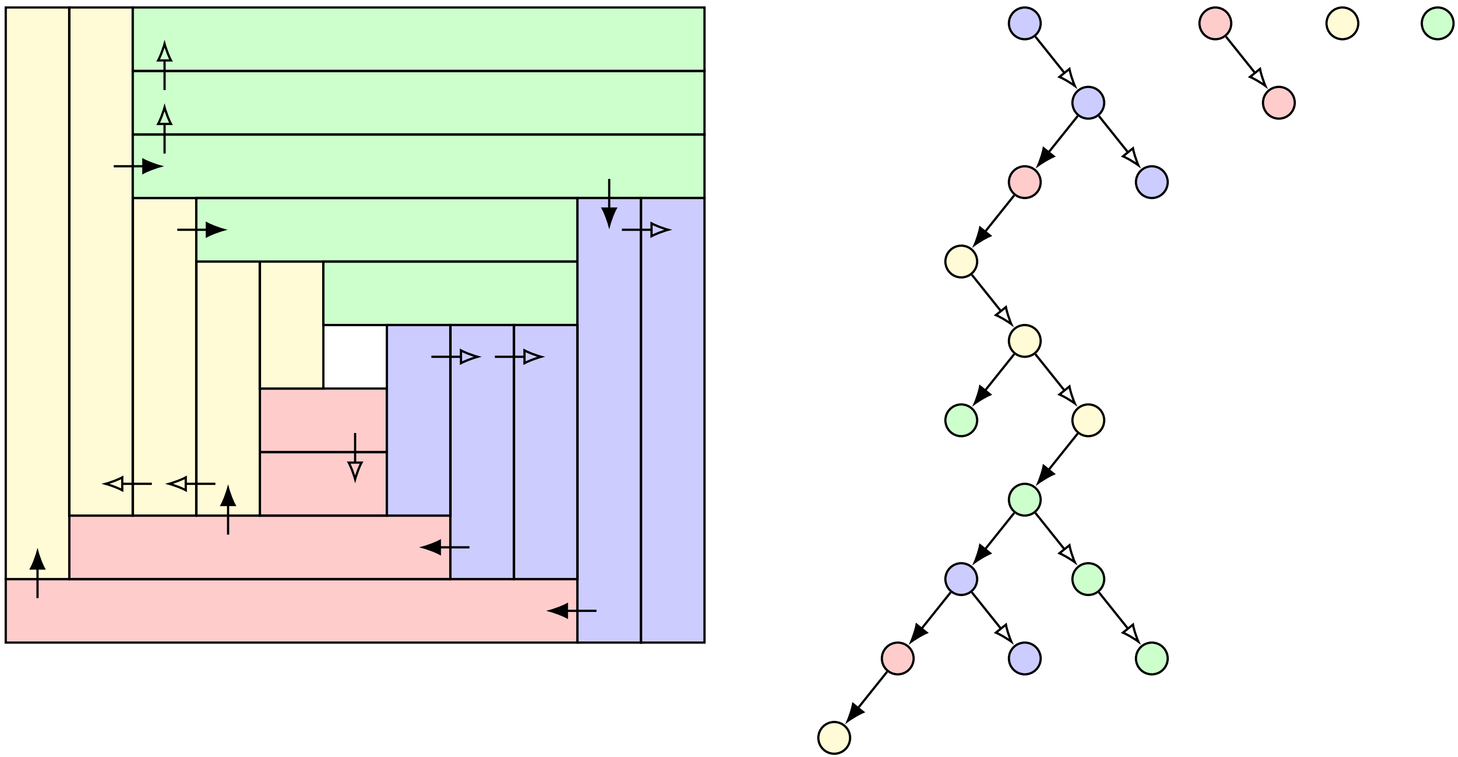

Counting simple whirls

Hugh Thomas
Université du Québec à Montréal, Département de mathématiques
thomas.hugh_r@uqam.ca

November 1, 2024. A rectangulation, like the one in the image, is a subdivision of a rectangle into subrectangles. There is a reasonable notion of what it means for two rectangulations to be combinatorially equivalent—**say a word about what this is?**—and rectangulations are considered up to this equivalence.

Various patterns arise in a rectangulation, for example, there is an occurrence of a windmill  in the image, centered at the blank square. A whirl is a rectangulation that avoids patterns of the form

A whirl is peelable if it contains a rectangle that stretches from the left boundary to the right boundary, or from the top boundary to the bottom boundary. A simple whirl is a non-peelable whirl containing exactly one  where the interior of the  is not further divided. The rectagulation on the obverse provides an example of a simple whirl.

Asinowski and Banderier [1] counted the simple whirls and showed that their generating function is $t^5C^4(t)$, where $C(t)$ is the well-known Catalan generating function

$$C(t) = \frac{1-\sqrt{1-4t}}{2t}$$

The Catalan generating function counts, in particular, planar binary trees. The image shows how a simple whirl can be decomposed into four planar binary trees, each of whose roots has no left child—these four trees are shown beside the rectangulation, **where two of the trees are single nodes (or childless roots).** The nodes on the right correspond to the rectangles on the left.

This gives a bijective proof of the result of Asinowski and Banderier, and is part of a joint project-in-process with these authors.

[1] Asinowski & Banderier. From geometry to generating functions: rectangulations and permutations. to appear.