

```
In[ ]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\Theta"];
Once[<< Theta.m]
Unset[ $\delta_{\alpha,\beta_-}$ ];
```

Loading KnotTheory` version of October 29, 2024, 10:29:52.1301.

Read more at <http://katlas.org/wiki/KnotTheory>.

```
In[ ]:=  $\delta_{\alpha_-,\alpha_-} = 1$ ;  $\delta_{\alpha_-^+,\beta_-^+} := \delta_{\alpha,\beta}$ ;  $\delta_{\alpha_-,\beta_-} /;$  OrderedQ@{ $\beta$ ,  $\alpha$ } :=  $\delta_{\beta,\alpha}$ ;
```

```
In[ ]:= R1 = Simplify[
  Residue[F1[{s0, i0, j0}], {T2, 1}] /. {g2, $\alpha_-,\alpha_- \rightarrow 1$ , g2,j0,i0  $\rightarrow 1 - g_{2,i0,j0}$ , g3, $\alpha_-,\beta_- \Rightarrow g_{1,\alpha,\beta}$ }]
```

```
Out[ ]:= - ( (-1 + T1^s0) g1,j0,i0 (g1,i0,i0 + 2 (-1 + T1^s0) g1,j0,i0 - g1,j0,j0) )
```

```
In[ ]:= R2 = Simplify[Residue[F2[{s0, i0, j0}], {s1, i1, j1}], {T2, 1}] /. g3, $\alpha_-,\beta_- \Rightarrow g_{1,\alpha,\beta}$ ]
```

```
Out[ ]:= (-1 + T1^s0) (-1 + T1^s1) g1,j0,i1 g1,j1,i0 (g2,i1,i0 - g2,i1,j0 - g2,j1,i0 + g2,j1,j0)
```

```
In[ ]:= D[{s-,i-,j-}[_]] := ( $\mathcal{E} / . \gamma \rightarrow i^+$ ) + ( $\mathcal{E} / . \gamma \rightarrow j^+$ ) - ( $\mathcal{E} / . \gamma \rightarrow i$ ) - ( $\mathcal{E} / . \gamma \rightarrow j$ );
lhs = D[{s1,i1,j1}][ (T1^s0 - 1) g2, $\gamma$ ,i0 g1, $\gamma$ ,i0 g1,j0 $^+,\gamma$  + (1 - T1^s0) g2, $\gamma$ ,j0 g1, $\gamma$ ,i0 g1,j0 $^+,\gamma$  ]
```

```
Out[ ]:= - ( (-1 + T1^s0) g1,i1,i0 g1,j0 $^+,i1$  g2,i1,i0 ) - (1 - T1^s0) g1,i1,i0 g1,j0 $^+,i1$  g2,i1,j0 -
  (-1 + T1^s0) g1,j1,i0 g1,j0 $^+,j1$  g2,j1,i0 - (1 - T1^s0) g1,j1,i0 g1,j0 $^+,j1$  g2,j1,j0 +
  (-1 + T1^s0) g1,i1 $^+,i0$  g1,j0 $^+,i1^+$  g2,i1 $^+,i0$  + (1 - T1^s0) g1,i1 $^+,i0$  g1,j0 $^+,i1^+$  g2,i1 $^+,j0$  +
  (-1 + T1^s0) g1,j0 $^+,j1^+$  g1,j1 $^+,i0$  g2,j1 $^+,i0$  + (1 - T1^s0) g1,j0 $^+,j1^+$  g1,j1 $^+,i0$  g2,j1 $^+,j0$ 
```

```
In[ ]:= rhs =  $\delta_{c0,c1}$  R1 + R2
```

```
Out[ ]:= - ( (-1 + T1^s0)  $\delta_{c0,c1}$  g1,j0,i0 (g1,i0,i0 + 2 (-1 + T1^s0) g1,j0,i0 - g1,j0,j0) ) +
  (-1 + T1^s0) (-1 + T1^s1) g1,j0,i1 g1,j1,i0 (g2,i1,i0 - g2,i1,j0 - g2,j1,i0 + g2,j1,j0)
```

```
In[ ]:= Expand[Expand[lhs == rhs /. gRules[{s0, i0, j0}], {s1, i1, j1}]] /. {
  T2  $\rightarrow 1$ ,  $\delta_{i0,j0} | \delta_{i1,j0} | \delta_{i0,j1} \rightarrow 0$ ,  $\delta_{i0,i1} | \delta_{j0,j1} | \delta_{c0,c1}^2 \rightarrow \delta_{c0,c1}$ ,
  g2, $\alpha_-^+,\alpha_- \rightarrow 0$ , g2,j0 $^+,i0 \rightarrow 1 - \delta_{i0,j0} - g_{2,i0^+,j0}$ ,
   $\delta_{c0,c1} \mathcal{E}_- \Rightarrow (\mathcal{E} / . \{s1 \rightarrow s0, i1 \rightarrow i0, j1 \rightarrow j0\})$ 
}]
```

```
Out[ ]:= True
```