

Pensieve Header: The separation powers of various invariants.

Loading the invariants

(Alt) In[]:=

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\Theta"];
```

(Alt) In[]:=

```
Do[
  NumberOfKnots[n, Alternating] =
    {0, 0, 1, 1, 2, 3, 7, 18, 41, 123, 367, 1288, 4878, 19536, 85263}[[n]];
  NumberOfKnots[n, NonAlternating] =
    {0, 0, 0, 0, 0, 0, 0, 3, 8, 42, 185, 888, 5110, 27436, 168030}[[n]],
  {n, 15}
];
NumberOfKnots[n_Integer] :=
  NumberOfKnots[n, Alternating] + NumberOfKnots[n, NonAlternating];
NumberOfKnots[{n_}] := Sum[NumberOfKnots[n1], {n1, n}];
{NumberOfKnots[15, Alternating],
  NumberOfKnots[15, NonAlternating], NumberOfKnots[15], NumberOfKnots[{15}]}
```

(Alt) Out[]:=

```
{85263, 168030, 253293, 313230}
```

The data files for H, Kh, and Θ are available at <https://drorbn.net/AcademicPensieve/Projects/HigherRank/Data>.

(Alt) In[]:=

```
Get["../.. /Projects/HigherRank/Data/HOMFLYPT3-15.m"] /. (K_ -> V_) -> (H[K] = Expand@V);
H[Knot[3, 1]]
```

(Alt) Out[]:=

$$2a^2 - a^4 + a^2z^2$$

(Alt) In[]:=

```
Get["../.. /Projects/HigherRank/Data/Kh3-15.m"] /. (K_ -> V_) -> (Kh[K] = Expand@V);
Kh[Knot[3, 1]]
```

(Alt) Out[]:=

$$\frac{1}{q^3} + \frac{1}{q} + \frac{1}{q^9 t^3} + \frac{1}{q^5 t^2}$$

(Alt) In[]:=

```
J[K_] := J[K] = Kh[K] /. t -> -1;
J[Knot[3, 1]]
```

(Alt) Out[]:=

$$-\frac{1}{q^9} + \frac{1}{q^5} + \frac{1}{q^3} + \frac{1}{q}$$

(Alt) In[]:=

```
Get["../.. /Projects/HigherRank/Data/theta3-15.m"] /. (K_ -> V_) := (Theta[K] = Expand@V);
Theta[Knot[3, 1]]
```

(Alt) Out[]:=

$$\left\{ -1 + \frac{1}{T} + T, -\frac{1}{T^2} - T^2 - \frac{1}{T^2} - \frac{1}{T^2 T^2} + \frac{1}{T^2 T^2} + \frac{1}{T^2 T^2} + \frac{T^2}{T^2} + \frac{T^2}{T^2} + T^2 T^2 - T^2^2 + T^2 T^2^2 - T^2^2 T^2^2 \right\}$$

(Alt) In[]:=

```
Delta[K_] := Delta[K] = Theta[K][[1]];
rho1[K_] := rho1[K] = -Theta[K][[2]] /. {T1 -> T, T2 -> 1};
Factor@{Delta[Knot[3, 1]], rho1[Knot[3, 1]]}
```

(Alt) Out[]:=

$$\left\{ \frac{1 - T + T^2}{T}, \frac{(-1 + T)^2 (1 + T^2)}{T^2} \right\}$$

The data files for ρ_2 are available at <https://drorbn.net/AcademicPensieve/Talks/KnotTheoryCongress-2502/Rho2Data>.

(Alt) In[]:=

```
(Join@@(Get["../.. /Talks/KnotTheoryCongress-2502/Rho2Data/" <> #] & /@
{"Rho2_3-10.m", "Rho2_11.m", "Rho2_12.m", "Rho2_13.m",
"Rho2_14.m", "Rho2_15.m"})) /. (K_ -> V_) := (rho2[K] = Expand@V);
rho2[Knot[3, 1]]
```

(Alt) Out[]:=

$$2 - 4 z^2 + 3 z^4 + 4 z^6 + z^8$$

The data file for Vol is available at <https://drorbn.net/AcademicPensieve/Projects/Theta/Vols/>.

(Alt) In[]:=

```
Get["../.. /Projects/Theta/Vols/Vols.m"] /. (K_ -> v_) := (Vol[K] = v);
Vol[Knot[4, 1]]
Vol[Knot[4, 1]] // Precision
```

(Alt) Out[]:=

2.02988321281930725004240510854904057188337861506059958403497821

(Alt) Out[]:=

63.3075

```
In[ ]:= LowVolKnots = Select[AllKnots[{3, 15}], Vol[#] < Vol[Knot[4, 1]] &]
```

Out[]:=

```
{Knot[3, 1], Knot[5, 1], Knot[7, 1], Knot[8, 19], Knot[9, 1],
Knot[10, 124], Knot[11, Alternating, 367], Knot[13, Alternating, 4878],
Knot[13, NonAlternating, 4587], Knot[13, NonAlternating, 4639],
Knot[14, NonAlternating, 21881], Knot[15, Alternating, 85263],
Knot[15, NonAlternating, 41185], Knot[15, NonAlternating, 124802]}
```

In[*]:= Vol /@ LowVolKnots

Out[*]=

```
{0, 0, 0, 0, 0, 0, 0, 0,
 3.5685265575250654054462215980785874491852334782548573710639971 × 10-48,
 1.7152356672329879527424751703536552551811797480236112575777653 × 10-47, 0,
 0, 0, 0.74809992574274203560797235226599115629979099639038686612626981}
```

Warning: σ was evaluated only up to 14 crossings!

(Alt) In[*]:=

```
fs = FileNames["../Talks/ICERM-2305/Data/*.m"];
ls = Table[f → Length[Get[f] /. (K_ → V_) :> (σ[K] = V)], {f, fs}]
Total[Last /@ ls] (* Expect 59937 *)
σ[Knot[3, 1]]
σ[Knot[14, NonAlternating, 27436]]
```

(Alt) Out[*]=

```
{../Talks/ICERM-2305/Data/14a10001-19536.m → 9536,
 ../Talks/ICERM-2305/Data/14a1-10000.m → 10000,
 ../Talks/ICERM-2305/Data/14n10001-20000.m → 10000,
 ../Talks/ICERM-2305/Data/14n1-10000.m → 10000,
 ../Talks/ICERM-2305/Data/14n20001-27436.m → 7436,
 ../Talks/ICERM-2305/Data/3-13.m → 12965}
```

(Alt) Out[*]=

59937

(Alt) Out[*]=

$$-2\theta\left[-\frac{\sqrt{3}}{2} + u\right] + 2\theta\left[\frac{\sqrt{3}}{2} + u\right]$$

(Alt) Out[*]=

$$4\theta\left[-\frac{\sqrt{3}}{2} + u\right] - 4\theta\left[\frac{\sqrt{3}}{2} + u\right] - 2\theta\left[u - \sqrt[6]{-0.992...}\right] - \\ 2\theta\left[u - \sqrt[6]{-0.782...}\right] + 2\theta\left[u - \sqrt[6]{0.782...}\right] + 2\theta\left[u - \sqrt[6]{0.992...}\right]$$

Stats

(Alt) In[]:=

```
Knots[n_Integer] := If[n ≤ 10,
  Table[Knot[n, k], {k, NumberOfKnots[n]}],
  Join[
    Table[Knot[n, Alternating, k], {k, NumberOfKnots[n, Alternating]}],
    Table[Knot[n, NonAlternating, k], {k, NumberOfKnots[n, NonAlternating]}]
  ]
];
Knots[{n_}] := Join@@Table[Knots[n1], {n1, n}];
Knots[{11}] // Short
```

(Alt) Out[]//Short=

```
{Knot[3, 1], Knot[4, 1], Knot[5, 1], Knot[5, 2], <<794>>, Knot[11, NonAlternating, 183],
Knot[11, NonAlternating, 184], Knot[11, NonAlternating, 185]}
```

(Alt) In[]:=

```
Stats[{n_}, ls_List] :=
  NumberOfKnots[{n}] - CountDistinct@Table[L[K], {K, Knots[{n}]}, {L, ls}];
Stats[ls_List] := Monitor[
  Table[def = Stats[{n}, ls], {n, 10, 14}],
  {n - 1, def}
];
```

(Alt) In[]:=

```
Stats[{Δ}]
```

(Alt) Out[]=

```
{38, 250, 1204, 7326, 39741}
```

Warning: σ was evaluated only up to 14 crossings!

(Alt) In[]:=

```
Stats[{σ}]
```

(Alt) Out[]=

```
{108, 356, 1525, 7736, 40101}
```

Warning: σ was evaluated only up to 14 crossings!

(Alt) In[]:=

```
Stats[{Δ, σ}]
```

(Alt) Out[]=

```
{25, 167, 907, 5927, 34132}
```

(Alt) In[]:=

```
Stats[{J}]
```

(Alt) Out[]=

```
{7, 70, 482, 3434, 21250}
```

Warning: σ was evaluated only up to 14 crossings!

```
(Alt) In[ ]:=
  Stats[{ $\sigma$ , J}]
```

```
(Alt) Out[ ]=
  {6, 43, 302, 2235, 13 527}
```

```
(Alt) In[ ]:=
  Stats[{Kh}]
```

```
(Alt) Out[ ]=
  {6, 65, 452, 3226, 19 754}
```

Warning: σ was evaluated only up to 14 crossings!

```
(Alt) In[ ]:=
  Stats[{ $\sigma$ , Kh}]
```

```
(Alt) Out[ ]=
  {6, 43, 300, 2196, 13 164}
```

```
(Alt) In[ ]:=
  Stats[{H}]
```

```
(Alt) Out[ ]=
  {2, 31, 222, 1839, 11 251}
```

Warning: σ was evaluated only up to 14 crossings!

```
(Alt) In[ ]:=
  Stats[{ $\sigma$ , H}]
```

```
(Alt) Out[ ]=
  {1, 30, 216, 1806, 11 097}
```

```
(Alt) In[ ]:=
  Stats[{H, Kh}]
```

```
(Alt) Out[ ]=
  {1, 30, 214, 1771, 10 788}
```

Warning: σ was evaluated only up to 14 crossings!

```
(Alt) In[ ]:=
  Stats[{ $\sigma$ , H, Kh}]
```

```
(Alt) Out[ ]=
  {1, 30, 214, 1771, 10 785}
```

```
(Alt) In[ ]:=
  Table[d → Stats[{If[Vol@# < 1, 0, N[Vol@#, d]] &}], {d, 0, 70}] // Column
```

```
(Alt) Out[ ]=
  0 → {247, 799, 2975, 12 963, 59 935}
  1 → {236, 787, 2962, 12 949, 59 921}
  2 → {182, 725, 2892, 12 873, 59 838}
  3 → {64, 456, 2538, 12 460, 59 360}
  4 → {13, 79, 851, 8652, 54 096}
  5 → {7, 35, 222, 3100, 34 925}
  6 → {7, 27, 126, 1306, 12 452}
  7 → {6, 25, 113, 1036, 6774}
  8 → {6, 25, 113, 1015, 6411}
  9 → {6, 25, 113, 1012, 6358}
  10 → {6, 25, 113, 1012, 6356}
  11 → {6, 25, 113, 1012, 6354}
```

12 $\rightarrow \{6, 25, 113, 1012, 6354\}$
13 $\rightarrow \{6, 25, 113, 1012, 6354\}$
14 $\rightarrow \{6, 25, 113, 1012, 6354\}$
15 $\rightarrow \{6, 25, 113, 1012, 6354\}$
16 $\rightarrow \{6, 25, 113, 1012, 6354\}$
17 $\rightarrow \{6, 25, 113, 1012, 6354\}$
18 $\rightarrow \{6, 25, 113, 1012, 6354\}$
19 $\rightarrow \{6, 25, 113, 1012, 6354\}$
20 $\rightarrow \{6, 25, 113, 1012, 6354\}$
21 $\rightarrow \{6, 25, 113, 1012, 6354\}$
22 $\rightarrow \{6, 25, 113, 1012, 6354\}$
23 $\rightarrow \{6, 25, 113, 1012, 6354\}$
24 $\rightarrow \{6, 25, 113, 1012, 6354\}$
25 $\rightarrow \{6, 25, 113, 1012, 6354\}$
26 $\rightarrow \{6, 25, 113, 1012, 6354\}$
27 $\rightarrow \{6, 25, 113, 1012, 6354\}$
28 $\rightarrow \{6, 25, 113, 1012, 6354\}$
29 $\rightarrow \{6, 25, 113, 1012, 6354\}$
30 $\rightarrow \{6, 25, 113, 1012, 6354\}$
31 $\rightarrow \{6, 25, 113, 1012, 6354\}$
32 $\rightarrow \{6, 25, 113, 1012, 6354\}$
33 $\rightarrow \{6, 25, 113, 1012, 6354\}$
34 $\rightarrow \{6, 25, 113, 1012, 6354\}$
35 $\rightarrow \{6, 25, 113, 1012, 6354\}$
36 $\rightarrow \{6, 25, 113, 1012, 6354\}$
37 $\rightarrow \{6, 25, 113, 1012, 6354\}$
38 $\rightarrow \{6, 25, 113, 1012, 6354\}$
39 $\rightarrow \{6, 25, 113, 1012, 6353\}$
40 $\rightarrow \{6, 25, 113, 1012, 6353\}$
41 $\rightarrow \{6, 25, 113, 1012, 6353\}$
42 $\rightarrow \{6, 25, 113, 1012, 6353\}$
43 $\rightarrow \{6, 25, 113, 1012, 6353\}$
44 $\rightarrow \{6, 25, 113, 1012, 6353\}$
45 $\rightarrow \{6, 25, 113, 1012, 6353\}$
46 $\rightarrow \{6, 25, 113, 1012, 6353\}$
47 $\rightarrow \{6, 25, 113, 1012, 6353\}$
48 $\rightarrow \{6, 25, 113, 1012, 6353\}$
49 $\rightarrow \{6, 25, 113, 1012, 6353\}$
50 $\rightarrow \{6, 25, 113, 1012, 6353\}$
51 $\rightarrow \{6, 25, 113, 1012, 6353\}$
52 $\rightarrow \{6, 25, 113, 1012, 6353\}$
53 $\rightarrow \{6, 25, 113, 1012, 6353\}$
54 $\rightarrow \{6, 25, 113, 1012, 6353\}$
55 $\rightarrow \{6, 25, 113, 1012, 6353\}$
56 $\rightarrow \{6, 25, 113, 1012, 6353\}$
57 $\rightarrow \{6, 25, 113, 1012, 6353\}$
58 $\rightarrow \{6, 25, 113, 1012, 6353\}$
59 $\rightarrow \{6, 25, 113, 1012, 6353\}$
60 $\rightarrow \{6, 25, 113, 1012, 6353\}$
61 $\rightarrow \{6, 25, 113, 1012, 6353\}$
62 $\rightarrow \{6, 25, 113, 1011, 6348\}$
63 $\rightarrow \{6, 22, 108, 990, 6216\}$
64 $\rightarrow \{5, 20, 86, 841, 5294\}$

$65 \rightarrow \{5, 19, 74, 690, 4394\}$
 $66 \rightarrow \{5, 19, 74, 690, 4394\}$
 $67 \rightarrow \{5, 19, 74, 690, 4394\}$
 $68 \rightarrow \{5, 19, 74, 690, 4394\}$
 $69 \rightarrow \{5, 19, 74, 690, 4394\}$
 $70 \rightarrow \{5, 19, 74, 690, 4394\}$

(Alt) In[]:=

```
Clear[TVol]
TVol[K_] := TVol[K] = If[Vol@K < 1, 0, N[Vol@K, 58]]
```

(Alt) In[]:=

```
Stats[{TVol}]
```

(Alt) Out[]=

```
{6, 25, 113, 1012, 6353}
```

Warning: σ was evaluated only up to 14 crossings!

(Alt) In[]:=

```
Stats[{ $\sigma$ , TVol}]
```

(Alt) Out[]=

```
{0, 14, 89, 928, 6016}
```

(Alt) In[]:=

```
Stats[{Kh, H, TVol}]
```

(Alt) Out[]=

```
{0, 14, 84, 911, 5917}
```

Warning: σ was evaluated only up to 14 crossings!

(Alt) In[]:=

```
Stats[{ $\sigma$ , Kh, H, TVol}]
```

(Alt) Out[]=

```
{0, 14, 84, 911, 5917}
```

(Alt) In[]:=

```
Stats[{ $\Delta$ ,  $\rho_1$ }]
```

(Alt) Out[]=

```
{0, 14, 95, 959, 6253}
```

(Alt) In[]:=

```
Stats[{ $\Delta$ ,  $\rho_1$ ,  $\rho_2$ }]
```

(Alt) Out[]=

```
{0, 14, 84, 911, 5926}
```

(Alt) In[]:=

```
Stats[{ $\rho_1$ ,  $\rho_2$ , Kh, H, TVol}]
```

(Alt) Out[]=

```
{0, 14, 84, 911, 5916}
```

Warning: σ was evaluated only up to 14 crossings!

```
(Alt) In[ ]:=
  Stats[{ $\sigma$ ,  $\rho_1$ ,  $\rho_2$ , Kh, H, TVol}]
```

```
(Alt) Out[ ]=
  {0, 14, 84, 911, 5916}
```

```
(Alt) In[ ]:=
  Stats[{ $\theta$ }]
```

```
(Alt) Out[ ]=
  {0, 3, 19, 194, 1118}
```

```
(Alt) In[ ]:=
  6758 / 41432 // N
```

```
(Alt) Out[ ]=
  0.163111
```

Warning: σ was evaluated only up to 14 crossings!

```
(Alt) In[ ]:=
  Stats[{ $\sigma$ ,  $\theta$ }]
```

```
(Alt) Out[ ]=
  {0, 3, 19, 194, 1118}
```

```
(Alt) In[ ]:=
  Stats[{ $\theta$ , Kh}]
```

```
(Alt) Out[ ]=
  {0, 3, 18, 185, 1062}
```

```
(Alt) In[ ]:=
  Stats[{ $\theta$ , H}]
```

```
(Alt) Out[ ]=
  {0, 3, 18, 185, 1064}
```

```
(Alt) In[ ]:=
  Stats[{ $\theta$ , H, Kh}]
```

```
(Alt) Out[ ]=
  {0, 3, 18, 185, 1062}
```

```
(Alt) In[ ]:=
  Stats[{ $\theta$ ,  $\rho_2$ }]
```

```
(Alt) Out[ ]=
  {0, 3, 10, 169, 982}
```

```
(Alt) In[ ]:=
  Stats[{ $\theta$ , TVol}]
```

```
(Alt) Out[ ]=
  {0, 3, 10, 169, 973}
```

```
(Alt) In[ ]:=
  Stats[{ $\theta$ ,  $\rho_2$ , TVol}]
```

```
(Alt) Out[ ]=
  {0, 3, 10, 169, 973}
```

```
(Alt) In[ ]:=
  Stats[{ $\Theta$ ,  $\rho_2$ , Kh}]
(Alt) Out[ ]=
  {0, 3, 10, 169, 981}
```

```
(Alt) In[ ]:=
  Stats[{ $\Theta$ ,  $\rho_2$ , H}]
(Alt) Out[ ]=
  {0, 3, 10, 169, 982}
```

Warning: σ was evaluated only up to 14 crossings!

```
(Alt) In[ ]:=
  Stats[{ $\sigma$ ,  $\Theta$ ,  $\rho_2$ }]
(Alt) Out[ ]=
  {0, 3, 10, 169, 982}
```

```
(Alt) In[ ]:=
  Stats[{ $\Theta$ ,  $\rho_2$ , H, Kh}]
(Alt) Out[ ]=
  {0, 3, 10, 169, 981}
```

```
(Alt) In[ ]:=
  Stats[{ $\Theta$ ,  $\rho_2$ , TVol}]
(Alt) Out[ ]=
  {0, 3, 10, 169, 973}
```

```
(Alt) In[ ]:=
  Stats[{ $\Theta$ ,  $\rho_2$ , H, Kh, TVol}]
(Alt) Out[ ]=
  {0, 3, 10, 169, 972}
```

Warning: σ was evaluated only up to 14 crossings!

```
(Alt) In[ ]:=
  Stats[{ $\sigma$ ,  $\Theta$ ,  $\rho_2$ , H, Kh, TVol}]
(Alt) Out[ ]=
  {0, 3, 10, 169, 972}
```