

Pensieve header: Playing with fibered knots.

```
In[ ]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\Theta"];
<< Theta.m
```

Loading the fibered-knots data obtained from <https://knotinfo.org/>:

```
In[ ]:= FiberedKnots = Cases [
  Import [
    "C:\\drorbn\\AcademicPensieve\\Projects\\Theta\\FiberedInfoFromKnotInfo.org.csv"],
  {K_, "Y"} :> Knot@K,
  ∞]
```

Out[]=



```
In[ ]:= MemberQ[Knot[12, NonAlternating, 57], FiberedKnots]
```

Out[]=

False

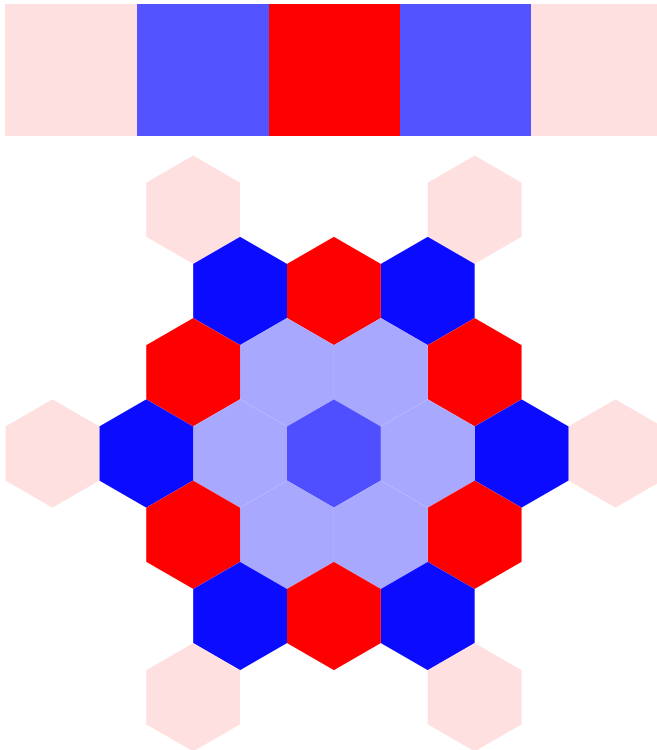
```
In[ ]:= @Knot[12, NonAlternating, 57]
```

Out[]=

$$\left\{ \frac{(1 - T + T^2)^2}{T^2}, \right. \\ \frac{1}{T_1^3 T_2^3} \left(2 (1 + T_1^3 - 14 T_1 T_2 + 16 T_1^2 T_2 - 14 T_1^3 T_2 + 16 T_1 T_2^2 - 2 T_1^2 T_2^2 - 2 T_1^3 T_2^2 + 16 T_1^4 T_2^2 + T_2^3 - 14 T_1 T_2^3 - \right. \\ \left. 2 T_1^2 T_2^3 - 6 T_1^3 T_2^3 - 2 T_1^4 T_2^3 - 14 T_1^5 T_2^3 + T_1^6 T_2^3 + 16 T_1^2 T_2^4 - 2 T_1^3 T_2^4 - \right. \\ \left. 2 T_1^4 T_2^4 + 16 T_1^5 T_2^4 - 14 T_1^3 T_2^5 + 16 T_1^4 T_2^5 - 14 T_1^5 T_2^5 + T_1^3 T_2^6 + T_1^6 T_2^6) \right\}$$

```
In[ ]:= PolyPlot@@@Knot[12, NonAlternating, 57]
```

```
Out[ ]:=
```



```
In[ ]:= Length@FiberedKnots
```

```
Out[ ]:=
```

```
5397
```

```
In[ ]:= Length@AllKnots[{3, 13}]
```

```
Out[ ]:=
```

```
12965
```

```
In[ ]:= K = Last[FiberedKnots]
```

```
Out[ ]:=
```

```
Knot[13, NonAlternating, 5106]
```

In[*]:= $\Theta[K]$

*** KnotTheory: Loading precomputed data in KnotTheory/13N.dts.

*** KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

Out[*]=

$$\left\{ \frac{(1 - T + T^2)^2 (1 + 3T - 7T^2 + 3T^3 + T^4)}{T^4}, \right. \\ \frac{1}{T_1^8 T_2^8} (1 - T_1 + T_1^2) (1 - T_2 + T_2^2) (1 - T_1 T_2 + T_1^2 T_2^2) (4 + 8T_1 - 36T_1^2 + 52T_1^3 - 36T_1^4 + 8T_1^5 + 4T_1^6 + 8T_2 + \\ 24T_1 T_2 - 56T_1^2 T_2 + 32T_1^3 T_2 + 32T_1^4 T_2 - 56T_1^5 T_2 + 24T_1^6 T_2 + 8T_1^7 T_2 - 36T_2^2 - 56T_1 T_2^2 + 305T_1^2 T_2^2 - \\ 586T_1^3 T_2^2 + 697T_1^4 T_2^2 - 586T_1^5 T_2^2 + 305T_1^6 T_2^2 - 56T_1^7 T_2^2 - 36T_1^8 T_2^2 + 52T_2^3 + 32T_1 T_2^3 - 586T_1^2 T_2^3 + \\ 922T_1^3 T_2^3 - 450T_1^4 T_2^3 - 450T_1^5 T_2^3 + 922T_1^6 T_2^3 - 586T_1^7 T_2^3 + 32T_1^8 T_2^3 + 52T_1^9 T_2^3 - 36T_2^4 + 32T_1 T_2^4 + \\ 697T_1^2 T_2^4 - 450T_1^3 T_2^4 - 875T_1^4 T_2^4 + 1697T_1^5 T_2^4 - 875T_1^6 T_2^4 - 450T_1^7 T_2^4 + 697T_1^8 T_2^4 + 32T_1^9 T_2^4 - \\ 36T_1^{10} T_2^4 + 8T_2^5 - 56T_1 T_2^5 - 586T_1^2 T_2^5 - 450T_1^3 T_2^5 + 1697T_1^4 T_2^5 - 1064T_1^5 T_2^5 - 1064T_1^6 T_2^5 + 1697T_1^7 T_2^5 - \\ 450T_1^8 T_2^5 - 586T_1^9 T_2^5 - 56T_1^{10} T_2^5 + 8T_1^{11} T_2^5 + 4T_2^6 + 24T_1 T_2^6 + 305T_1^2 T_2^6 + 922T_1^3 T_2^6 - 875T_1^4 T_2^6 - \\ 1064T_1^5 T_2^6 + 2484T_1^6 T_2^6 - 1064T_1^7 T_2^6 - 875T_1^8 T_2^6 + 922T_1^9 T_2^6 + 305T_1^{10} T_2^6 + 24T_1^{11} T_2^6 + 4T_1^{12} T_2^6 + \\ 8T_1 T_2^7 - 56T_1^2 T_2^7 - 586T_1^3 T_2^7 - 450T_1^4 T_2^7 + 1697T_1^5 T_2^7 - 1064T_1^6 T_2^7 - 1064T_1^7 T_2^7 + 1697T_1^8 T_2^7 - \\ 450T_1^9 T_2^7 - 586T_1^{10} T_2^7 - 56T_1^{11} T_2^7 + 8T_1^{12} T_2^7 - 36T_2^8 + 32T_1 T_2^8 + 697T_1^2 T_2^8 - 450T_1^3 T_2^8 - \\ 875T_1^4 T_2^8 + 1697T_1^5 T_2^8 - 875T_1^6 T_2^8 - 450T_1^7 T_2^8 + 697T_1^8 T_2^8 + 32T_1^9 T_2^8 - 36T_1^{10} T_2^8 + 52T_1^{11} T_2^8 + \\ 32T_1^{12} T_2^8 - 586T_1^5 T_2^9 + 922T_1^6 T_2^9 - 450T_1^7 T_2^9 - 450T_1^8 T_2^9 + 922T_1^9 T_2^9 - 586T_1^{10} T_2^9 + 32T_1^{11} T_2^9 + \\ 52T_1^{12} T_2^9 - 36T_1^4 T_2^{10} - 56T_1^5 T_2^{10} + 305T_1^6 T_2^{10} - 586T_1^7 T_2^{10} + 697T_1^8 T_2^{10} - 586T_1^9 T_2^{10} + 305T_1^{10} T_2^{10} - \\ 56T_1^{11} T_2^{10} - 36T_1^{12} T_2^{10} + 8T_1^5 T_2^{11} + 24T_1^6 T_2^{11} - 56T_1^7 T_2^{11} + 32T_1^8 T_2^{11} + 32T_1^9 T_2^{11} - 56T_1^{10} T_2^{11} + \\ 24T_1^{11} T_2^{11} + 8T_1^{12} T_2^{11} + 4T_1^6 T_2^{12} + 8T_1^7 T_2^{12} - 36T_1^8 T_2^{12} + 52T_1^9 T_2^{12} - 36T_1^{10} T_2^{12} + 8T_1^{11} T_2^{12} + 4T_1^{12} T_2^{12} \Big) \Big\}$$

In[*]:= $d = \text{Exponent}[\Theta[K][[1]], T]$

Out[*]=

4

In[*]:= $\text{Factor}\left[\frac{\text{Coefficient}[\Theta[K][[2]], T_1^{2d}]}{\Theta[K][[1]] T^d}\right]$

Out[*]=

4

In[*]:= $\text{Monitor}\left[\begin{array}{l} \text{Union@Table}\left[\right. \\ \quad g = \text{Exponent}[\Theta[K][[1]], T]; \quad q = \text{Coefficient}[\Theta[K][[1]], T^g], \\ \quad \{K, \text{FiberedKnots} \cap \text{AllKnots}[\{3, 13\}]\} \\ \left. \right], \{K, q\} \end{array}\right]$

Out[*]=

$\{-1, 1\}$

```

In[*]:= Monitor[
  Table[
    d = Exponent[Θ[K][[1]], T]; q = Factor[ $\frac{\text{Coefficient}[\Theta[K][[2]], T_2^d]}{\Theta[K][[1]] T^d}$ ],
    {K, FiberedKnots ∩ AllKnots[{3, 13}]}
  ], {K, q}]

```

⋯ KnotTheory: Loading precomputed data in KnotTheory/13A.dts.

Out[*]=

```

{-1, 0, -2, -1, 0, -3, -1, 0, -2, 2, 1, 0, 1, 0, -1, 0, 0, 3, 0, -1, -4, 2, -1, -2, 1, 0, 1, 0,
-1, -1, 0, -1, 1, 0, 0, 2, -1, 1, 2, 0, -1, 1, 1, -3, 2, 1, 0, -1, -1, 0, 0, -1, 0, 3, 2,
0, 1, 0, 2, 1, 1, 1, 0, -1, 0, -2, 0, 0, -1, -2, 0, -1, 0, 1, 0, 0, -2, 0, 1, 1, 0, 0, -1,
-1, 0, -1, 0, 0, 4, 1, -1, -2, 0, -1, 1, 0, 1, 4, 0, 0, -1, -2, -1, -2, 2, 1, -4, -1, 3, 0,
-1, 2, 0, -1, 2, -3, 1, -1, 0, -1, 2, 0, -1, 1, 1, 2, 0, 1, 0, 0, -1, 1, 0, 2, 1, 0, 0, -1,
-2, -1, -1, -3, -1, 1, 1, 0, 0, 2, 0, -1, 0, 1, -1, 2, 1, 0, -1, 0, 1, 0, -1, 0, 0, -2, -1,
1, 0, 2, 0, -2, -1, 1, -3, -1, 1, -1, -1, -1, -1, 1, 0, -1, 1, 0, 0, 1, -1, 0, 1, 2, 2, 0,
-2, -1, 0, -2, -1, 3, -3, 0, 2, 1, -1, -1, -1, 3, 0, -1, -1, 0, 1, -1, 2, 0, 1, 0, 1, -1,
0, 3, -2, 1, 0, -1, 1, 1, 0, 1, 1, -1, 1, 1, 0, 0, 0, -2, 0, -1, -2, -1, -2, 3, 1, 0, 0, 0,
-2, 0, 2, 2, 0, 0, 5, -1, 0, 1, 0, 1, -2, 3, -2, 1, 0, 3, -2, 1, -2, 0, 1, 2, 1, 1, 0, 3, 0,
2, 0, 1, 1, 0, 1, 2, 1, 2, 0, 1, 0, 1, 0, 1, 0, 1, 3, -1, 2, 1, 2, 1, 2, 0, -2, 4, 2, -2, 3,
1, 0, 0, -1, 3, -2, -1, -1, 0, 2, 1, 0, -1, -2, 3, 2, -1, 2, -2, -2, 0, 2, 1, -1, 0, -1,
2, 0, 1, -2, 1, 1, -1, 1, -1, 0, -1, 2, 2, -2, 0, 0, 2, 1, 1, 2, 1, 2, 0, 1, 0, 0, 2, -1, 1,
1, 2, 0, 1, 1, 1, 0, 2, -1, 1, 0, 0, 3, 0, 1, 0, 1, -1, 0, -2, 0, -1, 0, 0, 1, 1, -1, 0, 2,
-2, -1, 0, 0, -1, 1, -1, 0, -1, 0, 1, 0, 1, 1, -1, 0, -1, -1, -2, 1, -1, 0, 0, 0, -1, -1,
1, 1, 2, -1, 1, 0, 0, 3, 1, 2, -1, 1, 0, -1, 2, 1, -1, -2, 0, 0, 1, -1, 0, -1, 0, -1, -1,
1, 0, 0, 2, -3, 1, 0, 3, 1, 1, -2, 0, -1, 1, 0, -1, 0, 1, -2, 0, 0, -2, -1, -2, -2, -1, 0,
-1, 1, 0, 1, 0, -1, 2, 0, -2, 1, 0, 1, -1, -1, 0, 1, 0, 2, 2, 0, 1, 0, 1, 1, 0, 0, 0, -1,
1, 2, 0, -1, 1, -1, -1, -1, -1, -1, 2, -2, 1, 0, 0, -1, 0, -1, -1, 1, -2, 0, 0, 1, 0, 0,
-2, 0, -1, 1, 0, 0, -2, -2, -1, -3, 3, 0, 0, -2, -1, -2, 0, 0, -1, 0, 2, 1, 1, 0, 1, -1,
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-1, 1, -2, 0, -1, 0, 1, -1, 0, -1, 0, -1, 1, 1, 0, 1, 0, 1, 1, 1, -1, 0, -1, 0, 1, 2, 2,
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0, 2, 1, 0, -2, -1, 1, 0, -1, -1, 4, 1, 1, 2, 2, -3, -1, 0, 0, -2, -1, 1, 1, -1, 0, 1, 0,
-1, -1, 0, 2, 1, -1, -1, 2, 0, 1, -1, 0, 1, -1, 0, 1, -1, 0, 0, -1, -3, 0, -2, 0, -1, -1,
0, 1, 0, 1, 1, -1, 0, 2, -1, -1, 0, 2, -1, 0, 1, 1, 2, 0, 2, 0, 2, 2, 0, 0, 2, 1, 1, -1, 0,
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4, 2, 0, -1, -3, 1, 0, 2, 0, -1, -1, 0, 1, 0, 2, 0, 1, 1, -1, 0, 0, -1, 0, 2, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, -1, 2, -1, 2, -1, -1, 0, 0, -1, 2, -1, 0, 1, -1, 0, 0, -1, 1, 0, 1, 0,
-1, 2, -1, 0, 1, -1, 1, 0, 1, 0, -1, -2, 3, 1, 1, 2, -3, 3, 0, 1, 1, 2, -1, 2, 1, 3, -2,
4, 0, 3, 0, 2, 0, 2, 3, -2, 4, 0, 3, -2, 3, -2, 2, -1, 0, 1, 2, 1, 3, -2, 4, 0, 3, 2, 1, 0,
1, 2, -1, 0, -3, 2, 1, 0, 2, 1, 0, 0, 1, 0, 3, 0, 1, 2, 3, 1, 0, 2, 0, 2, 1, 2, -1, 3, -2,
4, 2, 1, -2, 3, -2, 1, -1, 2, 2, -1, 3, 1, 0, 1, 3, 2, -1, 3, 3, 1, -2, 3, -2, 5, 2, 0, 0,
-1, 2, -3, 1, 0, 0, 1, 0, 1, -1, 2, 2, -1, 2, 0, 0, 1, 1, 2, -1, 0, -1, 0, 2, -1, 2, -2,
4, -3, -2, -1, 0, -2, 1, 0, -1, -1, -2, -1, 1, 1, -1, -1, -1, 0, 1, -1, -1, -3, 0, 3, 1,

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0, 1, 1, 2, 2, 1, -1, 0, 3, 0, 1, 1, 1, -2, -1, 0, -2, 0, -1, 1, -3, -2, 2, 2, 0, -1, 2,
 4, 3, 3, 0, -1, 1, -2, 4, 2, 3, 0, -2, 0, 0, -2, -1, 2, -1, 2, 2, 0, 1, 1, -1, -1, -1, 0,
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 2, -2, -1, 0, 4, 1, -1, 0, 2, -3, 1, 0, 1, 0, 2, 1, -1, -1, 0, 1, 0, -1, 1, -2, -1, 5, 1,
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 0, 3, -1, 0, 3, 0, 5, 0, 3, -2, -2, 2, 5, 1, 3, 4, 0, 3, 2, 2, 0, 0, -2, 0, 0, 0, -1, 1, 0,
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 -1, 0, 2, 1, 1, -3, -1, 1, 1, 1, -1, 1, 1, 0, 0, -2, -1, 0, 1, 2, 4, 0, -1, 0, 1, -1, 0, 0,
 1, -1, 0, 1, 1, -1, 1, -1, -3, 2, 5, 0, 1, -1, 0, 0, 0, 1, -2, 2, 0, -1, 0, -1, -1, 0, 0,
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 -2, 1, 1, 0, 0, 0, 1, 2, -1, 1, 2, 3, -1, 1, -1, 2, 0, 1, 0, -1, 1, -1, 1, 0, -1, -1, 1, 0,
 0, -1, -2, 0, 1, 0, -1, 0, 2, 0, -1, 0, 1, 0, -1, 1, -1, 0, 1, 1, 0, 0, 0, 2, 1, 2, 0, 1,
 1, 1, 2, 0, 2, 1, 1, 2, 1, 1, 3, 0, 1, 0, 2, 0, 1, 2, 1, 1, 1, 1, 1, -1, 0, 2, 2, 1, 0, 0, 1,
 -1, 1, 1, 1, 1, 0, 1, 1, 1, -2, -2, -2, 0, -1, 0, 0, -2, -1, 0, 0, -1, -1, -1, 1, -2, 0,
 2, -1, 1, 1, 0, 0, 0, -1, 0, 2, -1, 1, 1, 2, 2, 1, 3, 0, 1, 0, 0, 0, 1, 1, 0, 2, 1, -1, 1,
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 -2, 0, -2, -1, 0, 0, -1, -1, 0, 0, -3, -2, -2, -2, -1, 1, -4, 0, -1, 2, 0, -2, 0, 2, 0,
 1, 2, -1, -1, 0, 1, 0, 3, 1, -2, 0, -1, 2, 1, 0, 1, 0, 0, 1, -1, -1, 1, 1, 0, 0, 1, -1, 0,
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 0, 1, -1, 1, 0, 0, 3, 1, -1, 0, 0, -2, 1, 2, -1, 0, 1, -2, 1, 0, 1, 1, -1, 1, -1, 0, -1,
 0, 0, 0, 0, 1, 0, 1, 1, 0, 1, 1, 1, 2, 0, -1, 1, 0, -1, 1, -1, 0, 1, 0, 3, 1, -1, 2, -1, 0,
 -2, 0, 1, 1, 0, 2, 0, -1, 1, 0, -1, 0, 1, -1, 0, 0, -1, 0, 2, -1, 1, -1, 2, 0, -2, -1, 1,
 0, -1, -2, -1, 0, 1, 1, -1, -3, -2, 0, -1, 1, -1, -1, -1, 0, 1, 2, -2, -2, 0, 1, -1, 2,
 1, 3, 1, -3, 0, 1, 0, 1, -1, -3, 0, -2, -2, 1, 0, 1, 1, -1, 2, -4, -1, -2, 2, -2, -1, 2,
 2, -2, 1, -1, 2, 2, 0, 1, -1, 3, 0, -2, 0, 2, -2, 0, 1, 0, -2, 0, 2, 0, 1, -2, 0, -1, -1,
 0, -2, 0, -2, -2, -2, 2, 2, -1, 0, -2, 0, 1, 2, -2, 1, -1, 1, 2, 0, -1, 1, -2, -1, 0, 2,
 0, 0, -1, 0, 0, 1, 1, -3, 0, -1, 0, 0, 1, 0, 1, 1, -2, 0, 0, -2, 0, 1, -1, 0, -1, -1, -1,
 -1, -2, 0, 0, 0, 0, 0, 1, -2, 0, -1, -1, 0, -2, -1, 0, 1, 2, 3, 3, 1, -1, -3, 0, -2, 1,
 -1, 1, 0, -1, 0, 1, 0, -1, 0, 1, 0, 0, 1, 0, 1, 1, 2, -1, 1, 0, -3, 0, -2, 4, -4, 1, 1, 1,
 -1, 1, -1, -1, -1, 0, 0, 2, -2, 1, -1, 1, 1, -2, -1, 0, 1, 3, 2, -2, 1, -2, 0, -2, 0, 2,
 4, 2, -1, -1, 1, -2, 0, 1, -1, 0, 0, 0, 1, 0, 0, -3, 0, 0, 1, 0, -1, 0, -2, 0, 1, -1, -3,
 0, 1, -2, 1, -1, -1, 0, 0, 0, 1, -1, 1, 0, -1, 1, 0, 2, -1, 1, 2, 0, 1, 0, 1, 2, -2, -2,
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```

```

In[*]:= Monitor[
  Total@Table[
    d = Exponent[Θ[K][[1]], T]; q = Factor[ $\frac{\text{Coefficient}[\Theta[K][[2]], T_2^{2d}]/. T_1 \rightarrow T}{\Theta[K][[1]] T^d}$ ];
    2 q == KnotSignature@K,
    {K, FiberedKnots ∩ AllKnots[{3, 13}]}
  ], {K, q}

```

```

Out[*]=
373 False + 5024 True

```

```

In[*]:= Length[Complement[AllKnots[{3, 13}], FiberedKnots]]

```

```

Out[*]=
7568

```

```

In[*]:= Length@Select[Complement[AllKnots[{3, 13}], FiberedKnots], (
  d = Exponent[Θ[#][[1]], T];
  ! NumberQ[Factor[ $\frac{\text{Coefficient}[\Theta[#][[2]], T_2, 2d]/. T_1 \rightarrow T}{\Theta[#][[1]] T^d}$ ]]
) &]

```

```

Out[*]=
7520

```

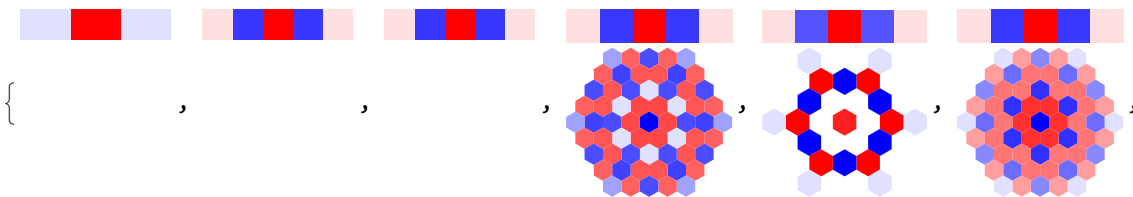
```
In[*]:= Select[Complement[AllKnots[{3, 13}], FiberedKnots], (
  d = Exponent[Θ[#][1], T];
  NumberQ[Factor[ $\frac{\text{Coefficient}[\Theta[\#][2], T_2, 2d]}{\Theta[\#][1] T^d}$ ]]]
) &]
```

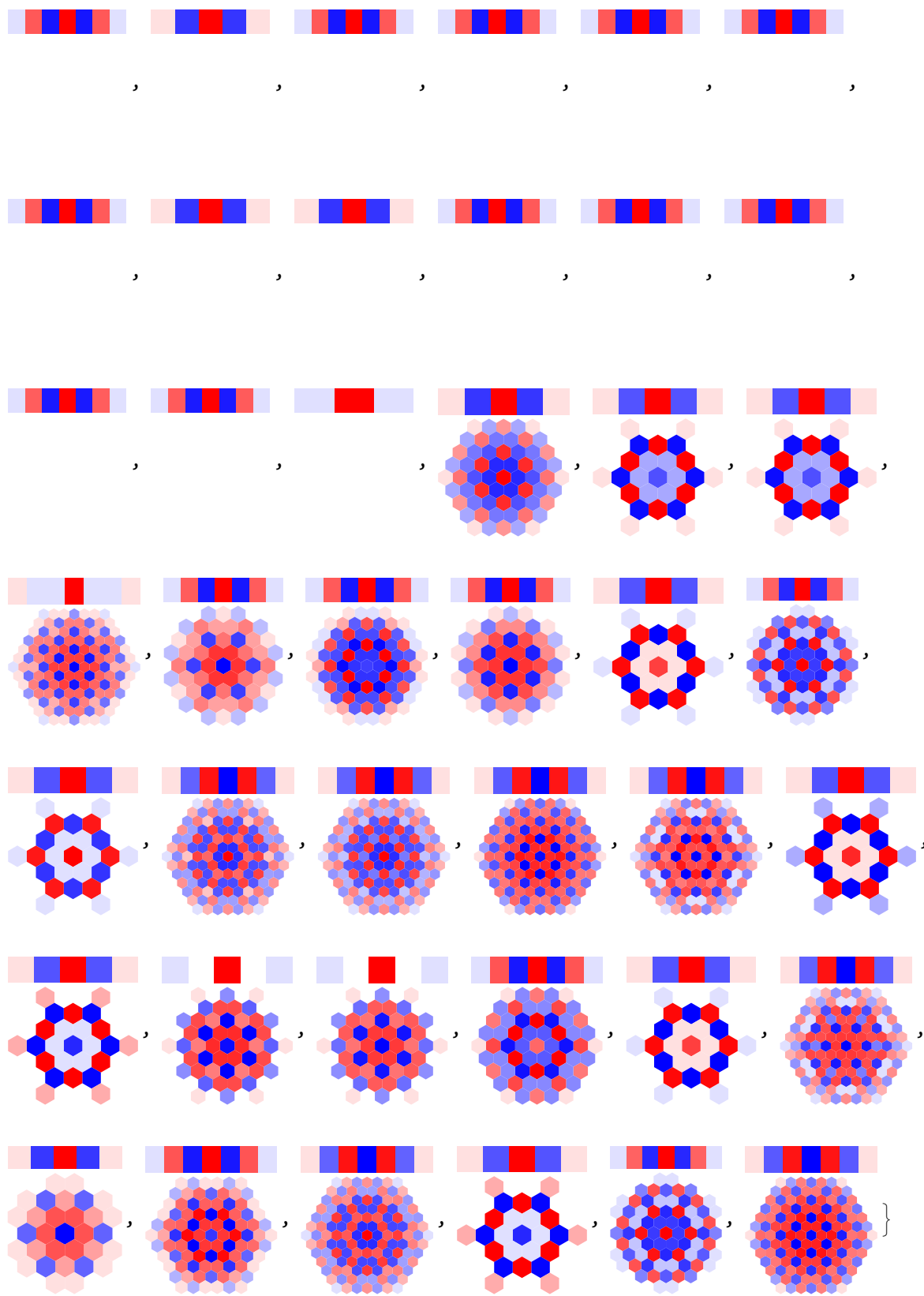
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```
In[*]:= PolyPlot[Θ[#], ImageSize → Tiny] & /@
  Select[Complement[AllKnots[{3, 13}], FiberedKnots], (
    d = Exponent[Θ[#][1], T];
    NumberQ[Factor[ $\frac{\text{Coefficient}[\Theta[\#][2], T_2, 2d]}{\Theta[\#][1] T^d}$ ]]]
  ) &]
```

Out[*]=





```

In[*]:= Monitor[
  Last@Reap@Table[
    g = Exponent[Θ[K][[1]], T];
    If[Abs[Coefficient[Θ[K][[1]], T, g]] == 1,
      q = Factor[ $\frac{\text{Coefficient}[\Theta[K][[2]], T_2, 2g] \cdot T_1 \rightarrow T}{\Theta[K][[1]] T^g}$ ];
      If[Not@NumberQ[q], Sow@Echo@K]
    ],
    {K, Complement[AllKnots[{3, 13}], FiberedKnots]}
  ], {K, q}]

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Out[*]=

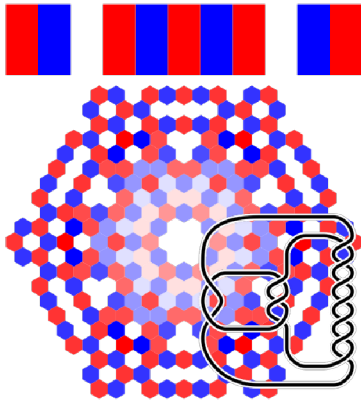
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Public domain $P(-2, 3, 7)$ image from [https://en.wikipedia.org/wiki/\(\%E2%88%922,3,7\)_pretzel_knot](https://en.wikipedia.org/wiki/(\%E2%88%922,3,7)_pretzel_knot)

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In[ ]:= ThetaK12n242 = ImageCompose[PolyPlot[ $\Theta$ [Knot["K12n242"]], ImageSize -> 480],
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```
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Out[ ]:=
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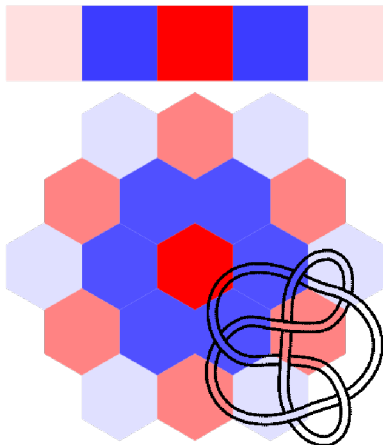
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In[ ]:= ThetaK7a7 = ImageCompose[PolyPlot[ $\Theta$ [Knot[7, 7]], ImageSize -> 480],
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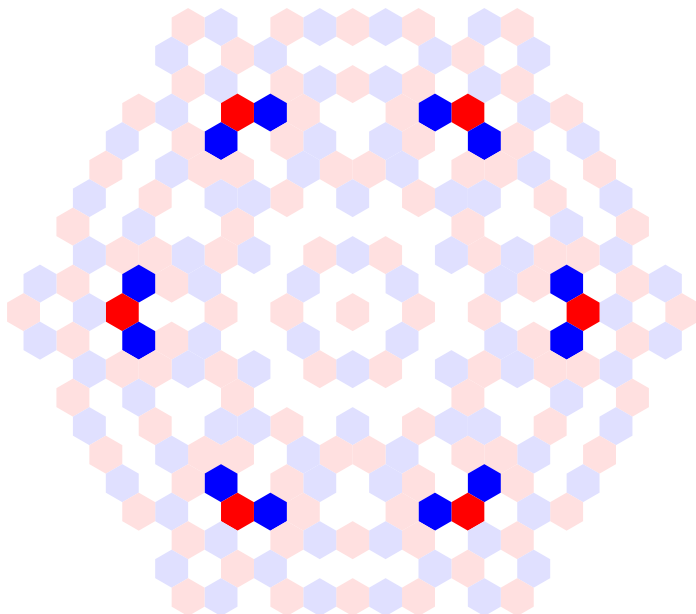
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```
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```

```
Out[ ]:=
```



```
In[ ]:= PolyPlot2[ $\theta\theta - 5$   $(\Delta\theta / . T \rightarrow T_1)$   $(\Delta\theta / . T \rightarrow T_2)$   $(\Delta\theta / . T \rightarrow T_3)$ ]
```

```
Out[ ]:=
```

