

```
In[*]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\Theta"];
Once[<< Theta.m];
```

Loading KnotTheory` version of October 29, 2024, 10:29:52.1301.

Read more at <http://katlas.org/wiki/KnotTheory>.

```
In[*]:= @Knot[3, 1]
```

 KnotTheory: Loading precomputed data in PD4Knots`.

```
Out[*]=
```

$$\left\{ \frac{1 - T + T^2}{T}, -\frac{1 - T_1 + T_1^2 - T_2 - T_1^3 T_2 + T_2^2 + T_1^4 T_2^2 - T_1 T_2^3 - T_1^4 T_2^3 + T_1^2 T_2^4 - T_1^3 T_2^4 + T_1^4 T_2^4}{T_1^2 T_2^2} \right\}$$

```

In[*]:= (*L should be a list of an odd number of odd integers.*)
OddPretzel[L_] := Module[{s = Total[Abs[#] & /@ L], x},
  PD @@ Flatten@Table[x = Sum[Abs[L[[j]]], {j, 1, u - 1}];

  If[EvenQ[u],
    If[L[[u]] > 0,
      Join[Table[X[s + x + Abs[L[[u]]] - 2 k, x + 2 + 2 k,
        s + x + Abs[L[[u]]] - 2 k - 1, x + 1 + 2 k], {k, 0,  $\frac{\text{Abs}[L[[u]]] - 1}{2}$ }],
        Table[X[x + 2 k, s + x + Abs[L[[u]]] + 2 - 2 k,
          x + 1 + 2 k, s + x + Abs[L[[u]]] + 1 - 2 k], {k, 1,  $\frac{\text{Abs}[L[[u]]] - 1}{2}$ }]],
      Join[Table[X[x + 1 + 2 k, s + x + Abs[L[[u]]] - 2 k,
        x + 2 + 2 k, s + x + Abs[L[[u]]] + 1 - 2 k], {k, 0,  $\frac{\text{Abs}[L[[u]]] - 1}{2}$ }],
        Table[X[s + x + Abs[L[[u]]] + 1 - 2 k, x + 2 k,
          s + x + Abs[L[[u]]] + 2 - 2 k, x + 1 + 2 k], {k, 1,  $\frac{\text{Abs}[L[[u]]] - 1}{2}$ }]],
      ],
    If[L[[u]] > 0,
      Join[Table[X[x + 1 + 2 k, s + x + Abs[L[[u]]] + 1 - 2 k,
        x + 2 + 2 k, s + x + Abs[L[[u]]] - 2 k], {k, 0,  $\frac{\text{Abs}[L[[u]]] - 1}{2}$ }],
        Table[X[s + x + Abs[L[[u]]] + 1 - 2 k, x + 1 + 2 k,
          s + x + Abs[L[[u]]] + 2 - 2 k, x + 2 k], {k, 1,  $\frac{\text{Abs}[L[[u]]] - 1}{2}$ }]],
      Join[Table[X[s + x + Abs[L[[u]]] - 2 k, x + 1 + 2 k,
        s + x + Abs[L[[u]]] + 1 - 2 k, x + 2 + 2 k], {k, 0,  $\frac{\text{Abs}[L[[u]]] - 1}{2}$ }],
        Table[X[x + 2 k, s + x + Abs[L[[u]]] + 1 - 2 k,
          x + 1 + 2 k, s + x + Abs[L[[u]]] + 2 - 2 k], {k, 1,  $\frac{\text{Abs}[L[[u]]] - 1}{2}$ }]],
      ],
    ],
    {u, Length[L]}]
]

```

In[*]:= **OddPretzel**[{1, 1, -1}]

Out[*]=

PD[X[1, 5, 2, 4], X[5, 3, 4, 2], X[6, 3, 7, 4]]

In[*]:= **Rot@OddPretzel**[{1, 1, -1}]

Out[*]=

{{{1, 4, 1}, {1, 2, 5}, {-1, 3, 6}}, {0, 0, 0, 1, 0, 0, 0}}

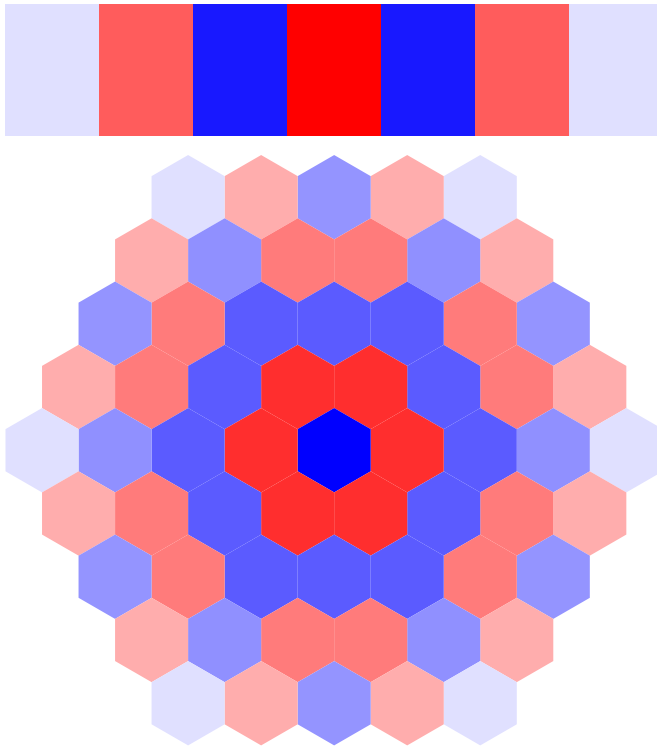
In[*]:= **Θ@OddPretzel**[{-1, -1, -1}]

Out[*]=

$$\left\{ \frac{1 - T + T^2}{T}, -\frac{1 - T_1 + T_1^2 - T_2 - T_1^3 T_2 + T_2^2 + T_1^4 T_2^2 - T_1 T_2^3 - T_1^4 T_2^3 + T_1^2 T_2^4 - T_1^3 T_2^4 + T_1^4 T_2^4}{T_1^2 T_2^2} \right\}$$

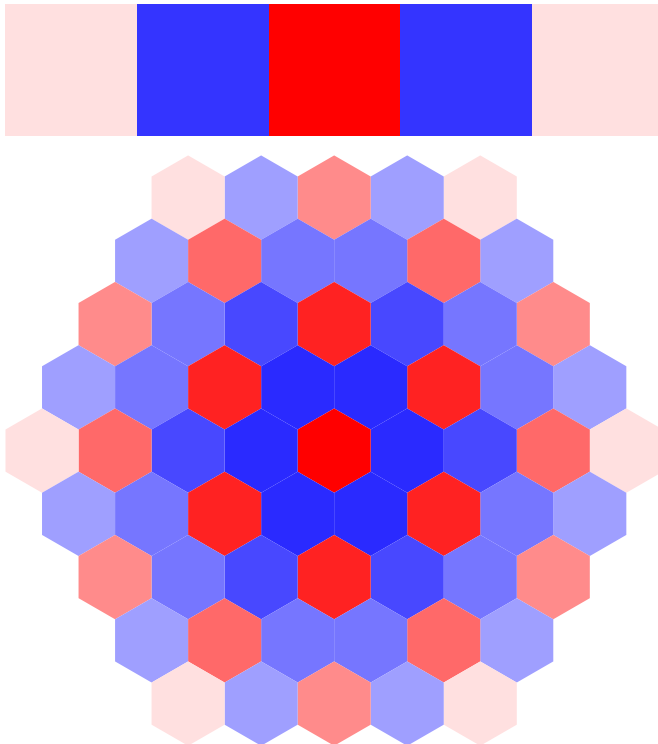
In[*]:= **PolyPlot**[**Θ@OddPretzel**[{1, 1, 1, 1, -3, -3, -3}]]

Out[*]=



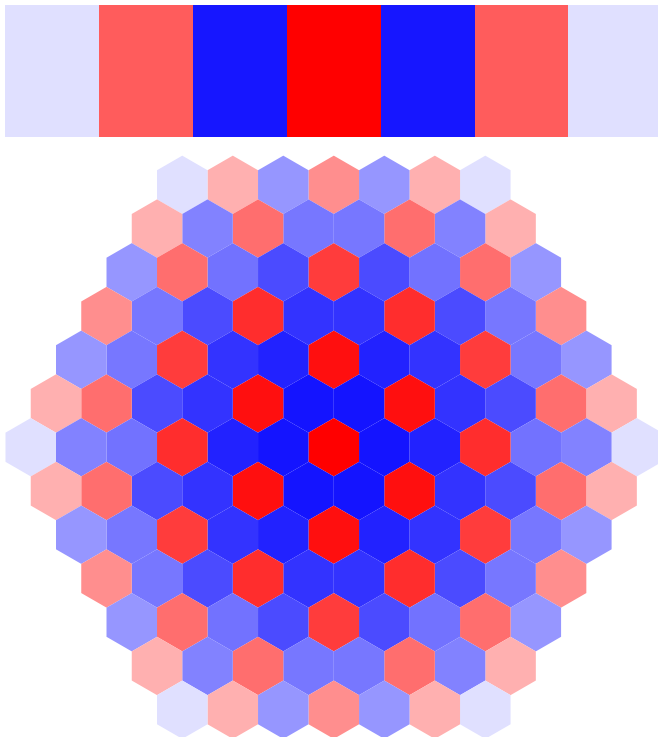
```
In[*]:= PolyPlot[OddPretzel[{11, 1, -3, 5, -3}]]
```

```
Out[*]=
```



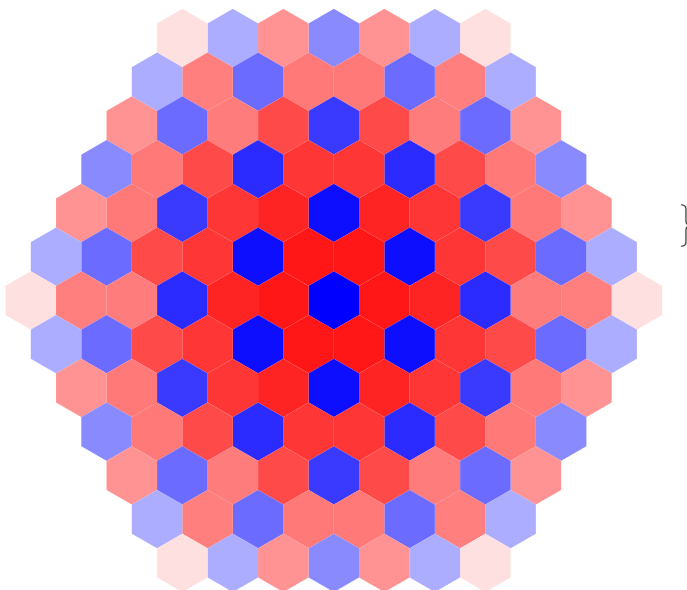
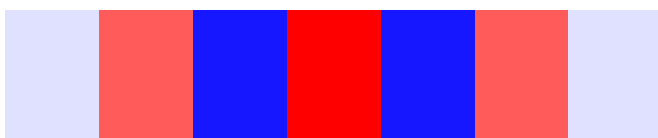
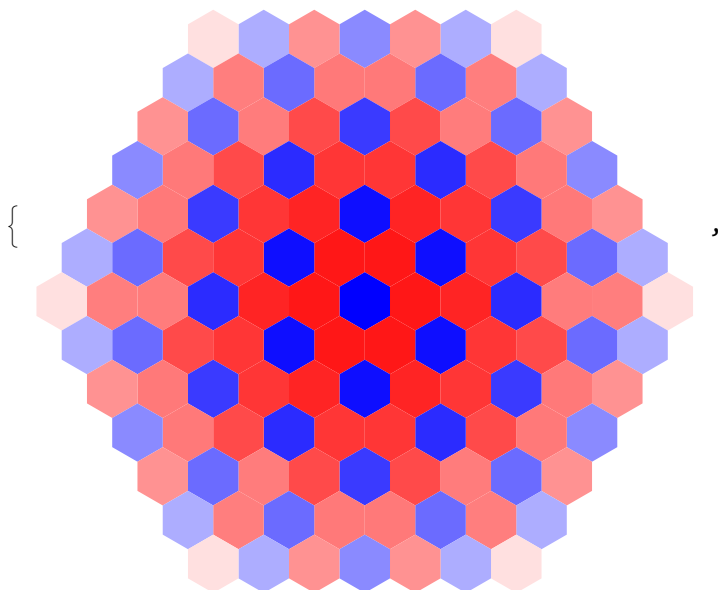
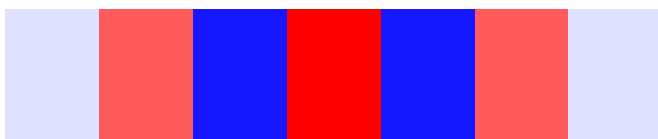
```
In[*]:= PolyPlot[OddPretzel[{5, -3, -7, 5, -3, 3, 9}]]
```

```
Out[*]=
```



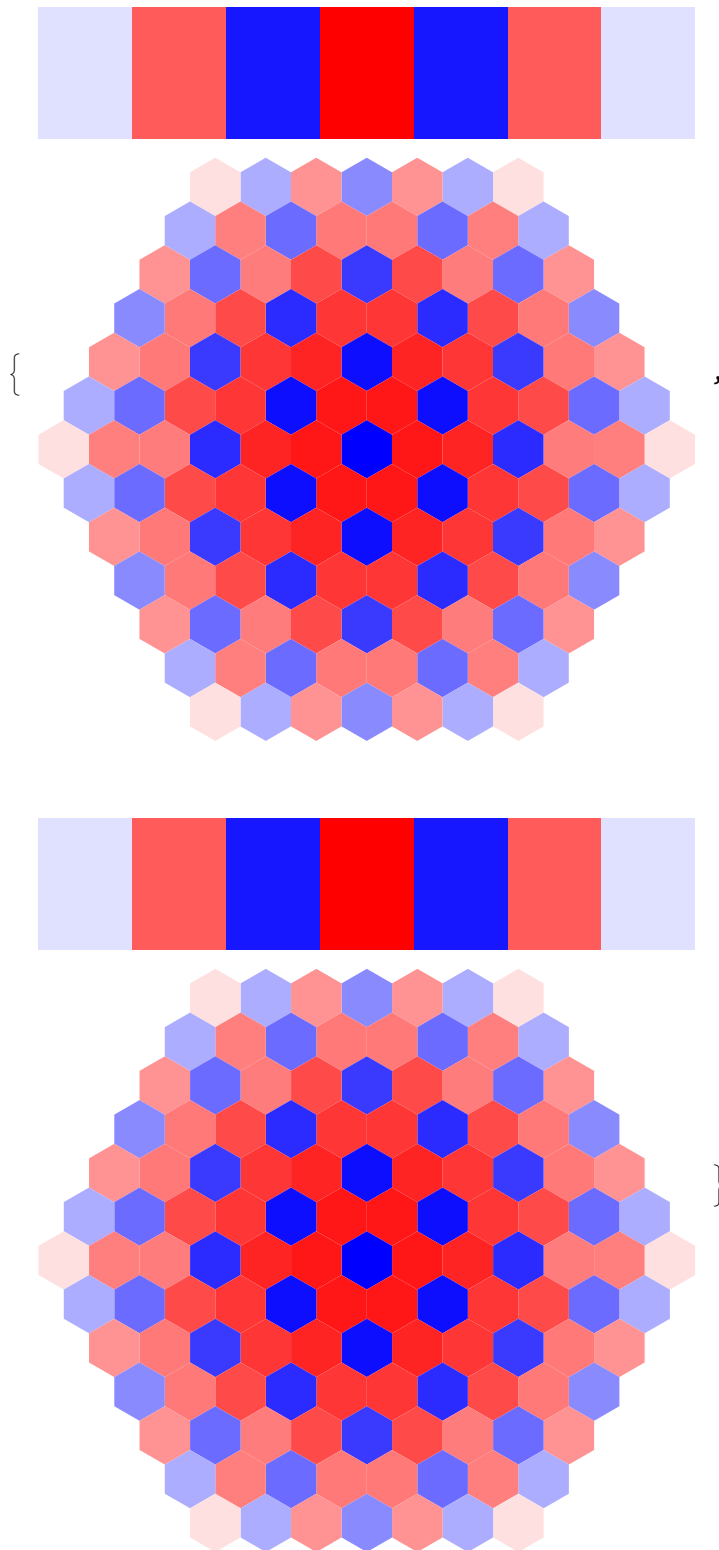
```
In[*]:= {PolyPlot[OddPretzel[{5, -3, -3, -5, -3, 3, 9}]]
, PolyPlot[OddPretzel[{5, -3, -3, 9, -3, 3, -5}]]}
```

```
Out[*]=
```



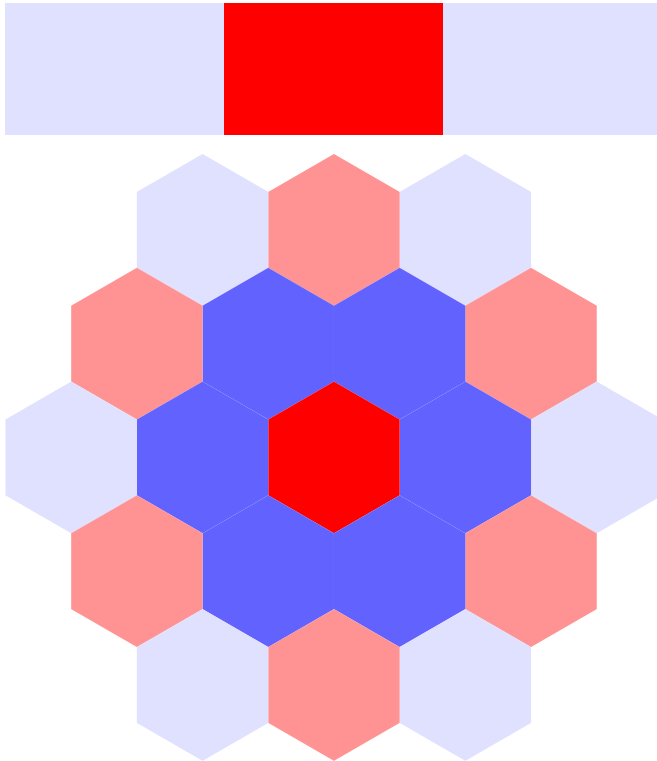
```
In[*]:= {PolyPlot[OddPretzel[{-3, 5, -3, -5, -3, 3, 9}]]
, PolyPlot[OddPretzel[{-3, 5, -5, 9, -3, 3, -3}]]}
```

```
Out[*]=
```



```
In[ ]:= PolyPlot[OddPretzel[{-3, 3, 7}]]
```

```
Out[ ]:=
```



```
In[ ]:= (*L should be a list L={L0,L1,..,Ln} where L0 and n≥0 are even and the Li, i>0 are odd integers.*)
```

```
EOddPretzel[LL_] := Module[{L = Rest@LL, s, x, y, n},
  s = 2 Abs[LL[[1]]] + Total[Abs /@ L]; n = Length[L];
  PD@@Flatten@
  Join[
    If[LL[[1]] > 0,
      Join[Table[X[1 + 2 k, s + 1 - 2 k, 2 + 2 k, s - 2 k], {k, 0,  $\frac{\text{Abs}[LL[[1]]]}{2} - 1$ }},
        Table[X[s + 1 - 2 k, 1 + 2 k, s + 2 - 2 k, 2 k], {k, 1,  $\frac{\text{Abs}[LL[[1]]]}{2}$ }}],
      Join[Table[X[s - 2 k, 1 + 2 k, s + 1 - 2 k, 2 + 2 k], {k, 0,  $\frac{\text{Abs}[LL[[1]]]}{2} - 1$ }},
        Table[X[2 k, s + 1 - 2 k, 1 + 2 k, s + 2 - 2 k], {k, 1,  $\frac{\text{Abs}[LL[[1]]]}{2}$ }}],
    ],
  Table[x = Sum[Abs[L[[j]]], {j, u + 1, n}];
  y = Abs[LL[[1]]] + Sum[Abs[L[[j]]], {j, 1, u - 1}];
```

```

If[EvenQ[u],
  If[L[[u]] > 0,
    Join[
      Table[X[s + x + 1 + 2 k, y + 1 + 2 k, s + x + 2 + 2 k, y + 2 + 2 k], {k, 0,  $\frac{\text{Abs}[L[[u]]] - 1}{2}$ }],
      Table[X[y + 2 k, s + x + 2 k, y + 1 + 2 k, s + x + 1 + 2 k], {k, 1,  $\frac{\text{Abs}[L[[u]]] - 1}{2}$ }]],
    Join[
      Table[X[y + 1 + 2 k, s + x + 2 + 2 k, y + 2 + 2 k, s + x + 1 + 2 k], {k, 0,  $\frac{\text{Abs}[L[[u]]] - 1}{2}$ }],
      Table[X[s + x + 2 k, y + 1 + 2 k, s + x + 1 + 2 k, y + 2 k], {k, 1,  $\frac{\text{Abs}[L[[u]]] - 1}{2}$ }]]],
  ,
  If[L[[u]] > 0,
    Join[
      Table[X[y + 1 + 2 k, s + x + 1 + 2 k, y + 2 + 2 k, s + x + 2 + 2 k], {k, 0,  $\frac{\text{Abs}[L[[u]]] - 1}{2}$ }],
      Table[X[s + x + 2 k, y + 2 k, s + x + 1 + 2 k, y + 1 + 2 k], {k, 1,  $\frac{\text{Abs}[L[[u]]] - 1}{2}$ }]],
    Join[
      Table[X[s + x + 1 + 2 k, y + 2 + 2 k, s + x + 2 + 2 k, y + 1 + 2 k], {k, 0,  $\frac{\text{Abs}[L[[u]]] - 1}{2}$ }],
      Table[X[y + 2 k, s + x + 1 + 2 k, y + 1 + 2 k, s + x + 2 k], {k, 1,  $\frac{\text{Abs}[L[[u]]] - 1}{2}$ }]]],
  ]
, {u, Length[L]}]
]

```

In[]:= EOddPretzel[{0, 1, 1, 1, 3}]

Out[]:=

PD[X[1, 12, 2, 13], X[11, 2, 12, 3],
X[3, 10, 4, 11], X[7, 4, 8, 5], X[9, 6, 10, 7], X[5, 8, 6, 9]]

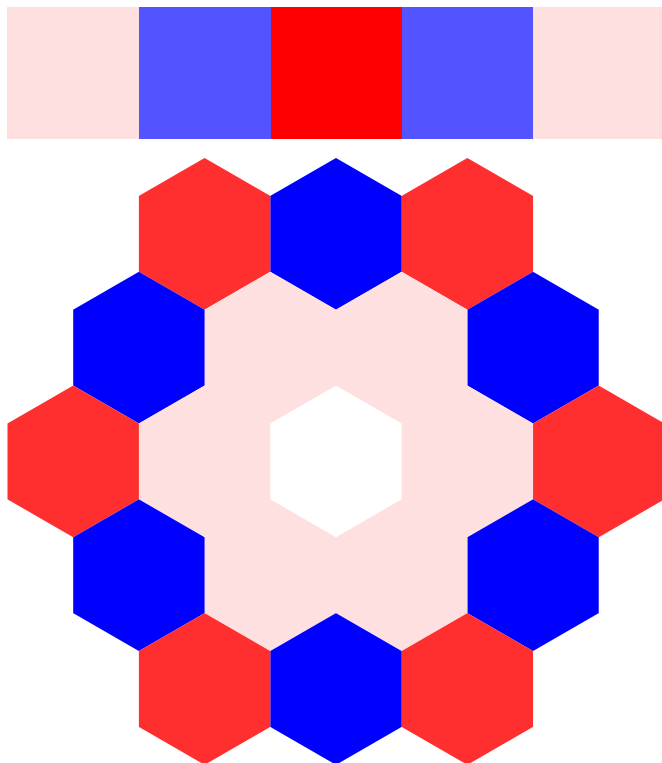
In[]:= @@EOddPretzel[{0, 1, 1, 1, 3}]

Out[]:=

$$\left\{ \frac{1 - T + T^2}{T}, -\frac{1 - T_1 + T_1^2 - T_2 - T_1^3 T_2 + T_2^2 + T_1^4 T_2^2 - T_1 T_2^3 - T_1^4 T_2^3 + T_1^2 T_2^4 - T_1^3 T_2^4 + T_1^4 T_2^4}{T_1^2 T_2^2} \right\}$$

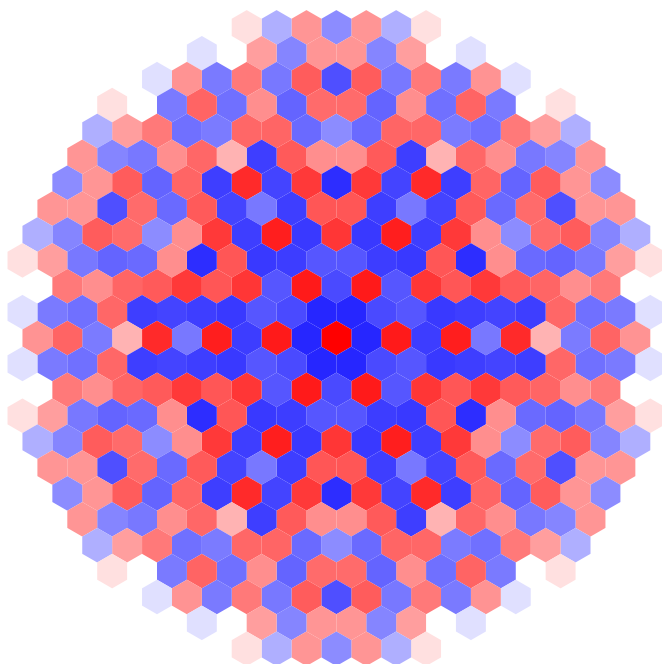
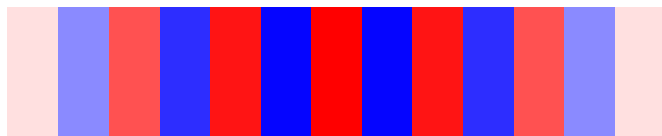

```
In[ ]:= PolyPlot[ $\Theta$ @EOddPretzel[{2, 1, 3, -1, -3}]]
```

```
Out[ ]:=
```



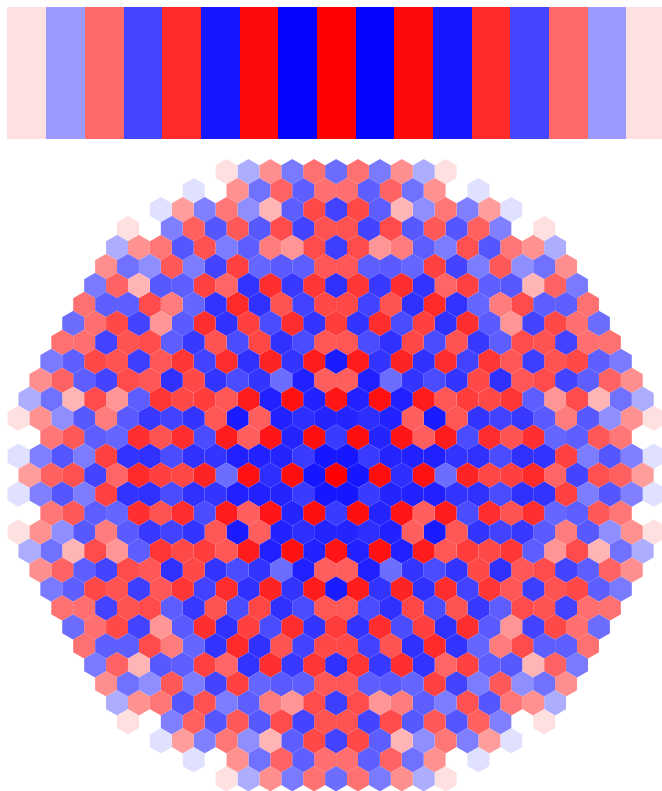
```
In[ ]:= PolyPlot[Theta@EOddPretzel[{4, 5, 3, -5, -3}]]
```

```
Out[ ]:=
```



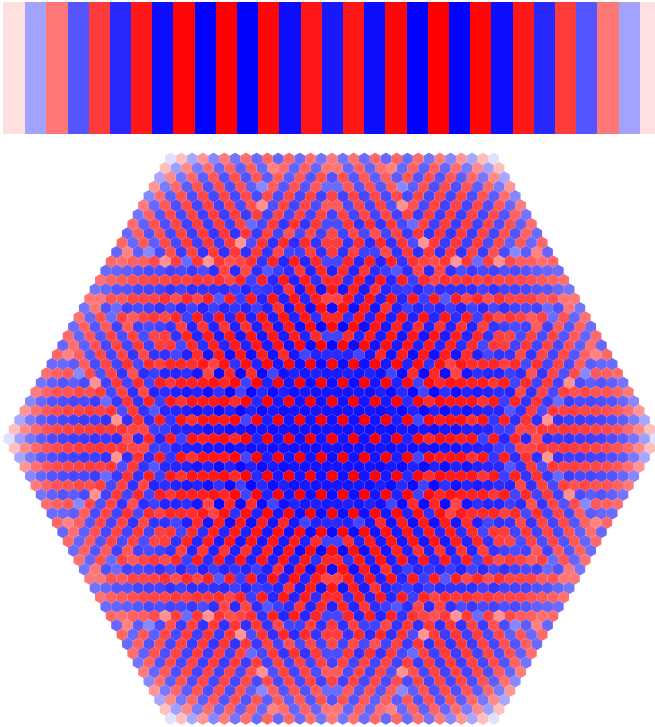
```
In[ ]:= PolyPlot[ $\Theta$ @EOddPretzel[{6, -3, 5, 3, 3, -5, -3}]]
```

```
Out[ ]:=
```



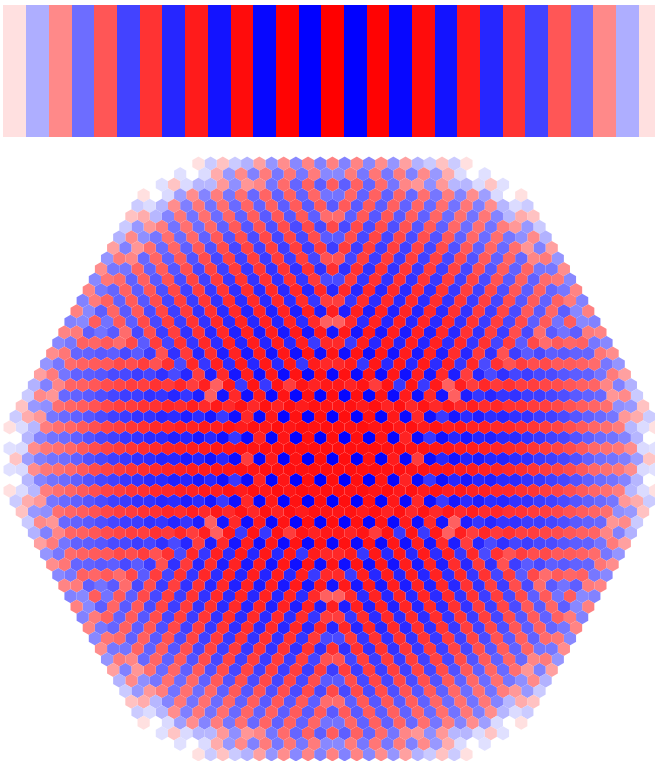
```
In[ ]:= PolyPlot[ $\Theta$ @EOddPretzel[{-6, 7, -3, 5, 3, 7, 3, -5, -3}]]
```

Out[]=

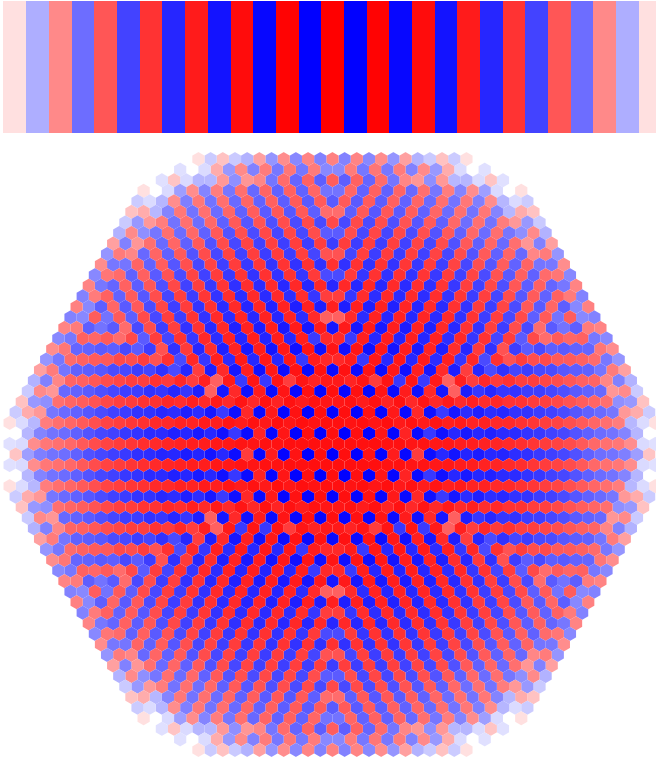


```
In[ ]:= PolyPlot[ $\Theta$ @EOddPretzel[{-6, 7, -3, 5, 3, -7, 3, -5, -3}]]
```

Out[]=

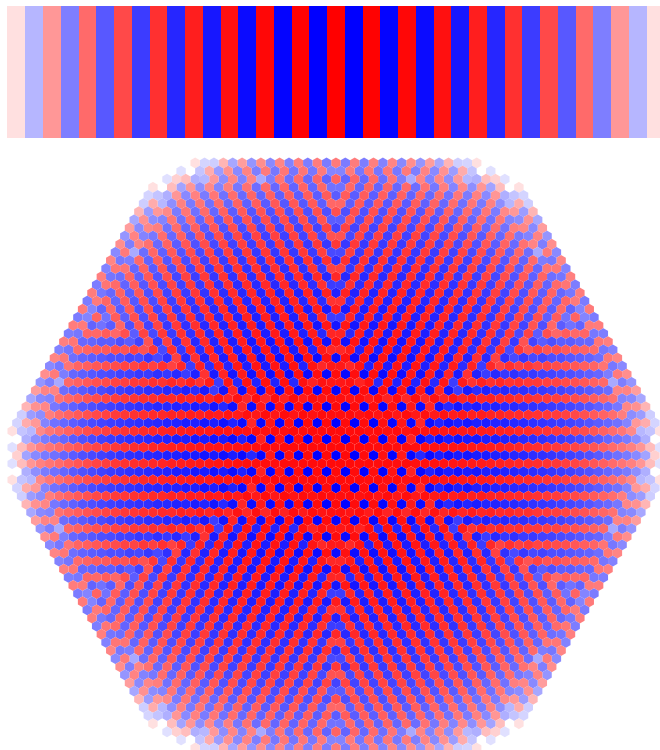


```
In[ ]:= PolyPlot[Theta@EOddPretzel[{-6, 7, -3, 5, 3, -1, -7, 3, -5, -3, 1}]]  
Out[ ]=
```



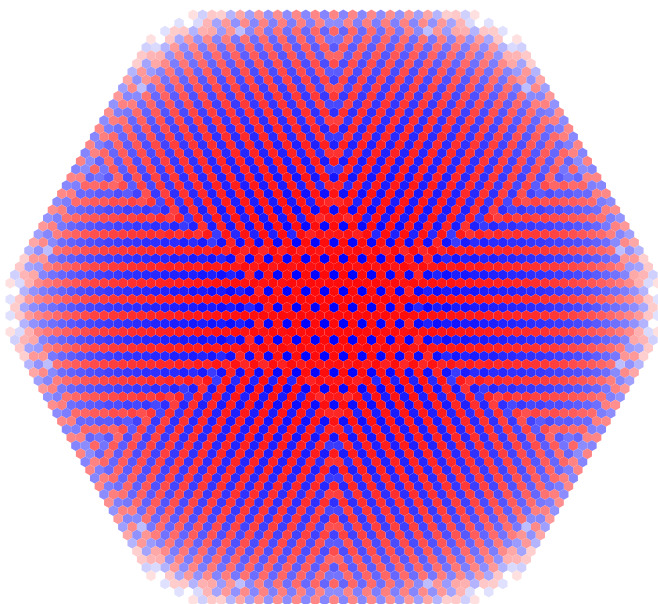
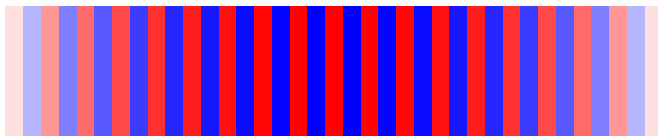
```
In[ ]:= PolyPlot[ $\Theta$ @EOddPretzel[{-6, 7, -3, 5, 3, -1, -5, -7, 3, -5, -3, 1, 5}]]
```

```
Out[ ]:=
```



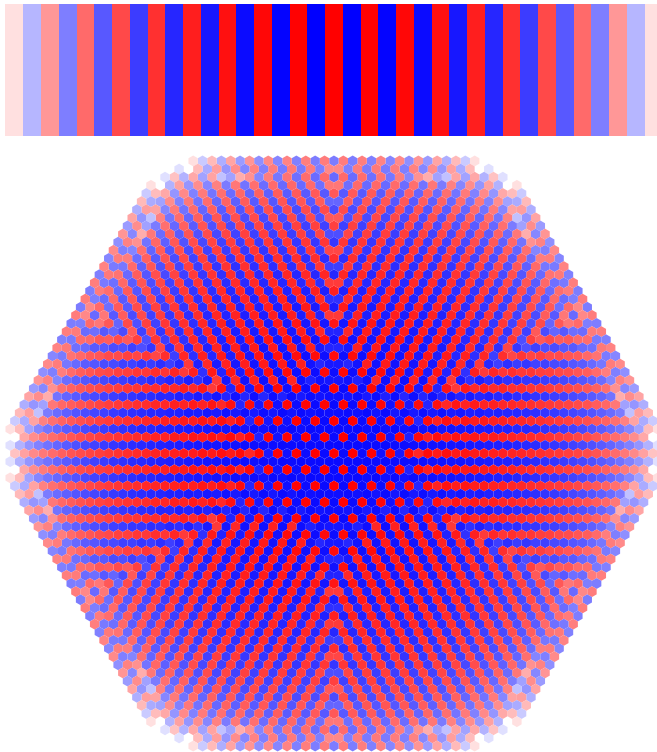
```
In[ ]:= PolyPlot[ $\Theta$ @EOddPretzel[{-4, 7, -3, 5, 3, -1, -5, -7, 3, -5, -3, 1, 5}]]
```

```
Out[ ]=
```



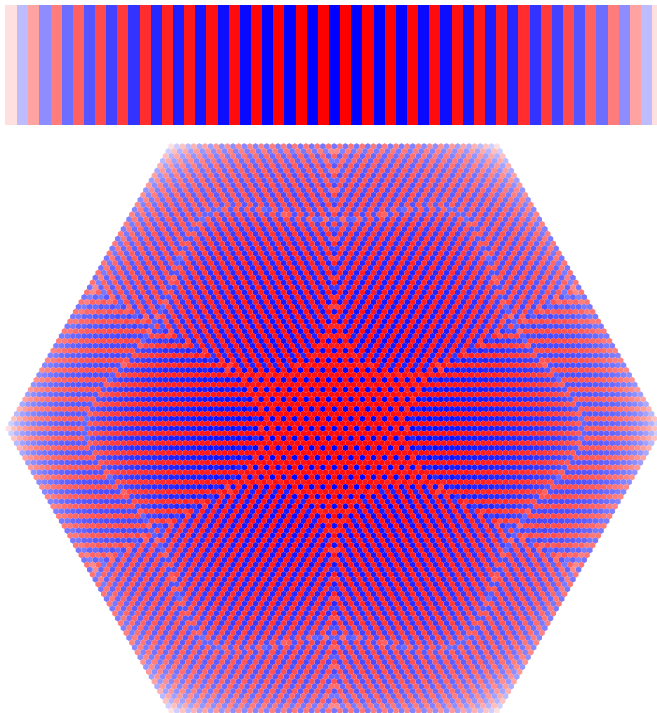
```
In[ ]:= PolyPlot[ $\Theta$ @EOddPretzel[{2, 7, -3, 5, 3, -1, -5, -7, 3, -5, -3, 1, 5}]]
```

Out[]=



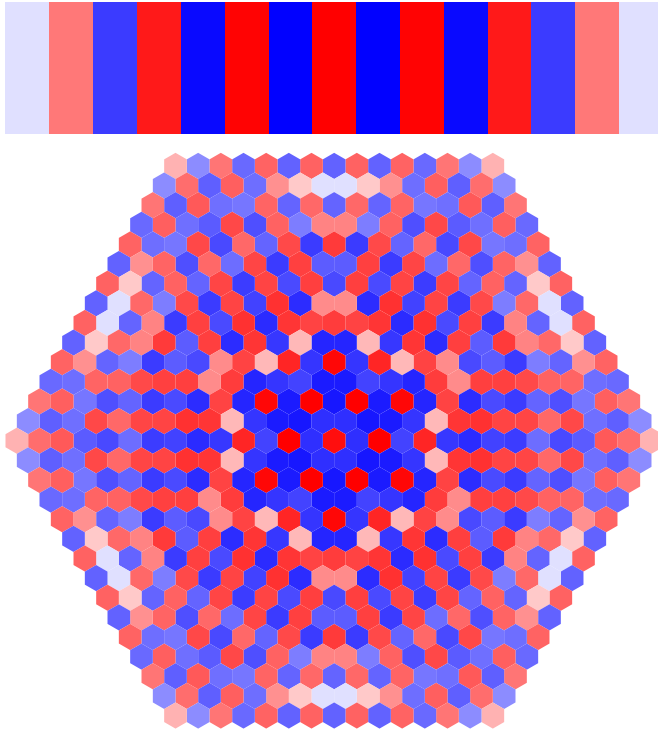
```
In[ ]:= PolyPlot[ $\Theta$ @EOddPretzel[{2, 7, -3, 5, 13, -1, -5, -7, -13, -5, -3, 1, 5}]]
```

Out[]=



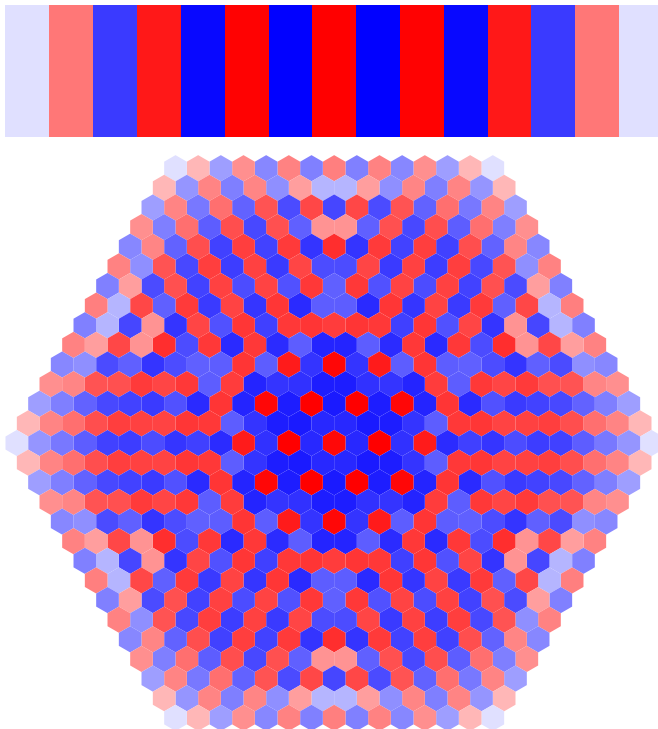

```
In[ ]:= PolyPlot[ $\Theta$ @EOddPretzel[{28, 3, -1, 5, -3, 1, 5}]]
```

```
Out[ ]:=
```



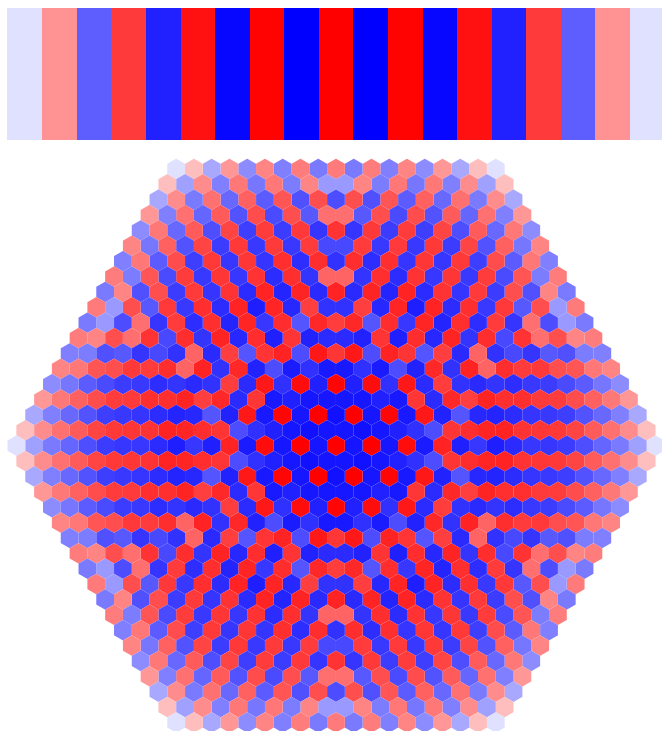
```
In[ ]:= PolyPlot[ $\Theta$ @EOddPretzel[{38, 3, -1, 5, -3, 1, 5}]]
```

```
Out[ ]:=
```



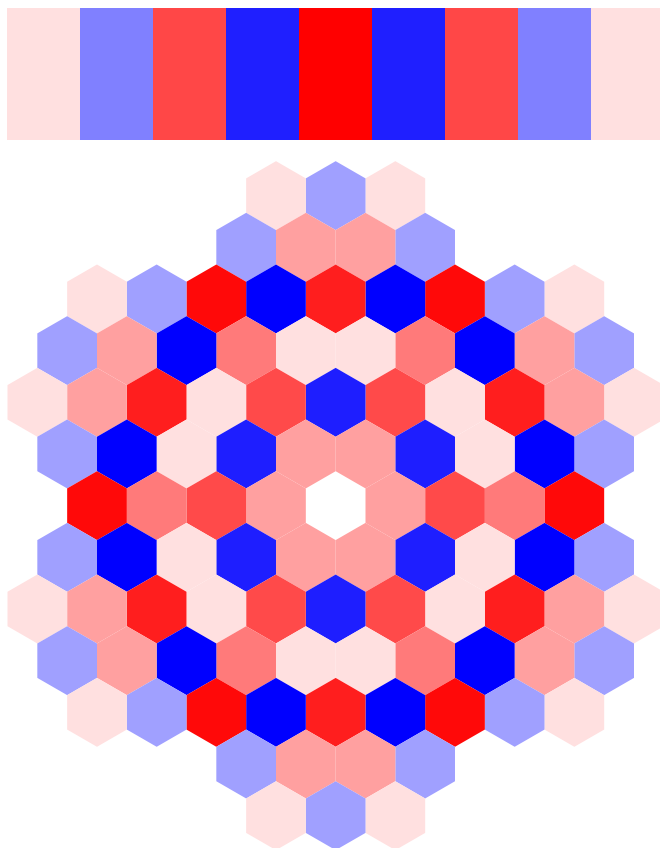
```
In[ ]:= PolyPlot[ $\Theta$ @EOddPretzel[{38, 3, 3, -1, 5, -3, -3, 1, 5}]]
```

```
Out[ ]=
```



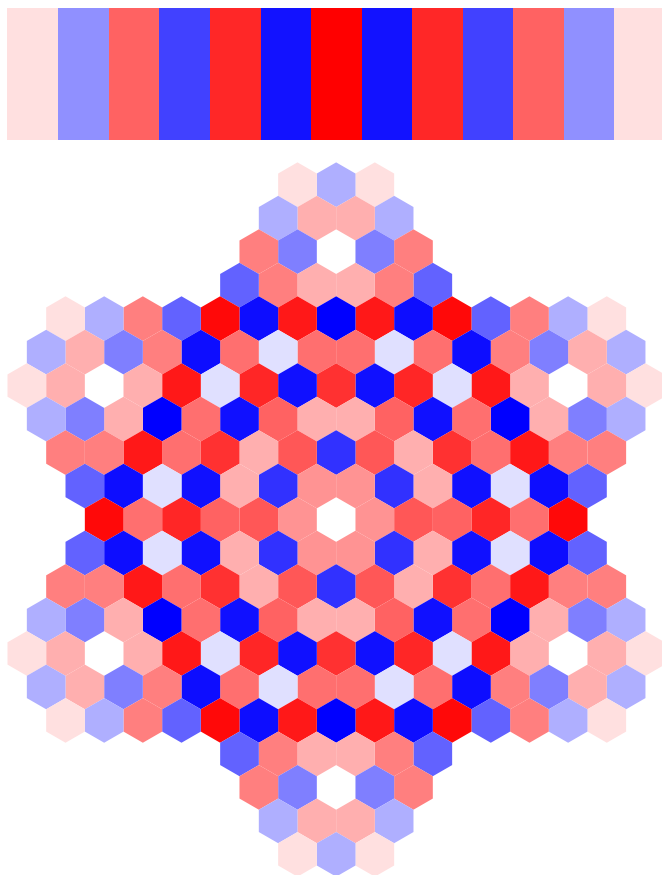
```
In[ ]:= PolyPlot[ $\Theta$ @EOddPretzel[{2, 5, -5}]]
```

```
Out[ ]:=
```



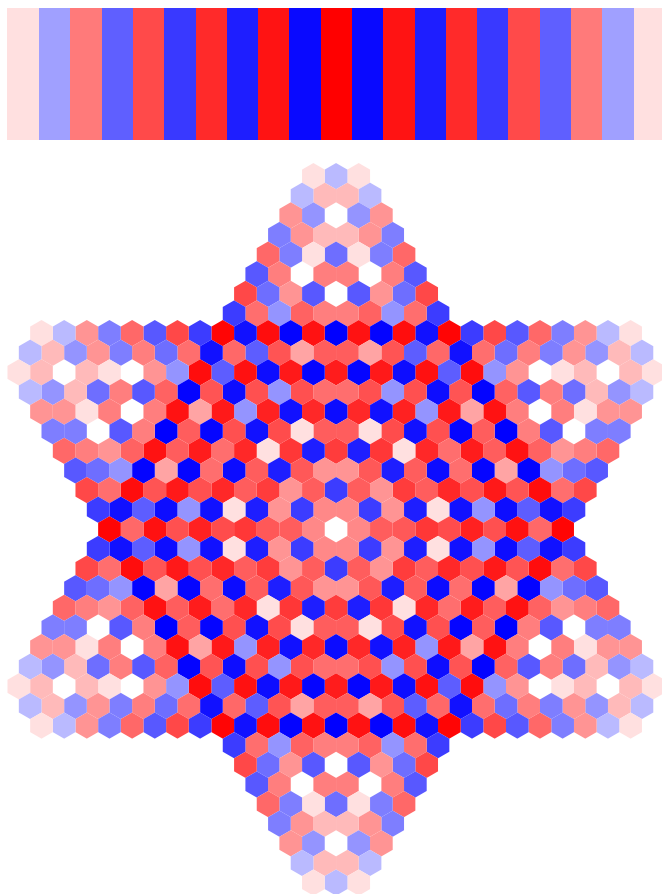
```
In[ ]:= PolyPlot[ $\Theta$ @EOddPretzel[{8, 7, -7}]]
```

```
Out[ ]:=
```



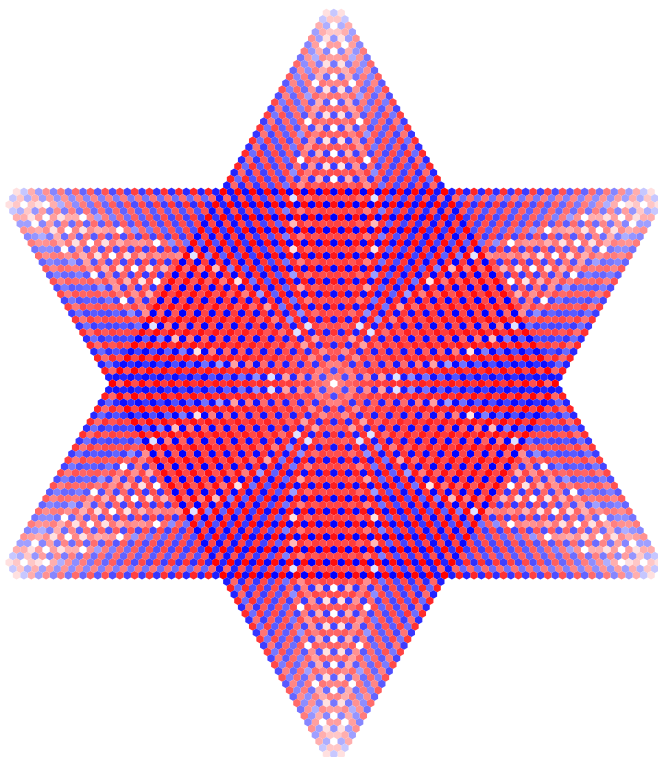
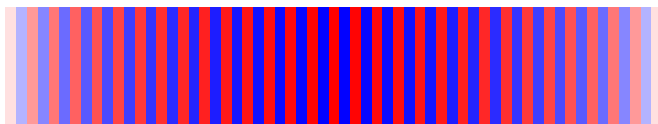
```
In[ ]:= PolyPlot[ $\Theta$ @EOddPretzel[{10, 11, -11}]]
```

```
Out[ ]:=
```



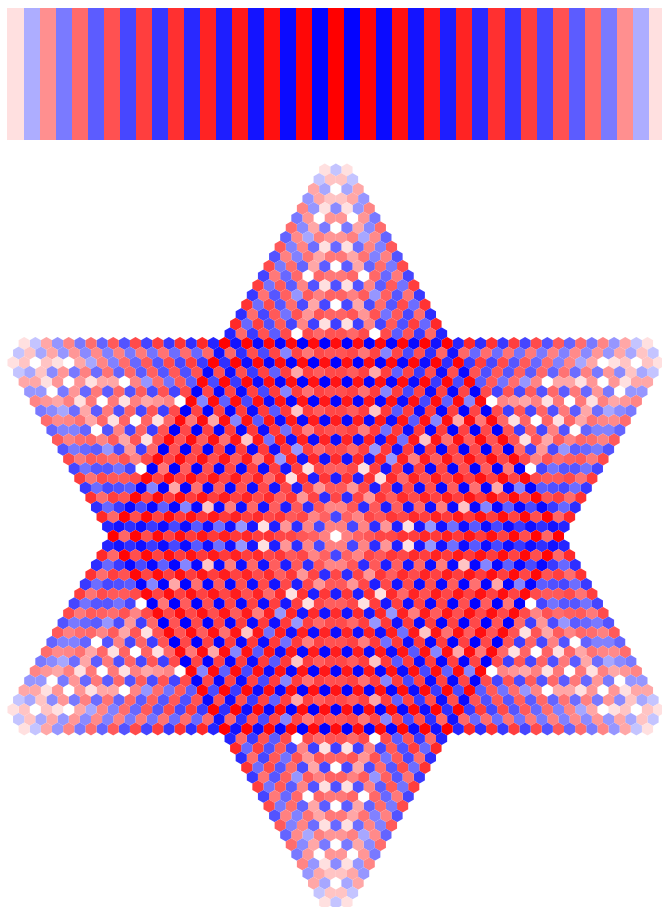
```
In[ ]:= PolyPlot[ $\Theta$ @EOddPretzel[{20, 31, -31}]]
```

```
Out[ ]:=
```



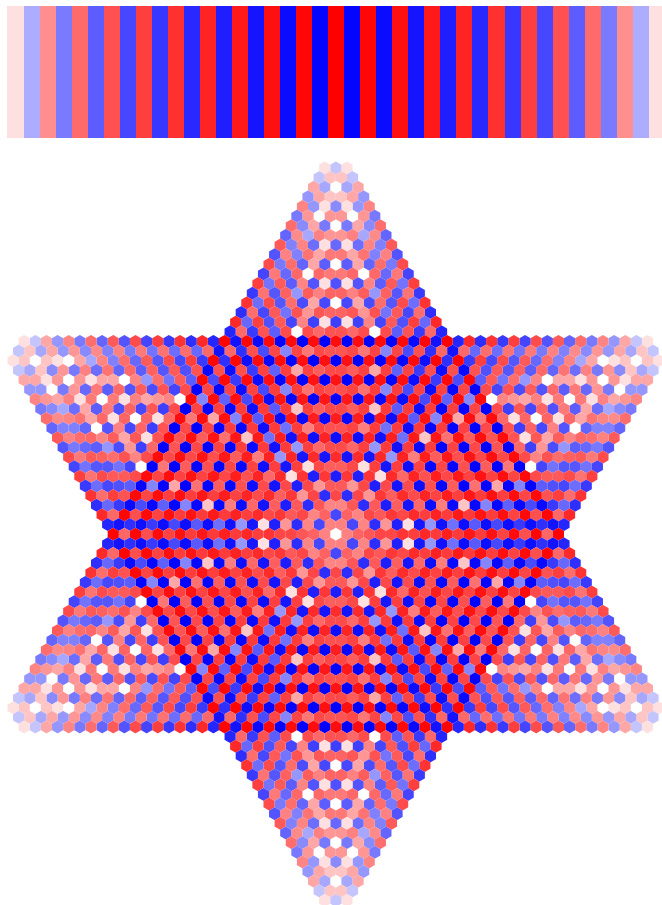
```
In[ ]:= PolyPlot[ $\Theta$ @EOddPretzel[{20, 21, -21}]]
```

```
Out[ ]=
```



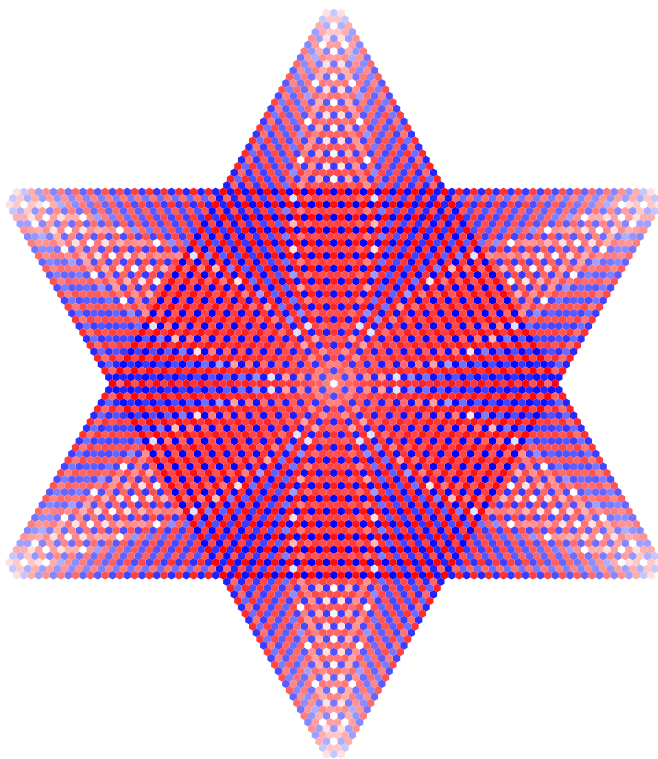
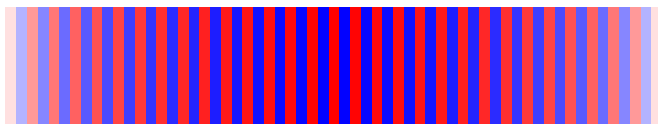
```
In[ ]:= PolyPlot[ $\Theta$ @EOddPretzel[{2, 21, -21}]]
```

```
Out[ ]=
```




```
In[ ]:= PolyPlot[ $\Theta$ @EOddPretzel[{2, 31, -31}]]
```

```
Out[ ]:=
```



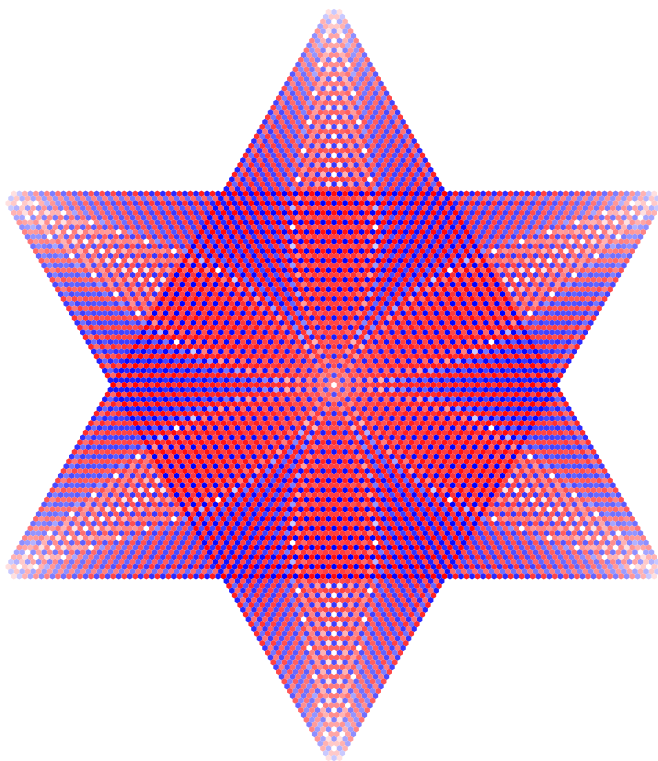
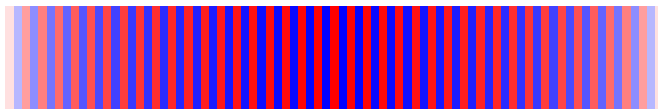
```
In[ ]:= Export["figs/Pretzel2.31.-31.pdf", PolyPlot2[Last@ $\Theta$ @EOddPretzel[{2, 31, -31}]]]
```

```
Out[ ]:=
```

figs/Pretzel2.31.-31.pdf

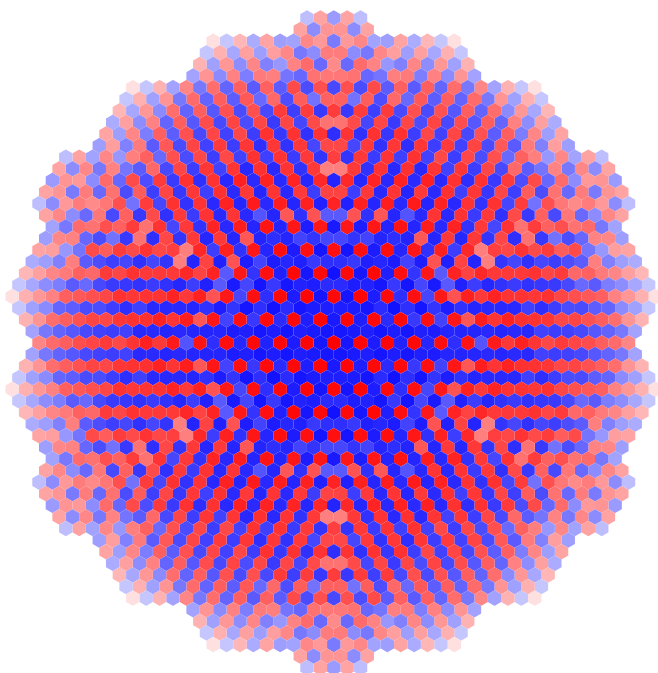
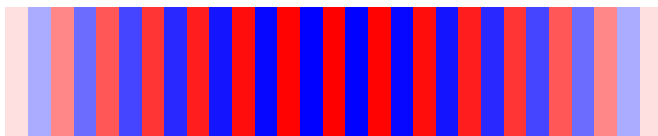
```
In[ ]:= PolyPlot[Theta@EOddPretzel[{2, 41, -41}]]
```

```
Out[ ]:=
```



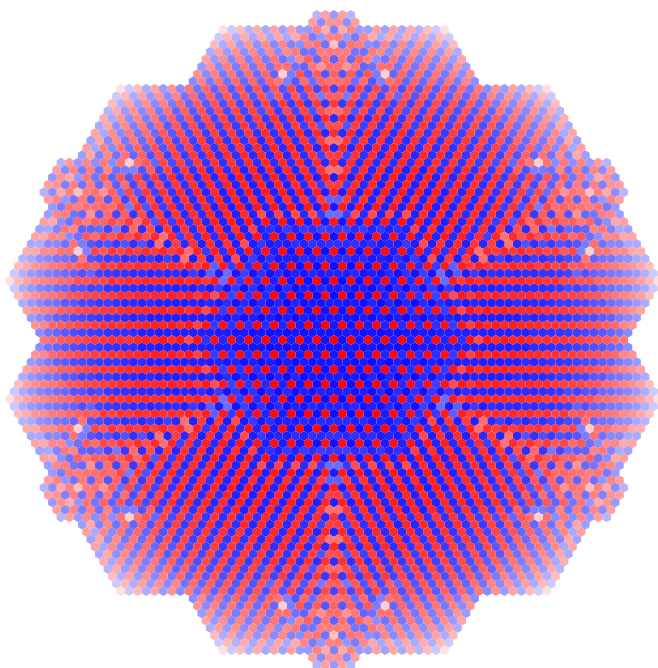
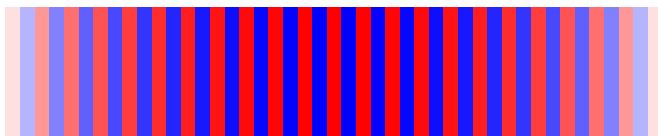
```
In[ ]:= PolyPlot[ $\Theta$ @EOddPretzel[{8, 7, -9, -7, 9}]]
```

```
Out[ ]:=
```



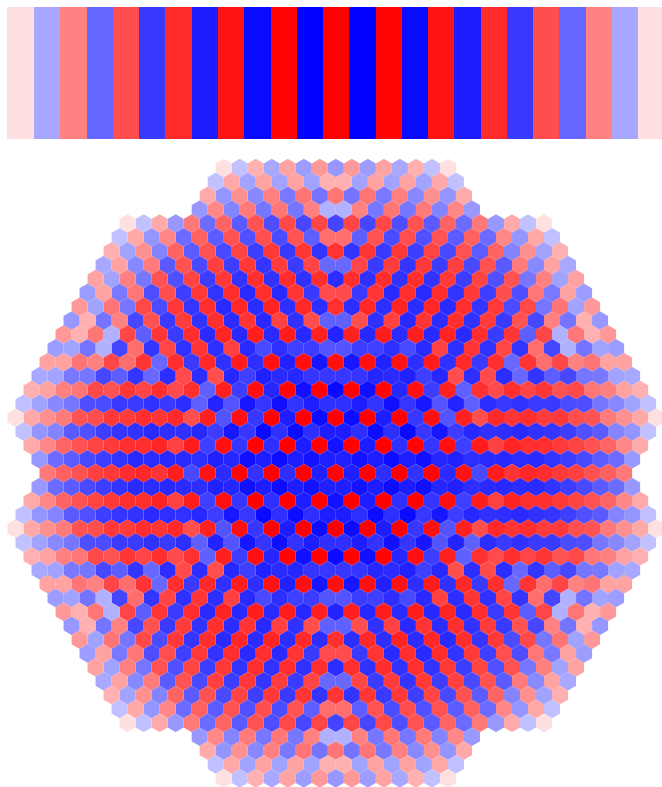
```
In[ ]:= PolyPlot[ $\Theta$ @EOddPretzel[{12, 13, -11, -13, 11}]]
```

```
Out[ ]=
```



```
In[ ]:= PolyPlot[ $\Theta$ @EOddPretzel[{12, 7, -7, -7, 7}]]
```

```
Out[ ]:=
```



```
In[ ]:= PolyPlot[ $\Theta$ @EOddPretzel[{12, 9, -9, -9, 11}]]
```

```
Out[ ]:=
```

