Pensieve header: Read and study the Snappy data in DTs.txt and in VolsLowPrecision.txt.

## Startup

```
In[•]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\Theta\\Vols"];
        << KnotTheory`
```

```
Loading KnotTheory` version of October 29, 2024, 10:29:52.1301.
Read more at http://katlas.org/wiki/KnotTheory.
```

```
In[•]:= Rot[pd_PD] := Module[{n, xs, x, rots, Xp, Xm, front = {1}, k},
          n = Length@pd; rots = Table[0, {2 n + 1}];
          xs = Cases[pd, x_X :> {Xp[x[[4]], x[[1]]] PositiveQ@x
                                 Xm[x[[2]], x[[1]]]      True   ];
          For[k = 1, k ≤ 2 n, ++k,
           If[FreeQ[front, -k],
            front = Flatten@Replace[front, k → (xs /. {
                    Xp[k, l_] | Xm[l_, k] :> {l + 1, k + 1, -l},
                    Xp[l_, k] | Xm[k, l_] :> (++rots[[l]]; {-l, k + 1, l + 1}),
                    _Xp | _Xm :> {}
                   }), {1}],
            Cases[front, k | -k] /. {k, -k} :> --rots[[k]];
           ]
          ];
          {xs /. {Xp[i_, j_] :> {+1, i, j}, Xm[i_, j_] :> {-1, i, j}}, rots}];
        Rot[K_] := Rot[PD[K]];
```

```
In[•]:= CF[ℰ_] := Expand@Collect[ℰ, g__, F] /. F → Factor;
```

```
In[•]:= T3 = T1 T2;
```

```
In[•]:= F1[{s_, i_, j_}] := CF[
          s (1/2 - g3ii + T2^s g1ii g2ji - g1ii g2jj - (T2^s - 1) g2ji g3ii + 2 g2jj g3ii - (1 - T3^s) g2ji g3ji -
             g2ii g3jj - T2^s g2ji g3jj + g1ii g3jj + ((T1^s - 1) g1ji (T2^2 s g2ji - T2^s g2jj + T2^s g3jj) +
                (T3^s - 1) g3ji (1 - T2^s g1ii + g2ij + (T2^s - 2) g2jj - (T1^s - 1) (T2^s + 1) g1ji)) / (T2^s - 1))]
```

```
In[•]:= F2[{s0_, i0_, j0_}, {s1_, i1_, j1_}] := CF[
          s1 (T1^s0 - 1) (T2^s1 - 1)^-1 (T3^s1 - 1) g1,j1,i0 g3,j0,i1 ((T2^s0 g2,i1,i0 - g2,i1,j0) - (T2^s0 g2,j1,i0 - g2,j1,j0))]
```

```
In[•]:= F3[φ_, k_] = φ g3kk - φ / 2;
```

```
In[•]:= Θ[K_] := Θ[K] = Module[{X, φ, n, A, Δ, G, ev, Θ},
    (* 01 *) {X, φ} = Rot[K]; n = Length[X];
    (* 02 *) A = IdentityMatrix[2 n + 1];
    (* 03 *) Cases[X, {s_, i_, j_} :> (A[[{i, j}, {i + 1, j + 1}]] += ({{-T^s, T^s - 1}, {0, -1}}))];
    (* 04 *) Δ = T^((-Total[φ] - Total[X[[All, 1]]])/2) Det[A];
    (* 05 *) G = Inverse[A];
    (* 06 *) ev[ℰ_] := Factor[ℰ /. g_{ν_, α_, β_} :> (G[[α, β]] /. T -> T_ν)];
    (* 07 *) Θ = ev[∑_{k=1}^{n} F1[X[[k]]]];
    (* 08 *) Θ += ev[∑_{k1=1}^{n} ∑_{k2=1}^{n} F2[X[[k1]], X[[k2]]]];
    (* 09 *) Θ += ev[∑_{k=1}^{2 n} F3[φ[[k]], k]];
    (* 10 *) Factor@{Δ, (Δ /. T -> T1) (Δ /. T -> T2) (Δ /. T -> T3) Θ}
   ];
```

```
In[•]:= {Θ[DTCode[4, 6, 2]], Θ[Knot[3, 1]]}
```

KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

KnotTheory: Loading precomputed data in PD4Knots`.

Out[•]=

$$\left\{\left\{\frac{1-T+T^2}{T},\ \frac{1-T_1+T_1^2-T_2-T_1^3T_2+T_2^2+T_1^4T_2^2-T_1T_2^3-T_1^4T_2^3+T_1^2T_2^4-T_1^3T_2^4+T_1^4T_2^4}{T_1^2T_2^2}\right\},\right.$$
$$\left.\left\{\frac{1-T+T^2}{T},\ -\frac{1-T_1+T_1^2-T_2-T_1^3T_2+T_2^2+T_1^4T_2^2-T_1T_2^3-T_1^4T_2^3+T_1^2T_2^4-T_1^3T_2^4+T_1^4T_2^4}{T_1^2T_2^2}\right\}\right\}$$

## Reading and Confirming the SnapPy DTs

```
In[•]:= f = OpenRead["C:\\drorbn\\AcademicPensieve\\Projects\\Theta\\Vols\\DTs.txt"];
DTs = Table[{Read[f, Word], Read[f, String]}, 313 230] /. {K_String, dt_String} :>
    (Knot[K] -> ToExpression@StringReplace[dt, {"[(" -> "DTCode[", ")]" -> "]"}]);
Close[f]; DTs
```

Out[•]=

```
In[•]:= Total[Drop[DTs, 249] /. {(K_ -> dt_) :> (DTCode[K] == dt)}]
```

KnotTheory: Loading precomputed data in DTCode4KnotsTo11`.

KnotTheory: Loading precomputed data in KnotTheory/12A.dts.

KnotTheory: Loading precomputed data in KnotTheory/12N.dts.

General: Further output of KnotTheory::loading will be suppressed during this calculation.

Out[•]=

312 981 True

```
In[•]:= Short@Thread[DTCode /@ AllKnots[{3, 10}] == Take[DTs, 249][[All, 2]]]
```

```
Out[•]//Short=
{True, <<247>>,
 DTCode[6, 8, 14, 18, 16, 4, -20, 10, 2, -12] == DTCode[6, -10, -18, <<4>>, 8, -4, 12]}
```

```
In[•]:= FindInRolfsen[K_] := Select[AllKnots[{3, 10}],
  (Θ[K][[1]] == Θ[#][[1]] ∧ (Θ[K][[2]] == Θ[#][[2]] ∨ Θ[K][[2]] == -Θ[#][[2]])) &]
```

```
In[•]:= FindInRolfsen[DTCode[4, 6, 2]]
```

```
Out[•]=
{Knot[3, 1]}
```

```
In[•]:= Table[1, 249] == Length /@ (FindInRolfsen /@ Take[DTs, 249][[All, 2]])
```

```
Out[•]=
True
```

```
In[•]:= Take[DTs, 249] /. (K_ → dt_) :> (K = First@FindInRolfsen[dt])
```

```
Out[•]=
{Knot[3, 1], Knot[4, 1], Knot[5, 2], Knot[5, 1], Knot[6, 3], Knot[6, 2], Knot[6, 1],
 Knot[7, 7], Knot[7, 6], Knot[7, 5], Knot[7, 2], Knot[7, 3], Knot[7, 4], Knot[7, 1],
 Knot[8, 14], Knot[8, 15], Knot[8, 10], Knot[8, 8], Knot[8, 12], Knot[8, 7], Knot[8, 13],
 Knot[8, 2], Knot[8, 11], Knot[8, 6], Knot[8, 1], Knot[8, 18], Knot[8, 5], Knot[8, 17],
 Knot[8, 16], Knot[8, 9], Knot[8, 4], Knot[8, 3], Knot[8, 20], Knot[8, 21], Knot[8, 19],
 Knot[9, 30], Knot[9, 22], Knot[9, 19], Knot[9, 25], Knot[9, 28], Knot[9, 32],
 Knot[9, 24], Knot[9, 8], Knot[9, 36], Knot[9, 15], Knot[9, 33], Knot[9, 27], Knot[9, 31],
 Knot[9, 17], Knot[9, 26], Knot[9, 23], Knot[9, 14], Knot[9, 37], Knot[9, 20],
 Knot[9, 11], Knot[9, 21], Knot[9, 12], Knot[9, 6], Knot[9, 18], Knot[9, 16], Knot[9, 7],
 Knot[9, 2], Knot[9, 34], Knot[9, 41], Knot[9, 38], Knot[9, 29], Knot[9, 39], Knot[9, 9],
 Knot[9, 13], Knot[9, 4], Knot[9, 5], Knot[9, 40], Knot[9, 3], Knot[9, 10], Knot[9, 35],
 Knot[9, 1], Knot[9, 44], Knot[9, 45], Knot[9, 43], Knot[9, 42], Knot[9, 46], Knot[9, 48],
 Knot[9, 47], Knot[9, 49], Knot[10, 60], Knot[10, 59], Knot[10, 73], Knot[10, 72],
 Knot[10, 36], Knot[10, 57], Knot[10, 81], Knot[10, 80], Knot[10, 55], Knot[10, 71],
 Knot[10, 88], Knot[10, 97], Knot[10, 49], Knot[10, 53], Knot[10, 47], Knot[10, 51],
 Knot[10, 78], Knot[10, 77], Knot[10, 34], Knot[10, 58], Knot[10, 89], Knot[10, 70],
 Knot[10, 35], Knot[10, 96], Knot[10, 45], Knot[10, 39], Knot[10, 75], Knot[10, 56],
 Knot[10, 38], Knot[10, 40], Knot[10, 42], Knot[10, 44], Knot[10, 14], Knot[10, 30],
 Knot[10, 41], Knot[10, 113], Knot[10, 67], Knot[10, 69], Knot[10, 87], Knot[10, 66],
 Knot[10, 62], Knot[10, 65], Knot[10, 12], Knot[10, 28], Knot[10, 101], Knot[10, 92],
 Knot[10, 95], Knot[10, 54], Knot[10, 37], Knot[10, 84], Knot[10, 63], Knot[10, 43],
 Knot[10, 29], Knot[10, 13], Knot[10, 32], Knot[10, 5], Knot[10, 23], Knot[10, 27],
 Knot[10, 2], Knot[10, 21], Knot[10, 25], Knot[10, 74], Knot[10, 18], Knot[10, 10],
 Knot[10, 7], Knot[10, 107], Knot[10, 68], Knot[10, 15], Knot[10, 31], Knot[10, 6],
 Knot[10, 24], Knot[10, 105], Knot[10, 76], Knot[10, 20], Knot[10, 1], Knot[10, 112],
 Knot[10, 114], Knot[10, 79], Knot[10, 48], Knot[10, 52], Knot[10, 46], Knot[10, 50],
 Knot[10, 82], Knot[10, 86], Knot[10, 119], Knot[10, 85], Knot[10, 83], Knot[10, 118],
 Knot[10, 122], Knot[10, 121], Knot[10, 94], Knot[10, 90], Knot[10, 109], Knot[10, 115],
 Knot[10, 106], Knot[10, 98], Knot[10, 102], Knot[10, 111], Knot[10, 117], Knot[10, 110],
 Knot[10, 93], Knot[10, 120], Knot[10, 99], Knot[10, 100], Knot[10, 103], Knot[10, 91],
 Knot[10, 17], Knot[10, 19], Knot[10, 33], Knot[10, 9], Knot[10, 26], Knot[10, 22],
 Knot[10, 4], Knot[10, 8], Knot[10, 16], Knot[10, 11], Knot[10, 3], Knot[10, 104],
 Knot[10, 108], Knot[10, 116], Knot[10, 123], Knot[10, 64], Knot[10, 61], Knot[10, 138],
 Knot[10, 137], Knot[10, 136], Knot[10, 133], Knot[10, 135], Knot[10, 134], Knot[10, 154],
 Knot[10, 151], Knot[10, 150], Knot[10, 153], Knot[10, 149], Knot[10, 148], Knot[10, 132],
 Knot[10, 145], Knot[10, 125], Knot[10, 127], Knot[10, 126], Knot[10, 129], Knot[10, 131],
 Knot[10, 130], Knot[10, 124], Knot[10, 128], Knot[10, 146], Knot[10, 147], Knot[10, 141],
 Knot[10, 143], Knot[10, 139], Knot[10, 144], Knot[10, 140], Knot[10, 142], Knot[10, 161],
 Knot[10, 156], Knot[10, 160], Knot[10, 159], Knot[10, 163], Knot[10, 152], Knot[10, 165],
 Knot[10, 164], Knot[10, 155], Knot[10, 162], Knot[10, 158], Knot[10, 157]}
```

```
In[•]:= f = OpenRead["C:\\drorbn\\AcademicPensieve\\Projects\\Theta\\Vols\\VolsLowPrecision.txt"];
        VolsLowPrecision = Table[{Read[f, Word], Read[f, String]}, 313 230] /.
          {K_String, v_String} :> (Knot[K] → ToExpression[StringReplace[v, " E" → "*^"]]);
        Close[f];
        VolsLowPrecision /. (K_ → v_) :> (VolLP[K] = v);
        VolLP[Knot[4, 1]]
Out[•]=
        2.02988
```

```
In[•]:= f = OpenRead["C:\\drorbn\\AcademicPensieve\\Projects\\Theta\\Vols\\Vols.txt"];
        Vols = Table[{Read[f, Word], Read[f, String]}, 313 230] /.
          {K_String, v_String} :> (Knot[K] → ToExpression[StringReplace[v, " E" → "*^"]]);
        Close[f];
        Vols /. (K_ → v_) :> (Vol[K] = v);
        Vol[Knot[4, 1]]
Out[•]=
        2.029883212819307250042405108549040571883378615060599584034978 21
```

```
In[•]:= 313 230 - CountDistinct[Last /@ VolsLowPrecision]
Out[•]=
        27 357
```

```
In[•]:= Table[
          d → 801 - CountDistinct[Round[Last /@ Take[VolsLowPrecision, 801], 10^-d]],
          {d, 0, 11}]
Out[•]=
        {0 → 781, 1 → 645, 2 → 221, 3 → 52, 4 → 27, 5 → 25, 6 → 25, 7 → 24, 8 → 25, 9 → 25, 10 → 20, 11 → 19}
```

```
In[•]:= Table[
          d → 801 - CountDistinct[Round[Last /@ Take[Vols, 801], 10^-d]],
          {d, 0, 30}]
Out[•]=
        {0 → 781, 1 → 645, 2 → 221, 3 → 52, 4 → 27, 5 → 25, 6 → 25, 7 → 25, 8 → 25, 9 → 25, 10 → 25,
         11 → 25, 12 → 25, 13 → 25, 14 → 25, 15 → 25, 16 → 25, 17 → 25, 18 → 25, 19 → 25, 20 → 25,
         21 → 25, 22 → 25, 23 → 25, 24 → 25, 25 → 25, 26 → 25, 27 → 25, 28 → 25, 29 → 25, 30 → 25}
```

```
In[•]:= Table[
          d → 313 230 - CountDistinct[Round[Last /@ VolsLowPrecision, 10^-d]],
          {d, 0, 15}]
Out[•]=
        {0 → 313 197, 1 → 312 934, 2 → 310 741, 3 → 293 679, 4 → 193 768, 5 → 70 519, 6 → 46 455, 7 → 43 801,
         8 → 42 813, 9 → 42 737, 10 → 27 370, 11 → 27 358, 12 → 27 358, 13 → 27 358, 14 → 27 358, 15 → 27 358}
```

```
In[•]:= Sort@Table[Abs[Vol[K] - VolLP[K]], {K, AllKnots[{3, 15}]}]
Out[•]=
```

```
In[•]:= SortBy[AllKnots[{3, 15}], -Abs[Vol[#] - VolLP[#]] &]
```

```
Out[•]=
```

```
In[•]:= Take[SortBy[AllKnots[{3, 15}], -Abs[Vol[#] - VolLP[#]] &], 10]
```

```
Out[•]= {Knot[15, NonAlternating, 40 211], Knot[15, NonAlternating, 59 184],
 Knot[15, NonAlternating, 124 802], Knot[15, NonAlternating, 156 076],
 Knot[13, NonAlternating, 4639], Knot[14, NonAlternating, 22 180],
 Knot[15, NonAlternating, 142 188], Knot[13, NonAlternating, 4587],
 Knot[15, NonAlternating, 115 646], Knot[14, NonAlternating, 26 039]}
```

```
In[•]:= Vol[Knot[15, NonAlternating, 40 211]]
```

```
Out[•]= 2.538502934788206090865315551220975540200511797239604498099072 63
```

```
In[•]:= VolLP[Knot[15, NonAlternating, 40 211]]
```

```
Out[•]= 0.0000829147
```