

Pensieve Header: The separation powers of various invariants.

Loading the invariants

```
In[ ]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\Theta"];
```

```
In[ ]:= Do[
  NumberOfKnots[n, Alternating] =
    {0, 0, 1, 1, 2, 3, 7, 18, 41, 123, 367, 1288, 4878, 19536, 85263}[[n]];
  NumberOfKnots[n, NonAlternating] =
    {0, 0, 0, 0, 0, 0, 0, 3, 8, 42, 185, 888, 5110, 27436, 168030}[[n]],
  {n, 15}
];
NumberOfKnots[n_Integer] :=
  NumberOfKnots[n, Alternating] + NumberOfKnots[n, NonAlternating];
NumberOfKnots[{n_}] := Sum[NumberOfKnots[n1], {n1, n}];
{NumberOfKnots[15, Alternating],
  NumberOfKnots[15, NonAlternating], NumberOfKnots[15], NumberOfKnots[{15}]}
```

```
Out[ ]:=
{85263, 168030, 253293, 313230}
```

The data files for H, Kh, and Θ are available at <https://drorbn.net/AcademicPensieve/Projects/HigherRank/Data>.

```
In[ ]:= Get["../..//Projects/HigherRank/Data/HOMFLYPT3-15.m"] /. (K_ -> V_) -> (H[K] = Expand@V);
H[Knot[3, 1]]
```

```
Out[ ]:=
2 a^2 - a^4 + a^2 z^2
```

```
In[ ]:= Get["../..//Projects/HigherRank/Data/Kh3-15.m"] /. (K_ -> V_) -> (Kh[K] = Expand@V);
Kh[Knot[3, 1]]
```

```
Out[ ]:=
1/q^3 + 1/q + 1/(q^9 t^3) + 1/(q^5 t^2)
```

```
In[ ]:= J[K_] := J[K] = Kh[K] /. t -> -1;
J[Knot[3, 1]]
```

```
Out[ ]:=
-1/q^9 + 1/q^5 + 1/q^3 + 1/q
```

```
In[*]:= Get["../.. /Projects/HigherRank/Data/theta3-15.m"] /. (K_ -> V_) :> (Theta[K] = Expand@V);
Theta[Knot[3, 1]]
```

```
Out[*]=
```

$$\left\{-1 + \frac{1}{T} + T, -\frac{1}{T^2} - T^2 - \frac{1}{T^2} - \frac{1}{T^2 T^2} + \frac{1}{T^2 T^2} + \frac{1}{T^2 T^2} + \frac{T^2}{T^2} + \frac{T^2}{T^2} + T^2 T^2 - T^2 + T^2 T^2 - T^2 T^2\right\}$$

```
In[*]:= Delta[K_] := Delta[K] = Theta[K][[1]];
rho1[K_] := rho1[K] = -Theta[K][[2]] /. {T1 -> T, T2 -> 1};
Factor@{Delta[Knot[3, 1]], rho1[Knot[3, 1]]}
```

```
Out[*]=
```

$$\left\{\frac{1 - T + T^2}{T}, \frac{(-1 + T)^2 (1 + T^2)}{T^2}\right\}$$

The data files for ρ_2 are available at <https://drorbn.net/AcademicPensieve/Talks/KnotTheoryCongress-2502/Rho2Data>.

```
In[*]:= (Join@@(Get["../.. /Talks/KnotTheoryCongress-2502/Rho2Data/" <> #] & /@
{"Rho2_3-10.m", "Rho2_11.m", "Rho2_12.m", "Rho2_13.m",
"Rho2_14.m", "Rho2_15.m"})) /. (K_ -> V_) :> (rho2[K] = Expand@V);
rho2[Knot[3, 1]]
```

```
Out[*]=
```

$$2 - 4 z^2 + 3 z^4 + 4 z^6 + z^8$$

The data file for Vol is available at <https://drorbn.net/AcademicPensieve/Projects/Theta/Vols/>.

```
In[*]:= Get["../.. /Projects/Theta/Vols/Vols.m"] /. (K_ -> v_) :> (Vol[K] = v);
Vol[Knot[4, 1]]
Vol[Knot[4, 1]] // Precision
```

```
Out[*]=
```

$$2.02988321281930725004240510854904057188337861506059958403497821$$

```
Out[*]=
```

$$63.3075$$

```
In[*]:= LowVolKnots = Select[AllKnots[{3, 15}], Vol[#] < Vol[Knot[4, 1]] &]
```

```
Out[*]=
```

$$\{\text{Knot}[3, 1], \text{Knot}[5, 1], \text{Knot}[7, 1], \text{Knot}[8, 19], \text{Knot}[9, 1], \\ \text{Knot}[10, 124], \text{Knot}[11, \text{Alternating}, 367], \text{Knot}[13, \text{Alternating}, 4878], \\ \text{Knot}[13, \text{NonAlternating}, 4587], \text{Knot}[13, \text{NonAlternating}, 4639], \\ \text{Knot}[14, \text{NonAlternating}, 21881], \text{Knot}[15, \text{Alternating}, 85263], \\ \text{Knot}[15, \text{NonAlternating}, 41185], \text{Knot}[15, \text{NonAlternating}, 124802]\}$$

```
In[*]:= Vol /@ LowVolKnots
```

```
Out[*]=
```

```
{0, 0, 0, 0, 0, 0, 0, 0,
 3.5685265575250654054462215980785874491852334782548573710639971 × 10-48,
 1.7152356672329879527424751703536552551811797480236112575777653 × 10-47, 0,
 0, 0, 0.74809992574274203560797235226599115629979099639038686612626981}
```

Warning: σ was evaluated only up to 13 crossings!

```
Get["../..\\Talks\\ICERM-2305/Data/3-13.m"] /. (K_ -> V_) -> (σ[K] = V);
σ[Knot[3, 1]]
```

```
Out[*]=
```

$$-2 \theta \left[-\frac{\sqrt{3}}{2} + u \right] + 2 \theta \left[\frac{\sqrt{3}}{2} + u \right]$$

Stats

```
In[*]:= Knots[n_Integer] := If[n ≤ 10,
  Table[Knot[n, k], {k, NumberOfKnots[n]}],
  Join[
    Table[Knot[n, Alternating, k], {k, NumberOfKnots[n, Alternating]}],
    Table[Knot[n, NonAlternating, k], {k, NumberOfKnots[n, NonAlternating]}]
  ];
Knots[{n_}] := Join@@Table[Knots[n1], {n1, n}];
Knots[{11}] // Short
```

```
Out[*]//Short=
```

```
{Knot[3, 1], Knot[4, 1], <<797>>,
 Knot[11, NonAlternating, 184], Knot[11, NonAlternating, 185]}
```

```
In[*]:= Stats[{n_}, {s_List}] :=
  NumberOfKnots[{n}] - CountDistinct@Table[σ[K], {K, Knots[{n]}], {σ, s}];
Stats[{s_List}] := Monitor[
  Table[def = Stats[{n}, {s}], {n, 10, 13}],
  {n - 1, def}
];
```

```
In[*]:= Stats[{Δ}]
```

```
Out[*]=
```

```
{38, 250, 1204, 7326}
```

Warning: σ was evaluated only up to 13 crossings!

```
In[*]:= Stats[{σ}]
```

```
Out[*]=
```

```
{108, 356, 1525, 7736}
```

Warning: σ was evaluated only up to 13 crossings!

```
In[ ]:= Stats[{Δ, σ}]
Out[ ]=
{25, 167, 907, 5927}
```

```
In[ ]:= Stats[{J}]
Out[ ]=
{7, 70, 482, 3434}
```

Warning: σ was evaluated only up to 13 crossings!

```
In[ ]:= Stats[{σ, J}]
Out[ ]=
{6, 43, 302, 2235}
```

```
In[ ]:= Stats[{Kh}]
Out[ ]=
{6, 65, 452, 3226}
```

Warning: σ was evaluated only up to 13 crossings!

```
In[ ]:= Stats[{σ, Kh}]
Out[ ]=
{6, 43, 300, 2196}
```

```
In[ ]:= Stats[{H}]
Out[ ]=
{2, 31, 222, 1839}
```

Warning: σ was evaluated only up to 13 crossings!

```
In[ ]:= Stats[{σ, H}]
Out[ ]=
{1, 30, 216, 1806}
```

```
In[ ]:= Stats[{H, Kh}]
Out[ ]=
{1, 30, 214, 1771}
```

```
In[ ]:= Table[d → Stats[{If[Vol@# < 1, 0, N[Vol@#, d]] &}], {d, 0, 70}] // Column
Out[ ]=
```

```
0 → {247, 799, 2975, 12 963}
1 → {236, 787, 2962, 12 949}
2 → {182, 725, 2892, 12 873}
3 → {64, 456, 2538, 12 460}
4 → {13, 79, 851, 8652}
5 → {7, 35, 222, 3100}
6 → {7, 27, 126, 1306}
7 → {6, 25, 113, 1036}
8 → {6, 25, 113, 1015}
9 → {6, 25, 113, 1012}
10 → {6, 25, 113, 1012}
```

11 $\rightarrow \{6, 25, 113, 1012\}$
12 $\rightarrow \{6, 25, 113, 1012\}$
13 $\rightarrow \{6, 25, 113, 1012\}$
14 $\rightarrow \{6, 25, 113, 1012\}$
15 $\rightarrow \{6, 25, 113, 1012\}$
16 $\rightarrow \{6, 25, 113, 1012\}$
17 $\rightarrow \{6, 25, 113, 1012\}$
18 $\rightarrow \{6, 25, 113, 1012\}$
19 $\rightarrow \{6, 25, 113, 1012\}$
20 $\rightarrow \{6, 25, 113, 1012\}$
21 $\rightarrow \{6, 25, 113, 1012\}$
22 $\rightarrow \{6, 25, 113, 1012\}$
23 $\rightarrow \{6, 25, 113, 1012\}$
24 $\rightarrow \{6, 25, 113, 1012\}$
25 $\rightarrow \{6, 25, 113, 1012\}$
26 $\rightarrow \{6, 25, 113, 1012\}$
27 $\rightarrow \{6, 25, 113, 1012\}$
28 $\rightarrow \{6, 25, 113, 1012\}$
29 $\rightarrow \{6, 25, 113, 1012\}$
30 $\rightarrow \{6, 25, 113, 1012\}$
31 $\rightarrow \{6, 25, 113, 1012\}$
32 $\rightarrow \{6, 25, 113, 1012\}$
33 $\rightarrow \{6, 25, 113, 1012\}$
34 $\rightarrow \{6, 25, 113, 1012\}$
35 $\rightarrow \{6, 25, 113, 1012\}$
36 $\rightarrow \{6, 25, 113, 1012\}$
37 $\rightarrow \{6, 25, 113, 1012\}$
38 $\rightarrow \{6, 25, 113, 1012\}$
39 $\rightarrow \{6, 25, 113, 1012\}$
40 $\rightarrow \{6, 25, 113, 1012\}$
41 $\rightarrow \{6, 25, 113, 1012\}$
42 $\rightarrow \{6, 25, 113, 1012\}$
43 $\rightarrow \{6, 25, 113, 1012\}$
44 $\rightarrow \{6, 25, 113, 1012\}$
45 $\rightarrow \{6, 25, 113, 1012\}$
46 $\rightarrow \{6, 25, 113, 1012\}$
47 $\rightarrow \{6, 25, 113, 1012\}$
48 $\rightarrow \{6, 25, 113, 1012\}$
49 $\rightarrow \{6, 25, 113, 1012\}$
50 $\rightarrow \{6, 25, 113, 1012\}$
51 $\rightarrow \{6, 25, 113, 1012\}$
52 $\rightarrow \{6, 25, 113, 1012\}$
53 $\rightarrow \{6, 25, 113, 1012\}$
54 $\rightarrow \{6, 25, 113, 1012\}$
55 $\rightarrow \{6, 25, 113, 1012\}$
56 $\rightarrow \{6, 25, 113, 1012\}$
57 $\rightarrow \{6, 25, 113, 1012\}$
58 $\rightarrow \{6, 25, 113, 1012\}$
59 $\rightarrow \{6, 25, 113, 1012\}$
60 $\rightarrow \{6, 25, 113, 1012\}$
61 $\rightarrow \{6, 25, 113, 1012\}$
62 $\rightarrow \{6, 25, 113, 1011\}$
63 $\rightarrow \{6, 22, 108, 990\}$

$64 \rightarrow \{5, 20, 86, 841\}$
 $65 \rightarrow \{5, 19, 74, 690\}$
 $66 \rightarrow \{5, 19, 74, 690\}$
 $67 \rightarrow \{5, 19, 74, 690\}$
 $68 \rightarrow \{5, 19, 74, 690\}$
 $69 \rightarrow \{5, 19, 74, 690\}$
 $70 \rightarrow \{5, 19, 74, 690\}$

```
In[ ]:= Clear[TVol]
TVol[K_] := TVol[K] = If[Vol@K < 1, 0, N[Vol@K, 58]]
```

```
In[ ]:= Stats[{TVol}]
```

```
Out[ ]=
{6, 25, 113, 1012}
```

Warning: σ was evaluated only up to 13 crossings!

```
In[ ]:= Stats[{ $\sigma$ , TVol}]
```

```
Out[ ]=
{0, 14, 89, 928}
```

```
In[ ]:= Stats[{Kh, H, TVol}]
```

```
Out[ ]=
{0, 14, 84, 911}
```

```
In[ ]:= Stats[{ $\Delta$ ,  $\rho_1$ ]}
```

```
Out[ ]=
{0, 14, 95, 959}
```

```
In[ ]:= Stats[{ $\Delta$ ,  $\rho_1$ ,  $\rho_2$ ]}
```

```
Out[ ]=
{0, 14, 84, 911}
```

```
In[ ]:= Stats[{ $\rho_1$ ,  $\rho_2$ , Kh, H, TVol}]
```

```
Out[ ]=
{0, 14, 84, 911}
```

Warning: σ was evaluated only up to 13 crossings!

```
In[ ]:= Stats[{ $\sigma$ ,  $\rho_1$ ,  $\rho_2$ , Kh, H, TVol}]
```

```
Out[ ]=
{0, 14, 84, 911}
```

```
In[ ]:= Stats[{0}]
```

```
Out[ ]=
{0, 3, 19, 194}
```

```
In[ ]:= 6758 / 41432 // N
```

```
Out[ ]=
0.163111
```

Warning: σ was evaluated only up to 13 crossings!

```
In[*]:= Stats[{ $\sigma$ ,  $\Theta$ }]
Out[*]= {0, 3, 19, 194}
```

```
In[*]:= Stats[{ $\Theta$ , Kh}]
Out[*]= {0, 3, 18, 185}
```

```
In[*]:= Stats[{ $\Theta$ , H}]
Out[*]= {0, 3, 18, 185}
```

```
In[*]:= Stats[{ $\Theta$ , H, Kh}]
Out[*]= {0, 3, 18, 185}
```

```
In[*]:= Stats[{ $\Theta$ ,  $\rho_2$ }]
Out[*]= {0, 3, 10, 169}
```

```
In[*]:= Stats[{ $\Theta$ , TVol}]
Out[*]= {0, 3, 10, 169}
```

```
In[*]:= Stats[{ $\Theta$ ,  $\rho_2$ , TVol}]
Out[*]= {0, 3, 10, 169}
```

```
In[*]:= Stats[{ $\Theta$ ,  $\rho_2$ , Kh}]
Out[*]= {0, 3, 10, 169}
```

```
In[*]:= Stats[{ $\Theta$ ,  $\rho_2$ , H}]
Out[*]= {0, 3, 10, 169}
```

Warning: σ was evaluated only up to 13 crossings!

```
In[*]:= Stats[{ $\sigma$ ,  $\Theta$ ,  $\rho_2$ }]
Out[*]= {0, 3, 10, 169}
```

```
In[*]:= Stats[{ $\Theta$ ,  $\rho_2$ , H, Kh}]
Out[*]= {0, 3, 10, 169}
```

```
In[*]:= Stats[{ $\Theta$ ,  $\rho_2$ , TVol}]
Out[*]= {0, 3, 10, 169}
```

```
In[ ]:= Stats[{ $\theta$ ,  $\rho_2$ , H, Kh, TVol}]
```

```
Out[ ]= {0, 3, 10, 169}
```

Warning: σ was evaluated only up to 13 crossings!

```
In[ ]:= Stats[{ $\sigma$ ,  $\theta$ ,  $\rho_2$ , H, Kh, TVol}]
```

```
Out[ ]= {0, 3, 10, 169}
```