```
In[•]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\Theta"];
        Once[<< Theta.m]
        SetOptions[PolyPlot, ImageSize → Tiny];
        Clear[Θ]
```

```
Loading KnotTheory` version of October 29, 2024, 10:29:52.1301.
Read more at http://katlas.org/wiki/KnotTheory.
```

```
In[•]:= eq = T2 - 1 == s0 T2^((1-s0)/2) (T2^s0 - 1); Simplify[{eq /. s0 → 1, eq /. s0 → -1}]
```

```
Out[•]=
{True, True}
```

```
In[•]:= RandomVK[n_] := {
          Prepend[#, 2 RandomInteger[1] - 1] & /@
           Partition[PermutationList[RandomPermutation[2 n], 2 n], 2],
          Table[RandomInteger[{-1, 1}], 2 n + 1]
        };
```

```
In[•]:= RandomVK[5]
```

```
Out[•]=
{{{1, 7, 3}, {-1, 6, 2}, {1, 4, 9}, {-1, 10, 1}, {1, 5, 8}},
 {-1, -1, 0, 1, 0, -1, 1, 1, -1, 1, 0}}
```

```
In[•]:= CF[ℰ_] := Expand@Collect[ℰ, g__ | χ__, F] /. F → Factor@*PowerExpand;
```

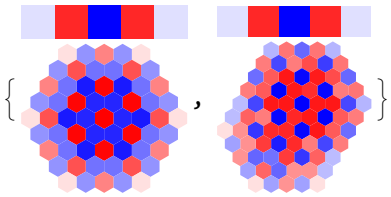
```
In[•]:= Short[Options[Θ] = {F1 → (F1i = F1[{s0, i0, j0}]),
          F2 → (F2i = F2[{s0, i0, j0}, {s1, i1, j1}]), F3 → (F3i = F3[φ, k])}]
```

```
Out[•]//Short=
{F1 → s0/2 + s0 T2^s0 g1,i0,i0 g2,j0,i0 + <<19>> + <<1>>/<<1>> - s0 g2,i0,i0 g3,j0,j0 - s0 T2^s0 g2,j0,i0 g3,j0,j0,
 F2 → <<1>>, F3 → -φ/2 + φ g<<1>>}
```

In[•]:=
```
Θ[K_, opts___Rule] := Module[{X, φφ, n, A, Δ, G, ev, ϴ, kk, k0, k1, f1, f2, f3},
  f1 = F1 /. {opts} /. Options[Θ];
  f2 = F2 /. {opts} /. Options[Θ];
  f3 = F3 /. {opts} /. Options[Θ];
  {X, φφ} = Rot[K]; n = Length[X]; A = IdentityMatrix[2 n + 1];
  Cases[X, {s_, i_, j_} :> (A[[{i, j}, {i + 1, j + 1}]] += ({{-T^s, T^s - 1}, {0, -1}}))];
  Δ = T^((-Total[φφ] - Total[X[[All, 1]]])/2) Det[A];
  G = Inverse[A];
  ev[ℰ_] := Factor[
    ℰ /. {k_^+ :> k + 1, $ -> 2 n + 1} /. {g_ν_,α_,β_ :> (G[[α, β]] /. T -> T_ν), χ_True -> 1, χ_False -> 0}];
  Θ = ev@Sum[f1 /. Thread[{s0, i0, j0} -> X[[kk]]] ⋃ Thread[{s1, i1, j1} -> X[[kk]]], {kk, n}];
  Θ += ev@Sum[f2 /.
      Thread[{s0, i0, j0} -> X[[k0]]] ⋃ Thread[{s1, i1, j1} -> X[[k1]]], {k0, n}, {k1, n}];
  Θ += ev@Sum[f3 /. {φ -> φφ[[kk]], k -> kk}, {kk, Length@φφ}];
  Factor@{Δ, (Δ /. T -> T_1) (Δ /. T -> T_2) (Δ /. T -> T_3) Θ}
 ];
```

In[•]:=
```
PolyPlot /@ {Θ[Knot[7, 6]], Θ[Knot[7, 6], F3 -> 0]}
```

Out[•]=



In[•]:=
```
δ_i_,j_ := χ_i==j;
χ_True = 1; χ_False = 0;
χ_α_==β_ /; OrderedQ[{β, α}] := χ_β==α;
χ^p_ph_ /; p > 1 ^:= χ_ph;
χ_i0==$ = χ_j0==$ = χ_i1==$ = χ_j1==$ = 0;
χ_α_^+==1 = χ_1==α_^+ = 0;
χ_α_^+==β_^+ := χ_α==β;
χ_α_≤α_ = 1;
χ_α_^+≤α_ = 0;
χ_α_^+≤β_ := χ_α≤β - χ_α==β;
```

In[•]:=
```
bRules[{s_, i_, j_}] := { (* b for "push indices backwards" *)
   g_ν_,j^+,β_ :> g_ν,j,β - δ_j,β, g_ν_,i^+,β_ :> T_ν^-s g_ν,i,β + (1 - T_ν^-s) g_ν,j,β - T_ν^-s δ_i,β - (1 - T_ν^-s) δ_j,β,
   g_ν_,α_,i^+ :> T_ν^s g_ν,α,i + δ_α,i^+, g_ν_,α_,j^+ :> g_ν,α,j + (1 - T_ν^s) g_ν,α,i + δ_α,j^+
  };
bRules[X___List] := Union @@ Table[bRules[c], {c, {X}}]
```

In[•]:=
```
Expand@{{g_{ν,i,β}, g_{ν,j,β}} /. gRules[{s, i, j}],
  {g_{ν,i,β}, g_{ν,j,β}} /. gRules[{s, i, j}] /. bRules[{s, i, j}]}
```

Out[•]=
$$\{\{\chi_{i==\beta} + T_\nu^s\, g_{\nu,i^+,\beta} + g_{\nu,j^+,\beta} - T_\nu^s\, g_{\nu,j^+,\beta},\ \chi_{j==\beta} + g_{\nu,j^+,\beta}\},\ \{g_{\nu,i,\beta},\ g_{\nu,j,\beta}\}\}$$

In[•]:=
```
sRules[ℰ_] := FixedPoint[CF[# /. bRules[{s0, i0, j0}]] ⋃ bRules[{s1, i1, j1}] ⋃ {
        χ_{i0==j1} → 0, χ_{i1==j0} → 0, χ_{i0==j0} → 0,
        χ_{j0==j1} → χ_{i0==i1},
        χ_{i1^+≤i0} → χ_{i1≤i0} - χ_{i0==i1}, χ_{i0^+==i1^+} → χ_{i0==i1},
        χ_{i0==i1} χ_{i1≤i0} → χ_{i0==i1}, χ_{i0==i1} χ_{j1≤j0} → χ_{i0==i1},
        χ_{j0≤i0} → 1 - χ_{i0≤j0}, χ_{i1≤i0} → 1 - χ_{i0≤i1} + χ_{i1==i0},
        χ_{j1≤i0} → 1 - χ_{i0≤j1} + χ_{j1==i0}, χ_{i1≤i0} → 1 - χ_{i0≤i1} + χ_{i1==i0}, χ_{j1≤j0} → 1 - χ_{j0≤j1} + χ_{j1==j0},
        χ_{i0==i1} γ_ /; ! FreeQ[γ, s1 | i1 | j1] :> χ_{i0==i1} (γ /. {s1 → s0, i1 → i0, j1 → j0})
       }] &,
  ℰ]
```

In[•]:=
```
D_{s_,i_,j_}[ℰ_] :=
  CF[((ℰ /. # → i^+) + (ℰ /. # → j^+) - (ℰ /. # → i) - (ℰ /. # → j)) //. bRules[{s, i, j}]];
B[ℰ_] := CF[((ℰ /. # → $) - (ℰ /. # → 1)) /. {g_{_,$,β_} → χ_{$==β}, g_{_,_,$} → 1, g_{_,α_,1} → χ_{α==1}}];
```

In[•]:=
```
D_{s1,i1,j1}[g_{1,#,i0}]
```

Out[•]=
$$-T_1^{-s1}\,\chi_{i0==i1} - T_1^{-s1}\left(-1 + 2\,T_1^{s1}\right)\chi_{i0==j1} - T_1^{-s1}\left(-1 + T_1^{s1}\right) g_{1,i1,i0} + T_1^{-s1}\left(-1 + T_1^{s1}\right) g_{1,j1,i0}$$

In[•]:=
```
D_{s1,i1,j1}[g_{3,j0,#}]
```

Out[•]=
$$\chi_{j0==i1^+} + \chi_{j0==j1^+}$$

In[•]:=
```
B[g_{3,j0,#}]
```

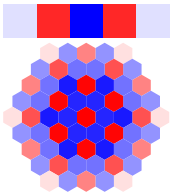
Out[•]=
$$-g_{3,j0,1} + g_{3,j0,\$}$$

In[•]:=
```
tw = g_{1,#,i0};
Θ[Knot[7, 6], F1 → F1i - B[tw], F2 → F2i + D_{s1,i1,j1}[tw]] // Echo // PolyPlot
```

» $\left\{-\frac{1-5\,T+7\,T^2-5\,T^3+T^4}{T^2},\right.$

$$\frac{1}{T_1^4\,T_2^4}\left(1-5\,T_1+7\,T_1^2-5\,T_1^3+T_1^4-5\,T_2+20\,T_1\,T_2-10\,T_1^2\,T_2-10\,T_1^3\,T_2+20\,T_1^4\,T_2-5\,T_1^5\,T_2+7\,T_2^2-\right.$$
$$10\,T_1\,T_2^2-64\,T_1^2\,T_2^2+98\,T_1^3\,T_2^2-64\,T_1^4\,T_2^2-10\,T_1^5\,T_2^2+7\,T_1^6\,T_2^2-5\,T_2^3-10\,T_1\,T_2^3+98\,T_1^2\,T_2^3-50\,T_1^3\,T_2^3-$$
$$50\,T_1^4\,T_2^3+98\,T_1^5\,T_2^3-10\,T_1^6\,T_2^3-5\,T_1^7\,T_2^3+T_2^4+20\,T_1\,T_2^4-64\,T_1^2\,T_2^4-50\,T_1^3\,T_2^4+108\,T_1^4\,T_2^4-50\,T_1^5\,T_2^4-$$
$$64\,T_1^6\,T_2^4+20\,T_1^7\,T_2^4+T_1^8\,T_2^4-5\,T_1\,T_2^5-10\,T_1^2\,T_2^5+98\,T_1^3\,T_2^5-50\,T_1^4\,T_2^5-50\,T_1^5\,T_2^5+98\,T_1^6\,T_2^5-$$
$$10\,T_1^7\,T_2^5-5\,T_1^8\,T_2^5+7\,T_1^2\,T_2^6-10\,T_1^3\,T_2^6-64\,T_1^4\,T_2^6+98\,T_1^5\,T_2^6-64\,T_1^6\,T_2^6-10\,T_1^7\,T_2^6+7\,T_1^8\,T_2^6-5\,T_1^3\,T_2^7+$$
$$\left.\left.20\,T_1^4\,T_2^7-10\,T_1^5\,T_2^7-10\,T_1^6\,T_2^7+20\,T_1^7\,T_2^7-5\,T_1^8\,T_2^7+T_1^4\,T_2^8-5\,T_1^5\,T_2^8+7\,T_1^6\,T_2^8-5\,T_1^7\,T_2^8+T_1^8\,T_2^8\right)\right\}$$

Out[•]=



In[•]:=
```
tw = g_{1,j0,#};
Θ[Knot[7, 6], F1 → -B[tw], F2 → D_{s1,i1,j1}[tw], F3 → 0] // Echo // PolyPlot
Θ[Knot[7, 6], F1 → F1i - B[tw], F2 → F2i + D_{s1,i1,j1}[tw]] // Echo // PolyPlot
```

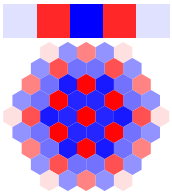
» $\left\{-\frac{1-5\,T+7\,T^2-5\,T^3+T^4}{T^2},\,0\right\}$

Out[•]=

» $\left\{-\frac{1-5\,T+7\,T^2-5\,T^3+T^4}{T^2},\right.$

$$\frac{1}{T_1^4\,T_2^4}\left(1-5\,T_1+7\,T_1^2-5\,T_1^3+T_1^4-5\,T_2+20\,T_1\,T_2-10\,T_1^2\,T_2-10\,T_1^3\,T_2+20\,T_1^4\,T_2-5\,T_1^5\,T_2+7\,T_2^2-\right.$$
$$10\,T_1\,T_2^2-64\,T_1^2\,T_2^2+98\,T_1^3\,T_2^2-64\,T_1^4\,T_2^2-10\,T_1^5\,T_2^2+7\,T_1^6\,T_2^2-5\,T_2^3-10\,T_1\,T_2^3+98\,T_1^2\,T_2^3-50\,T_1^3\,T_2^3-$$
$$50\,T_1^4\,T_2^3+98\,T_1^5\,T_2^3-10\,T_1^6\,T_2^3-5\,T_1^7\,T_2^3+T_2^4+20\,T_1\,T_2^4-64\,T_1^2\,T_2^4-50\,T_1^3\,T_2^4+108\,T_1^4\,T_2^4-50\,T_1^5\,T_2^4-$$
$$64\,T_1^6\,T_2^4+20\,T_1^7\,T_2^4+T_1^8\,T_2^4-5\,T_1\,T_2^5-10\,T_1^2\,T_2^5+98\,T_1^3\,T_2^5-50\,T_1^4\,T_2^5-50\,T_1^5\,T_2^5+98\,T_1^6\,T_2^5-$$
$$10\,T_1^7\,T_2^5-5\,T_1^8\,T_2^5+7\,T_1^2\,T_2^6-10\,T_1^3\,T_2^6-64\,T_1^4\,T_2^6+98\,T_1^5\,T_2^6-64\,T_1^6\,T_2^6-10\,T_1^7\,T_2^6+7\,T_1^8\,T_2^6-5\,T_1^3\,T_2^7+$$
$$\left.\left.20\,T_1^4\,T_2^7-10\,T_1^5\,T_2^7-10\,T_1^6\,T_2^7+20\,T_1^7\,T_2^7-5\,T_1^8\,T_2^7+T_1^4\,T_2^8-5\,T_1^5\,T_2^8+7\,T_1^6\,T_2^8-5\,T_1^7\,T_2^8+T_1^8\,T_2^8\right)\right\}$$

Out[•]=



In[•]:=
```
bas0 = List @@ Expand[(g1,i1⁺,# + g1,j1⁺,#) (g2,i1⁺,# + g2,j1⁺,#) (g3,#,i1 + g3,#,j1)]
```

Out[•]=
$$\{g_{1,i1^+,\#1}\,g_{2,i1^+,\#1}\,g_{3,\#1,i1},\ g_{1,j1^+,\#1}\,g_{2,i1^+,\#1}\,g_{3,\#1,i1},\ g_{1,i1^+,\#1}\,g_{2,j1^+,\#1}\,g_{3,\#1,i1},\ g_{1,j1^+,\#1}\,g_{2,j1^+,\#1}\,g_{3,\#1,i1},$$
$$g_{1,i1^+,\#1}\,g_{2,i1^+,\#1}\,g_{3,\#1,j1},\ g_{1,j1^+,\#1}\,g_{2,i1^+,\#1}\,g_{3,\#1,j1},\ g_{1,i1^+,\#1}\,g_{2,j1^+,\#1}\,g_{3,\#1,j1},\ g_{1,j1^+,\#1}\,g_{2,j1^+,\#1}\,g_{3,\#1,j1}\}$$

In[•]:=
```
B /@ bas0
```

Out[•]=
$$\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}$$

In[•]:=
```
Column[(# → Θ[Knot[7, 6], F1 → -B[#], F2 → D{s0,i0,j0}[#], F3 → 0][[2]]) & /@ bas0]
```

Out[•]=
$$g_{1,i1^+,\#1}\,g_{2,i1^+,\#1}\,g_{3,\#1,i1} \to 0$$
$$g_{1,j1^+,\#1}\,g_{2,i1^+,\#1}\,g_{3,\#1,i1} \to 0$$
$$g_{1,i1^+,\#1}\,g_{2,j1^+,\#1}\,g_{3,\#1,i1} \to 0$$
$$g_{1,j1^+,\#1}\,g_{2,j1^+,\#1}\,g_{3,\#1,i1} \to 0$$
$$g_{1,i1^+,\#1}\,g_{2,i1^+,\#1}\,g_{3,\#1,j1} \to 0$$
$$g_{1,j1^+,\#1}\,g_{2,i1^+,\#1}\,g_{3,\#1,j1} \to 0$$
$$g_{1,i1^+,\#1}\,g_{2,j1^+,\#1}\,g_{3,\#1,j1} \to 0$$
$$g_{1,j1^+,\#1}\,g_{2,j1^+,\#1}\,g_{3,\#1,j1} \to 0$$

In[•]:=
```
bas1 = List @@ Expand[(g1,#,i0 + g1,#,j0) (g2,#,i0 + g2,#,j0) (g3,i0⁺,# + g3,j0⁺,#)]
```

Out[•]=
$$\{g_{1,\#1,i0}\,g_{2,\#1,i0}\,g_{3,i0^+,\#1},\ g_{1,\#1,j0}\,g_{2,\#1,i0}\,g_{3,i0^+,\#1},\ g_{1,\#1,i0}\,g_{2,\#1,j0}\,g_{3,i0^+,\#1},\ g_{1,\#1,j0}\,g_{2,\#1,j0}\,g_{3,i0^+,\#1},$$
$$g_{1,\#1,i0}\,g_{2,\#1,i0}\,g_{3,j0^+,\#1},\ g_{1,\#1,j0}\,g_{2,\#1,i0}\,g_{3,j0^+,\#1},\ g_{1,\#1,i0}\,g_{2,\#1,j0}\,g_{3,j0^+,\#1},\ g_{1,\#1,j0}\,g_{2,\#1,j0}\,g_{3,j0^+,\#1}\}$$

In[•]:=
```
B /@ bas1
```

Out[•]=
$$\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}$$

In[•]:=
```
Column[(# → Θ[Knot[7, 6], F1 → -B[#], F2 → D{s1,i1,j1}[#], F3 → 0][[2]]) & /@ bas1]
```

Out[•]=
$$g_{1,\#1,i0}\,g_{2,\#1,i0}\,g_{3,i0^+,\#1} \to 0$$
$$g_{1,\#1,j0}\,g_{2,\#1,i0}\,g_{3,i0^+,\#1} \to 0$$
$$g_{1,\#1,i0}\,g_{2,\#1,j0}\,g_{3,i0^+,\#1} \to 0$$
$$g_{1,\#1,j0}\,g_{2,\#1,j0}\,g_{3,i0^+,\#1} \to 0$$
$$g_{1,\#1,i0}\,g_{2,\#1,i0}\,g_{3,j0^+,\#1} \to 0$$
$$g_{1,\#1,j0}\,g_{2,\#1,i0}\,g_{3,j0^+,\#1} \to 0$$
$$g_{1,\#1,i0}\,g_{2,\#1,j0}\,g_{3,j0^+,\#1} \to 0$$
$$g_{1,\#1,j0}\,g_{2,\#1,j0}\,g_{3,j0^+,\#1} \to 0$$

In[•]:=
```
Column[(# → sRules@D{s0,i0,j0}[#]) & /@ bas0]
```

Out[•]=
$$g_{1,i1^+,\#1}\,g_{2,i1^+,\#1}\,g_{3,\#1,i1} \to -T_1^{-s0}\,T_2^{-s0}\,\chi_{i0==i1}\,g_{1,i0,i0}\,g_{2,i0,i0} - T_1^{-s0}\left(-1+T_1^{s0}\right)T_2^{-s0}\,\chi_{i0==i1}\,g_{1,j0,i0}\,g_{2,i0,i0} -$$
$$T_1^{-s0}\,T_2^{-s0}\left(-1+T_2^{s0}\right)\chi_{i0==i1}\,g_{1,i0,i0}\,g_{2,j0,i0} - T_1^{-s0}\left(-1+T_1^{s0}\right)T_2^{-s0}\left(-1+T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,i0}\,g_{2,j0,i0} -$$
$$T_1^{-s0}\,T_2^{-s0}\,\chi_{i0==i1}\,g_{3,i0,i0} + T_1^{-s0}\,T_2^{-s0}\,\chi_{i0==i1}\,g_{1,i0,i0}\,g_{3,i0,i0} +$$
$$T_1^{-s0}\left(-1+T_1^{s0}\right)T_2^{-s0}\,\chi_{i0==i1}\,g_{1,j0,i0}\,g_{3,i0,i0} + T_1^{-s0}\,T_2^{-s0}\,\chi_{i0==i1}\,g_{2,i0,i0}\,g_{3,i0,i0} +$$
$$T_1^{-s0}\,T_2^{-s0}\left(-1+T_2^{s0}\right)\chi_{i0==i1}\,g_{2,j0,i0}\,g_{3,i0,i0} - T_1^{-s0}\left(-1+T_1^{s0}\right)T_2^{-s0}\left(-1+T_2^{s0}\right)\chi_{i0==i1}\,g_{3,j0,i0} +$$
$$T_1^{-s0}\,T_2^{-s0}\left(-1+T_2^{s0}\right)\chi_{i0==i1}\,g_{1,i0,j0}\,g_{3,j0,i0} + T_1^{-s0}\left(-1+T_1^{s0}\right)T_2^{-s0}\left(-1+T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,j0}\,g_{3,j0,i0} +$$
$$T_1^{-s0}\left(-1+T_1^{s0}\right)T_2^{-s0}\,\chi_{i0==i1}\,g_{2,i0,j0}\,g_{3,j0,i0} + T_1^{-s0}\left(-1+T_1^{s0}\right)T_2^{-s0}\left(-1+T_2^{s0}\right)\chi_{i0==i1}\,g_{2,j0,j0}\,g_{3,j0,i0} +$$
$$T_1^{-s1}\,T_2^{-s1}\left(-T_1^{s0}-T_2^{s0}+2\,T_1^{s0}\,T_2^{s0}\right)g_{1,i1,i0}\,g_{2,i1,i0}\,g_{3,j0,i1} - T_1^{-s1}\,T_2^{-s1}\left(-1+T_2^{s0}\right)g_{1,i1,j0}\,g_{2,i1,i0}\,g_{3,j0,i1} +$$
$$T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\left(-T_1^{s0}-T_2^{s0}+2\,T_1^{s0}\,T_2^{s0}\right)g_{1,j1,i0}\,g_{2,i1,i0}\,g_{3,j0,i1} -$$
$$T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\left(-1+T_2^{s0}\right)g_{1,j1,j0}\,g_{2,i1,i0}\,g_{3,j0,i1} -$$
$$T_1^{-s1}\left(-1+T_1^{s0}\right)T_2^{-s1}\,g_{1,i1,i0}\,g_{2,i1,j0}\,g_{3,j0,i1} - T_1^{-s1}\left(-1+T_1^{s0}\right)T_2^{-s1}\left(-1+T_2^{s1}\right)T_2^{-s1}\,g_{1,i1,i0}\,g_{2,i1,i0}\,g_{3,j0,i1} +$$

$$g_{1,j1^+,\#1}\,g_{2,i1^+,\#1}\,g_{3,\#1,i1}\to$$
$$-T_2^{-s0}\,\chi_{i0==i1}\,g_{1,j0,i0}\,g_{2,i0,i0}-T_2^{-s0}\left(-1+T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,i0}\,g_{2,j0,i0}+T_2^{-s0}\,\chi_{i0==i1}\,g_{1,j0,i0}\,g_{3,i0,i0}-$$
$$T_2^{-s0}\left(-1+T_2^{s0}\right)\chi_{i0==i1}\,g_{3,j0,i0}+T_2^{-s0}\left(-1+T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,j0}\,g_{3,j0,i0}+T_2^{-s0}\,\chi_{i0==i1}\,g_{2,i0,j0}\,g_{3,j0,i0}+$$
$$T_2^{-s0}\left(-1+T_2^{s0}\right)\chi_{i0==i1}\,g_{2,j0,j0}\,g_{3,j0,i0}+T_2^{-s1}\left(-T_1^{s0}-T_2^{s0}+2\,T_1^{s0}\,T_2^{s0}\right)g_{1,j1,i0}\,g_{2,i1,i0}\,g_{3,j0,i1}-$$
$$T_2^{-s1}\left(-1+T_2^{s0}\right)g_{1,j1,j0}\,g_{2,i1,i0}\,g_{3,j0,i1}-\left(-1+T_1^{s0}\right)T_2^{-s1}\,g_{1,j1,i0}\,g_{2,i1,j0}\,g_{3,j0,i1}+$$
$$T_2^{-s1}\left(-T_1^{s0}-T_2^{s0}+2\,T_1^{s0}\,T_2^{s0}\right)\left(-1+T_2^{s1}\right)g_{1,j1,i0}\,g_{2,j1,i0}\,g_{3,j0,i1}-$$
$$T_2^{-s1}\left(-1+T_2^{s0}\right)\left(-1+T_2^{s1}\right)g_{1,j1,j0}\,g_{2,j1,i0}\,g_{3,j0,i1}-\left(-1+T_1^{s0}\right)T_2^{-s1}\left(-1+T_2^{s1}\right)g_{1,j1,i0}\,g_{2,j1,j0}\,g_{3,j0,i1}$$

$$g_{1,i1^+,\#1}\,g_{2,j1^+,\#1}\,g_{3,\#1,i1}\to$$
$$-T_1^{-s0}\,\chi_{i0==i1}\,g_{1,i0,i0}\,g_{2,j0,i0}-T_1^{-s0}\left(-1+T_1^{s0}\right)\chi_{i0==i1}\,g_{1,j0,i0}\,g_{2,j0,i0}+T_1^{-s0}\,\chi_{i0==i1}\,g_{2,j0,i0}\,g_{3,i0,i0}-$$
$$T_1^{-s0}\left(-1+T_1^{s0}\right)\chi_{i0==i1}\,g_{3,j0,i0}+T_1^{-s0}\,\chi_{i0==i1}\,g_{1,i0,j0}\,g_{3,j0,i0}+T_1^{-s0}\left(-1+T_1^{s0}\right)\chi_{i0==i1}\,g_{1,j0,j0}\,g_{3,j0,i0}+$$
$$T_1^{-s0}\left(-1+T_1^{s0}\right)\chi_{i0==i1}\,g_{2,j0,j0}\,g_{3,j0,i0}+T_1^{-s1}\left(-T_1^{s0}-T_2^{s0}+2\,T_1^{s0}\,T_2^{s0}\right)g_{1,i1,i0}\,g_{2,j1,i0}\,g_{3,j0,i1}-$$
$$T_1^{-s1}\left(-1+T_2^{s0}\right)g_{1,i1,j0}\,g_{2,j1,i0}\,g_{3,j0,i1}+T_1^{-s1}\left(-1+T_1^{s1}\right)\left(-T_1^{s0}-T_2^{s0}+2\,T_1^{s0}\,T_2^{s0}\right)g_{1,j1,i0}\,g_{2,j1,i0}\,g_{3,j0,i1}-$$
$$T_1^{-s1}\left(-1+T_1^{s1}\right)\left(-1+T_2^{s0}\right)g_{1,j1,j0}\,g_{2,j1,i0}\,g_{3,j0,i1}-$$
$$T_1^{-s1}\left(-1+T_1^{s0}\right)g_{1,i1,i0}\,g_{2,j1,j0}\,g_{3,j0,i1}-T_1^{-s1}\left(-1+T_1^{s0}\right)\left(-1+T_1^{s1}\right)g_{1,j1,i0}\,g_{2,j1,j0}\,g_{3,j0,i1}$$

$$g_{1,j1^+,\#1}\,g_{2,j1^+,\#1}\,g_{3,\#1,i1}\to-\chi_{i0==i1}\,g_{1,j0,i0}\,g_{2,j0,i0}-\chi_{i0==i1}\,g_{3,j0,i0}+$$
$$\chi_{i0==i1}\,g_{1,j0,j0}\,g_{3,j0,i0}+\chi_{i0==i1}\,g_{2,j0,j0}\,g_{3,j0,i0}+\left(-T_1^{s0}-T_2^{s0}+2\,T_1^{s0}\,T_2^{s0}\right)g_{1,j1,i0}\,g_{2,j1,i0}\,g_{3,j0,i1}+$$
$$\left(1-T_2^{s0}\right)g_{1,j1,j0}\,g_{2,j1,i0}\,g_{3,j0,i1}+\left(1-T_1^{s0}\right)g_{1,j1,i0}\,g_{2,j1,j0}\,g_{3,j0,i1}$$

$$
\begin{aligned}
&g_{1,i1^+,\#1}\, g_{2,i1^+,\#1}\, g_{3,\#1,j1} \to \\
&\quad -T_1^{-s0}\, T_2^{-s0} \left(-T_1^{s0} - T_2^{s0} + 2\, T_1^{s0}\, T_2^{s0}\right) \chi_{i0==i1}\, g_{1,i0,i0}\, g_{2,i0,i0} + T_1^{-s0}\, T_2^{-s0} \left(-1 + T_2^{s0}\right) \chi_{i0==i1}\, g_{1,i0,j0}\, g_{2,i0,i0} - \\
&\quad T_1^{-s0} \left(-1 + T_1^{s0}\right) T_2^{-s0} \left(-T_1^{s0} - T_2^{s0} + 2\, T_1^{s0}\, T_2^{s0}\right) \chi_{i0==i1}\, g_{1,j0,i0}\, g_{2,i0,i0} + \\
&\quad T_1^{-s0} \left(-1 + T_1^{s0}\right) T_2^{-s0} \left(-1 + T_2^{s0}\right) \chi_{i0==i1}\, g_{1,j0,j0}\, g_{2,i0,i0} + \\
&\quad T_1^{-s0} \left(-1 + T_1^{s0}\right) T_2^{-s0}\, \chi_{i0==i1}\, g_{1,i0,i0}\, g_{2,i0,j0} - T_1^{-s0}\, T_2^{-s0}\, \chi_{i0==i1}\, g_{1,i0,j0}\, g_{2,i0,j0} + \\
&\quad T_1^{-s0} \left(-1 + T_1^{s0}\right)^2 T_2^{-s0}\, \chi_{i0==i1}\, g_{1,j0,i0}\, g_{2,i0,j0} - T_1^{-s0} \left(-1 + T_1^{s0}\right) T_2^{-s0}\, \chi_{i0==i1}\, g_{1,j0,j0}\, g_{2,i0,j0} - \\
&\quad T_1^{-s0}\, T_2^{-s0} \left(-1 + T_2^{s0}\right) \left(-T_1^{s0} - T_2^{s0} + 2\, T_1^{s0}\, T_2^{s0}\right) \chi_{i0==i1}\, g_{1,i0,i0}\, g_{2,j0,i0} + \\
&\quad T_1^{-s0}\, T_2^{-s0} \left(-1 + T_2^{s0}\right)^2 \chi_{i0==i1}\, g_{1,i0,j0}\, g_{2,j0,i0} - \\
&\quad T_1^{-s0} \left(-1 + T_1^{s0}\right) T_2^{-s0} \left(-1 + T_2^{s0}\right) \left(-T_1^{s0} - T_2^{s0} + 2\, T_1^{s0}\, T_2^{s0}\right) \chi_{i0==i1}\, g_{1,j0,i0}\, g_{2,j0,i0} + \\
&\quad T_1^{-s0} \left(-1 + T_1^{s0}\right) T_2^{-s0} \left(-1 + T_2^{s0}\right)^2 \chi_{i0==i1}\, g_{1,j0,j0}\, g_{2,j0,i0} + \\
&\quad T_1^{-s0} \left(-1 + T_1^{s0}\right) T_2^{-s0} \left(-1 + T_2^{s0}\right) \chi_{i0==i1}\, g_{1,i0,i0}\, g_{2,j0,j0} - T_1^{-s0}\, T_2^{-s0} \left(-1 + T_2^{s0}\right) \chi_{i0==i1}\, g_{1,i0,j0}\, g_{2,j0,j0} + \\
&\quad T_1^{-s0} \left(-1 + T_1^{s0}\right)^2 T_2^{-s0} \left(-1 + T_2^{s0}\right) \chi_{i0==i1}\, g_{1,j0,i0}\, g_{2,j0,j0} - \\
&\quad T_1^{-s0} \left(-1 + T_1^{s0}\right) T_2^{-s0} \left(-1 + T_2^{s0}\right) \chi_{i0==i1}\, g_{1,j0,j0}\, g_{2,j0,j0} - T_1^{-s0}\, T_2^{-s0}\, \chi_{i0==i1}\, g_{3,i0,j0} + \\
&\quad T_1^{-s0}\, T_2^{-s0}\, \chi_{i0==i1}\, g_{1,i0,i0}\, g_{3,i0,j0} + T_1^{-s0} \left(-1 + T_1^{s0}\right) T_2^{-s0}\, \chi_{i0==i1}\, g_{1,j0,i0}\, g_{3,i0,j0} + \\
&\quad T_1^{-s0}\, T_2^{-s0}\, \chi_{i0==i1}\, g_{2,i0,i0}\, g_{3,i0,j0} + T_1^{-s0}\, T_2^{-s0} \left(-1 + T_2^{s0}\right) \chi_{i0==i1}\, g_{2,j0,i0}\, g_{3,i0,j0} - \\
&\quad T_1^{-s0} \left(-1 + T_1^{s0}\right) T_2^{-s0} \left(-1 + T_2^{s0}\right) \chi_{i0==i1}\, g_{3,j0,j0} + T_1^{-s0}\, T_2^{-s0} \left(-1 + T_2^{s0}\right) \chi_{i0==i1}\, g_{1,i0,j0}\, g_{3,j0,j0} + \\
&\quad T_1^{-s0} \left(-1 + T_1^{s0}\right) T_2^{-s0} \left(-1 + T_2^{s0}\right) \chi_{i0==i1}\, g_{1,j0,j0}\, g_{3,j0,j0} + \\
&\quad T_1^{-s0} \left(-1 + T_1^{s0}\right) T_2^{-s0}\, \chi_{i0==i1}\, g_{2,i0,j0}\, g_{3,j0,j0} + T_1^{-s0} \left(-1 + T_1^{s0}\right) T_2^{-s0} \left(-1 + T_2^{s0}\right) \chi_{i0==i1}\, g_{2,j0,j0}\, g_{3,j0,j0} + \\
&\quad T_1^{-s1}\, T_2^{-s1} \left(-T_1^{s0} - T_2^{s0} + 2\, T_1^{s0}\, T_2^{s0}\right) g_{1,i1,i0}\, g_{2,i1,i0}\, g_{3,j0,j1} - T_1^{-s1}\, T_2^{-s1} \left(-1 + T_2^{s0}\right) g_{1,i1,j0}\, g_{2,i1,i0}\, g_{3,j0,j1} + \\
&\quad T_1^{-s1} \left(-1 + T_1^{s1}\right) T_2^{-s1} \left(-T_1^{s0} - T_2^{s0} + 2\, T_1^{s0}\, T_2^{s0}\right) g_{1,j1,i0}\, g_{2,i1,i0}\, g_{3,j0,j1} - \\
&\quad T_1^{-s1} \left(-1 + T_1^{s1}\right) T_2^{-s1} \left(-1 + T_2^{s0}\right) g_{1,j1,j0}\, g_{2,i1,i0}\, g_{3,j0,j1} - \\
&\quad T_1^{-s1} \left(-1 + T_1^{s0}\right) T_2^{-s1}\, g_{1,i1,i0}\, g_{2,i1,j0}\, g_{3,j0,j1} - T_1^{-s1} \left(-1 + T_1^{s0}\right) \left(-1 + T_1^{s1}\right) T_2^{-s1}\, g_{1,j1,i0}\, g_{2,i1,j0}\, g_{3,j0,j1} + \\
&\quad T_1^{-s1}\, T_2^{-s1} \left(-T_1^{s0} - T_2^{s0} + 2\, T_1^{s0}\, T_2^{s0}\right) \left(-1 + T_2^{s1}\right) g_{1,i1,i0}\, g_{2,j1,i0}\, g_{3,j0,j1} - \\
&\quad T_1^{-s1}\, T_2^{-s1} \left(-1 + T_2^{s0}\right) \left(-1 + T_2^{s1}\right) g_{1,i1,j0}\, g_{2,j1,i0}\, g_{3,j0,j1} + \\
&\quad T_1^{-s1} \left(-1 + T_1^{s1}\right) T_2^{-s1} \left(-T_1^{s0} - T_2^{s0} + 2\, T_1^{s0}\, T_2^{s0}\right) \left(-1 + T_2^{s1}\right) g_{1,j1,i0}\, g_{2,j1,i0}\, g_{3,j0,j1} - \\
&\quad T_1^{-s1} \left(-1 + T_1^{s1}\right) T_2^{-s1} \left(-1 + T_2^{s0}\right) \left(-1 + T_2^{s1}\right) g_{1,j1,j0}\, g_{2,j1,i0}\, g_{3,j0,j1} - \\
&\quad T_1^{-s1} \left(-1 + T_1^{s0}\right) T_2^{-s1} \left(-1 + T_2^{s1}\right) g_{1,i1,i0}\, g_{2,j1,j0}\, g_{3,j0,j1} - \\
&\quad T_1^{-s1} \left(-1 + T_1^{s0}\right) \left(-1 + T_1^{s1}\right) T_2^{-s1} \left(-1 + T_2^{s1}\right) g_{1,j1,i0}\, g_{2,j1,j0}\, g_{3,j0,j1}
\end{aligned}
$$

$$
\begin{aligned}
&g_{1,j1^+,\#1}\, g_{2,i1^+,\#1}\, g_{3,\#1,j1} \to \\
&\quad -T_2^{-s0} \left(-T_1^{s0} - T_2^{s0} + 2\, T_1^{s0}\, T_2^{s0}\right) \chi_{i0==i1}\, g_{1,j0,i0}\, g_{2,i0,i0} + T_2^{-s0} \left(-1 + T_2^{s0}\right) \chi_{i0==i1}\, g_{1,j0,j0}\, g_{2,i0,i0} + \\
&\quad \left(-1 + T_1^{s0}\right) T_2^{-s0}\, \chi_{i0==i1}\, g_{1,j0,i0}\, g_{2,i0,j0} - T_2^{-s0}\, \chi_{i0==i1}\, g_{1,j0,j0}\, g_{2,i0,j0} - \\
&\quad T_2^{-s0} \left(-1 + T_2^{s0}\right) \left(-T_1^{s0} - T_2^{s0} + 2\, T_1^{s0}\, T_2^{s0}\right) \chi_{i0==i1}\, g_{1,j0,i0}\, g_{2,j0,i0} + T_2^{-s0} \left(-1 + T_2^{s0}\right)^2 \chi_{i0==i1}\, g_{1,j0,j0}\, g_{2,j0,i0} + \\
&\quad \left(-1 + T_1^{s0}\right) T_2^{-s0} \left(-1 + T_2^{s0}\right) \chi_{i0==i1}\, g_{1,j0,i0}\, g_{2,j0,j0} - T_2^{-s0} \left(-1 + T_2^{s0}\right) \chi_{i0==i1}\, g_{1,j0,j0}\, g_{2,j0,j0} + \\
&\quad T_2^{-s0}\, \chi_{i0==i1}\, g_{1,j0,i0}\, g_{3,i0,j0} - T_2^{-s0} \left(-1 + T_2^{s0}\right) \chi_{i0==i1}\, g_{3,j0,j0} + T_2^{-s0} \left(-1 + T_2^{s0}\right) \chi_{i0==i1}\, g_{1,j0,j0}\, g_{3,j0,j0} + \\
&\quad T_2^{-s0}\, \chi_{i0==i1}\, g_{2,j0,i0}\, g_{3,j0,j0} + T_2^{-s0} \left(-1 + T_2^{s0}\right) \chi_{i0==i1}\, g_{2,j0,j0}\, g_{3,j0,j0} + \\
&\quad T_2^{-s1} \left(-T_1^{s0} - T_2^{s0} + 2\, T_1^{s0}\, T_2^{s0}\right) g_{1,j1,i0}\, g_{2,i1,i0}\, g_{3,j0,j1} - T_2^{-s1} \left(-1 + T_2^{s0}\right) g_{1,j1,j0}\, g_{2,i1,i0}\, g_{3,j0,j1} - \\
&\quad \left(-1 + T_1^{s0}\right) T_2^{-s1}\, g_{1,j1,i0}\, g_{2,i1,j0}\, g_{3,j0,j1} + T_2^{-s1} \left(-T_1^{s0} - T_2^{s0} + 2\, T_1^{s0}\, T_2^{s0}\right) \left(-1 + T_2^{s1}\right) g_{1,j1,i0}\, g_{2,j1,i0}\, g_{3,j0,j1} - \\
&\quad T_2^{-s1} \left(-1 + T_2^{s0}\right) \left(-1 + T_2^{s1}\right) g_{1,j1,j0}\, g_{2,j1,i0}\, g_{3,j0,j1} - \left(-1 + T_1^{s0}\right) T_2^{-s1} \left(-1 + T_2^{s1}\right) g_{1,j1,i0}\, g_{2,j1,j0}\, g_{3,j0,j1}
\end{aligned}
$$

$$g_{1,i1^+,\#1}\,g_{2,j1^+,\#1}\,g_{3,\#1,j1} \to -T_1^{-s0}\left(-T_1^{s0}-T_2^{s0}+2\,T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{1,i0,i0}\,g_{2,j0,i0}+ T_1^{-s0}\left(-1+T_2^{s0}\right)\chi_{i0==i1}\,g_{1,i0,j0}\,g_{2,j0,i0}-T_1^{-s0}\left(-1+T_1^{s0}\right)\left(-T_1^{s0}-T_2^{s0}+2\,T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,i0}\,g_{2,j0,i0}+ T_1^{-s0}\left(-1+T_1^{s0}\right)\left(-1+T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,j0}\,g_{2,j0,i0}+T_1^{-s0}\left(-1+T_1^{s0}\right)\chi_{i0==i1}\,g_{1,i0,i0}\,g_{2,j0,j0}- T_1^{-s0}\chi_{i0==i1}\,g_{1,i0,j0}\,g_{2,j0,j0}+T_1^{-s0}\left(-1+T_1^{s0}\right)^2\chi_{i0==i1}\,g_{1,j0,i0}\,g_{2,j0,j0}- T_1^{-s0}\left(-1+T_1^{s0}\right)\chi_{i0==i1}\,g_{1,j0,j0}\,g_{2,j0,j0}+T_1^{-s0}\chi_{i0==i1}\,g_{2,j0,i0}\,g_{3,i0,j0}- T_1^{-s0}\left(-1+T_1^{s0}\right)\chi_{i0==i1}\,g_{3,j0,j0}+T_1^{-s0}\chi_{i0==i1}\,g_{1,i0,j0}\,g_{3,j0,j0}+T_1^{-s0}\left(-1+T_1^{s0}\right)\chi_{i0==i1}\,g_{1,j0,j0}\,g_{3,j0,j0}+ T_1^{-s0}\left(-1+T_1^{s0}\right)\chi_{i0==i1}\,g_{2,j0,j0}\,g_{3,j0,j0}+T_1^{-s1}\left(-T_1^{s0}-T_2^{s0}+2\,T_1^{s0}\,T_2^{s0}\right)g_{1,i1,i0}\,g_{2,j1,i0}\,g_{3,j0,j1}- T_1^{-s1}\left(-1+T_2^{s0}\right)g_{1,i1,j0}\,g_{2,j1,i0}\,g_{3,j0,j1}+T_1^{-s1}\left(-1+T_1^{s1}\right)\left(-T_1^{s0}-T_2^{s0}+2\,T_1^{s0}\,T_2^{s0}\right)g_{1,j1,i0}\,g_{2,j1,i0}\,g_{3,j0,j1}- T_1^{-s1}\left(-1+T_1^{s1}\right)\left(-1+T_2^{s0}\right)g_{1,j1,j0}\,g_{2,j1,i0}\,g_{3,j0,j1}- T_1^{-s1}\left(-1+T_1^{s0}\right)g_{1,i1,i0}\,g_{2,j1,j0}\,g_{3,j0,j1}-T_1^{-s1}\left(-1+T_1^{s0}\right)\left(-1+T_1^{s1}\right)g_{1,j1,i0}\,g_{2,j1,j0}\,g_{3,j0,j1}$$

$$g_{1,j1^+,\#1}\,g_{2,j1^+,\#1}\,g_{3,\#1,j1} \to \left(T_1^{s0}+T_2^{s0}-2\,T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,i0}\,g_{2,j0,i0}+\left(-1+T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,j0}\,g_{2,j0,i0}+ \left(-1+T_1^{s0}\right)\chi_{i0==i1}\,g_{1,j0,i0}\,g_{2,j0,j0}-\chi_{i0==i1}\,g_{1,j0,j0}\,g_{2,j0,j0}-\chi_{i0==i1}\,g_{3,j0,j0}+ \chi_{i0==i1}\,g_{1,j0,j0}\,g_{3,j0,j0}+\chi_{i0==i1}\,g_{2,j0,j0}\,g_{3,j0,j0}+\left(-T_1^{s0}-T_2^{s0}+2\,T_1^{s0}\,T_2^{s0}\right)g_{1,j1,i0}\,g_{2,j1,i0}\,g_{3,j0,j1}+ \left(1-T_2^{s0}\right)g_{1,j1,j0}\,g_{2,j1,i0}\,g_{3,j0,j1}+\left(1-T_1^{s0}\right)g_{1,j1,i0}\,g_{2,j1,j0}\,g_{3,j0,j1}$$

*In[•]:=*
```
Column[(# → sRules@D_{s1,i1,j1}[#]) & /@ bas1]
```

*Out[•]=*

$$g_{1,\#1,i0}\,g_{2,\#1,i0}\,g_{3,i0^+,\#1} \to T_1^{-s0}\,T_2^{-s0}\,\chi_{i0==i1}\,g_{1,i0,i0}\,g_{2,i0,i0}+T_1^{-s0}\,T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,i0}\,g_{2,j0,i0}+ T_1^{-s0}\,T_2^{-s0}\,\chi_{i0==i1}\,g_{3,i0,i0}-T_1^{-s0}\,T_2^{-s0}\,\chi_{i0==i1}\,g_{1,i0,i0}\,g_{3,i0,i0}-T_1^{-s0}\left(-1+T_1^{s0}\right)T_2^{-s0}\,\chi_{i0==i1}\,g_{1,j0,i0}\,g_{3,i0,i0}- T_1^{-s0}\,T_2^{-s0}\,\chi_{i0==i1}\,g_{2,i0,i0}\,g_{3,i0,i0}-T_1^{-s0}\,T_2^{-s0}\left(-1+T_2^{s0}\right)\chi_{i0==i1}\,g_{2,j0,i0}\,g_{3,i0,i0}+ T_1^{-s0}\left(-1+T_1^{s1}\right)T_2^{-s0}\,g_{1,j1,i0}\,g_{2,i1,i0}\,g_{3,i0,i1}+T_1^{-s0}\,T_2^{-s0}\left(-1+T_2^{s1}\right)g_{1,i1,i0}\,g_{2,j1,i0}\,g_{3,i0,i1}- T_1^{-s0}\,T_2^{-s0}\left(-2+T_1^{s1}+T_2^{s1}\right)g_{1,j1,i0}\,g_{2,j1,i0}\,g_{3,i0,i1}+T_1^{-s0}\,T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{3,j0,i0}- T_1^{-s0}\,T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{1,i0,i0}\,g_{3,j0,i0}-T_1^{-s0}\left(-1+T_1^{s0}\right)T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,i0}\,g_{3,j0,i0}- T_1^{-s0}\,T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{2,i0,i0}\,g_{3,j0,i0}-T_1^{-s0}\,T_2^{-s0}\left(-1+T_2^{s0}\right)\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{2,j0,i0}\,g_{3,j0,i0}+ T_1^{-s0}\left(-1+T_1^{s1}\right)T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)g_{1,j1,i0}\,g_{2,i1,i0}\,g_{3,j0,i1}+ T_1^{-s0}\,T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\left(-1+T_2^{s1}\right)g_{1,i1,i0}\,g_{2,j1,i0}\,g_{3,j0,i1}- T_1^{-s0}\,T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\left(-2+T_1^{s1}+T_2^{s1}\right)g_{1,j1,i0}\,g_{2,j1,i0}\,g_{3,j0,i1}$$

$$g_{1,\#1,j0}\,g_{2,\#1,i0}\,g_{3,i0^+,\#1} \to T_1^{-s0}\,T_2^{-s0}\,\chi_{i0==i1}\,g_{1,i0,j0}\,g_{2,i0,i0}+T_1^{-s0}\,T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,j0}\,g_{2,j0,i0}+ T_1^{-s0}\left(-1+T_1^{s0}\right)T_2^{-s0}\,\chi_{i0==i1}\,g_{3,i0,i0}-T_1^{-s0}\,T_2^{-s0}\,\chi_{i0==i1}\,g_{1,i0,j0}\,g_{3,i0,i0}- T_1^{-s0}\left(-1+T_1^{s0}\right)T_2^{-s0}\,\chi_{i0==i1}\,g_{1,j0,j0}\,g_{3,i0,i0}-T_1^{-s0}\left(-1+T_1^{s0}\right)T_2^{-s0}\,\chi_{i0==i1}\,g_{2,i0,i0}\,g_{3,i0,i0}+ T_1^{-s0}\,T_2^{-s0}\left(-2+T_1^{s0}+T_2^{s0}\right)\chi_{i0==i1}\,g_{2,j0,i0}\,g_{3,i0,i0}+T_1^{-s0}\left(-1+T_1^{s1}\right)T_2^{-s0}\,g_{1,j1,j0}\,g_{2,i1,i0}\,g_{3,i0,i1}+ T_1^{-s0}\,T_2^{-s0}\left(-1+T_2^{s1}\right)g_{1,i1,j0}\,g_{2,j1,i0}\,g_{3,i0,i1}-T_1^{-s0}\,T_2^{-s0}\left(-2+T_1^{s1}+T_2^{s1}\right)g_{1,j1,j0}\,g_{2,j1,i0}\,g_{3,i0,i1}- T_1^{-s0}\,T_2^{-s0}\,\chi_{i0==i1}\,g_{2,j0,i0}\,g_{3,i0,j0}+T_1^{-s0}\left(-1+T_1^{s0}\right)T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{3,j0,i0}- T_1^{-s0}\,T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{1,i0,j0}\,g_{3,j0,i0}-T_1^{-s0}\left(-1+T_1^{s0}\right)T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,j0}\,g_{3,j0,i0}- T_1^{-s0}\left(-1+T_1^{s0}\right)T_2^{-s0}\left(-1+T_2^{s0}\right)\chi_{i0==i1}\,g_{2,i0,i0}\,g_{3,j0,i0}+ T_1^{-s0}\,T_2^{-s0}\left(-2+T_1^{s0}+T_2^{s0}\right)\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{2,j0,i0}\,g_{3,j0,i0}+ T_1^{-s0}\left(-1+T_1^{s1}\right)T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)g_{1,j1,j0}\,g_{2,i1,i0}\,g_{3,j0,i1}+ T_1^{-s0}\,T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\left(-1+T_2^{s1}\right)g_{1,i1,j0}\,g_{2,j1,i0}\,g_{3,j0,i1}- T_1^{-s0}\,T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\left(-2+T_1^{s1}+T_2^{s1}\right)g_{1,j1,j0}\,g_{2,j1,i0}\,g_{3,j0,i1}- T_1^{-s0}\,T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{2,j0,i0}\,g_{3,j0,j0}$$

$$g_{1,\#1,i0}\,g_{2,\#1,j0}\,g_{3,i0^+,\#1} \to T_1^{-s0}\,T_2^{-s0}\,\chi_{i0==i1}\,g_{1,i0,i0}\,g_{2,i0,j0} + T_1^{-s0}\,T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,i0}\,g_{2,j0,j0} +$$
$$T_1^{-s0}\,T_2^{-s0}\left(-1+T_2^{s0}\right)\chi_{i0==i1}\,g_{3,i0,i0} - T_1^{-s0}\,T_2^{-s0}\left(-1+T_2^{s0}\right)\chi_{i0==i1}\,g_{1,i0,i0}\,g_{3,i0,i0} +$$
$$T_1^{-s0}\,T_2^{-s0}\left(-2+T_1^{s0}+T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,i0}\,g_{3,i0,i0} - T_1^{-s0}\,T_2^{-s0}\,\chi_{i0==i1}\,g_{2,i0,j0}\,g_{3,i0,i0} -$$
$$T_1^{-s0}\,T_2^{-s0}\left(-1+T_2^{s0}\right)\chi_{i0==i1}\,g_{2,j0,j0}\,g_{3,i0,i0} + T_1^{-s0}\left(-1+T_1^{s1}\right)T_2^{-s0}\,g_{1,j1,i0}\,g_{2,i1,j0}\,g_{3,i0,i1} +$$
$$T_1^{-s0}\,T_2^{-s0}\left(-1+T_2^{s1}\right)g_{1,i1,i0}\,g_{2,j1,j0}\,g_{3,i0,i1} - T_1^{-s0}\,T_2^{-s0}\left(-2+T_1^{s1}+T_2^{s1}\right)g_{1,j1,i0}\,g_{2,j1,j0}\,g_{3,i0,i1} -$$
$$T_1^{-s0}\,T_2^{-s0}\,\chi_{i0==i1}\,g_{1,j0,i0}\,g_{3,i0,j0} + T_1^{-s0}\,T_2^{-s0}\left(-1+T_2^{s0}\right)\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{3,j0,i0} -$$
$$T_1^{-s0}\,T_2^{-s0}\left(-1+T_2^{s0}\right)\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{1,i0,i0}\,g_{3,j0,i0} +$$
$$T_1^{-s0}\,T_2^{-s0}\left(-2+T_1^{s0}+T_2^{s0}\right)\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,i0}\,g_{3,j0,i0} -$$
$$T_1^{-s0}\,T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{2,i0,j0}\,g_{3,j0,i0} - T_1^{-s0}\,T_2^{-s0}\left(-1+T_2^{s0}\right)\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{2,j0,j0}\,g_{3,j0,i0} +$$
$$T_1^{-s0}\left(-1+T_1^{s1}\right)T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)g_{1,j1,i0}\,g_{2,i1,j0}\,g_{3,j0,i1} +$$
$$T_1^{-s0}\,T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\left(-1+T_2^{s1}\right)g_{1,i1,i0}\,g_{2,j1,j0}\,g_{3,j0,i1} -$$
$$T_1^{-s0}\,T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\left(-2+T_1^{s1}+T_2^{s1}\right)g_{1,j1,i0}\,g_{2,j1,j0}\,g_{3,j0,i1} -$$
$$T_1^{-s0}\,T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,i0}\,g_{3,j0,j0}$$

$$g_{1,\#1,j0}\,g_{2,\#1,j0}\,g_{3,i0^+,\#1} \to T_1^{-s0}\,T_2^{-s0}\,\chi_{i0==i1}\,g_{1,i0,j0}\,g_{2,i0,j0} + T_1^{-s0}\,T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,j0}\,g_{2,j0,j0} -$$
$$T_1^{-s0}\,T_2^{-s0}\left(-2+T_1^{s0}+T_2^{s0}\right)\chi_{i0==i1}\,g_{3,i0,i0} - T_1^{-s0}\,T_2^{-s0}\left(-1+T_2^{s0}\right)\chi_{i0==i1}\,g_{1,i0,j0}\,g_{3,i0,i0} +$$
$$T_1^{-s0}\,T_2^{-s0}\left(-2+T_1^{s0}+T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,j0}\,g_{3,i0,i0} - T_1^{-s0}\left(-1+T_1^{s0}\right)T_2^{-s0}\,\chi_{i0==i1}\,g_{2,i0,j0}\,g_{3,i0,i0} +$$
$$T_1^{-s0}\,T_2^{-s0}\left(-2+T_1^{s0}+T_2^{s0}\right)\chi_{i0==i1}\,g_{2,j0,j0}\,g_{3,i0,i0} + T_1^{-s0}\left(-1+T_1^{s1}\right)T_2^{-s0}\,g_{1,j1,j0}\,g_{2,i1,j0}\,g_{3,i0,i1} +$$
$$T_1^{-s0}\,T_2^{-s0}\left(-1+T_2^{s1}\right)g_{1,i1,j0}\,g_{2,j1,j0}\,g_{3,i0,i1} - T_1^{-s0}\,T_2^{-s0}\left(-2+T_1^{s1}+T_2^{s1}\right)g_{1,j1,j0}\,g_{2,j1,j0}\,g_{3,i0,i1} +$$
$$T_1^{-s0}\,T_2^{-s0}\,\chi_{i0==i1}\,g_{3,i0,j0} - T_1^{-s0}\,T_2^{-s0}\,\chi_{i0==i1}\,g_{1,j0,j0}\,g_{3,i0,j0} -$$
$$T_1^{-s0}\,T_2^{-s0}\,\chi_{i0==i1}\,g_{2,j0,j0}\,g_{3,i0,j0} - T_1^{-s0}\,T_2^{-s0}\left(-2+T_1^{s0}+T_2^{s0}\right)\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{3,j0,i0} -$$
$$T_1^{-s0}\,T_2^{-s0}\left(-1+T_2^{s0}\right)\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{1,i0,j0}\,g_{3,j0,i0} +$$
$$T_1^{-s0}\,T_2^{-s0}\left(-2+T_1^{s0}+T_2^{s0}\right)\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,j0}\,g_{3,j0,i0} -$$
$$T_1^{-s0}\left(-1+T_1^{s0}\right)T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{2,i0,j0}\,g_{3,j0,i0} +$$
$$T_1^{-s0}\,T_2^{-s0}\left(-2+T_1^{s0}+T_2^{s0}\right)\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{2,j0,j0}\,g_{3,j0,i0} +$$
$$T_1^{-s0}\left(-1+T_1^{s1}\right)T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)g_{1,j1,j0}\,g_{2,i1,j0}\,g_{3,j0,i1} +$$
$$T_1^{-s0}\,T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\left(-1+T_2^{s1}\right)g_{1,i1,j0}\,g_{2,j1,j0}\,g_{3,j0,i1} -$$
$$T_1^{-s0}\,T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\left(-2+T_1^{s1}+T_2^{s1}\right)g_{1,j1,j0}\,g_{2,j1,j0}\,g_{3,j0,i1} + T_1^{-s0}\,T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{3,j0,j0} -$$
$$T_1^{-s0}\,T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,j0}\,g_{3,j0,j0} - T_1^{-s0}\,T_2^{-s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{2,j0,j0}\,g_{3,j0,j0}$$

$$g_{1,\#1,i0}\,g_{2,\#1,i0}\,g_{3,j0^+,\#1} \to$$
$$\chi_{i0==i1}\,g_{1,j0,i0}\,g_{2,j0,i0} + \chi_{i0==i1}\,g_{3,j0,i0} - \chi_{i0==i1}\,g_{1,i0,i0}\,g_{3,j0,i0} + \left(1-T_1^{s0}\right)\chi_{i0==i1}\,g_{1,j0,i0}\,g_{3,j0,i0} -$$
$$\chi_{i0==i1}\,g_{2,i0,i0}\,g_{3,j0,i0} + \left(1-T_2^{s0}\right)\chi_{i0==i1}\,g_{2,j0,i0}\,g_{3,j0,i0} + \left(-1+T_1^{s1}\right)g_{1,j1,i0}\,g_{2,i1,i0}\,g_{3,j0,i1} +$$
$$\left(-1+T_2^{s1}\right)g_{1,i1,i0}\,g_{2,j1,i0}\,g_{3,j0,i1} + \left(2-T_1^{s1}-T_2^{s1}\right)g_{1,j1,i0}\,g_{2,j1,i0}\,g_{3,j0,i1}$$

$$g_{1,\#1,j0}\,g_{2,\#1,i0}\,g_{3,j0^+,\#1} \to \chi_{i0==i1}\,g_{1,j0,j0}\,g_{2,j0,i0} + \left(-1+T_1^{s0}\right)\chi_{i0==i1}\,g_{3,j0,i0} -$$
$$\chi_{i0==i1}\,g_{1,i0,j0}\,g_{3,j0,i0} + \left(1-T_1^{s0}\right)\chi_{i0==i1}\,g_{1,j0,j0}\,g_{3,j0,i0} + \left(1-T_1^{s0}\right)\chi_{i0==i1}\,g_{2,i0,i0}\,g_{3,j0,i0} +$$
$$\left(-2+T_1^{s0}+T_2^{s0}\right)\chi_{i0==i1}\,g_{2,j0,i0}\,g_{3,j0,i0} + \left(-1+T_1^{s1}\right)g_{1,j1,j0}\,g_{2,i1,i0}\,g_{3,j0,i1} +$$
$$\left(-1+T_2^{s1}\right)g_{1,i1,j0}\,g_{2,j1,i0}\,g_{3,j0,i1} + \left(2-T_1^{s1}-T_2^{s1}\right)g_{1,j1,j0}\,g_{2,j1,i0}\,g_{3,j0,i1} - \chi_{i0==i1}\,g_{2,j0,i0}\,g_{3,j0,j0}$$

$$g_{1,\#1,i0}\,g_{2,\#1,j0}\,g_{3,j0^+,\#1} \to \chi_{i0==i1}\,g_{1,j0,i0}\,g_{2,j0,j0} + \left(-1+T_2^{s0}\right)\chi_{i0==i1}\,g_{3,j0,i0} +$$
$$\left(1-T_2^{s0}\right)\chi_{i0==i1}\,g_{1,i0,i0}\,g_{3,j0,i0} + \left(-2+T_1^{s0}+T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,i0}\,g_{3,j0,i0} -$$
$$\chi_{i0==i1}\,g_{2,i0,j0}\,g_{3,j0,i0} + \left(1-T_2^{s0}\right)\chi_{i0==i1}\,g_{2,j0,j0}\,g_{3,j0,i0} + \left(-1+T_1^{s1}\right)g_{1,j1,i0}\,g_{2,i1,j0}\,g_{3,j0,i1} +$$
$$\left(-1+T_2^{s1}\right)g_{1,i1,i0}\,g_{2,j1,j0}\,g_{3,j0,i1} + \left(2-T_1^{s1}-T_2^{s1}\right)g_{1,j1,i0}\,g_{2,j1,j0}\,g_{3,j0,i1} - \chi_{i0==i1}\,g_{1,j0,i0}\,g_{3,j0,j0}$$

$$g_{1,\#1,j0}\,g_{2,\#1,j0}\,g_{3,j0^+,\#1} \to \chi_{i0==i1}\,g_{1,j0,j0}\,g_{2,j0,j0} + \left(2-T_1^{s0}-T_2^{s0}\right)\chi_{i0==i1}\,g_{3,j0,i0} + \left(1-T_2^{s0}\right)\chi_{i0==i1}\,g_{1,i0,j0}\,g_{3,j0,i0} + \left(-2+T_1^{s0}+T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,j0}\,g_{3,j0,i0} + \left(1-T_1^{s0}\right)\chi_{i0==i1}\,g_{2,i0,j0}\,g_{3,j0,i0} + \left(-2+T_1^{s0}+T_2^{s0}\right)\chi_{i0==i1}\,g_{2,j0,j0}\,g_{3,j0,i0} + \left(-1+T_1^{s1}\right)g_{1,j1,j0}\,g_{2,i1,j0}\,g_{3,j0,i1} + \left(-1+T_2^{s1}\right)g_{1,i1,j0}\,g_{2,j1,j0}\,g_{3,j0,i1} + \left(2-T_1^{s1}-T_2^{s1}\right)g_{1,j1,j0}\,g_{2,j1,j0}\,g_{3,j0,i1} + \chi_{i0==i1}\,g_{3,j0,j0} - \chi_{i0==i1}\,g_{1,j0,j0}\,g_{3,j0,j0} - \chi_{i0==i1}\,g_{2,j0,j0}\,g_{3,j0,j0}$$

In[•]:=
```
tw0 = Table[a_i, {i, 8}].bas0 / (T_2 - 1);
tw1 = Table[b_i, {i, 8}].bas1 / (T_2 - 1);
{D_{s0,i0,j0}[tw0], D_{s1,i1,j1}[tw1]}
Θ[Knot[7, 6], F1 → 0, F2 → D_{s0,i0,j0}[tw0] + D_{s1,i1,j1}[tw1], F3 → 0] // Echo // PolyPlot
Θ[Knot[7, 6], F2 → F2i + D_{s0,i0,j0}[(T_2 - 1) tw0] + D_{s1,i1,j1}[(T_2 - 1) tw1]] // Echo // PolyPlot
```

Out[•]=

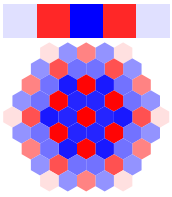
» $\left\{-\dfrac{1-5\,T+7\,T^2-5\,T^3+T^4}{T^2},\ 0\right\}$

Out[•]=

» $\left\{-\dfrac{1-5\,T+7\,T^2-5\,T^3+T^4}{T^2},\right.$

$$\left.\frac{1}{T_1^4\,T_2^4}\left(1-5\,T_1+7\,T_1^2-5\,T_1^3+T_1^4-5\,T_2+20\,T_1\,T_2-10\,T_1^2\,T_2-10\,T_1^3\,T_2+20\,T_1^4\,T_2-5\,T_1^5\,T_2+7\,T_2^2-10\,T_1\,T_2^2-64\,T_1^2\,T_2^2+98\,T_1^3\,T_2^2-64\,T_1^4\,T_2^2-10\,T_1^5\,T_2^2+7\,T_1^6\,T_2^2-5\,T_2^3-10\,T_1\,T_2^3+98\,T_1^2\,T_2^3-50\,T_1^3\,T_2^3-50\,T_1^4\,T_2^3+98\,T_1^5\,T_2^3-10\,T_1^6\,T_2^3-5\,T_1^7\,T_2^3+T_2^4+20\,T_1\,T_2^4-64\,T_1^2\,T_2^4-50\,T_1^3\,T_2^4+108\,T_1^4\,T_2^4-50\,T_1^5\,T_2^4-64\,T_1^6\,T_2^4+20\,T_1^7\,T_2^4+T_1^8\,T_2^4-5\,T_1\,T_2^5-10\,T_1^2\,T_2^5+98\,T_1^3\,T_2^5-50\,T_1^4\,T_2^5-50\,T_1^5\,T_2^5+98\,T_1^6\,T_2^5-10\,T_1^7\,T_2^5-5\,T_1^8\,T_2^5+7\,T_1^2\,T_2^6-10\,T_1^3\,T_2^6-64\,T_1^4\,T_2^6+98\,T_1^5\,T_2^6-64\,T_1^6\,T_2^6-10\,T_1^7\,T_2^6+7\,T_1^8\,T_2^6-5\,T_1^3\,T_2^7+20\,T_1^4\,T_2^7-10\,T_1^5\,T_2^7-10\,T_1^6\,T_2^7+20\,T_1^7\,T_2^7-5\,T_1^8\,T_2^7+T_1^4\,T_2^8-5\,T_1^5\,T_2^8+7\,T_1^6\,T_2^8-5\,T_1^7\,T_2^8+T_1^8\,T_2^8\right)\right\}$$

Out[•]=



In[•]:=
```
res = sRules[Residue[CF[χ_{i0==i1} F1i + F2i + D_{s0,i0,j0}[tw0] + D_{s1,i1,j1}[tw1]], {T_2, 1}] /.
    {g_{3,α_,β_} :↦ g_{1,α,β}, g_{2,α_,β_} :↦ χ_{α≤β}} ];
```

In[•]:=
```
sRules[res /. {b_5 → 1 - T_1^{s0}, b_7 → T_1^{s0} - 1} /. (a | b)_ → 0]
```

Out[•]=

0

```
In[•]:= sRules[res /. {a1 → b1, a2 → a4, a3 → 0, a5 → b2, a6 → 0, a7 → b4, a8 → 0} /. b1|2|3|4|6|8 → 0 /.
          a4 → (b5 + b7) / 2 /. b5 → 2 + b7 - 2 T1^s0 /. b7 → T1^s0 - 1]
```

```
Out[•]=
0
```

```
In[•]:= nF2 = CF[CF[(χi0==i1 F1i + F2i + D{s0,i0,j0}[tw0] + D{s1,i1,j1}[tw1]) /.
                {a1 → b1, a2 → a4, a3 → 0, a5 → b2, a6 → 0, a7 → b4, a8 → 0} /. b1|2|3|4|6|8 → 0 /.
              a4 → (b5 + b7) / 2 /. b5 → 2 + b7 - 2 T1^s0 /. b7 → T1^s0 - 1]];
        {nF1, nF2} = Simplify@{Coefficient[nF2, χi0==i1], nF2 /. χi0==i1 → 0}
        Table[Θ[K] == Θ[K, F1 → nF1, F2 → nF2], {K, AllKnots[{3, 8}]}]
```

```
Out[•]=
{s0/2 - T1^-s1 (-1 + T1^s0) T2^-s1 (-2 + T1^s1 + T2^s1) χi0==j1 χi1==j0 / (-1 + T2) +
 T1^-s1 (-1 + T1^s0) T2^-s1 (-1 + T2^s1) χi1==j0 χj0==j1 / (-1 + T2) +
 T1^-s1 (-1 + T1^s0) T2^-s1 χi1==j0 g1,i1,i0 / (-1 + T2) +
 T1^-s1 (-1 + T1^s0) (-1 + T1^s1) T2^-s1 χi1==j0 g1,j1,i0 / (-1 + T2) +
 T1^-s1 (-1 + T1^s0) T2^-s1 χi1==j0 g2,i1,i0 / (-1 + T2) -
 T1^-s1 (-1 + T1^s0) T2^-s1 χi1==j0 g2,i1,j0 / (-1 + T2) +
 s0 T2^s0 g1,i0,i0 g2,j0,i0 + s0 (-1 + T1^s0) T2^(2 s0) g1,j0,i0 g2,j0,i0 / (-1 + T2^s0) - s0 g1,i0,i0 g2,j0,j0 -
 s0 (-1 + T1^s0) T2^s0 g1,j0,i0 g2,j0,j0 / (-1 + T2^s0) + T1^-s1 (-1 + T1^s0) T2^-s1 (-1 + T2^s1) χi1==j0 g2,j1,i0 / (-1 + T2) -
 T1^-s1 (-1 + T1^s0) T2^-s1 (-1 + T2^s1) χi1==j0 g2,j1,j0 / (-1 + T2) - s0 g3,i0,i0 - s0 (-1 + T2^s0) g2,j0,i0 g3,i0,i0 +
 2 s0 g2,j0,j0 g3,i0,i0 + s0 (-1 + T1^s0 T2^s0) g3,j0,i0 / (-1 + T2^s0) - s0 T2^s0 (-1 + T1^s0 T2^s0) g1,i0,i0 g3,j0,i0 / (-1 + T2^s0) -
 s0 (-1 + T1^s0) (1 + T2^s0) (-1 + T1^s0 T2^s0) g1,j0,i0 g3,j0,i0 / (-1 + T2^s0) + s0 (-1 + T1^s0 T2^s0) g2,i0,j0 g3,j0,i0 / (-1 + T2^s0) +
 s0 (-1 + T1^s0 T2^s0) g2,j0,i0 g3,j0,i0 + s0 (-2 + T2^s0) (-1 + T1^s0 T2^s0) g2,j0,j0 g3,j0,i0 / (-1 + T2^s0) +
 s0 g1,i0,i0 g3,j0,j0 + s0 (-1 + T1^s0) T2^s0 g1,j0,i0 g3,j0,j0 / (-1 + T2^s0) - s0 g2,i0,i0 g3,j0,j0 -
 s0 T2^s0 g2,j0,i0 g3,j0,j0 - (-1 + T1^s0) g3,j0^+,i1 / (-1 + T2) - (-1 + T1^s0) (-2 + T1^s1 + T2^s1) χi0==j1 g3,j0^+,i1 / (-1 + T2) +
 (-1 + T1^s0) χi1==j0 g3,j0^+,i1 / (-1 + T2) + (-1 + T1^s0) (-1 + T2^s1) χj0==j1 g3,j0^+,i1 / (-1 + T2) + (-1 + T1^s0) g1,i1,i0 g3,j0^+,i1 / (-1 + T2) +
 (-1 + T1^s0) (-1 + T1^s1) g1,j1,i0 g3,j0^+,i1 / (-1 + T2) + (-1 + T1^s0) g2,i1,i0 g3,j0^+,i1 / (-1 + T2) - (-1 + T1^s0) g2,i1,j0 g3,j0^+,i1 / (-1 + T2) +
```

$$\frac{\left(-1+T_1^{s0}\right)\left(-1+T_2^{s1}\right) g_{2,j1,i0}\, g_{3,j0^+,i1}}{-1+T_2} - \frac{\left(-1+T_1^{s0}\right)\left(-1+T_2^{s1}\right) g_{2,j1,j0}\, g_{3,j0^+,i1}}{-1+T_2},$$

$$\frac{1}{-1+T_2}\left(-1+T_1^{s0}\right)\left(-T_1^{-s1}\,T_2^{-s1}\,\chi_{i1==j0}\left(g_{1,i1,i0}+\left(-1+T_1^{s1}\right) g_{1,j1,i0}\right)\left(1+\left(-1+T_2^{s1}\right)\chi_{j0==j1}+\right.\right.$$
$$g_{2,i1,i0}-g_{2,i1,j0}-g_{2,j1,i0}+T_2^{s1}\,g_{2,j1,i0}+g_{2,j1,j0}-T_2^{s1}\,g_{2,j1,j0}+T_1^{s1}\,T_2^{s1}\,g_{3,j0^+,i1}\Big)+$$
$$\frac{1}{-1+T_2^{s1}}\left(-s1\,g_{1,j1,i0}\,g_{2,i1,j0}\,g_{3,j0,i1}+s1\,T_2^{s0}\,g_{1,j1,i0}\left(g_{2,i1,i0}-g_{2,j1,i0}\right)g_{3,j0,i1}-\right.$$
$$s1\,T_2^{1+s0}\,g_{1,j1,i0}\left(g_{2,i1,i0}-g_{2,j1,i0}\right)g_{3,j0,i1}-s1\,T_1^{s1}\,T_2^{s0+s1}\,g_{1,j1,i0}\left(g_{2,i1,i0}-g_{2,j1,i0}\right)g_{3,j0,i1}+$$
$$s1\,T_1^{s1}\,T_2^{1+s0+s1}\,g_{1,j1,i0}\left(g_{2,i1,i0}-g_{2,j1,i0}\right)g_{3,j0,i1}+s1\,T_2\,g_{1,j1,i0}\left(g_{2,i1,j0}-g_{2,j1,j0}\right)g_{3,j0,i1}-$$
$$s1\,T_1^{s1}\,T_2^{1+s1}\,g_{1,j1,i0}\left(g_{2,i1,j0}-g_{2,j1,j0}\right)g_{3,j0,i1}+s1\,g_{1,j1,i0}\,g_{2,j1,j0}\,g_{3,j0,i1}-$$
$$g_{1,j1,i0}\,g_{2,i1,i0}\,g_{3,j0^+,i1}+T_1^{s1}\,g_{1,j1,i0}\,g_{2,i1,i0}\,g_{3,j0^+,i1}+g_{1,j1,i0}\,g_{2,i1,j0}\,g_{3,j0^+,i1}-$$
$$T_1^{s1}\,g_{1,j1,i0}\,g_{2,i1,j0}\,g_{3,j0^+,i1}-g_{1,i1,i0}\,g_{2,j1,i0}\,g_{3,j0^+,i1}+2\,g_{1,j1,i0}\,g_{2,j1,i0}\,g_{3,j0^+,i1}-$$
$$T_1^{s1}\,g_{1,j1,i0}\,g_{2,j1,i0}\,g_{3,j0^+,i1}-T_2^{2\,s1}\left(g_{1,i1,i0}-g_{1,j1,i0}\right)\left(g_{2,j1,i0}-g_{2,j1,j0}\right)g_{3,j0^+,i1}+$$
$$g_{1,i1,i0}\,g_{2,j1,j0}\,g_{3,j0^+,i1}-2\,g_{1,j1,i0}\,g_{2,j1,j0}\,g_{3,j0^+,i1}+T_1^{s1}\,g_{1,j1,i0}\,g_{2,j1,j0}\,g_{3,j0^+,i1}+$$
$$T_2^{s1}\left(\left(2\,g_{1,i1,i0}\left(g_{2,j1,i0}-g_{2,j1,j0}\right)+g_{1,j1,i0}\left(g_{2,i1,i0}-g_{2,i1,j0}-3\,g_{2,j1,i0}+3\,g_{2,j1,j0}\right)\right)\right.$$
$$g_{3,j0^+,i1}+T_1^{s1}\,g_{1,j1,i0}\left(\left(-g_{2,i1,i0}+g_{2,j1,i0}\right)g_{3,j0^+,i1}+g_{2,i1,j0}\left(s1\,g_{3,j0,i1}+g_{3,j0^+,i1}\right)-\right.$$
$$\left.\left.g_{2,j1,j0}\left(s1\,g_{3,j0,i1}+g_{3,j0^+,i1}\right)\right)\right)+\left(-1+T_2^{s1}\right)\chi_{j0==j1}\left(-\left(\left(-1+T_2^{s1}\right)g_{1,i1,i0}\,g_{3,j0^+,i1}\right)+\right.$$
$$\left.\left.g_{1,j1,i0}\left(-1-g_{2,j1,i0}+g_{2,j1,j0}-2\,g_{3,j0^+,i1}+T_1^{s1}\,g_{3,j0^+,i1}+T_2^{s1}\,g_{3,j0^+,i1}-g_{3,j0^+,j1}\right)\right)\right)+$$
$$\chi_{i0==j1}\left(\left(-2-g_{1,i1,i0}+2\,g_{1,j1,i0}-g_{2,i1,i0}+g_{2,i1,j0}+2\,g_{2,j1,i0}-T_1^{s1}\left(-1+g_{1,j1,i0}-g_{2,i1,i0}+\right.\right.\right.$$
$$\left.\left.\left.g_{2,i1,j0}+g_{2,j1,i0}-g_{2,j1,j0}\right)-2\,g_{2,j1,j0}+T_2^{s1}\left(1+g_{1,i1,i0}-g_{1,j1,i0}-g_{2,j1,i0}+g_{2,j1,j0}\right)\right)$$
$$g_{3,j0^+,i1}+T_1^{-s1}\,T_2^{-s1}\,\chi_{i1==j0}\left(-2+\chi_{j0==j1}-g_{1,i1,i0}+g_{1,j1,i0}-g_{2,i1,i0}+g_{2,i1,j0}+g_{2,j1,i0}-\right.$$
$$T_2^{s1}\left(-1+\chi_{j0==j1}-g_{1,i1,i0}+g_{1,j1,i0}+g_{2,j1,i0}-g_{2,j1,j0}\right)-g_{2,j1,j0}+T_1^{s1}\left(2-\chi_{j0==j1}-g_{1,j1,i0}+\right.$$
$$\left.\left.g_{2,i1,i0}-g_{2,i1,j0}-g_{2,j1,i0}+g_{2,j1,j0}+T_2^{s1}\left(-1+\chi_{j0==j1}+g_{1,j1,i0}+g_{2,j1,i0}-g_{2,j1,j0}-\right.\right.\right.$$
$$\left.\left.\left.g_{3,j0^+,i1}\right)\right)+T_1^{2\,s1}\,T_2^{s1}\,g_{3,j0^+,i1}\Big)+\left(-1+g_{1,j1,i0}+g_{2,j1,i0}-g_{2,j1,j0}\right)g_{3,j0^+,j1}+$$
$$\left.\left.\chi_{j0==j1}\left(g_{1,j1,i0}+g_{2,j1,i0}-g_{2,j1,j0}+2\,g_{3,j0^+,i1}-T_1^{s1}\,g_{3,j0^+,i1}-T_2^{s1}\,g_{3,j0^+,i1}+g_{3,j0^+,j1}\right)\right)\right)\Bigg\}$$

*Out[•]=*

{True, True, True, True, True, True, True, True, True, True, True,
True, True, True, True, True, True, True, True, True, True, True, True,
True, True, True, True, True, True, True, True, True, True, True, True}

In[•]:=
```
Table[Θ[K] == Θ[K,
 F1 → s0/2 + s0 T2^s0 g1,i0,i0 g2,j0,i0 + ((-1 + T1^s0) (1 - T2^s0 - s0 T2^(2 s0) + s0 T2^(1+2 s0)) g1,j0,i0 g2,j0,i0)/((-1 + T2) (-1 + T2^s0)) -
  s0 g1,i0,i0 g2,j0,j0 - ((-1 + T1^s0) (1 - (1 + s0) T2^s0 + s0 T2^(1+s0)) g1,j0,i0 g2,j0,j0)/((-1 + T2) (-1 + T2^s0)) -
  s0 g3,i0,i0 - s0 (-1 + T2^s0) g2,j0,i0 g3,i0,i0 + 2 s0 g2,j0,j0 g3,i0,i0 +
  ((-2 + s0 - s0 T2 + 3 T2^s0 - T2^(2 s0) + T1^s0 (2 - (3 + s0) T2^s0 + T2^(2 s0) + s0 T2^(1+s0))) g3,j0,i0)/((-1 + T2) (-1 + T2^s0)) -
  ((-2 + (3 + s0) T2^s0 - T2^(2 s0) - s0 T2^(1+s0) + T1^s0 (2 - 3 T2^s0 - (-1 + s0) T2^(2 s0) + s0 T2^(1+2 s0))) g1,i0,i0 g3,j0,i0)/((-1 + T2) (-1 + T2^s0)) -
  1/((-1 + T2) (-1 + T2^s0)) (-1 + T1^s0) (-3 + s0 - s0 T2 + (4 + s0) T2^s0 - T2^(2 s0) - s0 T2^(1+s0) + T1^s0
      (2 - (2 + s0) T2^s0 - s0 T2^(2 s0) + s0 T2^(1+s0) + s0 T2^(1+2 s0))) g1,j0,i0 g3,j0,i0 + ((-1 + T1^s0) g2,i0,i0 g3,j0,i0)/(-1 + T2) +
  ((-1 + s0 - s0 T2 + T2^s0 + T1^s0 (1 - (1 + s0) T2^s0 + s0 T2^(1+s0))) g2,i0,j0 g3,j0,i0)/((-1 + T2) (-1 + T2^s0)) +
  ((1 + s0 - s0 T2 - T2^s0 + T1^s0 (-1 - (-1 + s0) T2^s0 + s0 T2^(1+s0))) g2,j0,i0 g3,j0,i0)/(-1 + T2) +
  1/((-1 + T2) (-1 + T2^s0)) (1 - 2 s0 + 2 s0 T2 + (-2 + s0) T2^s0 + T2^(2 s0) - s0 T2^(1+s0) +
     T1^s0 (-1 + 2 (1 + s0) T2^s0 - (1 + s0) T2^(2 s0) - 2 s0 T2^(1+s0) + s0 T2^(1+2 s0))) g2,j0,j0 g3,j0,i0 +
  s0 g1,i0,i0 g3,j0,j0 + ((-1 + T1^s0) (1 - (1 + s0) T2^s0 + s0 T2^(1+s0)) g1,j0,i0 g3,j0,j0)/((-1 + T2) (-1 + T2^s0)) -
  s0 g2,i0,i0 g3,j0,j0 - s0 T2^s0 g2,j0,i0 g3,j0,j0,
 F2 → 1/((-1 + T2) (-1 + T2^s1)) (-1 + T1^s0)
   ((-1 + s1 T2^s0 - s1 T2^(1+s0) + T2^s1 + T1^s1 (1 - T2^s1 - s1 T2^(s0+s1) + s1 T2^(1+s0+s1))) g1,j1,i0 g2,i1,i0 -
     (-1 + s1 - s1 T2 + T2^s1 + T1^s1 (1 - (1 + s1) T2^s1 + s1 T2^(1+s1))) g1,j1,i0 g2,i1,j0 - (-1 + T2^s1)^2 g1,i1,i0
      g2,j1,i0 + (2 - s1 T2^s0 + s1 T2^(1+s0) - 3 T2^s1 + T2^(2 s1) + T1^s1 (-1 + T2^s1 + s1 T2^(s0+s1) - s1 T2^(1+s0+s1)))
      g1,j1,i0 g2,j1,i0 + (-1 + T2^s1)^2 g1,i1,i0 g2,j1,j0 +
     (-2 + s1 - s1 T2 + 3 T2^s1 - T2^(2 s1) + T1^s1 (1 - (1 + s1) T2^s1 + s1 T2^(1+s1))) g1,j1,i0 g2,j1,j0) g3,j0,i1
 ], {K, AllKnots[{3, 8}]}]
```

Out[•]=
```
{True, True, True, True, True, True, True, True, True, True, True,
 True, True, True, True, True, True, True, True, True, True, True,
 True, True, True, True, True, True, True, True, True, True, True, True}
```

In[•]:=
```
CF[ s0/2 + s0 T2^s0 g1,i0,i0 g2,j0,i0 + ((-1 + T1^s0) (1 - T2^s0 - s0 T2^(2 s0) + s0 T2^(1+2 s0)) g1,j0,i0 g2,j0,i0)/((-1 + T2) (-1 + T2^s0)) -
 s0 g1,i0,i0 g2,j0,j0 - ((-1 + T1^s0) (1 - (1 + s0) T2^s0 + s0 T2^(1+s0)) g1,j0,i0 g2,j0,j0)/((-1 + T2) (-1 + T2^s0)) -
 s0 g3,i0,i0 - s0 (-1 + T2^s0) g2,j0,i0 g3,i0,i0 + 2 s0 g2,j0,j0 g3,i0,i0 +
 ((-2 + s0 - s0 T2 + 3 T2^s0 - T2^(2 s0) + T1^s0 (2 - (3 + s0) T2^s0 + T2^(2 s0) + s0 T2^(1+s0))) g3,j0,i0)/((-1 + T2) (-1 + T2^s0)) -
 ((-2 + (3 + s0) T2^s0 - T2^(2 s0) - s0 T2^(1+s0) + T1^s0 (2 - 3 T2^s0 - (-1 + s0) T2^(2 s0) + s0 T2^(1+2 s0))) g1,i0,i0 g3,j0,i0)/((-1 + T2) (-1 + T2^s0)) -
 1/((-1 + T2) (-1 + T2^s0)) (-1 + T1^s0) (-3 + s0 - s0 T2 + (4 + s0) T2^s0 - T2^(2 s0) - s0 T2^(1+s0) +
    T1^s0 (2 - (2 + s0) T2^s0 - s0 T2^(2 s0) + s0 T2^(1+s0) + s0 T2^(1+2 s0))) g1,j0,i0 g3,j0,i0 + ((-1 + T1^s0) g2,i0,i0 g3,j0,i0)/(-1 + T2) +
 ((-1 + s0 - s0 T2 + T2^s0 + T1^s0 (1 - (1 + s0) T2^s0 + s0 T2^(1+s0))) g2,i0,j0 g3,j0,i0)/((-1 + T2) (-1 + T2^s0)) +
 ((1 + s0 - s0 T2 - T2^s0 + T1^s0 (-1 - (-1 + s0) T2^s0 + s0 T2^(1+s0))) g2,j0,i0 g3,j0,i0)/(-1 + T2) +
 1/((-1 + T2) (-1 + T2^s0)) (1 - 2 s0 + 2 s0 T2 + (-2 + s0) T2^s0 + T2^(2 s0) - s0 T2^(1+s0) +
    T1^s0 (-1 + 2 (1 + s0) T2^s0 - (1 + s0) T2^(2 s0) - 2 s0 T2^(1+s0) + s0 T2^(1+2 s0))) g2,j0,j0 g3,j0,i0 +
 s0 g1,i0,i0 g3,j0,j0 + ((-1 + T1^s0) (1 - (1 + s0) T2^s0 + s0 T2^(1+s0)) g1,j0,i0 g3,j0,j0)/((-1 + T2) (-1 + T2^s0)) -
 s0 g2,i0,i0 g3,j0,j0 - s0 T2^s0 g2,j0,i0 g3,j0,j0 /. s0 → 1]
```

Out[•]=
```
1/2 + T2 g1,i0,i0 g2,j0,i0 + (-1 + T1) (1 + T2) g1,j0,i0 g2,j0,i0 -
 g1,i0,i0 g2,j0,j0 + (1 - T1) g1,j0,i0 g2,j0,j0 - g3,i0,i0 + (1 - T2) g2,j0,i0 g3,i0,i0 +
 2 g2,j0,j0 g3,i0,i0 + (-1 + 2 T1) g3,j0,i0 + (2 - 2 T1 - T1 T2) g1,i0,i0 g3,j0,i0 -
 (-1 + T1) (-2 + 2 T1 + T1 T2) g1,j0,i0 g3,j0,i0 + ((-1 + T1) g2,i0,i0 g3,j0,i0)/(-1 + T2) + T1 g2,i0,j0 g3,j0,i0 +
 (-2 + T1 + T1 T2) g2,j0,i0 g3,j0,i0 + ((1 + T1 - 3 T1 T2 + T1 T2^2) g2,j0,j0 g3,j0,i0)/(-1 + T2) +
 g1,i0,i0 g3,j0,j0 + (-1 + T1) g1,j0,i0 g3,j0,j0 - g2,i0,i0 g3,j0,j0 - T2 g2,j0,i0 g3,j0,j0
```

In[•]:=
```
Factor[ (-1 + T1) + (1 + T1 - 3 T1 T2 + T1 T2^2) ]
```

Out[•]=
```
T1 (-2 + T2) (-1 + T2)
```

In[•]:=
```
CF[s0/2 + s0 T2^s0 g1,i0,i0 g2,j0,i0 + ((-1 + T1^s0) (1 - T2^s0 - s0 T2^(2 s0) + s0 T2^(1+2 s0)) g1,j0,i0 g2,j0,i0)/((-1 + T2) (-1 + T2^s0)) -
 s0 g1,i0,i0 g2,j0,j0 - ((-1 + T1^s0) (1 - (1 + s0) T2^s0 + s0 T2^(1+s0)) g1,j0,i0 g2,j0,j0)/((-1 + T2) (-1 + T2^s0)) -
 s0 g3,i0,i0 - s0 (-1 + T2^s0) g2,j0,i0 g3,i0,i0 + 2 s0 g2,j0,j0 g3,i0,i0 +
 ((-2 + s0 - s0 T2 + 3 T2^s0 - T2^(2 s0) + T1^s0 (2 - (3 + s0) T2^s0 + T2^(2 s0) + s0 T2^(1+s0))) g3,j0,i0)/((-1 + T2) (-1 + T2^s0)) -
 ((-2 + (3 + s0) T2^s0 - T2^(2 s0) - s0 T2^(1+s0) + T1^s0 (2 - 3 T2^s0 - (-1 + s0) T2^(2 s0) + s0 T2^(1+2 s0))) g1,i0,i0 g3,j0,i0)/((-1 + T2) (-1 + T2^s0)) -
 1/((-1 + T2) (-1 + T2^s0)) (-1 + T1^s0) (-3 + s0 - s0 T2 + (4 + s0) T2^s0 - T2^(2 s0) - s0 T2^(1+s0) +
    T1^s0 (2 - (2 + s0) T2^s0 - s0 T2^(2 s0) + s0 T2^(1+s0) + s0 T2^(1+2 s0))) g1,j0,i0 g3,j0,i0 + ((-1 + T1^s0) g2,i0,i0 g3,j0,i0)/(-1 + T2) +
 ((-1 + s0 - s0 T2 + T2^s0 + T1^s0 (1 - (1 + s0) T2^s0 + s0 T2^(1+s0))) g2,i0,j0 g3,j0,i0)/((-1 + T2) (-1 + T2^s0)) +
 ((1 + s0 - s0 T2 - T2^s0 + T1^s0 (-1 - (-1 + s0) T2^s0 + s0 T2^(1+s0))) g2,j0,i0 g3,j0,i0)/(-1 + T2) +
 1/((-1 + T2) (-1 + T2^s0)) (1 - 2 s0 + 2 s0 T2 + (-2 + s0) T2^s0 + T2^(2 s0) - s0 T2^(1+s0) +
    T1^s0 (-1 + 2 (1 + s0) T2^s0 - (1 + s0) T2^(2 s0) - 2 s0 T2^(1+s0) + s0 T2^(1+2 s0))) g2,j0,j0 g3,j0,i0 +
 s0 g1,i0,i0 g3,j0,j0 + ((-1 + T1^s0) (1 - (1 + s0) T2^s0 + s0 T2^(1+s0)) g1,j0,i0 g3,j0,j0)/((-1 + T2) (-1 + T2^s0)) -
 s0 g2,i0,i0 g3,j0,j0 - s0 T2^s0 g2,j0,i0 g3,j0,j0 /. s0 → -1]
```

Out[•]=
$$-\frac{1}{2}-\frac{g_{1,i0,i0}\,g_{2,j0,i0}}{T_2}+\frac{(-1+T_1)\,g_{1,j0,i0}\,g_{2,j0,i0}}{T_1\,T_2}+g_{1,i0,i0}\,g_{2,j0,j0}+g_{3,i0,i0}-\frac{(-1+T_2)\,g_{2,j0,i0}\,g_{3,i0,i0}}{T_2}-2\,g_{2,j0,j0}\,g_{3,i0,i0}-\frac{(1-T_1+T_1\,T_2)\,g_{3,j0,i0}}{T_1\,T_2}-\frac{(-2+T_1)\,g_{1,i0,i0}\,g_{3,j0,i0}}{T_1\,T_2}-\frac{(-1+T_1)\,(1-T_1+T_1\,T_2)\,g_{1,j0,i0}\,g_{3,j0,i0}}{T_1^2\,T_2}-\frac{(-1+T_1)\,g_{2,i0,i0}\,g_{3,j0,i0}}{T_1\,(-1+T_2)}-g_{2,i0,j0}\,g_{3,j0,i0}+\frac{(-2+T_1+T_1\,T_2)\,g_{2,j0,i0}\,g_{3,j0,i0}}{T_1\,T_2}+\frac{\left(T_1-T_2-2\,T_1\,T_2+2\,T_1\,T_2^2\right)\,g_{2,j0,j0}\,g_{3,j0,i0}}{T_1\,(-1+T_2)\,T_2}-g_{1,i0,i0}\,g_{3,j0,j0}+g_{2,i0,i0}\,g_{3,j0,j0}+\frac{g_{2,j0,i0}\,g_{3,j0,j0}}{T_2}$$

In[•]:=
```
Factor[-T2 (T1 - 1) + (T1 - T2 - 2 T1 T2 + 2 T1 T2^2)]
```

Out[•]=
$$T_1\,(-1+T_2)\,(-1+2\,T_2)$$