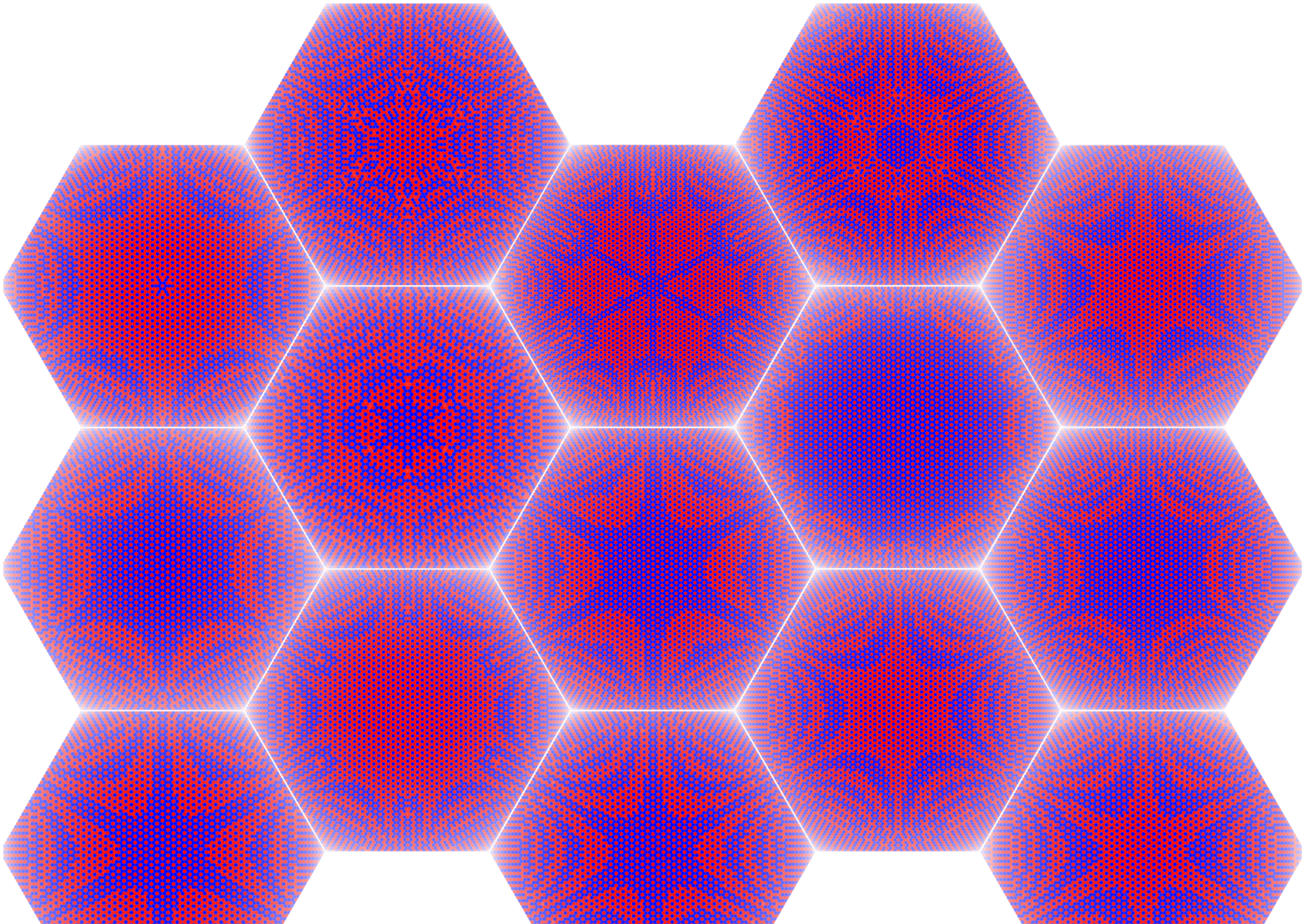

A Very Fast and Very Strong Knot Invariant

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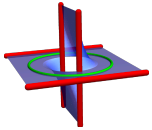
There is a knot invariant Θ that can go by a fancy name "the two loop contribution to the Kontsevich integral" [1–4].

Theorem [6]. There is a down to earth algorithm to compute Θ which makes it computable for knots with hundreds of crossings.

Fact. Θ is much stronger than the HOMFLY-PT polynomial H and Khovanov homology KH taken together. On the 313,230 prime knots with up to 15 crossings it attains 306,472 distinct values – a deficit of 6,758 – whereas H and KH together have a deficit of 70,245, about 10 times the worse.

Strongly Supported Conjecture. $\Theta(K)$ has a "Seifert Formula": It can be presented as a perturbed Gaussian integral of a Lagrangian on (6 copies of) the first homology H_1 of a Seifert surface Σ of a knot K, itself defined using low degree finite type invariants of links representing classes in H_1. Thus Θ bounds the genus of K.

Dream. Pretty Seifert surfaces will lead to pretty formulas, and in particular, Θ may say something about ribbon knots, whose Seifert surfaces, as on the right, are pretty.



Θ is a two variable polynomial. A two variable polynomial is a 2D array of coefficients, which can be interpreted as directing the colours of a 2D array of pixels, which can be viewed as a picture. On the other side are the 15 pictures corresponding to Θ of 15 random knots with 101–115 crossings. There are patterns there; we don't understand them yet.

[1] L. Rozansky, *A Universal $U(1)$-RCC Invariant of Links and Rationality Conjecture*, arXiv:math/0201139.
[2] S. Garoufaldis, L. Rozansky, *The Loop Expansion of the Kontsevich Integral, the Null-Move, and S-Equivalence*, arXiv:math.GT/0003187.
[3] A. Kricker, *The Lines of the Kontsevich Integral and Rozansky's Rationality Conjecture*, arXiv:math/0005284.
[4] T. Ohtsuki, *On the 2–loop Polynomial of Knots*, Geom. Top. **11** (2007) 1357–1475.
[5] DBN, R. van der Veen, *A Very Fast, Very Strong, Topologically Meaningful and Fun Knot Invariant*, in preparation at https://drorbn.net/Theta.