In[•]:=
```
SetDirectory["C:\\Users\\T15Roland\\Wiskunde\\Bn\\HigherRank"];
Once[<< KnotTheory`]
```

ParentDirectory: Argument File should be a positive machine-size integer, a nonempty string, or a File specification.

ParentDirectory: Argument File should be a positive machine-size integer, a nonempty string, or a File specification.

ToFileName: String or list of strings expected at position 1 in ToFileName[{File, WikiLink, mathematica}].

ToFileName: String or list of strings expected at position 1 in ToFileName[{File, QuantumGroups}].

```
Loading KnotTheory` version of September 6, 2014, 13:37:37.2841.
Read more at http://katlas.org/wiki/KnotTheory.
```

SetDelayed: Tag Diff in Diff[K_PD, rut_, ag_, n_, m_] is Protected.

In[•]:=
```
Rot[pd_PD] := Module[{n, xs, x, rots, Xp, Xm, front = {1}, k},
   n = Length@pd; rots = Table[0, {2 n}];
   xs = Cases[pd, x_X :> { Xp[x[[4]], x[[1]]] PositiveQ@x
                           Xm[x[[2]], x[[1]]]     True   ];
   For[k = 1, k ≤ 2 n, ++k,
    If[FreeQ[front, -k],
     front = Flatten@Replace[front, k → (xs /. {
             Xp[k, l_] | Xm[l_, k] :> {l + 1, k + 1, -l},
             Xp[l_, k] | Xm[k, l_] :> (++rots[[l]]; {-l, k + 1, l + 1}),
             _Xp | _Xm :> {}
            }), {1}],
     Cases[front, k | -k] /. {k, -k} :> --rots[[k]];
    ]
   ];
   {xs /. {Xp[i_, j_] :> {+1, i, j}, Xm[i_, j_] :> {-1, i, j}}, rots}];
Rot[K_] := Rot[PD[K]];
```

In[•]:=
```
CF[ℰ_] := Module[{vs = Union@Cases[ℰ, g__, ∞], ps, c},
   Total[CoefficientRules[Expand[ℰ], vs] /. (ps_ → c_) :> Factor[c] (Times @@ vs^ps)]];
```

In[•]:=
```
T3 = T1 T2;
```

In[•]:=
```
R1[s_, i_, j_] =
  CF[s (1/2 - g3ii + T2^s g1ii g2ji - g1ii g2jj - (T2^s - 1) g2ji g3ii + 2 g2jj g3ii - (1 - T3^s) g2ji g3ji -
      g2ii g3jj - T2^s g2ji g3jj + g1ii g3jj + ((T1^s - 1) g1ji (T2^(2 s) g2ji - T2^s g2jj + T2^s g3jj) +
        (T3^s - 1) g3ji (1 - T2^s g1ii - (T1^s - 1) (T2^s + 1) g1ji + (T2^s - 2) g2jj + g2ij)) / (T2^s - 1))];
```

In[•]:=
```
Θ[{s0_, i0_, j0_}, {s1_, i1_, j1_}] := CF[
  s1 (T1^s0 - 1) (T2^s1 - 1)^-1 (T3^s1 - 1) g1,j1,i0 g3,j0,i1 ((T2^s0 g2,i1,i0 - g2,i1,j0) - (T2^s0 g2,j1,i0 - g2,j1,j0))]
```

```
In[•]:= Γ1[φ_, k_] = -φ / 2 + φ g3kk;
```

```
In[•]:= Θ[K_] := Θ[K] = Module[{Cs, φ, n, A, s, i, j, k, Δ, G, ν, α, β, gEval, c, z},
  {Cs, φ} = Rot[K]; n = Length[Cs];
  A = IdentityMatrix[2 n + 1];
  Cases[Cs, {s_, i_, j_} :> (A[[{i, j}, {i + 1, j + 1}]] += ({{-T^s, T^s - 1}, {0, -1}}))];
  (*A//Echo;*)
  Δ = T^((-Total[φ] - Total[Cs[[All, 1]]])/2) Det[A];
  G = Inverse[A];
  gEval[ℰ_] := Factor[ℰ /. g_ν_,α_,β_ :> (G[[α, β]] /. T -> T_ν)];
  z = gEval[Sum[Sum[Θ[Cs[[k1]], Cs[[k2]]], {k2, 1, n}], {k1, 1, n}]];
  z += gEval[Sum[R1 @@ Cs[[k]], {k, 1, n}]];
  z += gEval[Sum[Γ1[φ[[k]], k], {k, 1, 2 n}]];
  {Δ, (Δ /. T -> T1) (Δ /. T -> T2) (Δ /. T -> T3) z} // Factor];
```

```
In[•]:= PolyPlot[{Δ_, Θ_}] := Module[{crs, m, m1, m2, maxc, minc, s, , rect, hex}, GraphicsColumn[{
     rect = {{0, 0}, {1, 0}, {1, 1}, {0, 1}};
     hex = Table[{Cos[α], Sin[α]} / Cos[2 π / 12] / 2, {α, 2 π / 12, 2 π, 2 π / 6}];
     If[Expand[Δ] === 0, Graphics[],
      crs = CoefficientRules[T^(m=-Exponent[Δ,T,Min]) Δ, {T}];
      maxc = N@Log@Max@Abs[Last /@ crs];
      minc = N@Log@Min@Select[Abs[Last /@ crs], # > 0 &];
      If[minc == maxc, s[_] = 0, s[c_] := s[c] = (maxc - Log@c) / (maxc - minc)];
      Graphics[crs /. ({x_} → c_) :> {
          Lighter[Which[c == 0, White, c > 0, Red, c < 0, Blue], 0.88 s[Abs@c]],
          Polygon[({x + m - 1 / 2, 0} + #) & /@ rect] }, AspectRatio → 1 / 5 ]
     ],
     If[Expand[Θ] === 0, Graphics[{White, Disk[]}],
      crs = CoefficientRules[T1^(m1=-Exponent[Θ,T1,Min]) T2^(m2=-Exponent[Θ,T2,Min]) Θ, {T1, T2}];
      maxc = N@Log@Max@Abs[Last /@ crs];
      minc = N@Log@Min@Select[Abs[Last /@ crs], # > 0 &];
      If[minc == maxc, s[_] = 0, s[c_] := s[c] = (maxc - Log@c) / (maxc - minc)];
      Graphics[{{{White, Disk[{0, 0}, 1 + Cos[2 π / 12] Norm[{m1, m2}] / √2 ]},
        crs /. ({x1_, x2_} → c_) :> {
            Lighter[Which[c == 0, White, c > 0, Red, c < 0, Blue], 0.88 s[Abs@c]],
            Polygon[(({{1, -1/2}, {0, √3/2}}.{x1 - m1, x2 - m2} + #) & /@ hex)] }
       }]
     ]
    }, Spacings → 0]];
```

```
In[•]:= Δ2[K_] := Δ2[K] = Module[{Cs, φ, n, A, s, i, j, k, Δ, G, ν, α, β, gEval, c, z},
     {Cs, φ} = Rot[K]; n = Length[Cs];
     A = IdentityMatrix[2 n + 1];
     Cases[Cs, {s_, i_, j_} :> (A[[{i, j}, {i + 1, j + 1}]] += ({{-T^s, T^s - 1}, {0, -1}}))];
     (*A//Echo;*)
     PolynomialGCD @@ Flatten[Minors[A]]
    ];
```

```
In[•]:= Δ2[Knot[8, 18]]
        Δ2[Knot[12, Alternating, 427]]
```

Out[•]=

$$\frac{1}{T^4}-\frac{1}{T^3}+\frac{1}{T^2}$$

Out[•]=

$$\frac{1}{T^6}-\frac{4}{T^5}+\frac{5}{T^4}-\frac{4}{T^3}+\frac{1}{T^2}$$

```
In[•]:= Union[Length[MonomialList[Denominator[Together[Θ[#]][[2]] /
            ((Δ2[#] /. T → T1) (Δ2[#] /. T → T2) (Δ2[#] /. T → T3))]]] & /@ AllKnots[{3, 12}]]
```

Out[•]=

```
{1}
```