

```
In[*]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\Theta"];
Once[<< Theta.m]
```

Loading KnotTheory` version of October 29, 2024, 10:29:52.1301.
Read more at <http://katlas.org/wiki/KnotTheory>.

```
In[*]:=  $\delta_{i-,i-} = 1$ ;  $\delta_{i-,j-} := \delta_{i,j}$ ;  $\delta_{i-,j-} := \chi_{\text{Sort}[i==j]}$ ;
```

```
In[*]:= D[{s-,i-,j-}][ $\mathcal{E}_-$ ] := ( $\mathcal{E} /. \gamma \rightarrow i^+$ ) + ( $\mathcal{E} /. \gamma \rightarrow j^+$ ) - ( $\mathcal{E} /. \gamma \rightarrow i$ ) - ( $\mathcal{E} /. \gamma \rightarrow j$ );
lhs = D[{s1,i1,j1}][ $T_1^{s0} - 1$ ]  $g_{2,\gamma,i0} g_{1,\gamma,i0} g_{1,j0^+,\gamma} + (1 - T_1^{s0}) g_{2,\gamma,j0} g_{1,\gamma,i0} g_{1,j0^+,\gamma}$ 
```

```
Out[*]:= -(( $-1 + T_1^{s0}$ )  $g_{1,i1,i0} g_{1,j0^+,i1} g_{2,i1,i0}$ ) - ( $1 - T_1^{s0}$ )  $g_{1,i1,i0} g_{1,j0^+,i1} g_{2,i1,j0}$  -
( $-1 + T_1^{s0}$ )  $g_{1,j1,i0} g_{1,j0^+,j1} g_{2,j1,i0}$  - ( $1 - T_1^{s0}$ )  $g_{1,j1,i0} g_{1,j0^+,j1} g_{2,j1,j0}$  +
( $-1 + T_1^{s0}$ )  $g_{1,i1^+,i0} g_{1,j0^+,i1^+} g_{2,i1^+,i0}$  + ( $1 - T_1^{s0}$ )  $g_{1,i1^+,i0} g_{1,j0^+,i1^+} g_{2,i1^+,j0}$  +
( $-1 + T_1^{s0}$ )  $g_{1,j0^+,j1^+} g_{1,j1^+,i0} g_{2,j1^+,i0}$  + ( $1 - T_1^{s0}$ )  $g_{1,j0^+,j1^+} g_{1,j1^+,i0} g_{2,j1^+,j0}$ 
```

```
In[*]:= R1 = Simplify[
Residue[F1[{s0, i0, j0}], {T2, 1}] /. { $g_{2,\alpha,-} \rightarrow 1$ ,  $g_{2,j0,i0} \rightarrow 1 - g_{2,i0,j0}$ ,  $g_{3,\alpha,-} \rightarrow g_{1,\alpha,\beta}$ }]
```

```
Out[*]:= -(( $-1 + T_1^{s0}$ )  $g_{1,j0,i0} (g_{1,i0,i0} + 2(-1 + T_1^{s0}) g_{1,j0,i0} - g_{1,j0,j0})$ )
```

```
In[*]:= R2 = Simplify[Residue[F2[{s0, i0, j0}], {s1, i1, j1}], {T2, 1}] /.  $g_{3,\alpha,-} \rightarrow g_{1,\alpha,\beta}$ 
```

```
Out[*]:= ( $-1 + T_1^{s0}$ ) ( $-1 + T_1^{s1}$ )  $g_{1,j0,i1} g_{1,j1,i0} (g_{2,i1,i0} - g_{2,i1,j0} - g_{2,j1,i0} + g_{2,j1,j0})$ 
```

```
In[*]:= rhs =  $\chi_{c0==c1}$  R1 + R2
```

```
Out[*]:= -(( $-1 + T_1^{s0}$ )  $\chi_{c0==c1} g_{1,j0,i0} (g_{1,i0,i0} + 2(-1 + T_1^{s0}) g_{1,j0,i0} - g_{1,j0,j0})$ ) +
( $-1 + T_1^{s0}$ ) ( $-1 + T_1^{s1}$ )  $g_{1,j0,i1} g_{1,j1,i0} (g_{2,i1,i0} - g_{2,i1,j0} - g_{2,j1,i0} + g_{2,j1,j0})$ 
```

```
In[*]:= Expand[Expand[lhs == rhs /. gRules[{s0, i0, j0}], {s1, i1, j1}]] /. {
T2  $\rightarrow 1$ ,  $\chi_{i0==j0} \mid \chi_{i1==j0} \mid \chi_{i0==j1} \rightarrow 0$ ,  $\chi_{i0==i1} \mid \chi_{j0==j1} \mid \chi_{c0==c1}^2 \rightarrow \chi_{c0==c1}$ ,
 $g_{2,\alpha^+,\alpha} \rightarrow 0$ ,  $g_{2,j0^+,i0} \rightarrow 1 - \chi_{i0==j0} - g_{2,i0^+,j0}$ ,
 $\chi_{c0==c1} \mathcal{E}_- \rightarrow (\mathcal{E} /. \{s1 \rightarrow s0, i1 \rightarrow i0, j1 \rightarrow j0\})$ 
}]
```

```
Out[*]:= True
```