```
In[•]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\Theta"];
        Once[<< Theta.m]
        SetOptions[PolyPlot, ImageSize → Tiny];
```

```
Loading KnotTheory` version of October 29, 2024, 10:29:52.1301.
```

```
In[•]:= eq = T[2] - 1 == s0 T[2]^((1-s0)/2) (T[2]^s0 - 1); Simplify[{eq /. s0 → 1, eq /. s0 → -1}]
Out[•]=
```

$$\{\text{True}, \text{True}\}$$

```
In[•]:= RandomVK[n_] := {
          Prepend[#, 2 RandomInteger[1] - 1] & /@
           Partition[PermutationList[RandomPermutation[2 n], 2 n], 2],
          Table[RandomInteger[{-1, 1}], 2 n + 1]
```

```
In[•]:= RandomVK[5]
Out[•]=
```

$$\{\{\{1, 7, 3\}, \{-1, 6, 2\}, \{1, 4, 9\}, \{-1, 10, 1\}, \{1, 5, 8\}\},$$

```
In[•]:= CF[ℰ_] := Expand@Collect[ℰ, g_ | χ_, F] /. F → Factor@*PowerExpand;
```

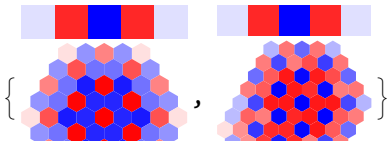
```
In[•]:= Short[Options[Θ] = {F1 → (F1i = F1[{s0, i0, j0}]),
```

```
Out[•]//Short=
```

$$\left\{F1 \to \frac{s0}{2} + s0\, T_2^{s0}\, g_{1,i0,i0}\, g_{2,j0,i0} + \text{<<19>>} + \frac{\text{<<1>>}}{\text{<<1>>}} - s0\, g_{2,i0,i0}\, g_{3,j0,j0} - s0\, T_2^{s0}\, g_{2,j0,i0}\, g_{3,j0,j0},\right.$$

```
In[•]:= Θ[K_, opts___Rule] := Module[{X, φφ, n, A, Δ, G, ev, Θ, kk, k0, k1, f1, f2, f3},
          f1 = F1 /. {opts} /. Options[Θ];
          f2 = F2 /. {opts} /. Options[Θ];
          f3 = F3 /. {opts} /. Options[Θ];
          {X, φφ} = Rot[K]; n = Length[X]; A = IdentityMatrix[2 n + 1];
          Cases[X, {s_, i_, j_} :> (A[[{i, j}, {i + 1, j + 1}]] += ({{-T^s, T^s - 1}, {0, -1}}))];
          Δ = T^((-Total[φφ]-Total[X[[All,1]]])/2) Det[A];
          G = Inverse[A];
          ev[ℰ_] := Factor[
            ℰ /. {k_^+ :> k + 1, $ → 2 n + 1} /. {g_ν_,α_,β_ :> (G[[α, β]] /. T → T_ν), χ_True → 1, χ_False → 0}];
          Θ = ev@Sum[f1 /. Thread[{s0, i0, j0} → X[[kk]]] ⋃ Thread[{s1, i1, j1} → X[[kk]]], {kk, n}];
```

```
In[•]:= PolyPlot /@ {Θ[Knot[7, 6]], Θ[Knot[7, 6], F3 → 0]}
Out[•]=
```



```
In[•]:= δ_{i_,j_} := χ_{i==j};
        χ_True = 1; χ_False = 0;
        χ_{α_==β_} /; OrderedQ[{β, α}] := χ_{β==α};
        χ^{p_}_{ph_} /; p > 1 ^:= χ_{ph};
        χ_{i0==$} = χ_{j0==$} = χ_{i1==$} = χ_{j1==$} = 0;
        χ_{α_^+==1} = χ_{1==α_^+} = 0;
        χ_{α_^+==β_^+} := χ_{α==β};
```

```
In[•]:= bRules[{s_, i_, j_}] := { (* b for "push indices backwards" *)
          g_{ν_,j^+,β_} :↦ g_{ν,j,β} - δ_{j,β}, g_{ν_,i^+,β_} :↦ T_ν^{-s} g_{ν,i,β} + (1 - T_ν^{-s}) g_{ν,j,β} - T_ν^{-s} δ_{i,β} - (1 - T_ν^{-s}) δ_{j,β},
          g_{ν_,α_,i^+} :↦ T_ν^s g_{ν,α,i} + δ_{α,i^+}, g_{ν_,α_,j^+} :↦ g_{ν,α,j} + (1 - T_ν^s) g_{ν,α,i} + δ_{α,j^+}
        }.
```

```
In[•]:= Expand@{{g_{ν,i,β}, g_{ν,j,β}} /. gRules[{s, i, j}],
```

Out[•]=

$$\{\{\chi_{i==\beta} + T_\nu^s\, g_{\nu,i^+,\beta} + g_{\nu,i^+,\beta} - T_\nu^s\, g_{\nu,i^+,\beta},\ \chi_{i==\beta} + g_{\nu,i^+,\beta}\},\ \{g_{\nu,i,\beta},\ g_{\nu,i,\beta}\}\}$$

```
In[•]:= sRules[ℰ_] := FixedPoint[CF[# /. bRules[{s0, i0, j0}] ⋃ bRules[{s1, i1, j1}] ⋃ {
            χ_{i0==j1} → 0, χ_{i1==j0} → 0, χ_{i0==j0} → 0,
            χ_{j0==j1} → χ_{i0==i1},
            χ_{i1^+≤i0} → χ_{i1≤i0} - χ_{i0==i1}, χ_{i0^+==i1^+} → χ_{i0==i1},
            χ_{i0==i1} χ_{i1≤i0} → χ_{i0==i1}, χ_{i0==i1} χ_{j1≤j0} → χ_{i0==i1},
            χ_{j0≤i0} → 1 - χ_{i0≤j0}, χ_{i1≤i0} → 1 - χ_{i0≤i1} + χ_{i1==i0},
            χ_{j1≤i0} → 1 - χ_{i0≤j1} + χ_{j1==i0}, χ_{i1≤i0} → 1 - χ_{i0≤i1} + χ_{i1==i0}, χ_{j1≤j0} → 1 - χ_{j0≤j1} + χ_{j1==j0},
```

```
In[•]:= D_{s_,i_,j_}[ℰ_] :=
          CF[((ℰ /. # → i^+) + (ℰ /. # → j^+) - (ℰ /. # → i) - (ℰ /. # → j)) //. bRules[{s, i, j}]];
```

```
In[•]:= D_{s1,i1,i1}[g_{1,#,i0}]
```

Out[•]=

$$-T_1^{-s1}\,\chi_{i0==i1} - T_1^{-s1}\left(-1 + 2\,T_1^{s1}\right)\chi_{i0==i1} - T_1^{-s1}\left(-1 + T_1^{s1}\right) g_{1,i1,i0} + T_1^{-s1}\left(-1 + T_1^{s1}\right) g_{1,i1,i0}$$

```
In[•]:= D_{s1,i1,i1}[g_{3,i0,#}]
```

Out[•]=

$$\chi_{i0==i1^+} + \chi_{i0==i1^+}$$

```
In[•]:= B[g_{3,i0,#}]
```

Out[•]=

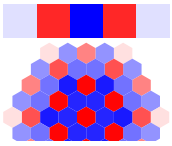
$$-g_{3,i0,1} + g_{3,i0,4}$$

In[•]:=
```
tw = g1,#,i0 ;
⊖[Knot[7, 6], F1 → -B[tw], F2 → D{s1,i1,j1}[tw], F3 → 0] // Echo // PolyPlot
```

» $\left\{-\frac{1-5\,T+7\,T^2-5\,T^3+T^4}{T^2},\right.$

$\left.\frac{1}{T_1^4\,T_2^4}\left(1-5\,T_1+7\,T_1^2-5\,T_1^3+T_1^4-5\,T_2+20\,T_1\,T_2-10\,T_1^2\,T_2-10\,T_1^3\,T_2+20\,T_1^4\,T_2-5\,T_1^5\,T_2+7\,T_2^2-\right.\right.$
$10\,T_1\,T_2^2-64\,T_1^2\,T_2^2+98\,T_1^3\,T_2^2-64\,T_1^4\,T_2^2-10\,T_1^5\,T_2^2+7\,T_1^6\,T_2^2-5\,T_2^3-10\,T_1\,T_2^3+98\,T_1^2\,T_2^3-50\,T_1^3\,T_2^3-$
$50\,T_1^4\,T_2^3+98\,T_1^5\,T_2^3-10\,T_1^6\,T_2^3-5\,T_1^7\,T_2^3+T_2^4+20\,T_1\,T_2^4-64\,T_1^2\,T_2^4-50\,T_1^3\,T_2^4+108\,T_1^4\,T_2^4-50\,T_1^5\,T_2^4-$
$64\,T_1^6\,T_2^4+20\,T_1^7\,T_2^4+T_1^8\,T_2^4-5\,T_1\,T_2^5-10\,T_1^2\,T_2^5+98\,T_1^3\,T_2^5-50\,T_1^4\,T_2^5-50\,T_1^5\,T_2^5+98\,T_1^6\,T_2^5-$

Out[•]=

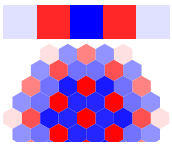


In[•]:=
```
tw = g1,j0,#;
⊖[Knot[7, 6], F1 → -B[tw], F2 → D{s1,i1,j1}[tw], F3 → 0] // Echo // PolyPlot
```

» $\left\{-\frac{1-5\,T+7\,T^2-5\,T^3+T^4}{T^2},\right.$

$\left.\frac{1}{T_1^4\,T_2^4}\left(1-5\,T_1+7\,T_1^2-5\,T_1^3+T_1^4-5\,T_2+20\,T_1\,T_2-10\,T_1^2\,T_2-10\,T_1^3\,T_2+20\,T_1^4\,T_2-5\,T_1^5\,T_2+7\,T_2^2-\right.\right.$
$10\,T_1\,T_2^2-64\,T_1^2\,T_2^2+98\,T_1^3\,T_2^2-64\,T_1^4\,T_2^2-10\,T_1^5\,T_2^2+7\,T_1^6\,T_2^2-5\,T_2^3-10\,T_1\,T_2^3+98\,T_1^2\,T_2^3-50\,T_1^3\,T_2^3-$
$50\,T_1^4\,T_2^3+98\,T_1^5\,T_2^3-10\,T_1^6\,T_2^3-5\,T_1^7\,T_2^3+T_2^4+20\,T_1\,T_2^4-64\,T_1^2\,T_2^4-50\,T_1^3\,T_2^4+108\,T_1^4\,T_2^4-50\,T_1^5\,T_2^4-$
$64\,T_1^6\,T_2^4+20\,T_1^7\,T_2^4+T_1^8\,T_2^4-5\,T_1\,T_2^5-10\,T_1^2\,T_2^5+98\,T_1^3\,T_2^5-50\,T_1^4\,T_2^5-50\,T_1^5\,T_2^5+98\,T_1^6\,T_2^5-$

Out[•]=



In[•]:=
```
bas0 = List @@ Expand[(g1,i1⁺,# + g1,j1⁺,#) (g2,i1⁺,# + g2,j1⁺,#) (g3,#,i1 + g3,#,j1)]
```

Out[•]=

$\{g_{1,i1^+,\#1}\,g_{2,i1^+,\#1}\,g_{3,\#1,i1},\ g_{1,j1^+,\#1}\,g_{2,i1^+,\#1}\,g_{3,\#1,i1},\ g_{1,i1^+,\#1}\,g_{2,j1^+,\#1}\,g_{3,\#1,i1},\ g_{1,j1^+,\#1}\,g_{2,j1^+,\#1}\,g_{3,\#1,i1},$

In[•]:=
```
B /@ bas0
```

Out[•]=

$\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}$

In[•]:=
```
Column[(# → Θ[Knot[7, 6], F1 → -B[#], F2 → D_{s0,i0,j0}[#], F3 → 0][[2]]) & /@ bas0]
```
Out[•]=

$$g_{1,i1^+,\#1}\, g_{2,i1^+,\#1}\, g_{3,\#1,i1} \to 0$$
$$g_{1,j1^+,\#1}\, g_{2,i1^+,\#1}\, g_{3,\#1,i1} \to 0$$
$$g_{1,i1^+,\#1}\, g_{2,j1^+,\#1}\, g_{3,\#1,i1} \to 0$$
$$g_{1,j1^+,\#1}\, g_{2,j1^+,\#1}\, g_{3,\#1,i1} \to 0$$
$$g_{1,i1^+,\#1}\, g_{2,i1^+,\#1}\, g_{3,\#1,j1} \to 0$$
$$g_{1,i1^+,\#1}\, g_{2,i1^+,\#1}\, g_{3,\#1,j1} \to 0$$

In[•]:=
```
bas1 = List @@ Expand[(g_{1,#,i0} + g_{1,#,j0}) (g_{2,#,i0} + g_{2,#,j0}) (g_{3,i0^+,#} + g_{3,j0^+,#})]
```
Out[•]=

$$\{g_{1,\#1,i0}\, g_{2,\#1,i0}\, g_{3,i0^+,\#1},\ g_{1,\#1,j0}\, g_{2,\#1,i0}\, g_{3,i0^+,\#1},\ g_{1,\#1,i0}\, g_{2,\#1,j0}\, g_{3,i0^+,\#1},\ g_{1,\#1,j0}\, g_{2,\#1,j0}\, g_{3,i0^+,\#1},$$

In[•]:=
```
B /@ bas1
```
Out[•]=

$$\{0, 0, 0, 0, 0, 0, 0, 0\}$$

In[•]:=
```
Column[(# → Θ[Knot[7, 6], F1 → -B[#], F2 → D_{s1,i1,i1}[#], F3 → 0][[2]]) & /@ bas1]
```
Out[•]=

$$g_{1,\#1,i0}\, g_{2,\#1,i0}\, g_{3,i0^+,\#1} \to 0$$
$$g_{1,\#1,j0}\, g_{2,\#1,i0}\, g_{3,i0^+,\#1} \to 0$$
$$g_{1,\#1,i0}\, g_{2,\#1,j0}\, g_{3,i0^+,\#1} \to 0$$
$$g_{1,\#1,j0}\, g_{2,\#1,j0}\, g_{3,i0^+,\#1} \to 0$$
$$g_{1,\#1,i0}\, g_{2,\#1,i0}\, g_{3,j0^+,\#1} \to 0$$
$$g_{1,\#1,i0}\, g_{2,\#1,i0}\, g_{3,j0^+,\#1} \to 0$$

In[•]:=
```
Column[(# → sRules@D_{s0,i0,j0}[#]) & /@ bas0]
```
Out[•]=

$$g_{1,i1^+,\#1}\, g_{2,i1^+,\#1}\, g_{3,\#1,i1} \to -T_1^{-s0}\, T_2^{-s0}\, \chi_{i0==i1}\, g_{1,i0,i0}\, g_{2,i0,i0} - T_1^{-s0}\left(-1+T_1^{s0}\right) T_2^{-s0}\, \chi_{i0==i1}\, g_{1,j0,i0}\, g_{2,i0,i0} -$$
$$T_1^{-s0}\, T_2^{-s0}\left(-1+T_2^{s0}\right) \chi_{i0==i1}\, g_{1,i0,i0}\, g_{2,j0,i0} - T_1^{-s0}\left(-1+T_1^{s0}\right) T_2^{-s0}\left(-1+T_2^{s0}\right) \chi_{i0==i1}\, g_{1,j0,i0}\, g_{2,j0,i0} -$$
$$T_1^{-s0}\, T_2^{-s0}\, \chi_{i0==i1}\, g_{3,i0,i0} + T_1^{-s0}\, T_2^{-s0}\, \chi_{i0==i1}\, g_{1,i0,i0}\, g_{3,i0,i0} +$$
$$T_1^{-s0}\left(-1+T_1^{s0}\right) T_2^{-s0}\, \chi_{i0==i1}\, g_{1,j0,i0}\, g_{3,i0,i0} + T_1^{-s0}\, T_2^{-s0}\, \chi_{i0==i1}\, g_{2,i0,i0}\, g_{3,i0,i0} +$$
$$T_1^{-s0}\, T_2^{-s0}\left(-1+T_2^{s0}\right) \chi_{i0==i1}\, g_{2,j0,i0}\, g_{3,i0,i0} - T_1^{-s0}\left(-1+T_1^{s0}\right) T_2^{-s0}\left(-1+T_2^{s0}\right) \chi_{i0==i1}\, g_{3,j0,i0} +$$
$$T_1^{-s0}\, T_2^{-s0}\left(-1+T_2^{s0}\right) \chi_{i0==i1}\, g_{1,i0,j0}\, g_{3,j0,i0} + T_1^{-s0}\left(-1+T_1^{s0}\right) T_2^{-s0}\left(-1+T_2^{s0}\right) \chi_{i0==i1}\, g_{1,j0,j0}\, g_{3,j0,i0} +$$
$$T_1^{-s0}\left(-1+T_1^{s0}\right) T_2^{-s0}\, \chi_{i0==i1}\, g_{2,i0,j0}\, g_{3,j0,i0} + T_1^{-s0}\left(-1+T_1^{s0}\right) T_2^{-s0}\left(-1+T_2^{s0}\right) \chi_{i0==i1}\, g_{2,j0,j0}\, g_{3,j0,i0} +$$
$$T_1^{-s1}\, T_2^{-s1}\left(-T_1^{s0} - T_2^{s0} + 2\, T_1^{s0}\, T_2^{s0}\right) g_{1,i1,i0}\, g_{2,i1,i0}\, g_{3,j0,i1} - T_1^{-s1}\, T_2^{-s1}\left(-1+T_2^{s0}\right) g_{1,i1,j0}\, g_{2,i1,i0}\, g_{3,j0,i1} +$$
$$T_1^{-s1}\left(-1+T_1^{s1}\right) T_2^{-s1}\left(-T_1^{s0} - T_2^{s0} + 2\, T_1^{s0}\, T_2^{s0}\right) g_{1,j1,i0}\, g_{2,i1,i0}\, g_{3,j0,i1} -$$
$$T_1^{-s1}\left(-1+T_1^{s1}\right) T_2^{-s1}\left(-1+T_2^{s0}\right) g_{1,j1,j0}\, g_{2,i1,i0}\, g_{3,j0,i1} -$$
$$T_1^{-s1}\left(-1+T_1^{s0}\right) T_2^{-s1}\, g_{1,i1,i0}\, g_{2,i1,j0}\, g_{3,j0,i1} - T_1^{-s1}\left(-1+T_1^{s0}\right)\left(-1+T_1^{s1}\right) T_2^{-s1}\, g_{1,j1,i0}\, g_{2,i1,j0}\, g_{3,j0,i1} +$$
$$T_1^{-s1}\, T_2^{-s1}\left(-T_1^{s0} - T_2^{s0} + 2\, T_1^{s0}\, T_2^{s0}\right)\left(-1+T_2^{s1}\right) g_{1,i1,i0}\, g_{2,j1,i0}\, g_{3,j0,i1} -$$
$$T_1^{-s1}\, T_2^{-s1}\left(-1+T_2^{s0}\right)\left(-1+T_2^{s1}\right) g_{1,i1,j0}\, g_{2,j1,i0}\, g_{3,j0,i1} +$$
$$T_1^{-s1}\left(-1+T_1^{s1}\right) T_2^{-s1}\left(-T_1^{s0} - T_2^{s0} + 2\, T_1^{s0}\, T_2^{s0}\right)\left(-1+T_2^{s1}\right) g_{1,j1,i0}\, g_{2,j1,i0}\, g_{3,j0,i1} -$$
$$T_1^{-s1}\left(-1+T_1^{s1}\right) T_2^{-s1}\left(-1+T_2^{s0}\right)\left(-1+T_2^{s1}\right) g_{1,j1,j0}\, g_{2,j1,i0}\, g_{3,j0,i1} -$$
$$T_1^{-s1}\left(-1+T_1^{s0}\right) T_2^{-s1}\left(-1+T_2^{s1}\right) g_{1,i1,i0}\, g_{2,j1,j0}\, g_{3,j0,i1} -$$
$$T_1^{-s1}\left(-1+T_1^{s0}\right)\left(-1+T_1^{s1}\right) T_2^{-s1}\left(-1+T_2^{s1}\right) g_{1,j1,i0}\, g_{2,j1,j0}\, g_{3,j0,i1}$$

$$g_{1,j1^+,\#1}\, g_{2,i1^+,\#1}\, g_{3,\#1,i1} \to$$
$$-T_2^{-s0}\, \chi_{i0==i1}\, g_{1,j0,i0}\, g_{2,i0,i0} - T_2^{-s0} \left(-1+T_2^{s0}\right) \chi_{i0==i1}\, g_{1,j0,i0}\, g_{2,j0,i0} + T_2^{-s0}\, \chi_{i0==i1}\, g_{1,j0,i0}\, g_{3,i0,i0} -$$
$$T_2^{-s0} \left(-1+T_2^{s0}\right) \chi_{i0==i1}\, g_{3,j0,i0} + T_2^{-s0} \left(-1+T_2^{s0}\right) \chi_{i0==i1}\, g_{1,j0,j0}\, g_{3,j0,i0} + T_2^{-s0}\, \chi_{i0==i1}\, g_{2,i0,j0}\, g_{3,j0,i0} +$$
$$T_2^{-s0} \left(-1+T_2^{s0}\right) \chi_{i0==i1}\, g_{2,j0,j0}\, g_{3,j0,i0} + T_2^{-s1} \left(-T_1^{s0} - T_2^{s0} + 2\, T_1^{s0}\, T_2^{s0}\right) g_{1,j1,i0}\, g_{2,i1,i0}\, g_{3,j0,i1} -$$
$$T_2^{-s1} \left(-1+T_2^{s0}\right) g_{1,j1,j0}\, g_{2,i1,i0}\, g_{3,j0,i1} - \left(-1+T_1^{s0}\right) T_2^{-s1}\, g_{1,j1,i0}\, g_{2,i1,j0}\, g_{3,j0,i1} +$$
$$T_2^{-s1} \left(-T_1^{s0} - T_2^{s0} + 2\, T_1^{s0}\, T_2^{s0}\right) \left(-1+T_2^{s1}\right) g_{1,j1,i0}\, g_{2,j1,i0}\, g_{3,j0,i1} -$$
$$T_2^{-s1} \left(-1+T_2^{s0}\right) \left(-1+T_2^{s1}\right) g_{1,j1,j0}\, g_{2,j1,i0}\, g_{3,j0,i1} - \left(-1+T_1^{s0}\right) T_2^{-s1} \left(-1+T_2^{s1}\right) g_{1,j1,i0}\, g_{2,j1,j0}\, g_{3,j0,i1}$$

$$g_{1,i1^+,\#1}\, g_{2,j1^+,\#1}\, g_{3,\#1,i1} \to$$
$$-T_1^{-s0}\, \chi_{i0==i1}\, g_{1,i0,i0}\, g_{2,j0,i0} - T_1^{-s0} \left(-1+T_1^{s0}\right) \chi_{i0==i1}\, g_{1,j0,i0}\, g_{2,j0,i0} + T_1^{-s0}\, \chi_{i0==i1}\, g_{2,j0,i0}\, g_{3,i0,i0} -$$
$$T_1^{-s0} \left(-1+T_1^{s0}\right) \chi_{i0==i1}\, g_{3,j0,i0} + T_1^{-s0}\, \chi_{i0==i1}\, g_{1,i0,j0}\, g_{3,j0,i0} + T_1^{-s0} \left(-1+T_1^{s0}\right) \chi_{i0==i1}\, g_{1,j0,j0}\, g_{3,j0,i0} +$$
$$T_1^{-s0} \left(-1+T_1^{s0}\right) \chi_{i0==i1}\, g_{2,j0,j0}\, g_{3,j0,i0} + T_1^{-s1} \left(-T_1^{s0} - T_2^{s0} + 2\, T_1^{s0}\, T_2^{s0}\right) g_{1,i1,i0}\, g_{2,j1,i0}\, g_{3,j0,i1} -$$
$$T_1^{-s1} \left(-1+T_2^{s0}\right) g_{1,i1,j0}\, g_{2,j1,i0}\, g_{3,j0,i1} + T_1^{-s1} \left(-1+T_1^{s1}\right) \left(-T_1^{s0} - T_2^{s0} + 2\, T_1^{s0}\, T_2^{s0}\right) g_{1,j1,i0}\, g_{2,j1,i0}\, g_{3,j0,i1} -$$
$$T_1^{-s1} \left(-1+T_1^{s1}\right) \left(-1+T_2^{s0}\right) g_{1,j1,j0}\, g_{2,j1,i0}\, g_{3,j0,i1} -$$
$$T_1^{-s1} \left(-1+T_1^{s0}\right) g_{1,i1,i0}\, g_{2,j1,j0}\, g_{3,j0,i1} - T_1^{-s1} \left(-1+T_1^{s0}\right) \left(-1+T_1^{s1}\right) g_{1,j1,i0}\, g_{2,j1,j0}\, g_{3,j0,i1}$$

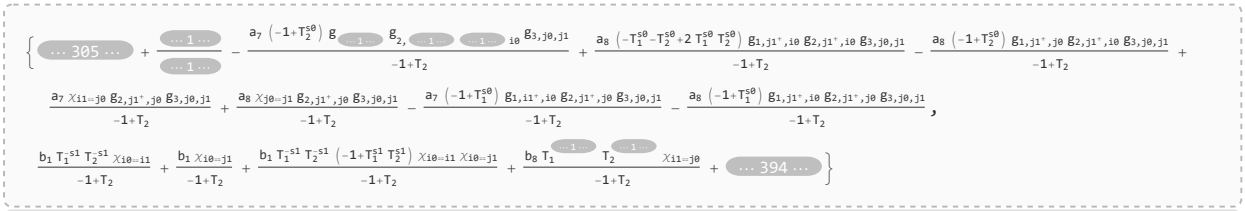
$$g_{1,j1^+,\#1}\, g_{2,j1^+,\#1}\, g_{3,\#1,i1} \to -\chi_{i0==i1}\, g_{1,j0,i0}\, g_{2,j0,i0} - \chi_{i0==i1}\, g_{3,j0,i0} +$$
$$\chi_{i0==i1}\, g_{1,j0,j0}\, g_{3,j0,i0} + \chi_{i0==i1}\, g_{2,j0,j0}\, g_{3,j0,i0} + \left(-T_1^{s0} - T_2^{s0} + 2\, T_1^{s0}\, T_2^{s0}\right) g_{1,j1,i0}\, g_{2,j1,i0}\, g_{3,j0,i1} +$$
$$\left(1-T_2^{s0}\right) g_{1,j1,j0}\, g_{2,j1,i0}\, g_{3,j0,i1} + \left(1-T_1^{s0}\right) g_{1,j1,i0}\, g_{2,j1,j0}\, g_{3,j0,i1}$$

$$
\begin{aligned}
&g_{1,i1^+,\#1}\, g_{2,i1^+,\#1}\, g_{3,\#1,j1} \to \\
&\quad -T_1^{-s0}\, T_2^{-s0} \left(-T_1^{s0} - T_2^{s0} + 2\, T_1^{s0}\, T_2^{s0}\right) \chi_{i0==i1}\, g_{1,i0,i0}\, g_{2,i0,i0} + T_1^{-s0}\, T_2^{-s0} \left(-1 + T_2^{s0}\right) \chi_{i0==i1}\, g_{1,i0,j0}\, g_{2,i0,i0} - \\
&\quad T_1^{-s0} \left(-1 + T_1^{s0}\right) T_2^{-s0} \left(-T_1^{s0} - T_2^{s0} + 2\, T_1^{s0}\, T_2^{s0}\right) \chi_{i0==i1}\, g_{1,j0,i0}\, g_{2,i0,i0} + \\
&\quad T_1^{-s0} \left(-1 + T_1^{s0}\right) T_2^{-s0} \left(-1 + T_2^{s0}\right) \chi_{i0==i1}\, g_{1,j0,j0}\, g_{2,i0,i0} + \\
&\quad T_1^{-s0} \left(-1 + T_1^{s0}\right) T_2^{-s0} \chi_{i0==i1}\, g_{1,i0,i0}\, g_{2,i0,j0} - T_1^{-s0}\, T_2^{-s0} \chi_{i0==i1}\, g_{1,i0,j0}\, g_{2,i0,j0} + \\
&\quad T_1^{-s0} \left(-1 + T_1^{s0}\right)^2 T_2^{-s0} \chi_{i0==i1}\, g_{1,j0,i0}\, g_{2,i0,j0} - T_1^{-s0} \left(-1 + T_1^{s0}\right) T_2^{-s0} \chi_{i0==i1}\, g_{1,j0,j0}\, g_{2,i0,j0} - \\
&\quad T_1^{-s0}\, T_2^{-s0} \left(-1 + T_2^{s0}\right) \left(-T_1^{s0} - T_2^{s0} + 2\, T_1^{s0}\, T_2^{s0}\right) \chi_{i0==i1}\, g_{1,i0,i0}\, g_{2,j0,i0} + \\
&\quad T_1^{-s0}\, T_2^{-s0} \left(-1 + T_2^{s0}\right)^2 \chi_{i0==i1}\, g_{1,i0,j0}\, g_{2,j0,i0} - \\
&\quad T_1^{-s0} \left(-1 + T_1^{s0}\right) T_2^{-s0} \left(-1 + T_2^{s0}\right) \left(-T_1^{s0} - T_2^{s0} + 2\, T_1^{s0}\, T_2^{s0}\right) \chi_{i0==i1}\, g_{1,j0,i0}\, g_{2,j0,i0} + \\
&\quad T_1^{-s0} \left(-1 + T_1^{s0}\right) T_2^{-s0} \left(-1 + T_2^{s0}\right)^2 \chi_{i0==i1}\, g_{1,j0,j0}\, g_{2,j0,i0} + \\
&\quad T_1^{-s0} \left(-1 + T_1^{s0}\right) T_2^{-s0} \left(-1 + T_2^{s0}\right) \chi_{i0==i1}\, g_{1,i0,i0}\, g_{2,j0,j0} - T_1^{-s0}\, T_2^{-s0} \left(-1 + T_2^{s0}\right) \chi_{i0==i1}\, g_{1,i0,j0}\, g_{2,j0,j0} + \\
&\quad T_1^{-s0} \left(-1 + T_1^{s0}\right)^2 T_2^{-s0} \left(-1 + T_2^{s0}\right) \chi_{i0==i1}\, g_{1,j0,i0}\, g_{2,j0,j0} - \\
&\quad T_1^{-s0} \left(-1 + T_1^{s0}\right) T_2^{-s0} \left(-1 + T_2^{s0}\right) \chi_{i0==i1}\, g_{1,j0,j0}\, g_{2,j0,j0} - T_1^{-s0}\, T_2^{-s0} \chi_{i0==i1}\, g_{3,i0,j0} + \\
&\quad T_1^{-s0}\, T_2^{-s0} \chi_{i0==i1}\, g_{1,i0,i0}\, g_{3,i0,j0} + T_1^{-s0} \left(-1 + T_1^{s0}\right) T_2^{-s0} \chi_{i0==i1}\, g_{1,j0,i0}\, g_{3,i0,j0} + \\
&\quad T_1^{-s0}\, T_2^{-s0} \chi_{i0==i1}\, g_{2,i0,i0}\, g_{3,i0,j0} + T_1^{-s0}\, T_2^{-s0} \left(-1 + T_2^{s0}\right) \chi_{i0==i1}\, g_{2,j0,i0}\, g_{3,i0,j0} - \\
&\quad T_1^{-s0} \left(-1 + T_1^{s0}\right) T_2^{-s0} \left(-1 + T_2^{s0}\right) \chi_{i0==i1}\, g_{3,j0,j0} + T_1^{-s0}\, T_2^{-s0} \left(-1 + T_2^{s0}\right) \chi_{i0==i1}\, g_{1,i0,j0}\, g_{3,j0,j0} + \\
&\quad T_1^{-s0} \left(-1 + T_1^{s0}\right) T_2^{-s0} \left(-1 + T_2^{s0}\right) \chi_{i0==i1}\, g_{1,j0,j0}\, g_{3,j0,j0} + \\
&\quad T_1^{-s0} \left(-1 + T_1^{s0}\right) T_2^{-s0} \chi_{i0==i1}\, g_{2,i0,j0}\, g_{3,j0,j0} + T_1^{-s0} \left(-1 + T_1^{s0}\right) T_2^{-s0} \left(-1 + T_2^{s0}\right) \chi_{i0==i1}\, g_{2,j0,j0}\, g_{3,j0,j0} + \\
&\quad T_1^{-s1}\, T_2^{-s1} \left(-T_1^{s0} - T_2^{s0} + 2\, T_1^{s0}\, T_2^{s0}\right) g_{1,i1,i0}\, g_{2,i1,i0}\, g_{3,j0,j1} - T_1^{-s1}\, T_2^{-s1} \left(-1 + T_2^{s0}\right) g_{1,i1,j0}\, g_{2,i1,i0}\, g_{3,j0,j1} + \\
&\quad T_1^{-s1} \left(-1 + T_1^{s1}\right) T_2^{-s1} \left(-T_1^{s0} - T_2^{s0} + 2\, T_1^{s0}\, T_2^{s0}\right) g_{1,j1,i0}\, g_{2,i1,i0}\, g_{3,j0,j1} - \\
&\quad T_1^{-s1} \left(-1 + T_1^{s1}\right) T_2^{-s1} \left(-1 + T_2^{s0}\right) g_{1,j1,j0}\, g_{2,i1,i0}\, g_{3,j0,j1} - \\
&\quad T_1^{-s1} \left(-1 + T_1^{s0}\right) T_2^{-s1} g_{1,i1,i0}\, g_{2,i1,j0}\, g_{3,j0,j1} - T_1^{-s1} \left(-1 + T_1^{s0}\right) \left(-1 + T_1^{s1}\right) T_2^{-s1} g_{1,j1,i0}\, g_{2,i1,j0}\, g_{3,j0,j1} + \\
&\quad T_1^{-s1}\, T_2^{-s1} \left(-T_1^{s0} - T_2^{s0} + 2\, T_1^{s0}\, T_2^{s0}\right) \left(-1 + T_2^{s1}\right) g_{1,i1,i0}\, g_{2,j1,i0}\, g_{3,j0,j1} - \\
&\quad T_1^{-s1}\, T_2^{-s1} \left(-1 + T_2^{s0}\right) \left(-1 + T_2^{s1}\right) g_{1,i1,j0}\, g_{2,j1,i0}\, g_{3,j0,j1} + \\
&\quad T_1^{-s1} \left(-1 + T_1^{s1}\right) T_2^{-s1} \left(-T_1^{s0} - T_2^{s0} + 2\, T_1^{s0}\, T_2^{s0}\right) \left(-1 + T_2^{s1}\right) g_{1,j1,i0}\, g_{2,j1,i0}\, g_{3,j0,j1} - \\
&\quad T_1^{-s1} \left(-1 + T_1^{s1}\right) T_2^{-s1} \left(-1 + T_2^{s0}\right) \left(-1 + T_2^{s1}\right) g_{1,j1,j0}\, g_{2,j1,i0}\, g_{3,j0,j1} - \\
&\quad T_1^{-s1} \left(-1 + T_1^{s0}\right) T_2^{-s1} \left(-1 + T_2^{s1}\right) g_{1,i1,i0}\, g_{2,j1,j0}\, g_{3,j0,j1} - \\
&\quad T_1^{-s1} \left(-1 + T_1^{s0}\right) \left(-1 + T_1^{s1}\right) T_2^{-s1} \left(-1 + T_2^{s1}\right) g_{1,j1,i0}\, g_{2,j1,j0}\, g_{3,j0,j1}
\end{aligned}
$$

$$
\begin{aligned}
&T_1^{-s0}\,T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\left(-2+T_1^{s1}+T_2^{s1}\right)g_{1,j1,i0}\,g_{2,j1,i0}\,g_{3,j0,i1}\\
g_{1,\#1,j0}\,g_{2,\#1,i0}\,g_{3,i0^+,\#1} \to\ & T_1^{-s0}\,T_2^{-s0}\,\chi_{i0==i1}\,g_{1,i0,j0}\,g_{2,i0,i0}+T_1^{-s0}\,T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,j0}\,g_{2,j0,i0}+\\
&T_1^{-s0}\left(-1+T_1^{s0}\right)T_2^{-s0}\,\chi_{i0==i1}\,g_{3,i0,i0}-T_1^{-s0}\,T_2^{-s0}\,\chi_{i0==i1}\,g_{1,i0,j0}\,g_{3,i0,i0}-\\
&T_1^{-s0}\left(-1+T_1^{s0}\right)T_2^{-s0}\,\chi_{i0==i1}\,g_{1,j0,j0}\,g_{3,i0,i0}-T_1^{-s0}\left(-1+T_1^{s0}\right)T_2^{-s0}\,\chi_{i0==i1}\,g_{2,i0,i0}\,g_{3,i0,i0}+\\
&T_1^{-s0}\,T_2^{-s0}\left(-2+T_1^{s0}+T_2^{s0}\right)\chi_{i0==i1}\,g_{2,j0,i0}\,g_{3,i0,i0}+T_1^{-s0}\left(-1+T_1^{s1}\right)T_2^{-s0}\,g_{1,j1,j0}\,g_{2,i1,i0}\,g_{3,i0,i1}+\\
&T_1^{-s0}\,T_2^{-s0}\left(-1+T_2^{s1}\right)g_{1,i1,j0}\,g_{2,j1,i0}\,g_{3,i0,i1}-T_1^{-s0}\,T_2^{-s0}\left(-2+T_1^{s1}+T_2^{s1}\right)g_{1,j1,j0}\,g_{2,j1,i0}\,g_{3,i0,i1}-\\
&T_1^{-s0}\,T_2^{-s0}\,\chi_{i0==i1}\,g_{2,j0,i0}\,g_{3,i0,j0}+T_1^{-s0}\left(-1+T_1^{s0}\right)T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{3,j0,i0}-\\
&T_1^{-s0}\,T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{1,i0,j0}\,g_{3,j0,i0}-T_1^{-s0}\left(-1+T_1^{s0}\right)T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,j0}\,g_{3,j0,i0}-\\
&T_1^{-s0}\left(-1+T_1^{s0}\right)T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{2,i0,i0}\,g_{3,j0,i0}+\\
&T_1^{-s0}\,T_2^{-s0}\left(-2+T_1^{s0}+T_2^{s0}\right)\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{2,j0,i0}\,g_{3,j0,i0}+\\
&T_1^{-s0}\left(-1+T_1^{s1}\right)T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)g_{1,j1,j0}\,g_{2,i1,i0}\,g_{3,j0,i1}+\\
&T_1^{-s0}\,T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\left(-1+T_2^{s1}\right)g_{1,i1,j0}\,g_{2,j1,i0}\,g_{3,j0,i1}-\\
&T_1^{-s0}\,T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\left(-2+T_1^{s1}+T_2^{s1}\right)g_{1,j1,j0}\,g_{2,j1,i0}\,g_{3,j0,i1}-\\
&T_1^{-s0}\,T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{2,j0,i0}\,g_{3,j0,j0}\\
g_{1,\#1,i0}\,g_{2,\#1,j0}\,g_{3,i0^+,\#1} \to\ & T_1^{-s0}\,T_2^{-s0}\,\chi_{i0==i1}\,g_{1,i0,i0}\,g_{2,i0,j0}+T_1^{-s0}\,T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,i0}\,g_{2,j0,j0}+\\
&T_1^{-s0}\,T_2^{-s0}\left(-1+T_2^{s0}\right)\chi_{i0==i1}\,g_{3,i0,i0}-T_1^{-s0}\,T_2^{-s0}\left(-1+T_2^{s0}\right)\chi_{i0==i1}\,g_{1,i0,i0}\,g_{3,i0,i0}+\\
&T_1^{-s0}\,T_2^{-s0}\left(-2+T_1^{s0}+T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,i0}\,g_{3,i0,i0}-T_1^{-s0}\,T_2^{-s0}\,\chi_{i0==i1}\,g_{2,i0,j0}\,g_{3,i0,i0}-\\
&T_1^{-s0}\,T_2^{-s0}\left(-1+T_2^{s0}\right)\chi_{i0==i1}\,g_{2,j0,j0}\,g_{3,i0,i0}+T_1^{-s0}\left(-1+T_1^{s1}\right)T_2^{-s0}\,g_{1,j1,i0}\,g_{2,i1,j0}\,g_{3,i0,i1}+\\
&T_1^{-s0}\,T_2^{-s0}\left(-1+T_2^{s1}\right)g_{1,i1,i0}\,g_{2,j1,j0}\,g_{3,i0,i1}-T_1^{-s0}\,T_2^{-s0}\left(-2+T_1^{s1}+T_2^{s1}\right)g_{1,j1,i0}\,g_{2,j1,j0}\,g_{3,i0,i1}-\\
&T_1^{-s0}\,T_2^{-s0}\,\chi_{i0==i1}\,g_{1,j0,i0}\,g_{3,i0,j0}+T_1^{-s0}\,T_2^{-s0}\left(-1+T_2^{s0}\right)\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{3,j0,i0}-\\
&T_1^{-s0}\,T_2^{-s0}\left(-1+T_2^{s0}\right)\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{1,i0,i0}\,g_{3,j0,i0}+\\
&T_1^{-s0}\,T_2^{-s0}\left(-2+T_1^{s0}+T_2^{s0}\right)\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,i0}\,g_{3,j0,i0}-\\
&T_1^{-s0}\,T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{2,i0,j0}\,g_{3,j0,i0}-T_1^{-s0}\,T_2^{-s0}\left(-1+T_2^{s0}\right)\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{2,j0,j0}\,g_{3,j0,i0}+\\
&T_1^{-s0}\left(-1+T_1^{s1}\right)T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)g_{1,j1,i0}\,g_{2,i1,j0}\,g_{3,j0,i1}+\\
&T_1^{-s0}\,T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\left(-1+T_2^{s1}\right)g_{1,i1,i0}\,g_{2,j1,j0}\,g_{3,j0,i1}-\\
&T_1^{-s0}\,T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\left(-2+T_1^{s1}+T_2^{s1}\right)g_{1,j1,i0}\,g_{2,j1,j0}\,g_{3,j0,i1}-\\
&T_1^{-s0}\,T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,i0}\,g_{3,j0,j0}\\
g_{1,\#1,j0}\,g_{2,\#1,j0}\,g_{3,i0^+,\#1} \to\ & T_1^{-s0}\,T_2^{-s0}\,\chi_{i0==i1}\,g_{1,i0,j0}\,g_{2,i0,j0}+T_1^{-s0}\,T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,j0}\,g_{2,j0,j0}-\\
&T_1^{-s0}\,T_2^{-s0}\left(-2+T_1^{s0}+T_2^{s0}\right)\chi_{i0==i1}\,g_{3,i0,i0}-T_1^{-s0}\,T_2^{-s0}\left(-1+T_2^{s0}\right)\chi_{i0==i1}\,g_{1,i0,j0}\,g_{3,i0,i0}+\\
&T_1^{-s0}\,T_2^{-s0}\left(-2+T_1^{s0}+T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,j0}\,g_{3,i0,i0}-T_1^{-s0}\left(-1+T_1^{s0}\right)T_2^{-s0}\,\chi_{i0==i1}\,g_{2,i0,j0}\,g_{3,i0,i0}+\\
&T_1^{-s0}\,T_2^{-s0}\left(-2+T_1^{s0}+T_2^{s0}\right)\chi_{i0==i1}\,g_{2,j0,j0}\,g_{3,i0,i0}+T_1^{-s0}\left(-1+T_1^{s1}\right)T_2^{-s0}\,g_{1,j1,j0}\,g_{2,i1,j0}\,g_{3,i0,i1}+\\
&T_1^{-s0}\,T_2^{-s0}\left(-1+T_2^{s1}\right)g_{1,i1,j0}\,g_{2,j1,j0}\,g_{3,i0,i1}-T_1^{-s0}\,T_2^{-s0}\left(-2+T_1^{s1}+T_2^{s1}\right)g_{1,j1,j0}\,g_{2,j1,j0}\,g_{3,i0,i1}+\\
&T_1^{-s0}\,T_2^{-s0}\,\chi_{i0==i1}\,g_{3,i0,j0}-T_1^{-s0}\,T_2^{-s0}\,\chi_{i0==i1}\,g_{1,j0,j0}\,g_{3,i0,j0}-\\
&T_1^{-s0}\,T_2^{-s0}\,\chi_{i0==i1}\,g_{2,j0,j0}\,g_{3,i0,j0}-T_1^{-s0}\,T_2^{-s0}\left(-2+T_1^{s0}+T_2^{s0}\right)\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{3,j0,i0}-\\
&T_1^{-s0}\,T_2^{-s0}\left(-1+T_2^{s0}\right)\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{1,i0,j0}\,g_{3,j0,i0}+\\
&T_1^{-s0}\,T_2^{-s0}\left(-2+T_1^{s0}+T_2^{s0}\right)\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,j0}\,g_{3,j0,i0}-\\
&T_1^{-s0}\left(-1+T_1^{s0}\right)T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{2,i0,j0}\,g_{3,j0,i0}+\\
&T_1^{-s0}\,T_2^{-s0}\left(-2+T_1^{s0}+T_2^{s0}\right)\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{2,j0,j0}\,g_{3,j0,i0}+\\
&T_1^{-s0}\left(-1+T_1^{s1}\right)T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)g_{1,j1,j0}\,g_{2,i1,j0}\,g_{3,j0,i1}+\\
&T_1^{-s0}\,T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\left(-1+T_2^{s1}\right)g_{1,i1,j0}\,g_{2,j1,j0}\,g_{3,j0,i1}-
\end{aligned}
$$

In[•]:=
```
tw0 = Table[a_i, {i, 8}].bas0 / (T_2 - 1);
tw1 = Table[b_i, {i, 8}].bas1 / (T_2 - 1);
{D_{s0,i0,j0}[tw0], D_{s1,i1,j1}[tw1]}
@[Knot[7, 6], F1 → 0, F2 → D_{s0,i0,j0}[tw0] + D_{s1,i1,j1}[tw1], F3 → 0] // Echo // PolyPlot
```

Out[•]=



Full expression not available (original memory size: 487.4 kB)

KnotTheory: Loading precomputed data in PD4Knots`.

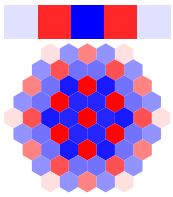
» $\left\{-\frac{1-5\,T+7\,T^2-5\,T^3+T^4}{T^2},\ 0\right\}$

Out[•]=

» $\left\{-\frac{1-5\,T+7\,T^2-5\,T^3+T^4}{T^2},\right.$

$\frac{1}{T_1^4\,T_2^4}\left(1-5\,T_1+7\,T_1^2-5\,T_1^3+T_1^4-5\,T_2+20\,T_1\,T_2-10\,T_1^2\,T_2-10\,T_1^3\,T_2+20\,T_1^4\,T_2-5\,T_1^5\,T_2+7\,T_2^2-\right.$
$10\,T_1\,T_2^2-64\,T_1^2\,T_2^2+98\,T_1^3\,T_2^2-64\,T_1^4\,T_2^2-10\,T_1^5\,T_2^2+7\,T_1^6\,T_2^2-5\,T_2^3-10\,T_1\,T_2^3+98\,T_1^2\,T_2^3-50\,T_1^3\,T_2^3-$
$50\,T_1^4\,T_2^3+98\,T_1^5\,T_2^3-10\,T_1^6\,T_2^3-5\,T_1^7\,T_2^3+T_2^4+20\,T_1\,T_2^4-64\,T_1^2\,T_2^4-50\,T_1^3\,T_2^4+108\,T_1^4\,T_2^4-50\,T_1^5\,T_2^4-$
$64\,T_1^6\,T_2^4+20\,T_1^7\,T_2^4+T_1^8\,T_2^4-5\,T_1\,T_2^5-10\,T_1^2\,T_2^5+98\,T_1^3\,T_2^5-50\,T_1^4\,T_2^5-50\,T_1^5\,T_2^5+98\,T_1^6\,T_2^5-$
$10\,T_1^7\,T_2^5-5\,T_1^8\,T_2^5+7\,T_1^2\,T_2^6-10\,T_1^3\,T_2^6-64\,T_1^4\,T_2^6+98\,T_1^5\,T_2^6-64\,T_1^6\,T_2^6-10\,T_1^7\,T_2^6+7\,T_1^8\,T_2^6-5\,T_1^3\,T_2^7+$
$\left.\left.20\,T_1^4\,T_2^7-10\,T_1^5\,T_2^7-10\,T_1^6\,T_2^7+20\,T_1^7\,T_2^7-5\,T_1^8\,T_2^7+T_1^4\,T_2^8-5\,T_1^5\,T_2^8+7\,T_1^6\,T_2^8-5\,T_1^7\,T_2^8+T_1^8\,T_2^8\right)\right\}$

Out[•]=



In[•]:=
```
res = sRules[Residue[CF[χ_{i0==i1} F1i + F2i + D_{s0,i0,j0}[tw0] + D_{s1,i1,j1}[tw1]], {T_2, 1}] /.
    {g_{3,α_,β_} :→ g_{1,α,β}, g_{2,α_,β_} :→ χ_{α≤β}} ];
```

In[•]:=
```
sRules[res /. {b_5 → 1 - T_1^{s0}, b_7 → T_1^{s0} - 1} /. (a | b)_ → 0]
```

Out[•]=

0

In[•]:=
```
vars = Union@Cases[res, a__ | b__, ∞]
```

Out[•]=

$$\{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8\}$$

In[•]:=
```
cvs = Union@Cases[res, g__ | χ__, ∞]
```

Out[•]=

$$\{\chi_{i0=i1}, \chi_{i0\le i1}, \chi_{i0\le j0}, \chi_{i0\le j1}, \chi_{i1\le j0}, \chi_{j0\le j1}, g_{1,i0,i0}, g_{1,i0,i1},\ g_{1,i0,j0}, g_{1,i1,i0}, g_{1,j0,i0}, g_{1,j0,i1}, g_{1,j0,j0}, g_{1,j0,j1}, g_{1,j1,i0}, g_{1,j1,j0}\}$$

In[•]:=
```
eqns = Sort[CoefficientRules[res, cvs] /. (_ → c_) :> (c == 0)]
```

Out[•]=

$$\Big\{a_5 - a_7 - a_5\,T_1^{-s0} + a_7\,T_1^{-s0} == 0,\ a_1\,T_1^{-s0} - b_1\,T_1^{-s0} == 0,\ a_5\,T_1^{-s0} - b_2\,T_1^{-s0} == 0,$$
$$-a_5 + b_2 - a_1\,T_1^{-s0} + a_5\,T_1^{-s0} + b_1\,T_1^{-s0} - b_2\,T_1^{-s0} == 0,\ a_5 + a_6 - b_2 - b_6 + a_3\,T_1^{-s0} - a_5\,T_1^{-s0} + b_2\,T_1^{-s0} - b_3\,T_1^{-s0} == 0,$$
$$a_7\,T_1^{-s0} - b_4\,T_1^{-s0} == 0,\ -a_7 - a_8 + b_4 + b_8 + a_7\,T_1^{-s0} - b_4\,T_1^{-s0} == 0,\ a_7 + a_8 - b_4 - b_8 - a_7\,T_1^{-s0} + b_4\,T_1^{-s0} == 0,$$
$$-a_5\,T_1^{-s0} - a_7\,T_1^{-s0} + b_2\,T_1^{-s0} + b_4\,T_1^{-s0} == 0,\ a_5 + a_7 - b_2 - b_4 - a_5\,T_1^{-s0} - a_7\,T_1^{-s0} + b_2\,T_1^{-s0} + b_4\,T_1^{-s0} == 0,$$
$$1 + 2\,a_1 + a_2 - a_3 - b_1 - b_5 - 2\,a_1\,T_1^{-s0} + a_3\,T_1^{-s0} + b_1\,T_1^{-s0} - T_1^{s0} == 0,$$
$$-2\,a_1 - a_2 + 2\,a_3 + a_4 + a_1\,T_1^{-s0} - a_3\,T_1^{-s0} + a_1\,T_1^{s0} + a_2\,T_1^{s0} - a_3\,T_1^{s0} - a_4\,T_1^{s0} == 0,$$
$$-1 + a_3 + a_4 - 2\,a_5 - a_6 + 2\,a_7 + a_8 - b_3 - b_7 - a_3\,T_1^{-s0} + a_5\,T_1^{-s0} - a_7\,T_1^{-s0} + b_3\,T_1^{-s0} + T_1^{s0} + a_5\,T_1^{s0} +$$
$$a_6\,T_1^{s0} - a_7\,T_1^{s0} - a_8\,T_1^{s0} == 0,\ -a_1 - a_2 - a_3 - a_4 + 2\,a_5 + a_6 + b_1 - 2\,b_2 + b_3 + b_5 - b_6 + b_7 +$$
$$a_1\,T_1^{-s0} + a_3\,T_1^{-s0} - a_5\,T_1^{-s0} - b_1\,T_1^{-s0} + b_2\,T_1^{-s0} - b_3\,T_1^{-s0} - a_5\,T_1^{s0} - a_6\,T_1^{s0} + b_2\,T_1^{s0} + b_6\,T_1^{s0} == 0,$$
$$a_1 + a_2 + a_3 + a_4 - 2\,a_5 - a_6 - 2\,a_7 - a_8 - b_1 + 2\,b_2 - b_3 + 2\,b_4 - b_5 + b_6 - b_7 + b_8 - a_1\,T_1^{-s0} -$$
$$a_3\,T_1^{-s0} + a_5\,T_1^{-s0} + a_7\,T_1^{-s0} + b_1\,T_1^{-s0} - b_2\,T_1^{-s0} + b_3\,T_1^{-s0} - b_4\,T_1^{-s0} + a_5\,T_1^{s0} + a_6\,T_1^{s0} +$$
$$a_7\,T_1^{s0} + a_8\,T_1^{s0} - b_2\,T_1^{s0} - b_4\,T_1^{s0} - b_6\,T_1^{s0} - b_8\,T_1^{s0} == 0,\ a_1\,T_1^{s0-s1} - a_1\,T_1^{-s1} == 0,$$
$$-a_1\,T_1^{s0-s1} + a_1\,T_1^{-s1} == 0,\ -a_1\,T_1^{s0-s1} + a_1\,T_1^{-s1} == 0,\ a_3\,T_1^{s0-s1} - a_3\,T_1^{-s1} == 0,\ -a_3\,T_1^{s0-s1} + a_3\,T_1^{-s1} == 0,$$
$$a_5\,T_1^{s0-s1} - a_5\,T_1^{-s1} == 0,\ a_5 + a_6 - a_5\,T_1^{s0} - a_6\,T_1^{s0} + a_5\,T_1^{s0-s1} - a_5\,T_1^{-s1} == 0,$$
$$a_5 + a_6 - a_5\,T_1^{s0} - a_6\,T_1^{s0} + a_5\,T_1^{s0-s1} - a_5\,T_1^{-s1} == 0,\ -a_5\,T_1^{s0-s1} + a_5\,T_1^{-s1} == 0,$$
$$-a_5\,T_1^{s0-s1} + a_5\,T_1^{-s1} == 0,\ -a_5 - a_6 + a_5\,T_1^{s0} + a_6\,T_1^{s0} - a_5\,T_1^{s0-s1} + a_5\,T_1^{-s1} == 0,$$
$$a_7\,T_1^{s0-s1} - a_7\,T_1^{-s1} == 0,\ a_7 + a_8 - a_7\,T_1^{s0} - a_8\,T_1^{s0} + a_7\,T_1^{s0-s1} - a_7\,T_1^{-s1} == 0,\ -a_7\,T_1^{s0-s1} + a_7\,T_1^{-s1} == 0,$$
$$-a_7 - a_8 + a_7\,T_1^{s0} + a_8\,T_1^{s0} - a_7\,T_1^{s0-s1} + a_7\,T_1^{-s1} == 0,\ b_1\,T_1^{-s0} - b_1\,T_1^{-s0+s1} == 0,\ -b_1\,T_1^{-s0} + b_1\,T_1^{-s0+s1} == 0,$$
$$b_2\,T_1^{-s0} - b_2\,T_1^{-s0+s1} == 0,\ -b_2 - b_6 + b_2\,T_1^{-s0} + b_2\,T_1^{s1} + b_6\,T_1^{s1} - b_2\,T_1^{-s0+s1} == 0,$$
$$-b_2\,T_1^{-s0} + b_2\,T_1^{-s0+s1} == 0,\ b_2 + b_6 - b_2\,T_1^{-s0} - b_2\,T_1^{s1} - b_6\,T_1^{s1} + b_2\,T_1^{-s0+s1} == 0,$$
$$b_3\,T_1^{-s0} - b_3\,T_1^{-s0+s1} == 0,\ -b_3\,T_1^{-s0} + b_3\,T_1^{-s0+s1} == 0,\ -b_3\,T_1^{-s0} + b_3\,T_1^{-s0+s1} == 0,$$
$$b_4\,T_1^{-s0} - b_4\,T_1^{-s0+s1} == 0,\ -b_4 - b_8 + b_4\,T_1^{-s0} + b_4\,T_1^{s1} + b_8\,T_1^{s1} - b_4\,T_1^{-s0+s1} == 0,$$
$$-b_4 - b_8 + b_4\,T_1^{-s0} + b_4\,T_1^{s1} + b_8\,T_1^{s1} - b_4\,T_1^{-s0+s1} == 0,\ -b_4\,T_1^{-s0} + b_4\,T_1^{-s0+s1} == 0,$$
$$-b_4\,T_1^{-s0} + b_4\,T_1^{-s0+s1} == 0,\ b_4 + b_8 - b_4\,T_1^{-s0} - b_4\,T_1^{s1} - b_8\,T_1^{s1} + b_4\,T_1^{-s0+s1} == 0,$$
$$-1 + a_1 + a_2 + b_1 + b_5 - b_1\,T_1^{-s0} + T_1^{s0} - a_1\,T_1^{s0} - a_2\,T_1^{s0} + a_1\,T_1^{s0-s1} - a_1\,T_1^{-s1} +$$
$$T_1^{s1} - b_1\,T_1^{s1} - b_5\,T_1^{s1} + b_1\,T_1^{-s0+s1} - T_1^{s0+s1} == 0,\ -1 + a_1 + a_2 - b_3 - b_7 + b_3\,T_1^{-s0} + T_1^{s0} -$$
$$a_1\,T_1^{s0} - a_2\,T_1^{s0} + a_1\,T_1^{s0-s1} - a_1\,T_1^{-s1} + T_1^{s1} + b_3\,T_1^{s1} + b_7\,T_1^{s1} - b_3\,T_1^{-s0+s1} - T_1^{s0+s1} == 0,$$
$$-1 - a_3 - a_4 - b_3 - b_7 + b_3\,T_1^{-s0} + T_1^{s0} + a_3\,T_1^{s0} + a_4\,T_1^{s0} - a_3\,T_1^{s0-s1} + a_3\,T_1^{-s1} + T_1^{s1} + b_3\,T_1^{s1} + b_7\,T_1^{s1} -$$
$$b_3\,T_1^{-s0+s1} - T_1^{s0+s1} == 0,\ 1 + a_3 + a_4 - b_1 - b_5 + b_1\,T_1^{-s0} - T_1^{s0} - a_3\,T_1^{s0} - a_4\,T_1^{s0} + a_3\,T_1^{s0-s1} -$$
$$a_3\,T_1^{-s1} - T_1^{s1} + b_1\,T_1^{s1} + b_5\,T_1^{s1} - b_1\,T_1^{-s0+s1} + T_1^{s0+s1} == 0,\ 1 - a_1 - a_2 + b_3 + b_7 - b_3\,T_1^{-s0} -$$
$$T_1^{s0} + a_1\,T_1^{s0} + a_2\,T_1^{s0} - a_1\,T_1^{s0-s1} + a_1\,T_1^{-s1} - T_1^{s1} - b_3\,T_1^{s1} - b_7\,T_1^{s1} + b_3\,T_1^{-s0+s1} + T_1^{s0+s1} == 0\Big\}$$

In[•]:=
```
{sol} = Solve[eqns, vars]
```

Out[•]=

$$\Big\{\{a_1 \to 0,\ a_2 \to 0,\ a_3 \to 0,\ a_4 \to 0,\ a_5 \to 0,\ a_6 \to 0,\ a_7 \to 0,\ a_8 \to 0,$$
$$b_1 \to 0,\ b_2 \to 0,\ b_3 \to 0,\ b_4 \to 0,\ b_5 \to 1 - T_1^{s0},\ b_6 \to 0,\ b_7 \to -1 + T_1^{s0},\ b_8 \to 0\}\Big\}$$

In[•]:=
```
nF2 = CF[CF[(χ[i0==i1] F1i + F2i + D[{s0,i0,j0}][tw0] + D[{s1,i1,j1}][tw1]) /. sol]];
{nF1, nF2} = Simplify@{Coefficient[nF2, χ[i0==i1]], nF2 /. χ[i0==i1] → 0}
Table[Θ[K] == Θ[K, F1 → nF1, F2 → nF2], {K, AllKnots[{3, 8}]}]
```

Out[•]=

$$\left\{\frac{s0}{2}-\frac{T_1^{-s1}\left(-1+T_1^{s0}\right)T_2^{-s1}\left(-2+T_1^{s1}+T_2^{s1}\right)\chi_{i0==j1}\chi_{i1==j0}}{-1+T_2}+\frac{T_1^{-s1}\left(-1+T_1^{s0}\right)T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{i1==j0}\chi_{j0==j1}}{-1+T_2}+\right.$$
$$\frac{T_1^{-s1}\left(-1+T_1^{s0}\right)T_2^{-s1}\chi_{i1==j0}\,g_{1,i1,i0}}{-1+T_2}+\frac{T_1^{-s1}\left(-1+T_1^{s0}\right)\left(-1+T_1^{s1}\right)T_2^{-s1}\chi_{i1==j0}\,g_{1,j1,i0}}{-1+T_2}+$$
$$\frac{T_1^{-s1}\left(-1+T_1^{s0}\right)T_2^{-s1}\chi_{i1==j0}\,g_{2,i1,i0}}{-1+T_2}-\frac{T_1^{-s1}\left(-1+T_1^{s0}\right)T_2^{-s1}\chi_{i1==j0}\,g_{2,i1,j0}}{-1+T_2}+$$
$$s0\,T_2^{s0}\,g_{1,i0,i0}\,g_{2,j0,i0}+\frac{s0\left(-1+T_1^{s0}\right)T_2^{2\,s0}\,g_{1,j0,i0}\,g_{2,j0,i0}}{-1+T_2^{s0}}-s0\,g_{1,i0,i0}\,g_{2,j0,j0}-$$
$$\frac{s0\left(-1+T_1^{s0}\right)T_2^{s0}\,g_{1,j0,i0}\,g_{2,j0,j0}}{-1+T_2^{s0}}+\frac{T_1^{-s1}\left(-1+T_1^{s0}\right)T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{i1==j0}\,g_{2,j1,i0}}{-1+T_2}-$$
$$\frac{T_1^{-s1}\left(-1+T_1^{s0}\right)T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{i1==j0}\,g_{2,j1,j0}}{-1+T_2}-s0\,g_{3,i0,i0}-s0\left(-1+T_2^{s0}\right)g_{2,j0,i0}\,g_{3,i0,i0}+$$
$$2\,s0\,g_{2,j0,j0}\,g_{3,i0,i0}+\frac{s0\left(-1+T_1^{s0}\,T_2^{s0}\right)g_{3,j0,i0}}{-1+T_2^{s0}}-\frac{s0\,T_2^{s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)g_{1,i0,i0}\,g_{3,j0,i0}}{-1+T_2^{s0}}-$$
$$\frac{s0\left(-1+T_1^{s0}\right)\left(1+T_2^{s0}\right)\left(-1+T_1^{s0}\,T_2^{s0}\right)g_{1,j0,i0}\,g_{3,j0,i0}}{-1+T_2^{s0}}+\frac{s0\left(-1+T_1^{s0}\,T_2^{s0}\right)g_{2,i0,j0}\,g_{3,j0,i0}}{-1+T_2^{s0}}+$$
$$s0\left(-1+T_1^{s0}\,T_2^{s0}\right)g_{2,j0,i0}\,g_{3,j0,i0}+\frac{s0\left(-2+T_2^{s0}\right)\left(-1+T_1^{s0}\,T_2^{s0}\right)g_{2,j0,j0}\,g_{3,j0,i0}}{-1+T_2^{s0}}+$$
$$s0\,g_{1,i0,i0}\,g_{3,j0,j0}+\frac{s0\left(-1+T_1^{s0}\right)T_2^{s0}\,g_{1,j0,i0}\,g_{3,j0,j0}}{-1+T_2^{s0}}-s0\,g_{2,i0,i0}\,g_{3,j0,j0}-$$
$$s0\,T_2^{s0}\,g_{2,j0,i0}\,g_{3,j0,j0}-\frac{\left(-1+T_1^{s0}\right)g_{3,j0^+,i1}}{-1+T_2}-\frac{\left(-1+T_1^{s0}\right)\left(-2+T_1^{s1}+T_2^{s1}\right)\chi_{i0==j1}\,g_{3,j0^+,i1}}{-1+T_2}+$$
$$\frac{\left(-1+T_1^{s0}\right)\chi_{i1==j0}\,g_{3,j0^+,i1}}{-1+T_2}+\frac{\left(-1+T_1^{s0}\right)\left(-1+T_2^{s1}\right)\chi_{j0==j1}\,g_{3,j0^+,i1}}{-1+T_2}+\frac{\left(-1+T_1^{s0}\right)g_{1,i1,i0}\,g_{3,j0^+,i1}}{-1+T_2}+$$
$$\frac{\left(-1+T_1^{s0}\right)\left(-1+T_1^{s1}\right)g_{1,j1,i0}\,g_{3,j0^+,i1}}{-1+T_2}+\frac{\left(-1+T_1^{s0}\right)g_{2,i1,i0}\,g_{3,j0^+,i1}}{-1+T_2}-\frac{\left(-1+T_1^{s0}\right)g_{2,i1,j0}\,g_{3,j0^+,i1}}{-1+T_2}+$$
$$\left.\frac{\left(-1+T_1^{s0}\right)\left(-1+T_2^{s1}\right)g_{2,j1,i0}\,g_{3,j0^+,i1}}{-1+T_2}-\frac{\left(-1+T_1^{s0}\right)\left(-1+T_2^{s1}\right)g_{2,j1,j0}\,g_{3,j0^+,i1}}{-1+T_2}\right\},$$

$$\frac{1}{-1+T_2}\left(-1+T_1^{s0}\right)\left(-T_1^{-s1}\,T_2^{-s1}\,\chi_{i1==j0}\left(g_{1,i1,i0}+\left(-1+T_1^{s1}\right)g_{1,j1,i0}\right)\left(1+\left(-1+T_2^{s1}\right)\chi_{j0==j1}+\right.\right.$$
$$\left.g_{2,i1,i0}-g_{2,i1,j0}-g_{2,j1,i0}+T_2^{s1}\,g_{2,j1,i0}+g_{2,j1,j0}-T_2^{s1}\,g_{2,j1,j0}+T_1^{s1}\,T_2^{s1}\,g_{3,j0^+,i1}\right)+$$
$$\frac{1}{-1+T_2^{s1}}\left(-s1\,g_{1,j1,i0}\,g_{2,i1,j0}\,g_{3,j0,i1}+s1\,T_2^{s0}\,g_{1,j1,i0}\left(g_{2,i1,i0}-g_{2,j1,i0}\right)g_{3,j0,i1}-\right.$$

$$\begin{aligned}
&s1\, T_2^{1+s0}\, g_{1,j1,i0}\,(g_{2,i1,i0}-g_{2,j1,i0})\, g_{3,j0,i1} - s1\, T_1^{s1}\, T_2^{s0+s1}\, g_{1,j1,i0}\,(g_{2,i1,i0}-g_{2,j1,i0})\, g_{3,j0,i1} + \\
&s1\, T_1^{s1}\, T_2^{1+s0+s1}\, g_{1,j1,i0}\,(g_{2,i1,i0}-g_{2,j1,i0})\, g_{3,j0,i1} + s1\, T_2\, g_{1,j1,i0}\,(g_{2,i1,j0}-g_{2,j1,j0})\, g_{3,j0,i1} - \\
&s1\, T_1^{s1}\, T_2^{1+s1}\, g_{1,j1,i0}\,(g_{2,i1,j0}-g_{2,j1,j0})\, g_{3,j0,i1} + s1\, g_{1,j1,i0}\, g_{2,j1,j0}\, g_{3,j0,i1} - \\
&g_{1,j1,i0}\, g_{2,i1,i0}\, g_{3,j0^+,i1} + T_1^{s1}\, g_{1,j1,i0}\, g_{2,i1,i0}\, g_{3,j0^+,i1} + g_{1,j1,i0}\, g_{2,i1,j0}\, g_{3,j0^+,i1} - \\
&T_1^{s1}\, g_{1,j1,i0}\, g_{2,i1,j0}\, g_{3,j0^+,i1} - g_{1,i1,i0}\, g_{2,j1,i0}\, g_{3,j0^+,i1} + 2\, g_{1,j1,i0}\, g_{2,j1,i0}\, g_{3,j0^+,i1} - \\
&T_1^{s1}\, g_{1,j1,i0}\, g_{2,j1,i0}\, g_{3,j0^+,i1} - T_2^{2\,s1}\,(g_{1,i1,i0}-g_{1,j1,i0})\,(g_{2,j1,i0}-g_{2,j1,j0})\, g_{3,j0^+,i1} + \\
&g_{1,i1,i0}\, g_{2,j1,j0}\, g_{3,j0^+,i1} - 2\, g_{1,j1,i0}\, g_{2,j1,j0}\, g_{3,j0^+,i1} + T_1^{s1}\, g_{1,j1,i0}\, g_{2,j1,j0}\, g_{3,j0^+,i1} + \\
&T_2^{s1}\Big((2\, g_{1,i1,i0}\,(g_{2,j1,i0}-g_{2,j1,j0}) + g_{1,j1,i0}\,(g_{2,i1,i0}-g_{2,i1,j0}-3\, g_{2,j1,i0}+3\, g_{2,j1,j0}))\\
&\quad g_{3,j0^+,i1} + T_1^{s1}\, g_{1,j1,i0}\,((-g_{2,i1,i0}+g_{2,j1,i0})\, g_{3,j0^+,i1} + g_{2,i1,j0}\,(s1\, g_{3,j0,i1}+g_{3,j0^+,i1}) - \\
&\quad g_{2,j1,j0}\,(s1\, g_{3,j0,i1}+g_{3,j0^+,i1}))\Big) + \Big(-1+T_2^{s1}\Big)\,\chi_{j0==j1}\Big(-\Big(\Big(-1+T_2^{s1}\Big)\, g_{1,i1,i0}\, g_{3,j0^+,i1}\Big) + \\
&\quad g_{1,j1,i0}\Big(-1-g_{2,j1,i0}+g_{2,j1,j0}-2\, g_{3,j0^+,i1}+T_1^{s1}\, g_{3,j0^+,i1}+T_2^{s1}\, g_{3,j0^+,i1}-g_{3,j0^+,j1}\Big)\Big)\Big) + \\
&\chi_{i0==j1}\Big(\Big(-2-g_{1,i1,i0}+2\, g_{1,j1,i0}-g_{2,i1,i0}+g_{2,i1,j0}+2\, g_{2,j1,i0}-T_1^{s1}\,(-1+g_{1,j1,i0}-g_{2,i1,i0}+ \\
&\quad g_{2,i1,j0}+g_{2,j1,i0}-g_{2,j1,j0})-2\, g_{2,j1,j0}+T_2^{s1}\,(1+g_{1,i1,i0}-g_{1,j1,i0}-g_{2,j1,i0}+g_{2,j1,j0})\Big) \\
&\quad g_{3,j0^+,i1} + T_1^{-s1}\, T_2^{-s1}\,\chi_{i1==j0}\Big(-2+\chi_{j0==j1}-g_{1,i1,i0}+g_{1,j1,i0}-g_{2,i1,i0}+g_{2,i1,j0}+g_{2,j1,i0}- \\
&\quad T_2^{s1}\,(-1+\chi_{j0==j1}-g_{1,i1,i0}+g_{1,j1,i0}+g_{2,j1,i0}-g_{2,j1,j0})-g_{2,j1,j0}+T_1^{s1}\Big(2-\chi_{j0==j1}-g_{1,j1,i0}+ \\
&\quad g_{2,i1,i0}-g_{2,i1,j0}-g_{2,j1,i0}+g_{2,j1,j0}+T_2^{s1}\,(-1+\chi_{j0==j1}+g_{1,j1,i0}+g_{2,j1,i0}-g_{2,j1,j0}- \\
&\quad g_{3,j0^+,i1})\Big)+T_1^{2\,s1}\, T_2^{s1}\, g_{3,j0^+,i1}\Big)+(-1+g_{1,j1,i0}+g_{2,j1,i0}-g_{2,j1,j0})\, g_{3,j0^+,j1}+ \\
&\chi_{j0==j1}\Big(g_{1,j1,i0}+g_{2,j1,i0}-g_{2,j1,j0}+2\, g_{3,j0^+,i1}-T_1^{s1}\, g_{3,j0^+,i1}-T_2^{s1}\, g_{3,j0^+,i1}+g_{3,j0^+,j1}\Big)\Big)\Big)\Big\}
\end{aligned}$$

Out[•]=

{True, True, True, True, True, True, True, True, True, True, True,
True, True, True, True, True, True, True, True, True, True, True, True,
True, True, True, True, True, True, True, True, True, True, True, True}