1 && IntegerQ[#]) &]]
FindNextCross[todo_, Nh_] := Module[{Cand =
     Select[todo, If[PositiveQ[#], #[[1]] == Nh || #[[4]] == Nh, #[[1]] == Nh || #[[2]] == Nh] &]},
   If[Length[Cand] == 0, False, First@Cand]
  ]
Bend[X[a_, b_, c_, d_], h_] := If[PositiveQ[X[a, b, c, d]],
   If[h == a, {I d, c, b}, {c, b, -I a}], If[h == a, {d, c, -I b}, { I a, d, c}]]
CheckCap[fr_, rots_] := Module[{i, newrots = rots, newfront = fr},
   Do[If[Abs[fr[[i]]] === Abs[fr[[i + 1]]],
     If[IntegerQ[fr[[i]]], newrots[[Abs[fr[[i]]]]] = (-I + fr[[i + 1]] / fr[[i]]) / (2 I),
      newrots[[Abs[fr[[i]]]]] = (I + fr[[i]] / fr[[i + 1]]) / (2 I)];
     newfront = Delete[newfront, {{i}, {i + 1}}]; Break[];
    ], {i, Length[fr] - 1}];
   Return[{newfront, newrots}]
  ]
FindComponents[pd_] := Module[{cl, L = Length[pd], comps},
   (*could have used KnotTheory's Skeleton function*)
   SetAttributes[S, Orderless];
   (*two edges are equivalent iff they belong to the same
    component. We start by writing down a list of the equivalence classes.*)
   cl = Sort[((S @@ Flatten[List @@ pd /. X[a_, b_, c_, d_] :→ {S[a, c], S[b, d]}]) //.
        {S[u___, S[a___, b_], S[b_, c___]] :→ S[u, S[a, b, c]]}) /. S → List];
   comps = Append[Table[First@First@Position[cl, j], {j, 1, 2 L}], 1]
   (*put a 1 at the end because we opened up component 1.*)
  ]
RotLink[pd_PD] := Module[{safety = 0, front = {1}, heads = {1},
    rots = ConstantArray[0, 2 Length[pd] + 1], comps = FindComponents[pd], todo = pd, cr},
   While[safety < 20 000 && ! (front === {1} && heads ≠ {1}),
    safety++;
    If[Length[SequenceCases[Abs /@ front, {a_, a_}]] > 0,
     {front, rots} = CheckCap[front, rots],
     heads = Sort@Select[front, IntegerQ[#] &];
     Do[cr = FindNextCross[todo, h];
      If[Head@cr === X, todo = DeleteCases[todo, cr];
       (*Print["h,cr= ", h,cr];*)
       front = Flatten[front /. h → Bend[cr, h]]; Break[]], {h, heads}]
    ];
   ];
   Return[{Cases[List @@ pd,
       X[a_, b_, c_, d_] → If[PositiveQ[X[a, b, c, d]], {1, d, a, b, c}, {-1, b, a, d, c}]] /.
      {{s_, k_, l_, 1, m_} :→ {s, k, l, 2 Length[pd] + 1, m},
       {s_, k_, l_, m_, 1} :→ {s, k, l, m, 2 Length[pd] + 1}}, rots, comps}];
  ]
RotLink[K_] := RotLink[PD[K]];
```

```
In[∘]:= CF[ℰ_] := Module[{vs = Union@Cases[ℰ, g__, ∞], ps, c},
           Total[CoefficientRules[Expand[ℰ], vs] /. (ps_ → c_) :> Factor[c] (Times @@ vs^ps)]];
```

```
In[∘]:= T3 = T1 T2;
```

```
In[∘]:= R1[s_, i_, j_, ip_, jp_] =
         CF[D1 D2 D3 (s (1/2 - D3^-1 g3ii + T2^s D1^-1 g1ii D2^-1 g2ji - D1^-1 g1ii D2^-1 g2jj -
             (T2^s - 1) D2^-1 g2ji D3^-1 g3ii + 2 D2^-1 g2jj D3^-1 g3ii - (1 - T3^s) D2^-1 g2ji D3^-1 g3ji -
             D2^-1 g2ii D3^-1 g3jj - T2^s D2^-1 g2ji D3^-1 g3jj + D1^-1 g1ii D3^-1 g3jj +
             ((T1^s - 1) D1^-1 g1ji (T2^(2 s) D2^-1 g2ji - T2^s D2^-1 g2jj + T2^s D3^-1 g3jj) + (T3^s - 1) D3^-1 g3ji (1 -
                T2^s D1^-1 g1ii - (T1^s - 1) (T2^s + 1) D1^-1 g1ji + (T2^s - 2) D2^-1 g2jj + D2^-1 g2ij)) / (T2^s - 1)))]
```

Out[∘]=

$$\frac{1}{2} D1\,D2\,D3\,s + D3\,s\,T_2^s g_{1,i,i} g_{2,j,i} + \frac{D3\,s\,(-1+T_1^s)\,T_2^{2s} g_{1,j,i} g_{2,j,i}}{-1+T_2^s} - D3\,s\,g_{1,i,i} g_{2,j,j} - \frac{D3\,s\,(-1+T_1^s)\,T_2^s g_{1,j,i} g_{2,j,j}}{-1+T_2^s} - D1\,D2\,s\,g_{3,i,i} - D1\,s\,(-1+T_2^s)\,g_{2,j,i} g_{3,i,i} + 2\,D1\,s\,g_{2,j,j} g_{3,i,i} + \frac{D1\,D2\,s\,(-1+(T_1 T_2)^s)\,g_{3,j,i}}{-1+T_2^s} - \frac{D2\,s\,T_2^s\,(-1+(T_1 T_2)^s)\,g_{1,i,i} g_{3,j,i}}{-1+T_2^s} - \frac{D2\,s\,(-1+T_1^s)\,(1+T_2^s)\,(-1+(T_1 T_2)^s)\,g_{1,j,i} g_{3,j,i}}{-1+T_2^s} + \frac{D1\,s\,(-1+(T_1 T_2)^s)\,g_{2,i,j} g_{3,j,i}}{-1+T_2^s} + D1\,s\,(-1+(T_1 T_2)^s)\,g_{2,j,i} g_{3,j,i} + \frac{D1\,s\,(-2+T_2^s)\,(-1+(T_1 T_2)^s)\,g_{2,j,j} g_{3,j,i}}{-1+T_2^s} + D2\,s\,g_{1,i,i} g_{3,j,j} + \frac{D2\,s\,(-1+T_1^s)\,T_2^s g_{1,j,i} g_{3,j,j}}{-1+T_2^s} - D1\,s\,g_{2,i,i} g_{3,j,j} - D1\,s\,T_2^s g_{2,j,i} g_{3,j,j}$$

```
In[∘]:= θ[Knot[3, 1]]
```

KnotTheory: Loading precomputed data in PD4Knots`.

Out[∘]=

$$\left\{\frac{1-T+T^2}{T},\ -\frac{1-T_1+T_1^2-T_2-T_1^3 T_2+T_2^2+T_1^4 T_2^2-T_1 T_2^3-T_1^4 T_2^3+T_1^2 T_2^4-T_1^3 T_2^4+T_1^4 T_2^4}{T_1^2 T_2^2}\right\}$$

```
In[∘]:= θ[{s0_, i0_, j0_, ip0_, jp0_}, {s1_, i1_, j1_, ip1_, jp1_}] :=
         CF[D1 D2 D3 (s1 (T1^s0 - 1) (T2^s1 - 1)^-1 (T3^s1 - 1) D1^-1 g1,j1,i0 D3^-1 g3,j0,i1
             ((T2^s0 D2^-1 g2,i1,i0 - D2^-1 g2,i1,j0) - (T2^s0 D2^-1 g2,j1,i0 - D2^-1 g2,j1,j0)))]
```

```
In[∘]:= Γ1[φ_, k_] = -D1 D2 D3 φ/2 + φ g3kk D1 D2
```

Out[∘]=

$$-\frac{1}{2} D1\,D2\,D3\,\varphi + D1\,D2\,\varphi\,g_{3,k,k}$$

In[∘]:=
```
adj[m_] := Map[Reverse, Minors[Transpose[m], Length[m] - 1], {0, 1}]
  Table[(-1)^(i + j), {i, Length[m]}, {j, Length[m]}]

θ[K_] := θ[K] =
   Module[{Cs, φ, n, A, s, i, j, k, jp, ip, Δ, Δ1, Δ2, Δ3, G, ν, α, β, gEval, c, z, comp},
    {Cs, φ, comp} = RotLink[K]; n = Length[Cs];
    A = IdentityMatrix[2 n + 1];
    Cases[Cs, {s_, i_, j_, ip_, jp_} :> (A[[{i, j}, {ip, jp}]] += (-T^s  T^s - 1
                                                                  0     -1   ))];
    (*Print[A//MatrixForm];*)
    Δ = T^((-Total[φ]-Total[Cs[[All,1]]])/2) Det[A];
    Δ1 = (Δ /. T → T_1); Δ2 = (Δ /. T → T_2); Δ3 = (Δ /. T → T_3);
    G = T^((-Total[φ]-Total[Cs[[All,1]]])/2) adj[A] / 1 (*Inverse[A]*);
    gEval[ℰ_] := Factor[ℰ /. {g_{ν_,α_,β_} :> (G[[α, β]] /. T → T_ν), D1 → Δ1, D2 → Δ2, D3 → Δ3}];
    z = gEval[Sum_{k1=1}^n Sum_{k2=1}^n θ[Cs[[k1]], Cs[[k2]]]];
    z += gEval[Sum_{k=1}^n R_1 @@ Cs[[k]]];
    z += gEval[Sum_{k=1}^{2 n} Γ_1[φ[[k]], k]];
    If[Length[Skeleton[K]] > 1,
     Expand@Together@PowerExpand[{T^(1/2)/(T - 1) Δ, T_1^(1/2)/(T_1 - 1) T_2^(1/2)/(T_2 - 1) T_3^(1/2)/(T_3 - 1) z} // Factor],
     {Δ, z} // Factor]
   ];
```

The first component is the MVA with T_i set to T (Except for Hopf link which is a bug)

In[∘]:=
```
CompareMVA[L_] := Simplify[
  {L, ((θ[L])[[1]])^-1 (-1)^(1+Length[Skeleton[L]]) MultivariableAlexander[L][T] /. {T[k_] :> T}} /.
   {T → T^2}, T > 0]
```

Checking that it doesnt matter where you cut the link. By default we cut at edge 1 so we rotate the labels in the PD code around to move the one to anywhere else and check.

In[∘]:=
```
Perm_{n_,s_}[K_] := Nest[PermutationReplace[#, Cycles[{Table[i, {i, 2 n}]}]] &, K, s]
 (*Rotate all labels around s times assuming n crossings.*)
```

In[∘]:=
```
CheckCut[L_] := Table[θ[PD[L] // Perm_{L[[1]],s}], {s, 1, 2 L[[1]]}] // Union // Length

CheckCut[Link[6, Alternating, 3]]
```

Out[∘]=
```
1
```

In[∘]:= **CheckCut[Link[6, NonAlternating, 1]]**

Out[∘]=

4

In[∘]:= **θ[PD[Link[6, NonAlternating, 1]]]**

Out[∘]=

$$\left\{\frac{1}{\sqrt{T}} - \sqrt{T},\ 0\right\}$$

In[∘]:= **θ[Perm$_{6,7}$[PD[Link[6, NonAlternating, 1]]]]**

Out[∘]=

$$\left\{\sqrt{T} - T^{3/2},\ 5 - \frac{1}{T_1} - 3 T_1 - T_1^2 - \frac{1}{T_2} + \frac{T_1}{T_2} - 5 T_2 + \frac{T_2}{T_1} + 2 T_1 T_2 + 2 T_1^2 T_2 - T_2^2 + 2 T_1 T_2^2 + T_1^2 T_2^2 - 2 T_1^3 T_2^2\right\}$$

In[∘]:= **Perm$_{6,7}$[PD[Link[6, NonAlternating, 1]]]**

Out[∘]=

PD[X[1, 8, 2, 9], X[7, 3, 4, 2], X[11, 7, 8, 6], X[12, 6, 1, 5], X[10, 3, 11, 12], X[4, 10, 5, 9]]

In[∘]:= **PD[Link[6, NonAlternating, 1]]**

Out[∘]=

PD[X[6, 1, 7, 2], X[12, 8, 9, 7], X[4, 12, 1, 11], X[5, 11, 6, 10], X[3, 8, 4, 5], X[9, 3, 10, 2]]

?????

θ[Perm$_{8,1}$[PD[Link[8, NonAlternating, 3]]]];

Table[θ[PD[Link[8, NonAlternating, 3]] // Perm$_{8,s}$], {s, 1, 2}];

In[∘]:= **θ[PD[Link[7, Alternating, 4]]]**

Out[∘]=

$$\left\{-4 + \frac{2}{T} + 2 T,\ -30 + \frac{5}{T_1^2} + \frac{10}{T_1} + 10 T_1 + 5 T_1^2 + \frac{5}{T_2^2} + \frac{5}{T_1^2 T_2^2} - \frac{10}{T_1 T_2^2} + \frac{10}{T_2} - \frac{10}{T_1^2 T_2} + \frac{10}{T_1 T_2} - \frac{10 T_1}{T_2} + 10 T_2 - \frac{10 T_2}{T_1} + 10 T_1 T_2 - 10 T_1^2 T_2 + 5 T_2^2 - 10 T_1 T_2^2 + 5 T_1^2 T_2^2\right\}$$

In[∘]:= **θ[PD[Link[7, Alternating, 4]] // Perm$_{7,1}$]**

Out[∘]=

$$\left\{-4 + \frac{2}{T} + 2 T,\ -30 + \frac{5}{T_1^2} + \frac{10}{T_1} + 10 T_1 + 5 T_1^2 + \frac{5}{T_2^2} + \frac{5}{T_1^2 T_2^2} - \frac{10}{T_1 T_2^2} + \frac{10}{T_2} - \frac{10}{T_1^2 T_2} + \frac{10}{T_1 T_2} - \frac{10 T_1}{T_2} + 10 T_2 - \frac{10 T_2}{T_1} + 10 T_1 T_2 - 10 T_1^2 T_2 + 5 T_2^2 - 10 T_1 T_2^2 + 5 T_1^2 T_2^2\right\}$$

In[∘]:= **Expand[θ[Knot[3, 1]]]**

Out[∘]=

$$\left\{-1 + \frac{1}{T} + T,\ -\frac{1}{T_1^2} - T_1^2 - \frac{1}{T_2^2} - \frac{1}{T_1^2 T_2^2} + \frac{1}{T_1 T_2^2} + \frac{1}{T_1^2 T_2} + \frac{T_1}{T_2} + \frac{T_2}{T_1} + T_1^2 T_2 - T_2^2 + T_1 T_2^2 - T_1^2 T_2^2\right\}$$

In[∘]:= `Expand[θ[Knot[10, 165]]]`

Out[∘]=

$$\left\{-15-\frac{2}{T^2}+\frac{10}{T}+10\,T-2\,T^2,\right.$$
$$-1404-\frac{9}{T_1^4}-\frac{178}{T_1^3}+\frac{607}{T_1^2}+\frac{624}{T_1}+624\,T_1+607\,T_1^2-178\,T_1^3-9\,T_1^4-\frac{9}{T_2^4}-\frac{9}{T_1^4 T_2^4}+\frac{44}{T_1^3 T_2^4}-\frac{65}{T_1^2 T_2^4}+$$
$$\frac{44}{T_1 T_2^4}-\frac{178}{T_2^3}+\frac{44}{T_1^4 T_2^3}-\frac{178}{T_1^3 T_2^3}+\frac{104}{T_1^2 T_2^3}+\frac{104}{T_1 T_2^3}+\frac{44\,T_1}{T_2^3}+\frac{607}{T_2^2}-\frac{65}{T_1^4 T_2^2}+\frac{104}{T_1^3 T_2^2}+\frac{607}{T_1^2 T_2^2}-\frac{1041}{T_1 T_2^2}+$$
$$\frac{104\,T_1}{T_2^2}-\frac{65\,T_1^2}{T_2^2}+\frac{624}{T_2}+\frac{44}{T_1^4 T_2}+\frac{104}{T_1^3 T_2}-\frac{1041}{T_1^2 T_2}+\frac{624}{T_1 T_2}-\frac{1041\,T_1}{T_2}+\frac{104\,T_1^2}{T_2}+\frac{44\,T_1^3}{T_2}+624\,T_2+$$
$$\frac{44\,T_2}{T_1^3}+\frac{104\,T_2}{T_1^2}-\frac{1041\,T_2}{T_1}+624\,T_1 T_2-1041\,T_1^2 T_2+104\,T_1^3 T_2+44\,T_1^4 T_2+607\,T_2^2-$$
$$\frac{65\,T_2^2}{T_1^2}+\frac{104\,T_2^2}{T_1}-1041\,T_1 T_2^2+607\,T_1^2 T_2^2+104\,T_1^3 T_2^2-65\,T_1^4 T_2^2-178\,T_2^3+\frac{44\,T_2^3}{T_1}+$$
$$\left.104\,T_1 T_2^3+104\,T_1^2 T_2^3-178\,T_1^3 T_2^3+44\,T_1^4 T_2^3-9\,T_2^4+44\,T_1 T_2^4-65\,T_1^2 T_2^4+44\,T_1^3 T_2^4-9\,T_1^4 T_2^4\right\}$$

In[∘]:=

```
Options[PolyPlot1] = {Labeled → False};
PolyPlot1[Δ_, OptionsPattern[]] := Module[{Δ1, crs, m, maxc, minc, s, rect},
  Δ1 = PowerExpand@Expand@Δ;
  rect = {{0, 0}, {1, 0}, {1, 1}, {0, 1}};
  If[Δ1 === 0, Graphics[{White, Polygon@rect}, AspectRatio → 1 / 5],
   m = Max[-Exponent[Δ1, T, Min], Exponent[Δ1, T, Max]];
   crs = CoefficientRules[T^m Δ1, {T}];
   maxc = N@Log@Max@Abs[Last /@ crs];
   minc = N@Log@Min@Select[Abs[Last /@ crs], # > 0 &];
   If[minc == maxc, s[_] = 0, s[c_] := s[c] = (maxc - Log@c) / (maxc - minc)];
   Graphics[crs /. ({x_} → c_) :→ {
       Lighter[Which[c == 0, White, c > 0, Red, c < 0, Blue], 0.88 s[Abs@c]],
       Tooltip[Polygon[({x + m - 1 / 2, 0} + #) & /@ rect], c T^(x-m)],
       If[Not@OptionValue[Labeled], {},
        {Black, FontSize → Scaled[1/((m + 1) (6 + maxc))], Text[c T^(x-m), {x + m, 0.5}]}]
      }, AspectRatio → Min[1 / 5 , 1/ Sqrt[m + 1]],
    ImagePadding → None, PlotRangePadding → None]
   ]];
Options[PolyPlot2] = {Labeled → False};
PolyPlot2[θ_, OptionsPattern[]] := Module[{θ1, crs, m1, m2, maxc, minc, s, hex, p},
   θ1 = PowerExpand@Expand@θ;
   If[θ1 === 0, Graphics[{White, Disk[]}],
```

```
    hex = Table[{Cos[α], Sin[α]} / Cos[2 π / 12] / 2, {α, 2 π / 12, 2 π, 2 π / 6}];
    m1 = Max[-Exponent[θ1, T1, Min], Exponent[θ1, T1, Max]];
    m2 = Max[-Exponent[θ1, T2, Min], Exponent[θ1, T2, Max]];
    crs = CoefficientRules[T1^m1 T2^m2 θ1, {T1, T2}];
    maxc = N@Log@Max@Abs[Last /@ crs];
    minc = N@Log@Min@Select[Abs[Last /@ crs], # > 0 &];
    If[minc == maxc, s[_] = 0, s[c_] := s[c] = (maxc - Log@c) / (maxc - minc)];
    Graphics[{(*{Yellow,Disk[{0,0},1+Cos[2π/12]Norm[{m1,m2}]/Sqrt[2]]},*)
      crs /. ({x1_, x2_} → c_) ⧴ {
        Lighter[Which[c == 0, White, c > 0, Red, c < 0, Blue], 0.88 s[Abs@c]],
        p = {{1, -1/2}, {0, Sqrt[3]/2}}.{x1 - m1, x2 - m2};
        Tooltip[Polygon[(p + #) & /@ hex], c T1^(x1-m1) T2^(x2-m2)],
        If[Not@OptionValue[Labeled], {},
         {Black, FontSize → Scaled[1/(Max[m1, m2] (10 + maxc))], Text[c T1^(x1-m1) T2^(x2-m2), p]}]
       }
     }, ImagePadding → None, PlotRangePadding → None]
   ]];
PolyPlot[{Δ_, θ_}, opts___Rule] := GraphicsColumn[
   {PolyPlot1[Δ, FilterRules[{opts}, Options[PolyPlot1]]],
    PolyPlot2[θ, FilterRules[{opts}, Options[PolyPlot2]]]},
   Spacings → Scaled@0.08, ImagePadding → None, PlotRangePadding → None,
   FilterRules[{opts}, Options[GraphicsColumn]]
  ];
```

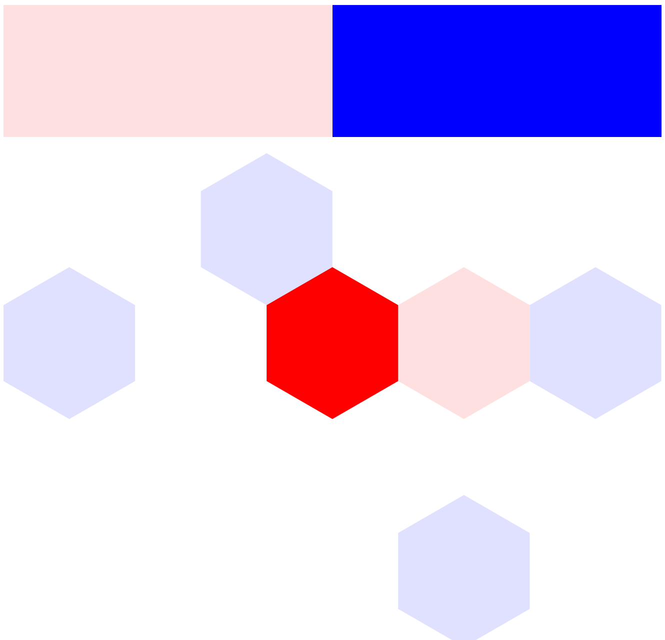
In[∘]:=
```
θ[Knot[3, 1]]〚2〛 // Expand
```

Out[∘]=

$$-\frac{1}{T_1^2}-T_1^2-\frac{1}{T_2^2}-\frac{1}{T_1^2 T_2^2}+\frac{1}{T_1 T_2^2}+\frac{1}{T_1^2 T_2}+\frac{T_1}{T_2}+\frac{T_2}{T_1}+T_1^2 T_2-T_2^2+T_1 T_2^2-T_1^2 T_2^2$$

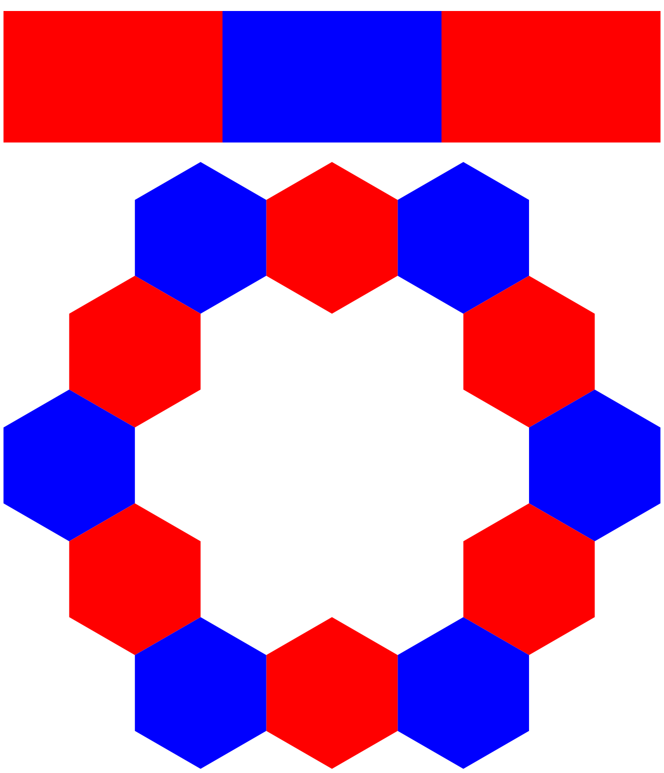
In[ ]:= PolyPlot$\left[\left\{T + -3\,T^2,\ -T_2 + T_1 + 11 - \frac{1}{T_1^2} - T_1^2 - \frac{1}{T_2^2}\right\}\right]$

Out[ ]=



In[ ]:= `PolyPlot[θ[Knot[3, 1]]]`

Out[ ]=



In[∘]:= θ[Link[7, Alternating, 4]]

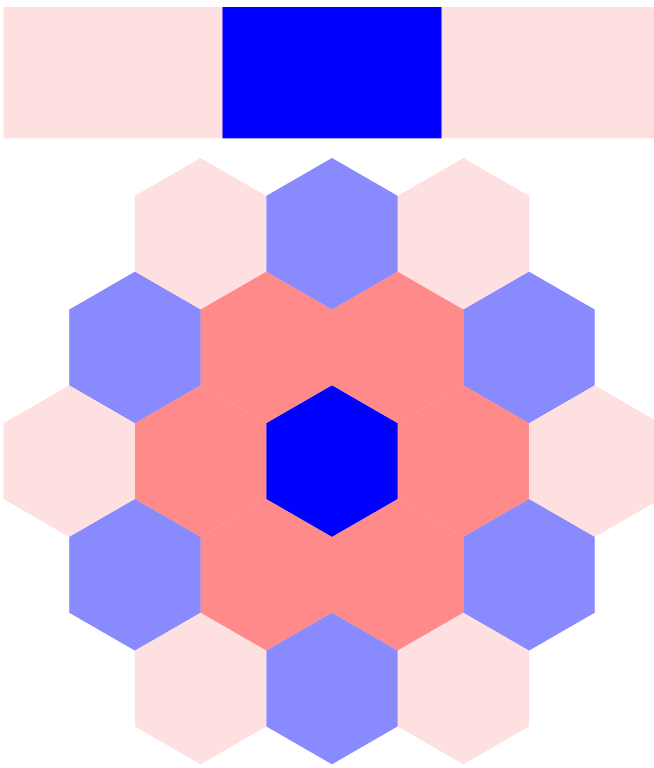
KnotTheory: Loading precomputed data in PD4Links`.

Out[∘]=

$$\left\{-4+\frac{2}{T}+2\,T,\ -30+\frac{5}{T_1^2}+\frac{10}{T_1}+10\,T_1+5\,T_1^2+\frac{5}{T_2^2}+\frac{5}{T_1^2\,T_2^2}-\frac{10}{T_1\,T_2^2}+\frac{10}{T_2}-\frac{10}{T_1^2\,T_2}+\frac{10}{T_1\,T_2}-\frac{10\,T_1}{T_2}+10\,T_2-\frac{10\,T_2}{T_1}+10\,T_1\,T_2-10\,T_1^2\,T_2+5\,T_2^2-10\,T_1\,T_2^2+5\,T_1^2\,T_2^2\right\}$$

In[∘]:= PolyPlot[θ[Link[7, Alternating, 4]]]

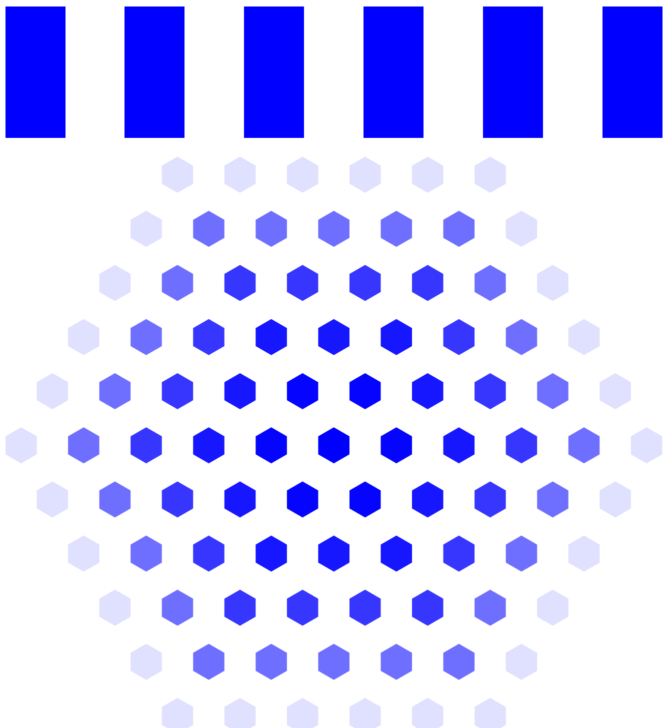
Out[∘]=



In[∘]:= θ[TorusKnot[12, 2]];

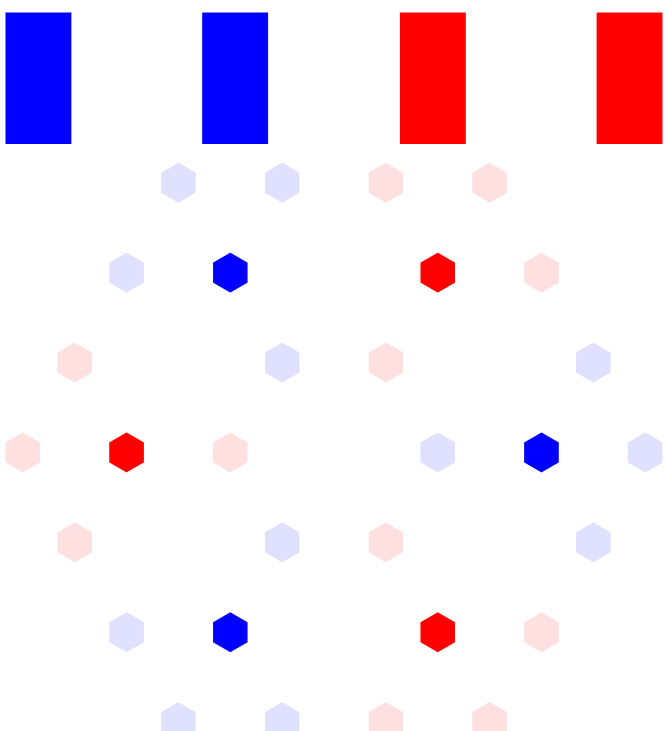
In[ ]:= PolyPlot[θ[TorusKnot[12, 2]]]

Out[ ]=



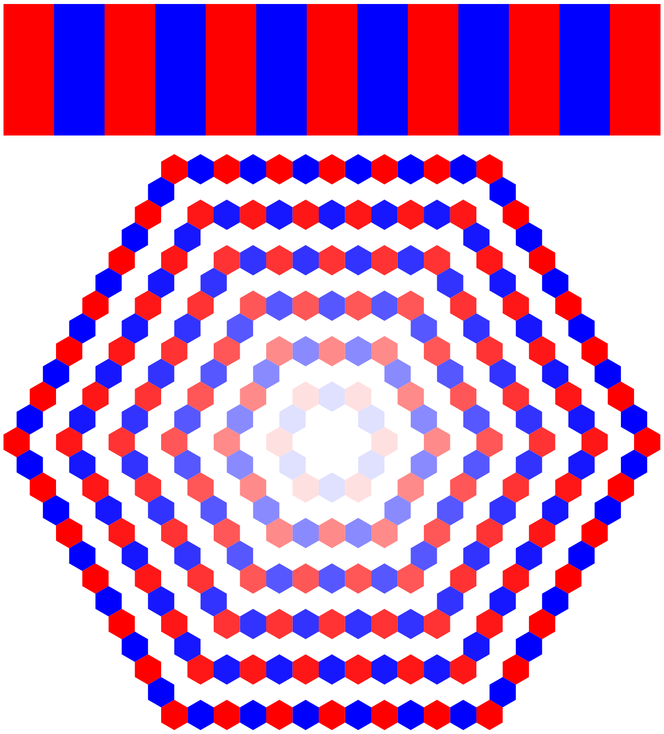
In[ ]:= PolyPlot[θ[TorusKnot[6, 3]]]

Out[ ]=



In[ ]:=
```
PolyPlot[θ[TorusKnot[13, 2]]]
```

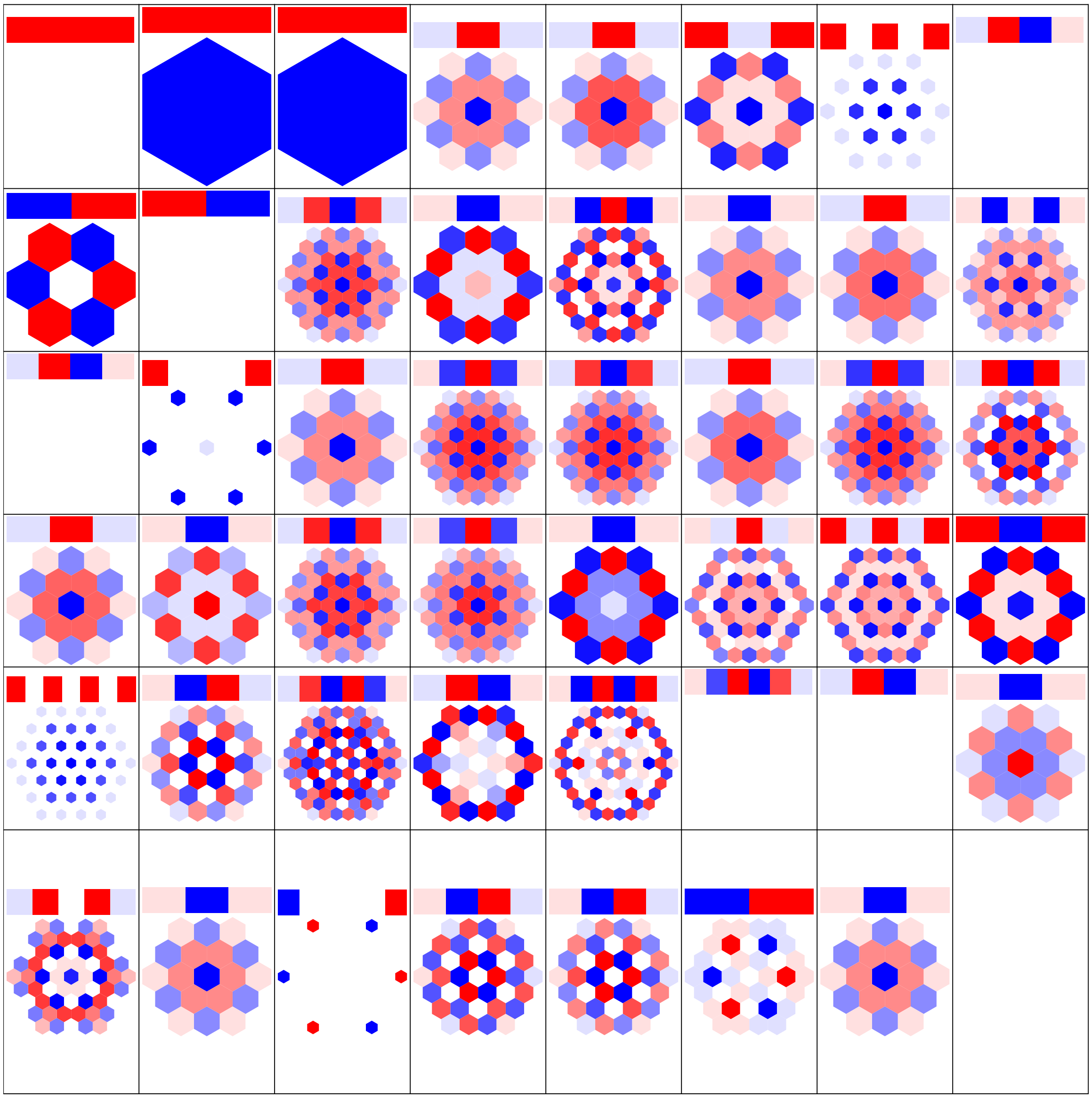
Out[ ]=



In[ ]:=
```
tab48L = {{1, 0}}~Join~Table[θ[K], {K, AllLinks[{2, 8}]}];
```

In[∘]:=
```
g48L = GraphicsGrid[Partition[PolyPlot /@ tab48L, 8],
  Spacings → 0, Dividers → All, ImagePadding → None, PlotRangePadding → None]
```

Out[∘]=



In[∘]:=
```
tab418L[[38]]
```

Part: Part specification tab418L⟦38⟧ is longer than depth of object.

Out[∘]=
```
tab418L⟦38⟧
```

In[∘]:=
```
AllLinks[{2, 10}][[42]]
θ@%
```

Out[∘]=

Link[8, NonAlternating, 3]

Out[∘]=

$$\left\{-\frac{1}{T^{5/2}}+T^{5/2},\ -\frac{3}{T_1^5}+3\,T_1^5-\frac{3}{T_2^5}+\frac{3}{T_1^5\,T_2^5}+3\,T_2^5-3\,T_1^5\,T_2^5\right\}$$

In[∘]:=
```
Length@AllLinks[{2, 9}]
```

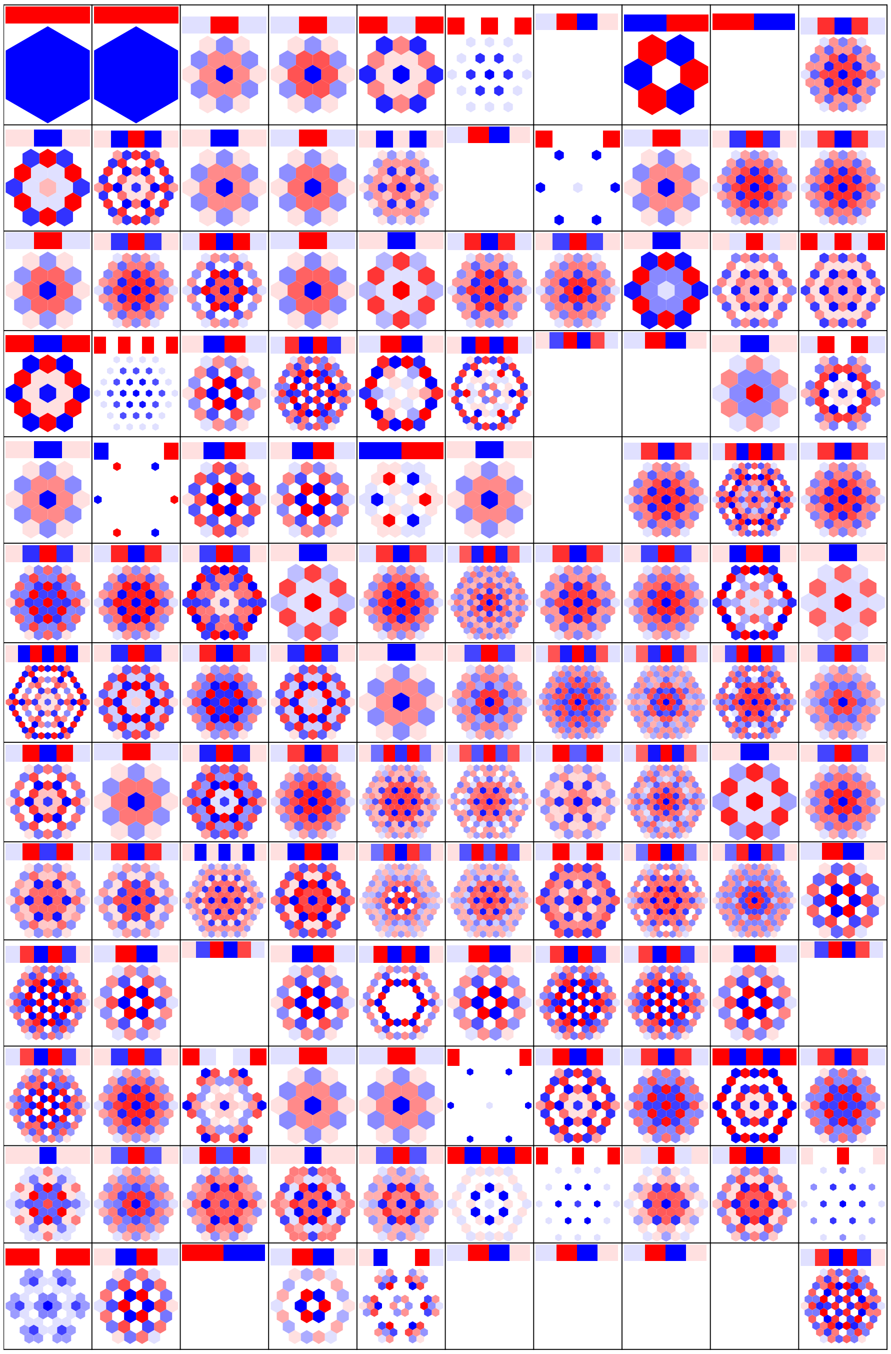
Out[∘]=

130

In[•]:=
```
StringJoin @@ (AllLinks[{2, 9}] /. Link[n_, t_, k_] :> StringJoin[
    "\\l{", ToString@n, "}{", t /. {Alternating → "a", NonAlternating → "n"},
    "}{", ToString[k], "}{", If[n ≥ 8 ∧ k ≤ 9, "0", ""], ToString[k], "} & "
   ])
```

Out[•]=
```
\l{2}{a}{1}{1} & \l{4}{a}{1}{1} & \l{5}{a}{1}{1} & \l{6}{a}{1}{1} & \l{6}{a}{2}{2}
 & \l{6}{a}{3}{3} & \l{6}{a}{4}{4} & \l{6}{a}{5}{5} & \l{6}{n}{1}{1} &
 \l{7}{a}{1}{1} & \l{7}{a}{2}{2} & \l{7}{a}{3}{3} & \l{7}{a}{4}{4} & \l{7}{a}{5}{5}
 & \l{7}{a}{6}{6} & \l{7}{a}{7}{7} & \l{7}{n}{1}{1} & \l{7}{n}{2}{2} &
 \l{8}{a}{1}{01} & \l{8}{a}{2}{02} & \l{8}{a}{3}{03} & \l{8}{a}{4}{04} &
 \l{8}{a}{5}{05} & \l{8}{a}{6}{06} & \l{8}{a}{7}{07} & \l{8}{a}{8}{08} &
 \l{8}{a}{9}{09} & \l{8}{a}{10}{10} & \l{8}{a}{11}{11} & \l{8}{a}{12}{12} &
 \l{8}{a}{13}{13} & \l{8}{a}{14}{14} & \l{8}{a}{15}{15} & \l{8}{a}{16}{16} &
 \l{8}{a}{17}{17} & \l{8}{a}{18}{18} & \l{8}{a}{19}{19} & \l{8}{a}{20}{20}
 & \l{8}{a}{21}{21} & \l{8}{n}{1}{01} & \l{8}{n}{2}{02} & \l{8}{n}{3}{03}
 & \l{8}{n}{4}{04} & \l{8}{n}{5}{05} & \l{8}{n}{6}{06} & \l{8}{n}{7}{07} &
 \l{8}{n}{8}{08} & \l{9}{a}{1}{01} & \l{9}{a}{2}{02} & \l{9}{a}{3}{03} &
 \l{9}{a}{4}{04} & \l{9}{a}{5}{05} & \l{9}{a}{6}{06} & \l{9}{a}{7}{07} &
 \l{9}{a}{8}{08} & \l{9}{a}{9}{09} & \l{9}{a}{10}{10} & \l{9}{a}{11}{11} &
 \l{9}{a}{12}{12} & \l{9}{a}{13}{13} & \l{9}{a}{14}{14} & \l{9}{a}{15}{15} &
 \l{9}{a}{16}{16} & \l{9}{a}{17}{17} & \l{9}{a}{18}{18} & \l{9}{a}{19}{19} &
 \l{9}{a}{20}{20} & \l{9}{a}{21}{21} & \l{9}{a}{22}{22} & \l{9}{a}{23}{23} &
 \l{9}{a}{24}{24} & \l{9}{a}{25}{25} & \l{9}{a}{26}{26} & \l{9}{a}{27}{27} &
 \l{9}{a}{28}{28} & \l{9}{a}{29}{29} & \l{9}{a}{30}{30} & \l{9}{a}{31}{31} &
 \l{9}{a}{32}{32} & \l{9}{a}{33}{33} & \l{9}{a}{34}{34} & \l{9}{a}{35}{35} &
 \l{9}{a}{36}{36} & \l{9}{a}{37}{37} & \l{9}{a}{38}{38} & \l{9}{a}{39}{39} &
 \l{9}{a}{40}{40} & \l{9}{a}{41}{41} & \l{9}{a}{42}{42} & \l{9}{a}{43}{43} &
 \l{9}{a}{44}{44} & \l{9}{a}{45}{45} & \l{9}{a}{46}{46} & \l{9}{a}{47}{47} &
 \l{9}{a}{48}{48} & \l{9}{a}{49}{49} & \l{9}{a}{50}{50} & \l{9}{a}{51}{51} &
 \l{9}{a}{52}{52} & \l{9}{a}{53}{53} & \l{9}{a}{54}{54} & \l{9}{a}{55}{55}
 & \l{9}{n}{1}{01} & \l{9}{n}{2}{02} & \l{9}{n}{3}{03} & \l{9}{n}{4}{04} &
 \l{9}{n}{5}{05} & \l{9}{n}{6}{06} & \l{9}{n}{7}{07} & \l{9}{n}{8}{08} &
 \l{9}{n}{9}{09} & \l{9}{n}{10}{10} & \l{9}{n}{11}{11} & \l{9}{n}{12}{12} &
 \l{9}{n}{13}{13} & \l{9}{n}{14}{14} & \l{9}{n}{15}{15} & \l{9}{n}{16}{16} &
 \l{9}{n}{17}{17} & \l{9}{n}{18}{18} & \l{9}{n}{19}{19} & \l{9}{n}{20}{20} &
 \l{9}{n}{21}{21} & \l{9}{n}{22}{22} & \l{9}{n}{23}{23} & \l{9}{n}{24}{24} &
 \l{9}{n}{25}{25} & \l{9}{n}{26}{26} & \l{9}{n}{27}{27} & \l{9}{n}{28}{28} &
```

In[∘]:=
```
g130 = GraphicsGrid[
  Partition[PolyPlot /@ Table[θ[K], {K, AllLinks[{2, 9}]}], 10],
  Spacings → 0, Dividers → All, ImagePadding → None, PlotRangePadding → None
 ]
```
Out[∘]=



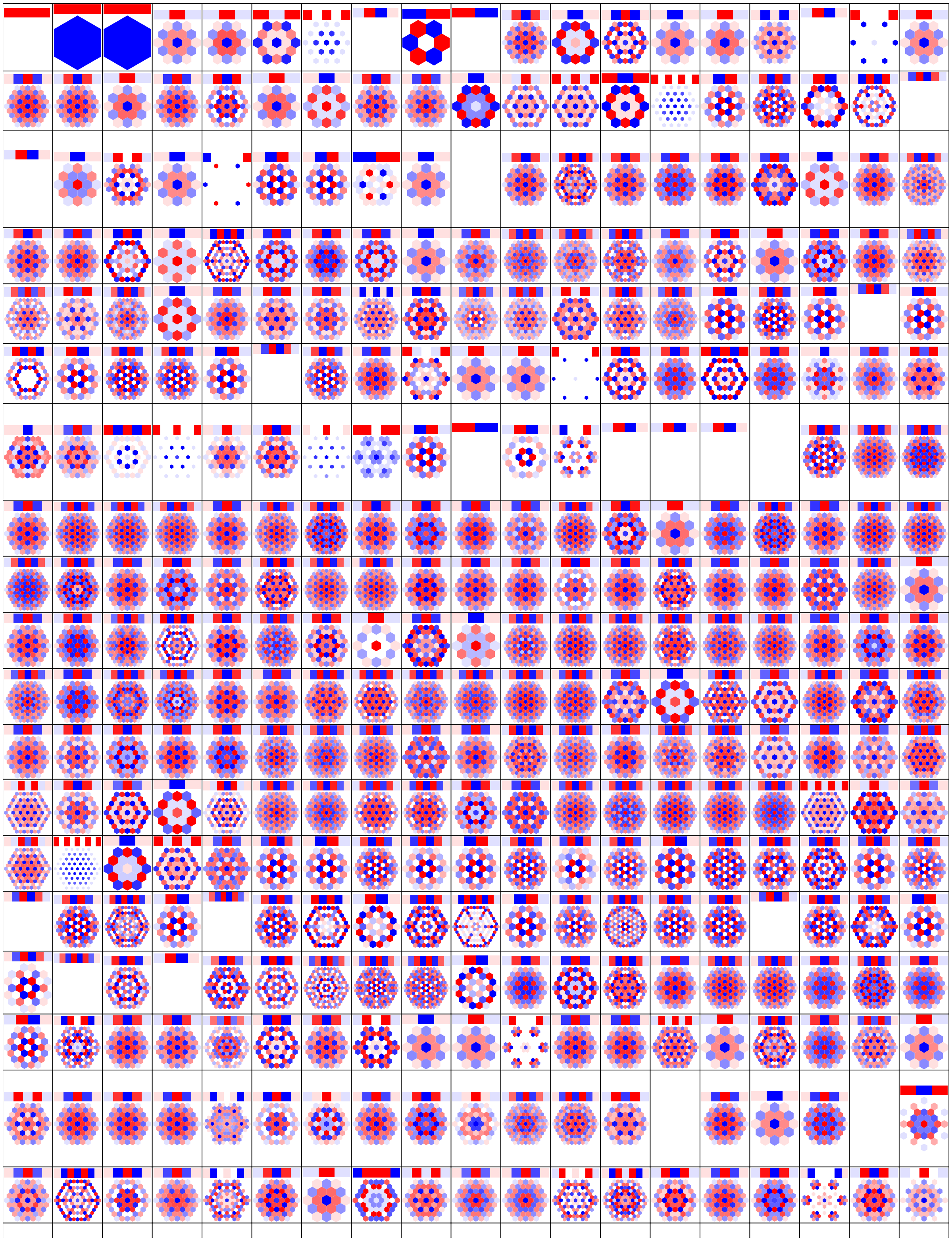
In[∘]:=
```
Export["Theta4Links2-9.pdf", g130]
```
Out[∘]=

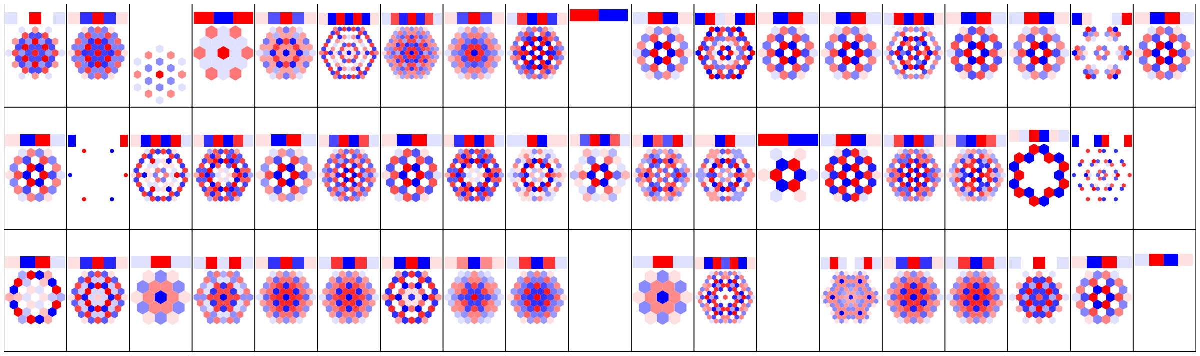
Theta4Links2-9.pdf

```
In[•]:= tab418L = {{1, 0}}~Join~Table[θ[K], {K, AllLinks[{2, 10}]}];

In[•]:= g418L = GraphicsGrid[Partition[PolyPlot /@ tab418L, 19],
          Spacings → 0, Dividers → All, ImagePadding → None, PlotRangePadding → None]
```

Out[•]=

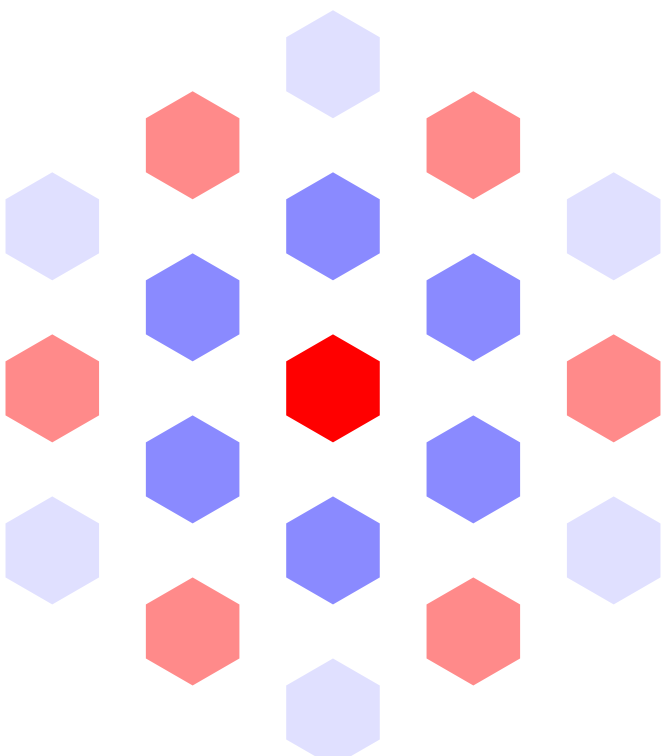




A link with vanishing Δ yet non-vanishing θ:

In[∘]:=
```
PolyPlot@θ@Link[10, NonAlternating, 59]
```

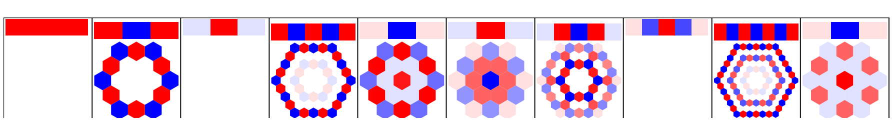
Out[∘]=

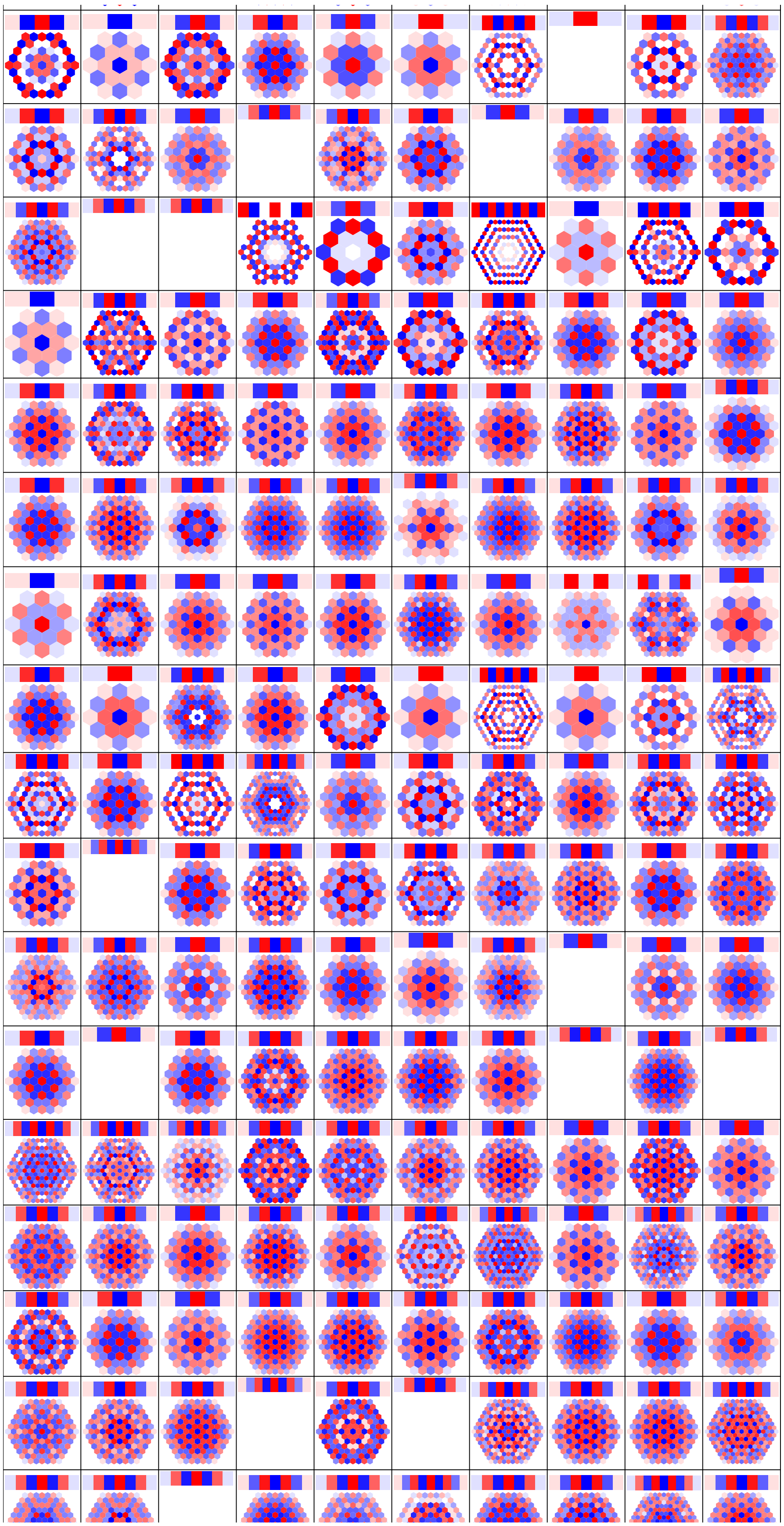


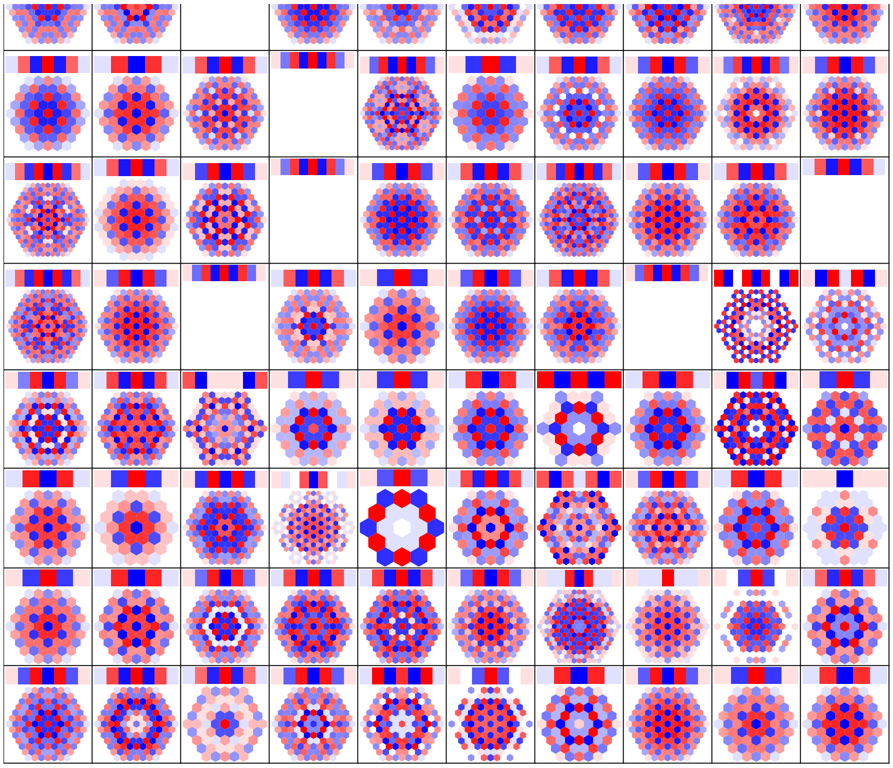
In[∘]:=
```
tab250 = {{1, 0}}~Join~Table[θ[K], {K, AllKnots[{3, 10}]}];
```

In[∘]:=
```
g250 = GraphicsGrid[Partition[PolyPlot /@ tab250, 10],
  Spacings → 0, Dividers → All, ImagePadding → None, PlotRangePadding → None]
```

Out[∘]=







In[ ]:= VanishingMVALinks =

```
{Link[9, NonAlternating, 27], Link[10, NonAlternating, 32], Link[10, NonAlternating, 36],
 Link[10, NonAlternating, 107], Link[11, NonAlternating, 244],
 Link[11, NonAlternating, 247], Link[11, NonAlternating, 334],
 Link[11, NonAlternating, 381], Link[11, NonAlternating, 396],
 Link[11, NonAlternating, 404], Link[11, NonAlternating, 406]};
```

In[ ]:= Θ /@ VanishingMVALinks

Out[ ]=

{{0, 0}, {0, 0}, {0, 0}, {0, 0}, {0, 0}, {0, 0}, {0, 0}, {0, 0}, {0, 0}, {0, 0}, {0, 0}}