

```
In[*]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\Theta"];
Once[<< Theta.m]
```

Loading KnotTheory` version of October 29, 2024, 10:29:52.1301.
Read more at <http://katlas.org/wiki/KnotTheory>.

```
In[*]:=  $\delta_{\alpha_-, \alpha_-} = 1$ ;  $\delta_{\alpha^-, \beta_-} := \delta_{\alpha, \beta}$ ;  $\delta_{\alpha_-, \beta_-} /;$  OrderedQ@{ $\beta$ ,  $\alpha$ } :=  $\delta_{\beta, \alpha}$ ;
```

```
In[*]:= R1 = Simplify[
  Residue[F1[{s0, i0, j0}], {T2, 1}] /. {g2, $\alpha_-, \alpha_-$  → 1, g2,j0,i0 → 1 - g2,i0,j0, g3, $\alpha_-, \beta_-$  → g1, $\alpha, \beta$ }]
```

```
Out[*]= - ( (-1 + T1^s0) g1,j0,i0 (g1,i0,i0 + 2 (-1 + T1^s0) g1,j0,i0 - g1,j0,j0) )
```

```
In[*]:= R2 = Simplify[Residue[F2[{s0, i0, j0}], {s1, i1, j1}], {T2, 1}] /. g3, $\alpha_-, \beta_-$  → g1, $\alpha, \beta$ ]
```

```
Out[*]= (-1 + T1^s0) (-1 + T1^s1) g1,j0,i1 g1,j1,i0 (g2,i1,i0 - g2,i1,j0 - g2,j1,i0 + g2,j1,j0)
```

```
In[*]:= D[s $_{-}$ , i $_{-}$ , j $_{-}$ ][ $\mathcal{E}_{-}$ ] := ( $\mathcal{E}$  /.  $\gamma$  → i $^{+}$ ) + ( $\mathcal{E}$  /.  $\gamma$  → j $^{+}$ ) - ( $\mathcal{E}$  /.  $\gamma$  → i) - ( $\mathcal{E}$  /.  $\gamma$  → j);
lhs = D[{s1, i1, j1}][ (T1^s0 - 1) g2, $\gamma$ ,i0 g1, $\gamma$ ,i0 g1,j0 $^{+}$ , $\gamma$  + (1 - T1^s0) g2, $\gamma$ ,j0 g1, $\gamma$ ,i0 g1,j0 $^{+}$ , $\gamma$  ]
```

```
Out[*]= - ( (-1 + T1^s0) g1,i1,i0 g1,j0 $^{+}$ ,i1 g2,i1,i0 ) - (1 - T1^s0) g1,i1,i0 g1,j0 $^{+}$ ,i1 g2,i1,j0 -
  (-1 + T1^s0) g1,j1,i0 g1,j0 $^{+}$ ,j1 g2,j1,i0 - (1 - T1^s0) g1,j1,i0 g1,j0 $^{+}$ ,j1 g2,j1,j0 +
  (-1 + T1^s0) g1,i1 $^{+}$ ,i0 g1,j0 $^{+}$ ,i1 $^{+}$  g2,i1 $^{+}$ ,i0 + (1 - T1^s0) g1,i1 $^{+}$ ,i0 g1,j0 $^{+}$ ,i1 $^{+}$  g2,i1 $^{+}$ ,j0 +
  (-1 + T1^s0) g1,j0 $^{+}$ ,j1 $^{+}$  g1,j1 $^{+}$ ,i0 g2,j1 $^{+}$ ,i0 + (1 - T1^s0) g1,j0 $^{+}$ ,j1 $^{+}$  g1,j1 $^{+}$ ,i0 g2,j1 $^{+}$ ,j0
```

```
In[*]:= rhs =  $\delta_{c0, c1}$  R1 + R2
```

```
Out[*]= - ( (-1 + T1^s0)  $\delta_{c0, c1}$  g1,j0,i0 (g1,i0,i0 + 2 (-1 + T1^s0) g1,j0,i0 - g1,j0,j0) ) +
  (-1 + T1^s0) (-1 + T1^s1) g1,j0,i1 g1,j1,i0 (g2,i1,i0 - g2,i1,j0 - g2,j1,i0 + g2,j1,j0)
```

```
In[*]:= Expand[Expand[lhs == rhs /. gRules[{s0, i0, j0}], {s1, i1, j1}]] /. {
  T2 → 1,  $\delta_{i0, j0}$  |  $\delta_{i1, j0}$  |  $\delta_{i0, j1}$  → 0,  $\delta_{i0, i1}$  |  $\delta_{j0, j1}$  |  $\delta_{c0, c1}^2$  →  $\delta_{c0, c1}$ ,
  g2, $\alpha_-, \alpha_-$  → 0, g2,j0 $^{+}$ ,i0 → 1 -  $\delta_{i0, j0}$  - g2,i0 $^{+}$ ,j0,
   $\delta_{c0, c1}$   $\mathcal{E}_{-}$  → ( $\mathcal{E}$  /. {s1 → s0, i1 → i0, j1 → j0})
}]
```

```
Out[*]=  $\delta_{i1, i0}^2$  g1,j0 $^{+}$ ,i1 - T1^s0  $\delta_{i1, i0}^2$  g1,j0 $^{+}$ ,i1 + T1^s1  $\delta_{i1, i0}$  g1,i1 $^{+}$ ,i0 g1,j0 $^{+}$ ,i1 -
  T1^s0+s1  $\delta_{i1, i0}$  g1,i1 $^{+}$ ,i0 g1,j0 $^{+}$ ,i1 +  $\delta_{j1, i0}^2$  g1,j0 $^{+}$ ,j1 - T1^s0  $\delta_{j1, i0}^2$  g1,j0 $^{+}$ ,j1 -  $\delta_{j1, i0}$   $\delta_{j1, j0}$  g1,j0 $^{+}$ ,j1 +
  T1^s0  $\delta_{j1, i0}$   $\delta_{j1, j0}$  g1,j0 $^{+}$ ,j1 +  $\delta_{i1, i0}$  g1,j0 $^{+}$ ,i1 g1,j1 $^{+}$ ,i0 - T1^s0  $\delta_{i1, i0}$  g1,j0 $^{+}$ ,i1 g1,j1 $^{+}$ ,i0 -
  T1^s1  $\delta_{i1, i0}$  g1,j0 $^{+}$ ,i1 g1,j1 $^{+}$ ,i0 + T1^s0+s1  $\delta_{i1, i0}$  g1,j0 $^{+}$ ,i1 g1,j1 $^{+}$ ,i0 +  $\delta_{j1, i0}$  g1,j0 $^{+}$ ,j1 g1,j1 $^{+}$ ,i0 -
  T1^s0  $\delta_{j1, i0}$  g1,j0 $^{+}$ ,j1 g1,j1 $^{+}$ ,i0 -  $\delta_{j1, j0}$  g1,j0 $^{+}$ ,j1 g1,j1 $^{+}$ ,i0 + T1^s0  $\delta_{j1, j0}$  g1,j0 $^{+}$ ,j1 g1,j1 $^{+}$ ,i0 -
   $\delta_{j0 $^{+}$ , i1 $^{+}$ }$  g1,i1 $^{+}$ ,i0 g2,i1 $^{+}$ ,i0 + T1^s0  $\delta_{j0 $^{+}$ , i1 $^{+}$ }$  g1,i1 $^{+}$ ,i0 g2,i1 $^{+}$ ,i0 +  $\delta_{i1, i0}$  g1,j0 $^{+}$ ,i1 g2,i1 $^{+}$ ,i0 -
  T1^s0  $\delta_{i1, i0}$  g1,j0 $^{+}$ ,i1 g2,i1 $^{+}$ ,i0 + g1,j0 $^{+}$ ,i1 g1,j1 $^{+}$ ,i0 g2,i1 $^{+}$ ,i0 - T1^s0 g1,j0 $^{+}$ ,i1 g1,j1 $^{+}$ ,i0 g2,i1 $^{+}$ ,i0 -
  T1^s1 g1,j0 $^{+}$ ,i1 g1,j1 $^{+}$ ,i0 g2,i1 $^{+}$ ,i0 + T1^s0+s1 g1,j0 $^{+}$ ,i1 g1,j1 $^{+}$ ,i0 g2,i1 $^{+}$ ,i0 +
   $\delta_{j0 $^{+}$ , i1 $^{+}$ }$  g1,i1 $^{+}$ ,i0 g2,i1 $^{+}$ ,j0 - T1^s0  $\delta_{j0 $^{+}$ , i1 $^{+}$ }$  g1,i1 $^{+}$ ,i0 g2,i1 $^{+}$ ,j0 -  $\delta_{i1, i0}$  g1,j0 $^{+}$ ,i1 g2,i1 $^{+}$ ,j0 +
  T1^s0  $\delta_{i1, i0}$  g1,j0 $^{+}$ ,i1 g2,i1 $^{+}$ ,j0 - g1,j0 $^{+}$ ,i1 g1,j1 $^{+}$ ,i0 g2,i1 $^{+}$ ,j0 + T1^s0 g1,j0 $^{+}$ ,i1 g1,j1 $^{+}$ ,i0 g2,i1 $^{+}$ ,j0 +
```

[illegible]