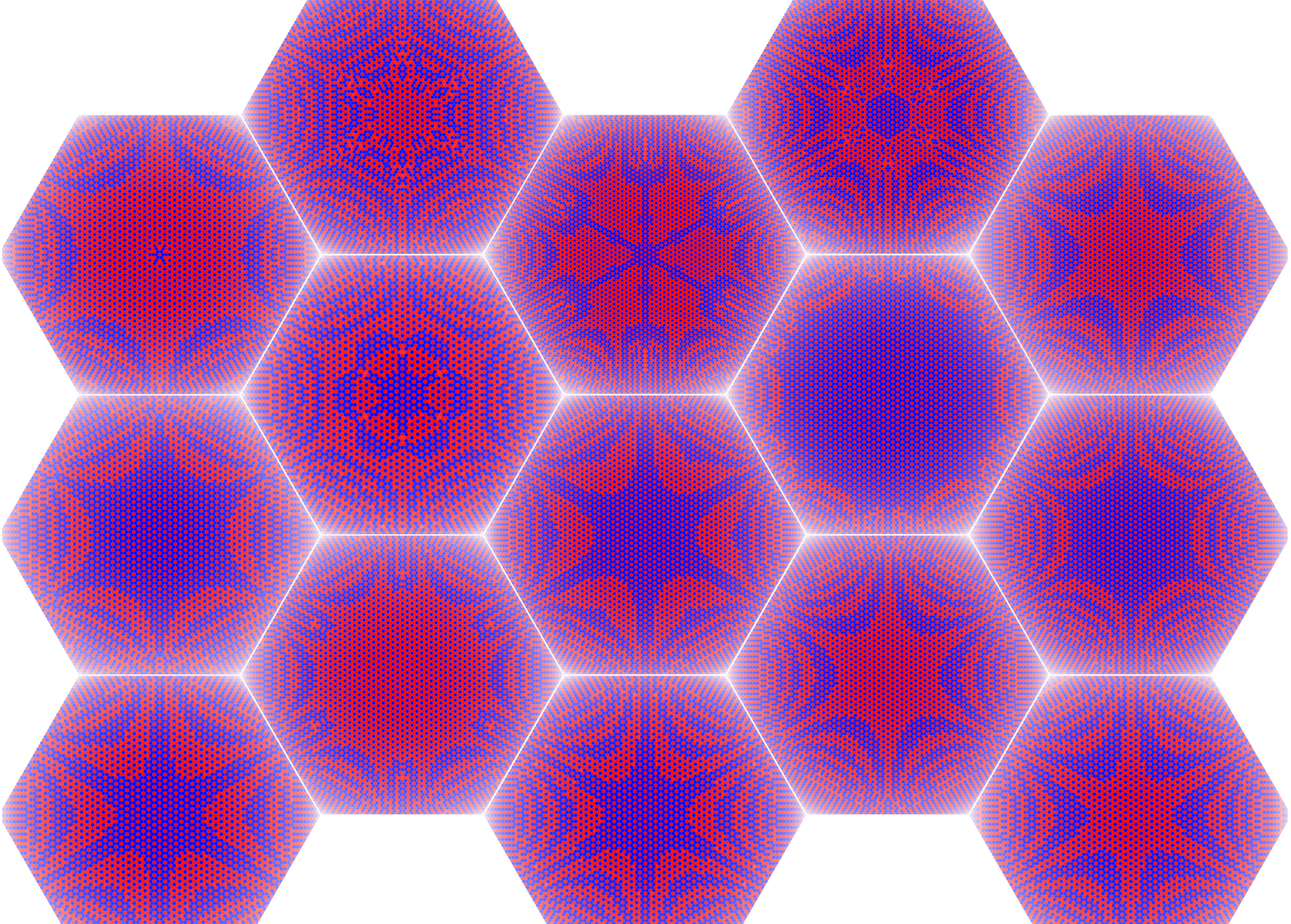

Very Fast and Very Strong

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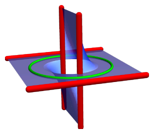
February 19, 2025. There is a knot invariant Θ that can go by a fancy name "the two loop contribution to the Kontsevich integral" [1–4].

Theorem. A down-to-earth algorithm for computing Θ exists that makes it computable for knots with hundreds of crossings [5].

Fact. On the 313,230 prime knots with up to 15 crossings Θ attains 306,472 distinct values—a deficit of 6,758—whereas the HOMFLY-PT polynomial and Khovanov homology, taken together, have a deficit of 70,245, about 10 times the worse.

Strongly Supported Conjecture. There is a *Seifert Formula*: Θ can be presented as a perturbed Gaussian integral of a Lagrangian on (6 copies of) the first homology H_1 of a Seifert surface Σ of a knot K, itself defined using low degree finite type invariants of links representing classes in H_1. Thus Θ bounds the genus of K.

Dream. Pretty Seifert surfaces will lead to pretty formulas. In particular, Θ may say something about ribbon knots, whose Seifert surfaces (right) are pretty.



As a two variable polynomial, Θ is a 2D array of coefficients, which can be interpreted as directing the colours of a 2D array of pixels, which can be viewed as a picture. On the obverse are the pictures corresponding to Θ for 15 random knots with 101–115 crossings. There are patterns there; we don't understand them yet.

[1] Rozansky. A Universal $U(1)$-RCC Invariant of Links and Rationality Conjecture. Preprint.
[2] Garoufaldis & Rozansky. The Loop Expansion of the Kontsevich Integral, the Null-Move, and S-Equivalence. Preprint.
[3] Kricker. The Lines of the Kontsevich Integral and Rozansky's Rationality Conjecture. Preprint.
[4] Ohtsuki. On the 2–loop Polynomial of Knots. Geom. Top., 2007.
[5] Bar-Natan & van der Veen. A Very Fast, Very Strong, Topologically Meaningful and Fun Knot Invariant. In preparation, `https://drorbn.net/Theta`.