```
In[•]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\Theta"];
        Once[<< Theta.m]
        SetOptions[PolyPlot, ImageSize → Tiny];
```

```
Loading KnotTheory` version of October 29, 2024, 10:29:52.1301.
```

```
In[•]:= eq = T, - 1 == s0 T,^((1-s0)/2) (T,^s0 - 1); Simplify[{eq /. s0 → 1, eq /. s0 → -1}]
Out[•]= {True, True}
```

```
In[•]:= RandomVK[n_] := {
          Prepend[#, 2 RandomInteger[1] - 1] & /@
           Partition[PermutationList[RandomPermutation[2 n], 2 n], 2],
```

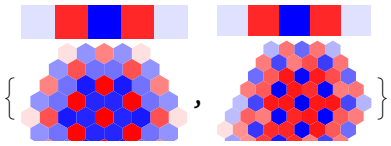
```
In[•]:= RandomVK[5]
Out[•]= {{{1, 7, 3}, {-1, 6, 2}, {1, 4, 9}, {-1, 10, 1}, {1, 5, 8}},
```

```
In[•]:= CF[ℰ_] := Expand@Collect[ℰ, g_ | χ_, F] /. F → Factor@*PowerExpand;
```

```
In[•]:= Short[Options[Θ] = {F1 → (F1i = F1[{s0, i0, j0}]),
```

```
Out[•]//Short= {F1 → s0/2 + s0 T2^s0 g1,i0,i0 g2,j0,i0 + <<19>> + <<1>>/<<1>> - s0 g2,i0,i0 g3,j0,j0 - s0 T2^s0 g2,j0,i0 g3,j0,j0,
```

```
In[•]:= Θ[K_, opts___Rule] := Module[{X, φφ, n, A, Δ, G, ev, Θ, kk, k0, k1, f1, f2, f3},
          f1 = F1 /. {opts} /. Options[Θ];
          f2 = F2 /. {opts} /. Options[Θ];
          f3 = F3 /. {opts} /. Options[Θ];
          {X, φφ} = Rot[K]; n = Length[X]; A = IdentityMatrix[2 n + 1];
          Cases[X, {s_, i_, j_} :> (A[[{i, j}, {i + 1, j + 1}]] += ((-T^s  T^s - 1)
                                                                    (  0      -1  )))];
          Δ = T^((-Total[φφ]-Total[X[[All,1]]])/2) Det[A];
          G = Inverse[A];
          ev[ℰ_] := Factor[
            ℰ /. {k_^+ :> k + 1, $ → 2 n + 1} /. {g_ν_,α_,β_ :> (G[[α, β]] /. T → T_ν), χ_True → 1, χ_False → 0}];
          Θ = ev@Sum[f1 /. Thread[{s0, i0, j0} → X[[kk]]] ⋃ Thread[{s1, i1, j1} → X[[kk]]], {kk, n}];
```

```
In[•]:= PolyPlot /@ {Θ[Knot[7, 6]], Θ[Knot[7, 6], F3 → 0]}
Out[•]= {  }
```

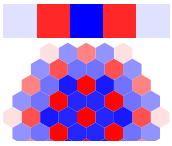
```
In[•]:= δ_i_,j_ := χ_i==j;
        χ_True = 1; χ_False = 0;
        χ_α_==β_ /; OrderedQ[{β, α}] := χ_β==α;
        χ_ph_^p_ /; p > 1 ^:= χ_ph;
        χ_i0==$ = χ_j0==$ = χ_i1==$ = χ_j1==$ = 0;
        χ_α_^+==1 = χ_1==α_^+ = 0;
        χ_α_^+==β_^+ := χ_α==β;

In[•]:= bRules[{s_, i_, j_}] := { (* b for "push indices backwards" *)
          g_ν_,j^+,β_ :→ g_ν,j,β - δ_j,β, g_ν_,i^+,β_ :→ T_ν^-s g_ν,i,β + (1 - T_ν^-s) g_ν,j,β - T_ν^-s δ_i,β - (1 - T_ν^-s) δ_j,β,
          g_ν_,α_,i^+ :→ T_ν^s g_ν,α,i + δ_α,i^+, g_ν_,α_,j^+ :→ g_ν,α,j + (1 - T_ν^s) g_ν,α,i + δ_α,j^+
        }.

In[•]:= Expand@{{g_ν,i,β, g_ν,j,β} /. gRules[{s, i, j}],

Out[•]= {{χ_i==β + T_ν^s g_ν,i^+,β + g_ν,i^+,β - T_ν^s g_ν,i^+,β, χ_i==β + g_ν,i^+,β}, {g_ν,i,β, g_ν,i,β}}

In[•]:= sRules[ℰ_] := FixedPoint[CF[# /. bRules[{s0, i0, j0}] ⋃ bRules[{s1, i1, j1}] ⋃ {
            χ_i0==j1 → 0, χ_i1==j0 → 0, χ_i0==j0 → 0,
            χ_j0==j1 → χ_i0==i1,
            χ_i1^+≤i0 → χ_i1≤i0 - χ_i0==i1, χ_i0^+==i1^+ → χ_i0==i1,
            χ_i0==i1 χ_i1≤i0 → χ_i0==i1, χ_i0==i1 χ_j1≤j0 → χ_i0==i1,
            χ_j0≤i0 → 1 - χ_i0≤j0, χ_i1≤i0 → 1 - χ_i0≤i1 + χ_i1==i0,
            χ_j1≤i0 → 1 - χ_i0≤j1 + χ_j1==i0, χ_i1≤i0 → 1 - χ_i0≤i1 + χ_i1==i0, χ_j1≤j0 → 1 - χ_j0≤j1 + χ_j1==j0,

In[•]:= D_{s_,i_,j_}[ℰ_] :=
          CF[((ℰ /. # → i^+) + (ℰ /. # → j^+) - (ℰ /. # → i) - (ℰ /. # → j)) //. bRules[{s, i, j}]];

In[•]:= D_s1,i1,i1[g_1,#,i0]

Out[•]= -T_1^-s1 χ_i0==i1 - T_1^-s1 (-1 + 2 T_1^s1) χ_i0==i1 - T_1^-s1 (-1 + T_1^s1) g_1,i1,i0 + T_1^-s1 (-1 + T_1^s1) g_1,i1,i0

In[•]:= D_s1,i1,i1[g_3,i0,#]

Out[•]= χ_i0==i1^+ + χ_i0==i1^+

In[•]:= B[g_3,i0,#]

Out[•]= -g_3,i0,1 + g_3,i0,4
```

In[•]:=
```
tw = g1,#,i0 ;
⊖[Knot[7, 6], F1 → -B[tw], F2 → D{s1,i1,j1}[tw], F3 → 0] // Echo // PolyPlot
```

» $\left\{-\frac{1-5\,T+7\,T^2-5\,T^3+T^4}{T^2},\right.$

$\frac{1}{T_1^4\,T_2^4}\left(1-5\,T_1+7\,T_1^2-5\,T_1^3+T_1^4-5\,T_2+20\,T_1\,T_2-10\,T_1^2\,T_2-10\,T_1^3\,T_2+20\,T_1^4\,T_2-5\,T_1^5\,T_2+7\,T_2^2-\right.$
$10\,T_1\,T_2^2-64\,T_1^2\,T_2^2+98\,T_1^3\,T_2^2-64\,T_1^4\,T_2^2-10\,T_1^5\,T_2^2+7\,T_1^6\,T_2^2-5\,T_2^3-10\,T_1\,T_2^3+98\,T_1^2\,T_2^3-50\,T_1^3\,T_2^3-$
$50\,T_1^4\,T_2^3+98\,T_1^5\,T_2^3-10\,T_1^6\,T_2^3-5\,T_1^7\,T_2^3+T_2^4+20\,T_1\,T_2^4-64\,T_1^2\,T_2^4-50\,T_1^3\,T_2^4+108\,T_1^4\,T_2^4-50\,T_1^5\,T_2^4-$
$64\,T_1^6\,T_2^4+20\,T_1^7\,T_2^4+T_1^8\,T_2^4-5\,T_1\,T_2^5-10\,T_1^2\,T_2^5+98\,T_1^3\,T_2^5-50\,T_1^4\,T_2^5-50\,T_1^5\,T_2^5+98\,T_1^6\,T_2^5-$

Out[•]=

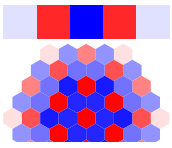


In[•]:=
```
tw = g1,j0,#;
⊖[Knot[7, 6], F1 → -B[tw], F2 → D{s1,i1,j1}[tw], F3 → 0] // Echo // PolyPlot
```

» $\left\{-\frac{1-5\,T+7\,T^2-5\,T^3+T^4}{T^2},\right.$

$\frac{1}{T_1^4\,T_2^4}\left(1-5\,T_1+7\,T_1^2-5\,T_1^3+T_1^4-5\,T_2+20\,T_1\,T_2-10\,T_1^2\,T_2-10\,T_1^3\,T_2+20\,T_1^4\,T_2-5\,T_1^5\,T_2+7\,T_2^2-\right.$
$10\,T_1\,T_2^2-64\,T_1^2\,T_2^2+98\,T_1^3\,T_2^2-64\,T_1^4\,T_2^2-10\,T_1^5\,T_2^2+7\,T_1^6\,T_2^2-5\,T_2^3-10\,T_1\,T_2^3+98\,T_1^2\,T_2^3-50\,T_1^3\,T_2^3-$
$50\,T_1^4\,T_2^3+98\,T_1^5\,T_2^3-10\,T_1^6\,T_2^3-5\,T_1^7\,T_2^3+T_2^4+20\,T_1\,T_2^4-64\,T_1^2\,T_2^4-50\,T_1^3\,T_2^4+108\,T_1^4\,T_2^4-50\,T_1^5\,T_2^4-$
$64\,T_1^6\,T_2^4+20\,T_1^7\,T_2^4+T_1^8\,T_2^4-5\,T_1\,T_2^5-10\,T_1^2\,T_2^5+98\,T_1^3\,T_2^5-50\,T_1^4\,T_2^5-50\,T_1^5\,T_2^5+98\,T_1^6\,T_2^5-$

Out[•]=



In[•]:=
```
bas0 = List @@ Expand[(g1,i1+,#+g1,j1+,#) (g2,i1+,#+g2,j1+,#) (g3,#,i1+g3,#,i1)]
```

Out[•]=

$\{g_{1,i1^+,\#1}\,g_{2,i1^+,\#1}\,g_{3,\#1,i1},\ g_{1,j1^+,\#1}\,g_{2,i1^+,\#1}\,g_{3,\#1,i1},\ g_{1,i1^+,\#1}\,g_{2,j1^+,\#1}\,g_{3,\#1,i1},\ g_{1,j1^+,\#1}\,g_{2,j1^+,\#1}\,g_{3,\#1,i1},$

In[•]:=
```
B /@ bas0
```

Out[•]=

$\{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\}$

In[•]:=
```
Column[(# → ⊖[Knot[7, 6], F1 → -B[#], F2 → D_{s0,i0,j0}[#], F3 → 0][[2]]) & /@ bas0]
```
Out[•]=

$$g_{1,i1^+,\#1}\, g_{2,i1^+,\#1}\, g_{3,\#1,i1} \to 0$$
$$g_{1,j1^+,\#1}\, g_{2,i1^+,\#1}\, g_{3,\#1,i1} \to 0$$
$$g_{1,i1^+,\#1}\, g_{2,j1^+,\#1}\, g_{3,\#1,i1} \to 0$$
$$g_{1,j1^+,\#1}\, g_{2,j1^+,\#1}\, g_{3,\#1,i1} \to 0$$
$$g_{1,i1^+,\#1}\, g_{2,i1^+,\#1}\, g_{3,\#1,j1} \to 0$$
$$g_{1,i1^+,\#1}\, g_{2,i1^+,\#1}\, g_{3,\#1,i1} \to 0$$

In[•]:=
```
bas1 = List @@ Expand[(g_{1,#,i0} + g_{1,#,j0}) (g_{2,#,i0} + g_{2,#,j0}) (g_{3,i0^+,#} + g_{3,j0^+,#})]
```
Out[•]=

$$\{g_{1,\#1,i0}\, g_{2,\#1,i0}\, g_{3,i0^+,\#1},\ g_{1,\#1,j0}\, g_{2,\#1,i0}\, g_{3,i0^+,\#1},\ g_{1,\#1,i0}\, g_{2,\#1,j0}\, g_{3,i0^+,\#1},\ g_{1,\#1,j0}\, g_{2,\#1,j0}\, g_{3,i0^+,\#1},$$

In[•]:=
```
B /@ bas1
```
Out[•]=

$$\{0, 0, 0, 0, 0, 0, 0, 0\}$$

In[•]:=
```
Column[(# → ⊖[Knot[7, 6], F1 → -B[#], F2 → D_{s1,i1,i1}[#], F3 → 0][[2]]) & /@ bas1]
```
Out[•]=

$$g_{1,\#1,i0}\, g_{2,\#1,i0}\, g_{3,i0^+,\#1} \to 0$$
$$g_{1,\#1,j0}\, g_{2,\#1,i0}\, g_{3,i0^+,\#1} \to 0$$
$$g_{1,\#1,i0}\, g_{2,\#1,j0}\, g_{3,i0^+,\#1} \to 0$$
$$g_{1,\#1,j0}\, g_{2,\#1,j0}\, g_{3,i0^+,\#1} \to 0$$
$$g_{1,\#1,i0}\, g_{2,\#1,i0}\, g_{3,j0^+,\#1} \to 0$$
$$g_{1,\#1,i0}\, g_{2,\#1,i0}\, g_{3,i0^+,\#1} \to 0$$

In[•]:=
```
Column[(# → sRules@D_{s0,i0,j0}[#]) & /@ bas0]
```
Out[•]=

$$\begin{aligned}
g_{1,i1^+,\#1}\, g_{2,i1^+,\#1}\, g_{3,\#1,i1} \to\ & -T_1^{-s0}\, T_2^{-s0}\, \chi_{i0==i1}\, g_{1,i0,i0}\, g_{2,i0,i0} - T_1^{-s0}\left(-1+T_1^{s0}\right) T_2^{-s0}\, \chi_{i0==i1}\, g_{1,j0,i0}\, g_{2,i0,i0} - \\
& T_1^{-s0}\, T_2^{-s0}\left(-1+T_2^{s0}\right) \chi_{i0==i1}\, g_{1,i0,i0}\, g_{2,j0,i0} - T_1^{-s0}\left(-1+T_1^{s0}\right) T_2^{-s0}\left(-1+T_2^{s0}\right) \chi_{i0==i1}\, g_{1,j0,i0}\, g_{2,j0,i0} - \\
& T_1^{-s0}\, T_2^{-s0}\, \chi_{i0==i1}\, g_{3,i0,i0} + T_1^{-s0}\, T_2^{-s0}\, \chi_{i0==i1}\, g_{1,i0,i0}\, g_{3,i0,i0} + \\
& T_1^{-s0}\left(-1+T_1^{s0}\right) T_2^{-s0}\, \chi_{i0==i1}\, g_{1,j0,i0}\, g_{3,i0,i0} + T_1^{-s0}\, T_2^{-s0}\, \chi_{i0==i1}\, g_{2,i0,i0}\, g_{3,i0,i0} + \\
& T_1^{-s0}\, T_2^{-s0}\left(-1+T_2^{s0}\right) \chi_{i0==i1}\, g_{2,j0,i0}\, g_{3,i0,i0} - T_1^{-s0}\left(-1+T_1^{s0}\right) T_2^{-s0}\left(-1+T_2^{s0}\right) \chi_{i0==i1}\, g_{3,j0,i0} + \\
& T_1^{-s0}\, T_2^{-s0}\left(-1+T_2^{s0}\right) \chi_{i0==i1}\, g_{1,i0,j0}\, g_{3,j0,i0} + T_1^{-s0}\left(-1+T_1^{s0}\right) T_2^{-s0}\left(-1+T_2^{s0}\right) \chi_{i0==i1}\, g_{1,j0,j0}\, g_{3,j0,i0} + \\
& T_1^{-s0}\left(-1+T_1^{s0}\right) T_2^{-s0}\, \chi_{i0==i1}\, g_{2,i0,j0}\, g_{3,j0,i0} + T_1^{-s0}\left(-1+T_1^{s0}\right) T_2^{-s0}\left(-1+T_2^{s0}\right) \chi_{i0==i1}\, g_{2,j0,j0}\, g_{3,j0,i0} + \\
& T_1^{-s1}\, T_2^{-s1}\left(-T_1^{s0}-T_2^{s0}+2\,T_1^{s0}\,T_2^{s0}\right) g_{1,i1,i0}\, g_{2,i1,i0}\, g_{3,j0,i1} - T_1^{-s1}\, T_2^{-s1}\left(-1+T_2^{s0}\right) g_{1,i1,j0}\, g_{2,i1,i0}\, g_{3,j0,i1} + \\
& T_1^{-s1}\left(-1+T_1^{s1}\right) T_2^{-s1}\left(-T_1^{s0}-T_2^{s0}+2\,T_1^{s0}\,T_2^{s0}\right) g_{1,j1,i0}\, g_{2,i1,i0}\, g_{3,j0,i1} - \\
& T_1^{-s1}\left(-1+T_1^{s1}\right) T_2^{-s1}\left(-1+T_2^{s0}\right) g_{1,j1,j0}\, g_{2,i1,i0}\, g_{3,j0,i1} - \\
& T_1^{-s1}\left(-1+T_1^{s0}\right) T_2^{-s1}\, g_{1,i1,i0}\, g_{2,i1,j0}\, g_{3,j0,i1} - T_1^{-s1}\left(-1+T_1^{s0}\right)\left(-1+T_1^{s1}\right) T_2^{-s1}\, g_{1,j1,i0}\, g_{2,i1,j0}\, g_{3,j0,i1} + \\
& T_1^{-s1}\, T_2^{-s1}\left(-T_1^{s0}-T_2^{s0}+2\,T_1^{s0}\,T_2^{s0}\right)\left(-1+T_2^{s1}\right) g_{1,i1,i0}\, g_{2,j1,i0}\, g_{3,j0,i1} - \\
& T_1^{-s1}\, T_2^{-s1}\left(-1+T_2^{s0}\right)\left(-1+T_2^{s1}\right) g_{1,i1,j0}\, g_{2,j1,i0}\, g_{3,j0,i1} + \\
& T_1^{-s1}\left(-1+T_1^{s1}\right) T_2^{-s1}\left(-T_1^{s0}-T_2^{s0}+2\,T_1^{s0}\,T_2^{s0}\right)\left(-1+T_2^{s1}\right) g_{1,j1,i0}\, g_{2,j1,i0}\, g_{3,j0,i1} - \\
& T_1^{-s1}\left(-1+T_1^{s1}\right) T_2^{-s1}\left(-1+T_2^{s0}\right)\left(-1+T_2^{s1}\right) g_{1,j1,j0}\, g_{2,j1,i0}\, g_{3,j0,i1} - \\
& T_1^{-s1}\left(-1+T_1^{s0}\right) T_2^{-s1}\left(-1+T_2^{s1}\right) g_{1,i1,i0}\, g_{2,j1,j0}\, g_{3,j0,i1} - \\
& T_1^{-s1}\left(-1+T_1^{s0}\right)\left(-1+T_1^{s1}\right) T_2^{-s1}\left(-1+T_2^{s1}\right) g_{1,j1,i0}\, g_{2,j1,j0}\, g_{3,j0,i1}
\end{aligned}$$

$$g_{1,j1^+,\#1}\,g_{2,i1^+,\#1}\,g_{3,\#1,i1}\to$$
$$-T_2^{-s0}\,\chi_{i0==i1}\,g_{1,j0,i0}\,g_{2,i0,i0}-T_2^{-s0}\left(-1+T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,i0}\,g_{2,j0,i0}+T_2^{-s0}\,\chi_{i0==i1}\,g_{1,j0,i0}\,g_{3,i0,i0}-$$
$$T_2^{-s0}\left(-1+T_2^{s0}\right)\chi_{i0==i1}\,g_{3,j0,i0}+T_2^{-s0}\left(-1+T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,j0}\,g_{3,j0,i0}+T_2^{-s0}\,\chi_{i0==i1}\,g_{2,i0,j0}\,g_{3,j0,i0}+$$
$$T_2^{-s0}\left(-1+T_2^{s0}\right)\chi_{i0==i1}\,g_{2,j0,j0}\,g_{3,j0,i0}+T_2^{-s1}\left(-T_1^{s0}-T_2^{s0}+2\,T_1^{s0}\,T_2^{s0}\right)g_{1,j1,i0}\,g_{2,i1,i0}\,g_{3,j0,i1}-$$
$$T_2^{-s1}\left(-1+T_2^{s0}\right)g_{1,j1,j0}\,g_{2,i1,i0}\,g_{3,j0,i1}-\left(-1+T_1^{s0}\right)T_2^{-s1}\,g_{1,j1,i0}\,g_{2,i1,j0}\,g_{3,j0,i1}+$$
$$T_2^{-s1}\left(-T_1^{s0}-T_2^{s0}+2\,T_1^{s0}\,T_2^{s0}\right)\left(-1+T_2^{s1}\right)g_{1,j1,i0}\,g_{2,j1,i0}\,g_{3,j0,i1}-$$
$$T_2^{-s1}\left(-1+T_2^{s0}\right)\left(-1+T_2^{s1}\right)g_{1,j1,j0}\,g_{2,j1,i0}\,g_{3,j0,i1}-\left(-1+T_1^{s0}\right)T_2^{-s1}\left(-1+T_2^{s1}\right)g_{1,j1,i0}\,g_{2,j1,j0}\,g_{3,j0,i1}$$

$$g_{1,i1^+,\#1}\,g_{2,j1^+,\#1}\,g_{3,\#1,i1}\to$$
$$-T_1^{-s0}\,\chi_{i0==i1}\,g_{1,i0,i0}\,g_{2,j0,i0}-T_1^{-s0}\left(-1+T_1^{s0}\right)\chi_{i0==i1}\,g_{1,j0,i0}\,g_{2,j0,i0}+T_1^{-s0}\,\chi_{i0==i1}\,g_{2,j0,i0}\,g_{3,i0,i0}-$$
$$T_1^{-s0}\left(-1+T_1^{s0}\right)\chi_{i0==i1}\,g_{3,j0,i0}+T_1^{-s0}\,\chi_{i0==i1}\,g_{1,i0,j0}\,g_{3,j0,i0}+T_1^{-s0}\left(-1+T_1^{s0}\right)\chi_{i0==i1}\,g_{1,j0,j0}\,g_{3,j0,i0}+$$
$$T_1^{-s0}\left(-1+T_1^{s0}\right)\chi_{i0==i1}\,g_{2,j0,j0}\,g_{3,j0,i0}+T_1^{-s1}\left(-T_1^{s0}-T_2^{s0}+2\,T_1^{s0}\,T_2^{s0}\right)g_{1,i1,i0}\,g_{2,j1,i0}\,g_{3,j0,i1}-$$
$$T_1^{-s1}\left(-1+T_2^{s0}\right)g_{1,i1,j0}\,g_{2,j1,i0}\,g_{3,j0,i1}+T_1^{-s1}\left(-1+T_1^{s1}\right)\left(-T_1^{s0}-T_2^{s0}+2\,T_1^{s0}\,T_2^{s0}\right)g_{1,j1,i0}\,g_{2,j1,i0}\,g_{3,j0,i1}-$$
$$T_1^{-s1}\left(-1+T_1^{s1}\right)\left(-1+T_2^{s0}\right)g_{1,j1,j0}\,g_{2,j1,i0}\,g_{3,j0,i1}-$$
$$T_1^{-s1}\left(-1+T_1^{s0}\right)g_{1,i1,i0}\,g_{2,j1,j0}\,g_{3,j0,i1}-T_1^{-s1}\left(-1+T_1^{s0}\right)\left(-1+T_1^{s1}\right)g_{1,j1,i0}\,g_{2,j1,j0}\,g_{3,j0,i1}$$

$$g_{1,j1^+,\#1}\,g_{2,j1^+,\#1}\,g_{3,\#1,i1}\to-\chi_{i0==i1}\,g_{1,j0,i0}\,g_{2,j0,i0}-\chi_{i0==i1}\,g_{3,j0,i0}+$$
$$\chi_{i0==i1}\,g_{1,j0,j0}\,g_{3,j0,i0}+\chi_{i0==i1}\,g_{2,j0,j0}\,g_{3,j0,i0}+\left(-T_1^{s0}-T_2^{s0}+2\,T_1^{s0}\,T_2^{s0}\right)g_{1,j1,i0}\,g_{2,j1,i0}\,g_{3,j0,i1}+$$
$$\left(1-T_2^{s0}\right)g_{1,j1,j0}\,g_{2,j1,i0}\,g_{3,j0,i1}+\left(1-T_1^{s0}\right)g_{1,j1,i0}\,g_{2,j1,j0}\,g_{3,j0,i1}$$

$$
\begin{aligned}
&g_{1,i1^+,\#1}\, g_{2,i1^+,\#1}\, g_{3,\#1,j1} \to \\
&\quad -T_1^{-s0}\, T_2^{-s0} \left(-T_1^{s0} - T_2^{s0} + 2\, T_1^{s0}\, T_2^{s0}\right) \chi_{i0==i1}\, g_{1,i0,i0}\, g_{2,i0,i0} + T_1^{-s0}\, T_2^{-s0} \left(-1 + T_2^{s0}\right) \chi_{i0==i1}\, g_{1,i0,j0}\, g_{2,i0,i0} - \\
&\quad T_1^{-s0} \left(-1 + T_1^{s0}\right) T_2^{-s0} \left(-T_1^{s0} - T_2^{s0} + 2\, T_1^{s0}\, T_2^{s0}\right) \chi_{i0==i1}\, g_{1,j0,i0}\, g_{2,i0,i0} + \\
&\quad T_1^{-s0} \left(-1 + T_1^{s0}\right) T_2^{-s0} \left(-1 + T_2^{s0}\right) \chi_{i0==i1}\, g_{1,j0,j0}\, g_{2,i0,i0} + \\
&\quad T_1^{-s0} \left(-1 + T_1^{s0}\right) T_2^{-s0}\, \chi_{i0==i1}\, g_{1,i0,i0}\, g_{2,i0,j0} - T_1^{-s0}\, T_2^{-s0}\, \chi_{i0==i1}\, g_{1,i0,j0}\, g_{2,i0,j0} + \\
&\quad T_1^{-s0} \left(-1 + T_1^{s0}\right)^2 T_2^{-s0}\, \chi_{i0==i1}\, g_{1,j0,i0}\, g_{2,i0,j0} - T_1^{-s0} \left(-1 + T_1^{s0}\right) T_2^{-s0}\, \chi_{i0==i1}\, g_{1,j0,j0}\, g_{2,i0,j0} - \\
&\quad T_1^{-s0}\, T_2^{-s0} \left(-1 + T_2^{s0}\right) \left(-T_1^{s0} - T_2^{s0} + 2\, T_1^{s0}\, T_2^{s0}\right) \chi_{i0==i1}\, g_{1,i0,i0}\, g_{2,j0,i0} + \\
&\quad T_1^{-s0}\, T_2^{-s0} \left(-1 + T_2^{s0}\right)^2 \chi_{i0==i1}\, g_{1,i0,j0}\, g_{2,j0,i0} - \\
&\quad T_1^{-s0} \left(-1 + T_1^{s0}\right) T_2^{-s0} \left(-1 + T_2^{s0}\right) \left(-T_1^{s0} - T_2^{s0} + 2\, T_1^{s0}\, T_2^{s0}\right) \chi_{i0==i1}\, g_{1,j0,i0}\, g_{2,j0,i0} + \\
&\quad T_1^{-s0} \left(-1 + T_1^{s0}\right) T_2^{-s0} \left(-1 + T_2^{s0}\right)^2 \chi_{i0==i1}\, g_{1,j0,j0}\, g_{2,j0,i0} + \\
&\quad T_1^{-s0} \left(-1 + T_1^{s0}\right) T_2^{-s0} \left(-1 + T_2^{s0}\right) \chi_{i0==i1}\, g_{1,i0,i0}\, g_{2,j0,j0} - T_1^{-s0}\, T_2^{-s0} \left(-1 + T_2^{s0}\right) \chi_{i0==i1}\, g_{1,i0,j0}\, g_{2,j0,j0} + \\
&\quad T_1^{-s0} \left(-1 + T_1^{s0}\right)^2 T_2^{-s0} \left(-1 + T_2^{s0}\right) \chi_{i0==i1}\, g_{1,j0,i0}\, g_{2,j0,j0} - \\
&\quad T_1^{-s0} \left(-1 + T_1^{s0}\right) T_2^{-s0} \left(-1 + T_2^{s0}\right) \chi_{i0==i1}\, g_{1,j0,j0}\, g_{2,j0,j0} - T_1^{-s0}\, T_2^{-s0}\, \chi_{i0==i1}\, g_{3,i0,j0} + \\
&\quad T_1^{-s0}\, T_2^{-s0}\, \chi_{i0==i1}\, g_{1,i0,i0}\, g_{3,i0,j0} + T_1^{-s0} \left(-1 + T_1^{s0}\right) T_2^{-s0}\, \chi_{i0==i1}\, g_{1,j0,i0}\, g_{3,i0,j0} + \\
&\quad T_1^{-s0}\, T_2^{-s0}\, \chi_{i0==i1}\, g_{2,i0,i0}\, g_{3,i0,j0} + T_1^{-s0}\, T_2^{-s0} \left(-1 + T_2^{s0}\right) \chi_{i0==i1}\, g_{2,j0,i0}\, g_{3,i0,j0} - \\
&\quad T_1^{-s0} \left(-1 + T_1^{s0}\right) T_2^{-s0} \left(-1 + T_2^{s0}\right) \chi_{i0==i1}\, g_{3,j0,j0} + T_1^{-s0}\, T_2^{-s0} \left(-1 + T_2^{s0}\right) \chi_{i0==i1}\, g_{1,i0,j0}\, g_{3,j0,j0} + \\
&\quad T_1^{-s0} \left(-1 + T_1^{s0}\right) T_2^{-s0} \left(-1 + T_2^{s0}\right) \chi_{i0==i1}\, g_{1,j0,j0}\, g_{3,j0,j0} + \\
&\quad T_1^{-s0} \left(-1 + T_1^{s0}\right) T_2^{-s0}\, \chi_{i0==i1}\, g_{2,i0,j0}\, g_{3,j0,j0} + T_1^{-s0} \left(-1 + T_1^{s0}\right) T_2^{-s0} \left(-1 + T_2^{s0}\right) \chi_{i0==i1}\, g_{2,j0,j0}\, g_{3,j0,j0} + \\
&\quad T_1^{-s1}\, T_2^{-s1} \left(-T_1^{s0} - T_2^{s0} + 2\, T_1^{s0}\, T_2^{s0}\right) g_{1,i1,i0}\, g_{2,i1,i0}\, g_{3,j0,j1} - T_1^{-s1}\, T_2^{-s1} \left(-1 + T_2^{s0}\right) g_{1,i1,j0}\, g_{2,i1,i0}\, g_{3,j0,j1} + \\
&\quad T_1^{-s1} \left(-1 + T_1^{s1}\right) T_2^{-s1} \left(-T_1^{s0} - T_2^{s0} + 2\, T_1^{s0}\, T_2^{s0}\right) g_{1,j1,i0}\, g_{2,i1,i0}\, g_{3,j0,j1} - \\
&\quad T_1^{-s1} \left(-1 + T_1^{s1}\right) T_2^{-s1} \left(-1 + T_2^{s0}\right) g_{1,j1,j0}\, g_{2,i1,i0}\, g_{3,j0,j1} - \\
&\quad T_1^{-s1} \left(-1 + T_1^{s0}\right) T_2^{-s1}\, g_{1,i1,i0}\, g_{2,i1,j0}\, g_{3,j0,j1} - T_1^{-s1} \left(-1 + T_1^{s0}\right) \left(-1 + T_1^{s1}\right) T_2^{-s1}\, g_{1,j1,i0}\, g_{2,i1,j0}\, g_{3,j0,j1} + \\
&\quad T_1^{-s1}\, T_2^{-s1} \left(-T_1^{s0} - T_2^{s0} + 2\, T_1^{s0}\, T_2^{s0}\right) \left(-1 + T_2^{s1}\right) g_{1,i1,i0}\, g_{2,j1,i0}\, g_{3,j0,j1} - \\
&\quad T_1^{-s1}\, T_2^{-s1} \left(-1 + T_2^{s0}\right) \left(-1 + T_2^{s1}\right) g_{1,i1,j0}\, g_{2,j1,i0}\, g_{3,j0,j1} + \\
&\quad T_1^{-s1} \left(-1 + T_1^{s1}\right) T_2^{-s1} \left(-T_1^{s0} - T_2^{s0} + 2\, T_1^{s0}\, T_2^{s0}\right) \left(-1 + T_2^{s1}\right) g_{1,j1,i0}\, g_{2,j1,i0}\, g_{3,j0,j1} - \\
&\quad T_1^{-s1} \left(-1 + T_1^{s1}\right) T_2^{-s1} \left(-1 + T_2^{s0}\right) \left(-1 + T_2^{s1}\right) g_{1,j1,j0}\, g_{2,j1,i0}\, g_{3,j0,j1} - \\
&\quad T_1^{-s1} \left(-1 + T_1^{s0}\right) T_2^{-s1} \left(-1 + T_2^{s1}\right) g_{1,i1,i0}\, g_{2,j1,j0}\, g_{3,j0,j1} - \\
&\quad T_1^{-s1} \left(-1 + T_1^{s0}\right) \left(-1 + T_1^{s1}\right) T_2^{-s1} \left(-1 + T_2^{s1}\right) g_{1,j1,i0}\, g_{2,j1,j0}\, g_{3,j0,j1}
\end{aligned}
$$

$$
\begin{aligned}
&T_1^{s0}\,T_2^{s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\left(-1+T_2^{s0}\right)g_{1,i1,i0}\,g_{2,j1,i0}\,g_{3,j0,i1}-\\
&T_1^{s0}\,T_2^{s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\left(-2+T_1^{s1}+T_2^{s1}\right)g_{1,j1,i0}\,g_{2,j1,i0}\,g_{3,j0,i1}
\end{aligned}
$$

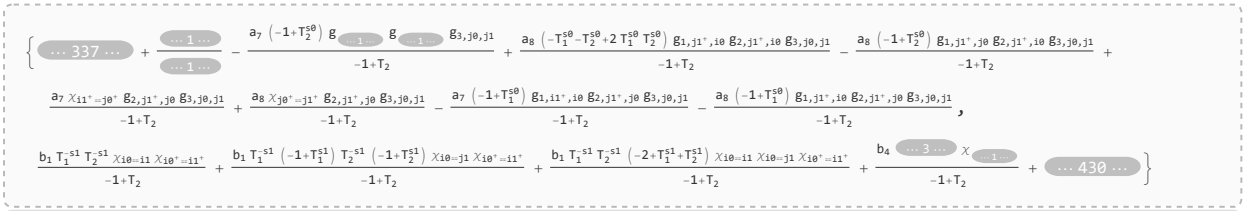
$$
\begin{aligned}
g_{1,\#1,j0}\,g_{2,\#1,i0}\,g_{3,i0^+,\#1} \to\;& T_1^{s0}\,T_2^{s0}\,\chi_{i0==i1}\,g_{1,i0,j0}\,g_{2,i0,i0}+T_1^{s0}\,T_2^{s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,j0}\,g_{2,j0,i0}+\\
&T_1^{s0}\left(-1+T_1^{s0}\right)T_2^{s0}\,\chi_{i0==i1}\,g_{3,i0,i0}-T_1^{s0}\,T_2^{s0}\,\chi_{i0==i1}\,g_{1,i0,j0}\,g_{3,i0,i0}-\\
&T_1^{s0}\left(-1+T_1^{s0}\right)T_2^{s0}\,\chi_{i0==i1}\,g_{1,j0,j0}\,g_{3,i0,i0}-T_1^{s0}\left(-1+T_1^{s0}\right)T_2^{s0}\,\chi_{i0==i1}\,g_{2,i0,i0}\,g_{3,i0,i0}+\\
&T_1^{s0}\,T_2^{s0}\left(-2+T_1^{s0}+T_2^{s0}\right)\chi_{i0==i1}\,g_{2,j0,i0}\,g_{3,i0,i0}+T_1^{s0}\left(-1+T_1^{s1}\right)T_2^{s0}\,g_{1,j1,j0}\,g_{2,i1,i0}\,g_{3,i0,i1}+\\
&T_1^{s0}\,T_2^{s0}\left(-1+T_2^{s1}\right)g_{1,i1,j0}\,g_{2,j1,i0}\,g_{3,i0,i1}-T_1^{s0}\,T_2^{s0}\left(-2+T_1^{s1}+T_2^{s1}\right)g_{1,j1,j0}\,g_{2,j1,i0}\,g_{3,i0,i1}-\\
&T_1^{s0}\,T_2^{s0}\,\chi_{i0==i1}\,g_{2,j0,i0}\,g_{3,i0,j0}+T_1^{s0}\left(-1+T_1^{s0}\right)T_2^{s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{3,j0,i0}-\\
&T_1^{s0}\,T_2^{s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{1,i0,j0}\,g_{3,j0,i0}-T_1^{s0}\left(-1+T_1^{s0}\right)T_2^{s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,j0}\,g_{3,j0,i0}-\\
&T_1^{s0}\left(-1+T_1^{s0}\right)T_2^{s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{2,i0,i0}\,g_{3,j0,i0}+\\
&T_1^{s0}\,T_2^{s0}\left(-2+T_1^{s0}+T_2^{s0}\right)\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{2,j0,i0}\,g_{3,j0,i0}+\\
&T_1^{s0}\left(-1+T_1^{s1}\right)T_2^{s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)g_{1,j1,j0}\,g_{2,i1,i0}\,g_{3,j0,i1}+\\
&T_1^{s0}\,T_2^{s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\left(-1+T_2^{s1}\right)g_{1,i1,j0}\,g_{2,j1,i0}\,g_{3,j0,i1}-\\
&T_1^{s0}\,T_2^{s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\left(-2+T_1^{s1}+T_2^{s1}\right)g_{1,j1,j0}\,g_{2,j1,i0}\,g_{3,j0,i1}-\\
&T_1^{s0}\,T_2^{s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{2,j0,i0}\,g_{3,j0,j0}
\end{aligned}
$$

$$
\begin{aligned}
g_{1,\#1,i0}\,g_{2,\#1,j0}\,g_{3,i0^+,\#1} \to\;& T_1^{s0}\,T_2^{s0}\,\chi_{i0==i1}\,g_{1,i0,i0}\,g_{2,i0,j0}+T_1^{s0}\,T_2^{s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,i0}\,g_{2,j0,j0}+\\
&T_1^{s0}\,T_2^{s0}\left(-1+T_2^{s0}\right)\chi_{i0==i1}\,g_{3,i0,i0}-T_1^{s0}\,T_2^{s0}\left(-1+T_2^{s0}\right)\chi_{i0==i1}\,g_{1,i0,i0}\,g_{3,i0,i0}+\\
&T_1^{s0}\,T_2^{s0}\left(-2+T_1^{s0}+T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,i0}\,g_{3,i0,i0}-T_1^{s0}\,T_2^{s0}\,\chi_{i0==i1}\,g_{2,i0,j0}\,g_{3,i0,i0}-\\
&T_1^{s0}\,T_2^{s0}\left(-1+T_2^{s0}\right)\chi_{i0==i1}\,g_{2,j0,j0}\,g_{3,i0,i0}+T_1^{s0}\left(-1+T_1^{s1}\right)T_2^{s0}\,g_{1,j1,i0}\,g_{2,i1,j0}\,g_{3,i0,i1}+\\
&T_1^{s0}\,T_2^{s0}\left(-1+T_2^{s1}\right)g_{1,i1,i0}\,g_{2,j1,j0}\,g_{3,i0,i1}-T_1^{s0}\,T_2^{s0}\left(-2+T_1^{s1}+T_2^{s1}\right)g_{1,j1,i0}\,g_{2,j1,j0}\,g_{3,i0,i1}-\\
&T_1^{s0}\,T_2^{s0}\,\chi_{i0==i1}\,g_{1,j0,i0}\,g_{3,i0,j0}+T_1^{s0}\,T_2^{s0}\left(-1+T_2^{s0}\right)\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{3,j0,i0}-\\
&T_1^{s0}\,T_2^{s0}\left(-1+T_2^{s0}\right)\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{1,i0,i0}\,g_{3,j0,i0}+\\
&T_1^{s0}\,T_2^{s0}\left(-2+T_1^{s0}+T_2^{s0}\right)\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,i0}\,g_{3,j0,i0}-\\
&T_1^{s0}\,T_2^{s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{2,i0,j0}\,g_{3,j0,i0}-T_1^{s0}\,T_2^{s0}\left(-1+T_2^{s0}\right)\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{2,j0,j0}\,g_{3,j0,i0}+\\
&T_1^{s0}\left(-1+T_1^{s1}\right)T_2^{s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)g_{1,j1,i0}\,g_{2,i1,j0}\,g_{3,j0,i1}+\\
&T_1^{s0}\,T_2^{s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\left(-1+T_2^{s1}\right)g_{1,i1,i0}\,g_{2,j1,j0}\,g_{3,j0,i1}-\\
&T_1^{s0}\,T_2^{s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\left(-2+T_1^{s1}+T_2^{s1}\right)g_{1,j1,i0}\,g_{2,j1,j0}\,g_{3,j0,i1}-\\
&T_1^{s0}\,T_2^{s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,i0}\,g_{3,j0,j0}
\end{aligned}
$$

$$
\begin{aligned}
g_{1,\#1,j0}\,g_{2,\#1,j0}\,g_{3,i0^+,\#1} \to\;& T_1^{s0}\,T_2^{s0}\,\chi_{i0==i1}\,g_{1,i0,j0}\,g_{2,i0,j0}+T_1^{s0}\,T_2^{s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,j0}\,g_{2,j0,j0}-\\
&T_1^{s0}\,T_2^{s0}\left(-2+T_1^{s0}+T_2^{s0}\right)\chi_{i0==i1}\,g_{3,i0,i0}-T_1^{s0}\,T_2^{s0}\left(-1+T_2^{s0}\right)\chi_{i0==i1}\,g_{1,i0,j0}\,g_{3,i0,i0}+\\
&T_1^{s0}\,T_2^{s0}\left(-2+T_1^{s0}+T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,j0}\,g_{3,i0,i0}-T_1^{s0}\left(-1+T_1^{s0}\right)T_2^{s0}\,\chi_{i0==i1}\,g_{2,i0,j0}\,g_{3,i0,i0}+\\
&T_1^{s0}\,T_2^{s0}\left(-2+T_1^{s0}+T_2^{s0}\right)\chi_{i0==i1}\,g_{2,j0,j0}\,g_{3,i0,i0}+T_1^{s0}\left(-1+T_1^{s1}\right)T_2^{s0}\,g_{1,j1,j0}\,g_{2,i1,j0}\,g_{3,i0,i1}+\\
&T_1^{s0}\,T_2^{s0}\left(-1+T_2^{s1}\right)g_{1,i1,j0}\,g_{2,j1,j0}\,g_{3,i0,i1}-T_1^{s0}\,T_2^{s0}\left(-2+T_1^{s1}+T_2^{s1}\right)g_{1,j1,j0}\,g_{2,j1,j0}\,g_{3,i0,i1}+\\
&T_1^{s0}\,T_2^{s0}\,\chi_{i0==i1}\,g_{3,i0,j0}-T_1^{s0}\,T_2^{s0}\,\chi_{i0==i1}\,g_{1,j0,j0}\,g_{3,i0,j0}-\\
&T_1^{s0}\,T_2^{s0}\,\chi_{i0==i1}\,g_{2,j0,j0}\,g_{3,i0,j0}-T_1^{s0}\,T_2^{s0}\left(-2+T_1^{s0}+T_2^{s0}\right)\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{3,j0,i0}-\\
&T_1^{s0}\,T_2^{s0}\left(-1+T_2^{s0}\right)\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{1,i0,j0}\,g_{3,j0,i0}+\\
&T_1^{s0}\,T_2^{s0}\left(-2+T_1^{s0}+T_2^{s0}\right)\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{1,j0,j0}\,g_{3,j0,i0}-\\
&T_1^{s0}\left(-1+T_1^{s0}\right)T_2^{s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{2,i0,j0}\,g_{3,j0,i0}+\\
&T_1^{s0}\,T_2^{s0}\left(-2+T_1^{s0}+T_2^{s0}\right)\left(-1+T_1^{s0}\,T_2^{s0}\right)\chi_{i0==i1}\,g_{2,j0,j0}\,g_{3,j0,i0}+\\
&T_1^{s0}\left(-1+T_1^{s1}\right)T_2^{s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)g_{1,j1,j0}\,g_{2,i1,j0}\,g_{3,j0,i1}+\\
&T_1^{s0}\,T_2^{s0}\left(-1+T_1^{s0}\,T_2^{s0}\right)\left(-1+T_2^{s1}\right)g_{1,i1,j0}\,g_{2,j1,j0}\,g_{3,j0,i1}-
\end{aligned}
$$

In[•]:=
```
tw0 = Table[a_i, {i, 8}].bas0 / (T_2 - 1);
tw1 = Table[b_i, {i, 8}].bas1 / (T_2 - 1);
{D_{s0,i0,j0}[tw0], D_{s1,i1,j1}[tw1]}
@[Knot[7, 6], F1 → 0, F2 → D_{s0,i0,j0}[tw0] + D_{s1,i1,j1}[tw1], F3 → 0] // Echo // PolyPlot
```

Out[•]=



Full expression not available (original memory size: 0.6 MB)

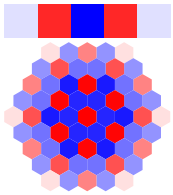
» $\left\{-\frac{1-5\,T+7\,T^2-5\,T^3+T^4}{T^2},\ 0\right\}$

Out[•]=

» $\left\{-\frac{1-5\,T+7\,T^2-5\,T^3+T^4}{T^2},\right.$

$$\frac{1}{T_1^4\,T_2^4}\left(1-5\,T_1+7\,T_1^2-5\,T_1^3+T_1^4-5\,T_2+20\,T_1\,T_2-10\,T_1^2\,T_2-10\,T_1^3\,T_2+20\,T_1^4\,T_2-5\,T_1^5\,T_2+7\,T_2^2-\right.$$
$$10\,T_1\,T_2^2-64\,T_1^2\,T_2^2+98\,T_1^3\,T_2^2-64\,T_1^4\,T_2^2-10\,T_1^5\,T_2^2+7\,T_1^6\,T_2^2-5\,T_2^3-10\,T_1\,T_2^3+98\,T_1^2\,T_2^3-50\,T_1^3\,T_2^3-$$
$$50\,T_1^4\,T_2^3+98\,T_1^5\,T_2^3-10\,T_1^6\,T_2^3-5\,T_1^7\,T_2^3+T_2^4+20\,T_1\,T_2^4-64\,T_1^2\,T_2^4-50\,T_1^3\,T_2^4+108\,T_1^4\,T_2^4-50\,T_1^5\,T_2^4-$$
$$64\,T_1^6\,T_2^4+20\,T_1^7\,T_2^4+T_1^8\,T_2^4-5\,T_1\,T_2^5-10\,T_1^2\,T_2^5+98\,T_1^3\,T_2^5-50\,T_1^4\,T_2^5-50\,T_1^5\,T_2^5+98\,T_1^6\,T_2^5-$$
$$\left.10\,T_1^7\,T_2^5-5\,T_1^8\,T_2^5+7\,T_2^6-10\,T_1^3\,T_2^6-64\,T_1^4\,T_2^6+98\,T_1^5\,T_2^6-64\,T_1^6\,T_2^6-10\,T_1^7\,T_2^6+7\,T_1^8\,T_2^6-5\,T_1^3\,T_2^7+\right.$$
$$\left.20\,T_1^4\,T_2^7-10\,T_1^5\,T_2^7-10\,T_1^6\,T_2^7+20\,T_1^7\,T_2^7-5\,T_1^8\,T_2^7+T_1^4\,T_2^8-5\,T_1^5\,T_2^8+7\,T_1^6\,T_2^8-5\,T_1^7\,T_2^8+T_1^8\,T_2^8\right)\Big\}$$

Out[•]=



In[•]:=
```
res = sRules[Residue[CF[χ_{i0=i1} F1i + F2i + D_{s0,i0,j0}[tw0] + D_{s1,i1,j1}[tw1]], {T_2, 1}] /.
    {g_{3,α_,β_} :→ g_{1,α,β}, g_{2,α_,β_} :→ χ_{α≤β}}];
```

In[•]:=
```
sRules[res /. {b_5 → 1 - T_1^{s0}, b_7 → T_1^{s0} - 1} /. (a | b)_ → 0]
```

Out[•]=

0

```
In[•]:= sRules[res /. {a1 → b1, a2 → a4, a3 → 0, a5 → b2, a6 → 0, a7 → b4, a8 → 0} /. b1|2|3|4|6|8 → 0 /.
          a4 → (b5 + b7) / 2 /. b5 → 2 + b7 - 2 T1^s0 /. b7 → T1^s0 - 1]
```

```
Out[•]=
0
```

```
In[•]:= nF2 = CF[CF[(χi0==i1 F1i + F2i + D{s0,i0,j0}[tw0] + D{s1,i1,j1}[tw1]) /.
                {a1 → b1, a2 → a4, a3 → 0, a5 → b2, a6 → 0, a7 → b4, a8 → 0} /. b1|2|3|4|6|8 → 0 /.
              a4 → (b5 + b7) / 2 /. b5 → 2 + b7 - 2 T1^s0 /. b7 → T1^s0 - 1]];
        {nF1, nF2} = Simplify@{Coefficient[nF2, χi0==i1], nF2 /. χi0==i1 → 0}
        Table[Θ[K] == Θ[K, F1 → nF1, F2 → nF2], {K, AllKnots[{3, 8}]}]
```

```
Out[•]=
{ s0/2 - T1^-s1 (-1 + T1^s0) T2^-s1 (-2 + T1^s1 + T2^s1) χi0==j1 χi1==j0 / (-1 + T2) + T1^-s1 (-1 + T1^s0) T2^-s1 (-1 + T2^s1) χi1==j0 χj0==j1 / (-1 + T2) +
  T1^-s1 (-1 + T1^s0) T2^-s1 χi1==j0 g1,i1,i0 / (-1 + T2) + T1^-s1 (-1 + T1^s0) (-1 + T1^s1) T2^-s1 χi1==j0 g1,j1,i0 / (-1 + T2) +
  T1^-s1 (-1 + T1^s0) T2^-s1 χi1==j0 g2,i1,i0 / (-1 + T2) - T1^-s1 (-1 + T1^s0) T2^-s1 χi1==j0 g2,i1,j0 / (-1 + T2) +
  s0 T2^s0 g1,i0,i0 g2,j0,i0 + s0 (-1 + T1^s0) T2^(2 s0) g1,j0,i0 g2,j0,i0 / (-1 + T2^s0) - s0 g1,i0,i0 g2,j0,j0 -
  s0 (-1 + T1^s0) T2^s0 g1,j0,i0 g2,j0,j0 / (-1 + T2^s0) + T1^-s1 (-1 + T1^s0) T2^-s1 (-1 + T2^s1) χi1==j0 g2,j1,i0 / (-1 + T2) -
  T1^-s1 (-1 + T1^s0) T2^-s1 (-1 + T2^s1) χi1==j0 g2,j1,j0 / (-1 + T2) - s0 g3,i0,i0 - s0 (-1 + T2^s0) g2,j0,i0 g3,i0,i0 +
  2 s0 g2,j0,j0 g3,i0,i0 + s0 (-1 + T1^s0 T2^s0) g3,j0,i0 / (-1 + T2^s0) - s0 T2^s0 (-1 + T1^s0 T2^s0) g1,i0,i0 g3,j0,i0 / (-1 + T2^s0) -
  s0 (-1 + T1^s0) (1 + T2^s0) (-1 + T1^s0 T2^s0) g1,j0,i0 g3,j0,i0 / (-1 + T2^s0) + s0 (-1 + T1^s0 T2^s0) g2,i0,j0 g3,j0,i0 / (-1 + T2^s0) +
  s0 (-1 + T1^s0 T2^s0) g2,j0,i0 g3,j0,i0 + s0 (-2 + T2^s0) (-1 + T1^s0 T2^s0) g2,j0,j0 g3,j0,i0 / (-1 + T2^s0) +
  s0 g1,i0,i0 g3,j0,j0 + s0 (-1 + T1^s0) T2^s0 g1,j0,i0 g3,j0,j0 / (-1 + T2^s0) - s0 g2,i0,i0 g3,j0,j0 -
  s0 T2^s0 g2,j0,i0 g3,j0,j0 - (-1 + T1^s0) g3,j0^+,i1 / (-1 + T2) - (-1 + T1^s0) (-2 + T1^s1 + T2^s1) χi0==j1 g3,j0^+,i1 / (-1 + T2) +
  (-1 + T1^s0) χi1==j0 g3,j0^+,i1 / (-1 + T2) + (-1 + T1^s0) (-1 + T2^s1) χj0==j1 g3,j0^+,i1 / (-1 + T2) + (-1 + T1^s0) g1,i1,i0 g3,j0^+,i1 / (-1 + T2) +
  (-1 + T1^s0) (-1 + T1^s1) g1,j1,i0 g3,j0^+,i1 / (-1 + T2) + (-1 + T1^s0) g2,i1,i0 g3,j0^+,i1 / (-1 + T2) - (-1 + T1^s0) g2,i1,j0 g3,j0^+,i1 / (-1 + T2) +
```

$$\frac{\left(-1+T_1^{s0}\right)\left(-1+T_2^{s1}\right) g_{2,j1,i0}\, g_{3,j0^+,i1}}{-1+T_2} - \frac{\left(-1+T_1^{s0}\right)\left(-1+T_2^{s1}\right) g_{2,j1,j0}\, g_{3,j0^+,i1}}{-1+T_2},$$

$$\frac{1}{-1+T_2}\left(-1+T_1^{s0}\right)\left(-T_1^{-s1}\,T_2^{-s1}\,\chi_{i1==j0}\left(g_{1,i1,i0}+\left(-1+T_1^{s1}\right)g_{1,j1,i0}\right)\left(1+\left(-1+T_2^{s1}\right)\chi_{j0==j1}+\right.\right.$$
$$g_{2,i1,i0}-g_{2,i1,j0}-g_{2,j1,i0}+T_2^{s1}\,g_{2,j1,i0}+g_{2,j1,j0}-T_2^{s1}\,g_{2,j1,j0}+T_1^{s1}\,T_2^{s1}\,g_{3,j0^+,i1}\Big)+$$
$$\frac{1}{-1+T_2^{s1}}\left(-s1\,g_{1,j1,i0}\,g_{2,i1,j0}\,g_{3,j0,i1}+s1\,T_2^{s0}\,g_{1,j1,i0}\left(g_{2,i1,i0}-g_{2,j1,i0}\right)g_{3,j0,i1}-\right.$$
$$s1\,T_2^{1+s0}\,g_{1,j1,i0}\left(g_{2,i1,i0}-g_{2,j1,i0}\right)g_{3,j0,i1}-s1\,T_1^{s1}\,T_2^{s0+s1}\,g_{1,j1,i0}\left(g_{2,i1,i0}-g_{2,j1,i0}\right)g_{3,j0,i1}+$$
$$s1\,T_1^{s1}\,T_2^{1+s0+s1}\,g_{1,j1,i0}\left(g_{2,i1,i0}-g_{2,j1,i0}\right)g_{3,j0,i1}+s1\,T_2\,g_{1,j1,i0}\left(g_{2,i1,j0}-g_{2,j1,j0}\right)g_{3,j0,i1}-$$
$$s1\,T_1^{s1}\,T_2^{1+s1}\,g_{1,j1,i0}\left(g_{2,i1,j0}-g_{2,j1,j0}\right)g_{3,j0,i1}+s1\,g_{1,j1,i0}\,g_{2,j1,j0}\,g_{3,j0,i1}-$$
$$g_{1,j1,i0}\,g_{2,i1,i0}\,g_{3,j0^+,i1}+T_1^{s1}\,g_{1,j1,i0}\,g_{2,i1,i0}\,g_{3,j0^+,i1}+g_{1,j1,i0}\,g_{2,i1,j0}\,g_{3,j0^+,i1}-$$
$$T_1^{s1}\,g_{1,j1,i0}\,g_{2,i1,j0}\,g_{3,j0^+,i1}-g_{1,i1,i0}\,g_{2,j1,i0}\,g_{3,j0^+,i1}+2\,g_{1,j1,i0}\,g_{2,j1,i0}\,g_{3,j0^+,i1}-$$
$$T_1^{s1}\,g_{1,j1,i0}\,g_{2,j1,i0}\,g_{3,j0^+,i1}-T_2^{2\,s1}\left(g_{1,i1,i0}-g_{1,j1,i0}\right)\left(g_{2,j1,i0}-g_{2,j1,j0}\right)g_{3,j0^+,i1}+$$
$$g_{1,i1,i0}\,g_{2,j1,j0}\,g_{3,j0^+,i1}-2\,g_{1,j1,i0}\,g_{2,j1,j0}\,g_{3,j0^+,i1}+T_1^{s1}\,g_{1,j1,i0}\,g_{2,j1,j0}\,g_{3,j0^+,i1}+$$
$$T_2^{s1}\left(\left(2\,g_{1,i1,i0}\left(g_{2,j1,i0}-g_{2,j1,j0}\right)+g_{1,j1,i0}\left(g_{2,i1,i0}-g_{2,i1,j0}-3\,g_{2,j1,i0}+3\,g_{2,j1,j0}\right)\right)\right.$$
$$g_{3,j0^+,i1}+T_1^{s1}\,g_{1,j1,i0}\left(\left(-g_{2,i1,i0}+g_{2,j1,i0}\right)g_{3,j0^+,i1}+g_{2,i1,j0}\left(s1\,g_{3,j0,i1}+g_{3,j0^+,i1}\right)-\right.$$
$$\left.\left.g_{2,j1,j0}\left(s1\,g_{3,j0,i1}+g_{3,j0^+,i1}\right)\right)\right)+\left(-1+T_2^{s1}\right)\chi_{j0==j1}\left(-\left(\left(-1+T_2^{s1}\right)g_{1,i1,i0}\,g_{3,j0^+,i1}\right)+\right.$$
$$\left.\left.g_{1,j1,i0}\left(-1-g_{2,j1,i0}+g_{2,j1,j0}-2\,g_{3,j0^+,i1}+T_1^{s1}\,g_{3,j0^+,i1}+T_2^{s1}\,g_{3,j0^+,i1}-g_{3,j0^+,j1}\right)\right)\right)+$$
$$\chi_{i0==j1}\left(\left(-2-g_{1,i1,i0}+2\,g_{1,j1,i0}-g_{2,i1,i0}+g_{2,i1,j0}+2\,g_{2,j1,i0}-T_1^{s1}\left(-1+g_{1,j1,i0}-g_{2,i1,i0}+\right.\right.\right.$$
$$\left.\left.\left.g_{2,i1,j0}+g_{2,j1,i0}-g_{2,j1,j0}\right)-2\,g_{2,j1,j0}+T_2^{s1}\left(1+g_{1,i1,i0}-g_{1,j1,i0}-g_{2,j1,i0}+g_{2,j1,j0}\right)\right)$$
$$g_{3,j0^+,i1}+T_1^{-s1}\,T_2^{-s1}\,\chi_{i1==j0}\left(-2+\chi_{j0==j1}-g_{1,i1,i0}+g_{1,j1,i0}-g_{2,i1,i0}+g_{2,i1,j0}+g_{2,j1,i0}-\right.$$
$$T_2^{s1}\left(-1+\chi_{j0==j1}-g_{1,i1,i0}+g_{1,j1,i0}+g_{2,j1,i0}-g_{2,j1,j0}\right)-g_{2,j1,j0}+T_1^{s1}\left(2-\chi_{j0==j1}-g_{1,j1,i0}+\right.$$
$$\left.\left.g_{2,i1,i0}-g_{2,i1,j0}-g_{2,j1,i0}+g_{2,j1,j0}+T_2^{s1}\left(-1+\chi_{j0==j1}+g_{1,j1,i0}+g_{2,j1,i0}-g_{2,j1,j0}-\right.\right.\right.$$
$$\left.\left.\left.g_{3,j0^+,i1}\right)\right)+T_1^{2\,s1}\,T_2^{s1}\,g_{3,j0^+,i1}\Big)+\left(-1+g_{1,j1,i0}+g_{2,j1,i0}-g_{2,j1,j0}\right)g_{3,j0^+,j1}+$$
$$\left.\left.\chi_{j0==j1}\left(g_{1,j1,i0}+g_{2,j1,i0}-g_{2,j1,j0}+2\,g_{3,j0^+,i1}-T_1^{s1}\,g_{3,j0^+,i1}-T_2^{s1}\,g_{3,j0^+,i1}+g_{3,j0^+,j1}\right)\right)\right)\Bigg\}$$

Out[•]=

{True, True, True, True, True, True, True, True, True, True, True,
True, True, True, True, True, True, True, True, True, True, True, True,
True, True, True, True, True, True, True, True, True, True, True, True}

In[•]:=
```
Table[Θ[K] == Θ[K,
 F1 → s0/2 + s0 T2^s0 g1,i0,i0 g2,j0,i0 + ((-1 + T1^s0) (1 - T2^s0 - s0 T2^(2 s0) + s0 T2^(1+2 s0)) g1,j0,i0 g2,j0,i0)/((-1 + T2) (-1 + T2^s0)) -
  s0 g1,i0,i0 g2,j0,j0 - ((-1 + T1^s0) (1 - (1 + s0) T2^s0 + s0 T2^(1+s0)) g1,j0,i0 g2,j0,j0)/((-1 + T2) (-1 + T2^s0)) -
  s0 g3,i0,i0 - s0 (-1 + T2^s0) g2,j0,i0 g3,i0,i0 + 2 s0 g2,j0,j0 g3,i0,i0 +
  ((-2 + s0 - s0 T2 + 3 T2^s0 - T2^(2 s0) + T1^s0 (2 - (3 + s0) T2^s0 + T2^(2 s0) + s0 T2^(1+s0))) g3,j0,i0)/((-1 + T2) (-1 + T2^s0)) -
  ((-2 + (3 + s0) T2^s0 - T2^(2 s0) - s0 T2^(1+s0) + T1^s0 (2 - 3 T2^s0 - (-1 + s0) T2^(2 s0) + s0 T2^(1+2 s0))) g1,i0,i0 g3,j0,i0)/((-1 + T2) (-1 + T2^s0)) -
  1/((-1 + T2) (-1 + T2^s0)) (-1 + T1^s0) (-3 + s0 - s0 T2 + (4 + s0) T2^s0 - T2^(2 s0) - s0 T2^(1+s0) + T1^s0
      (2 - (2 + s0) T2^s0 - s0 T2^(2 s0) + s0 T2^(1+s0) + s0 T2^(1+2 s0))) g1,j0,i0 g3,j0,i0 + ((-1 + T1^s0) g2,i0,i0 g3,j0,i0)/(-1 + T2) +
  ((-1 + s0 - s0 T2 + T2^s0 + T1^s0 (1 - (1 + s0) T2^s0 + s0 T2^(1+s0))) g2,i0,j0 g3,j0,i0)/((-1 + T2) (-1 + T2^s0)) +
  ((1 + s0 - s0 T2 - T2^s0 + T1^s0 (-1 - (-1 + s0) T2^s0 + s0 T2^(1+s0))) g2,j0,i0 g3,j0,i0)/(-1 + T2) +
  1/((-1 + T2) (-1 + T2^s0)) (1 - 2 s0 + 2 s0 T2 + (-2 + s0) T2^s0 + T2^(2 s0) - s0 T2^(1+s0) +
     T1^s0 (-1 + 2 (1 + s0) T2^s0 - (1 + s0) T2^(2 s0) - 2 s0 T2^(1+s0) + s0 T2^(1+2 s0))) g2,j0,j0 g3,j0,i0 +
  s0 g1,i0,i0 g3,j0,j0 + ((-1 + T1^s0) (1 - (1 + s0) T2^s0 + s0 T2^(1+s0)) g1,j0,i0 g3,j0,j0)/((-1 + T2) (-1 + T2^s0)) -
  s0 g2,i0,i0 g3,j0,j0 - s0 T2^s0 g2,j0,i0 g3,j0,j0,
 F2 → 1/((-1 + T2) (-1 + T2^s1)) (-1 + T1^s0)
   ((-1 + s1 T2^s0 - s1 T2^(1+s0) + T2^s1 + T1^s1 (1 - T2^s1 - s1 T2^(s0+s1) + s1 T2^(1+s0+s1))) g1,j1,i0 g2,i1,i0 -
     (-1 + s1 - s1 T2 + T2^s1 + T1^s1 (1 - (1 + s1) T2^s1 + s1 T2^(1+s1))) g1,j1,i0 g2,i1,j0 - (-1 + T2^s1)^2 g1,i1,i0
      g2,j1,i0 + (2 - s1 T2^s0 + s1 T2^(1+s0) - 3 T2^s1 + T2^(2 s1) + T1^s1 (-1 + T2^s1 + s1 T2^(s0+s1) - s1 T2^(1+s0+s1)))
      g1,j1,i0 g2,j1,i0 + (-1 + T2^s1)^2 g1,i1,i0 g2,j1,j0 +
     (-2 + s1 - s1 T2 + 3 T2^s1 - T2^(2 s1) + T1^s1 (1 - (1 + s1) T2^s1 + s1 T2^(1+s1))) g1,j1,i0 g2,j1,j0) g3,j0,i1
 ], {K, AllKnots[{3, 8}]}]
```

Out[•]=
```
{True, True, True, True, True, True, True, True, True, True, True,
 True, True, True, True, True, True, True, True, True, True, True,
 True, True, True, True, True, True, True, True, True, True, True, True}
```

In[•]:=
```
CF[s0/2 + s0 T2^s0 g1,i0,i0 g2,j0,i0 + ((-1 + T1^s0) (1 - T2^s0 - s0 T2^(2 s0) + s0 T2^(1+2 s0)) g1,j0,i0 g2,j0,i0)/((-1 + T2) (-1 + T2^s0)) -
 s0 g1,i0,i0 g2,j0,j0 - ((-1 + T1^s0) (1 - (1 + s0) T2^s0 + s0 T2^(1+s0)) g1,j0,i0 g2,j0,j0)/((-1 + T2) (-1 + T2^s0)) -
 s0 g3,i0,i0 - s0 (-1 + T2^s0) g2,j0,i0 g3,i0,i0 + 2 s0 g2,j0,j0 g3,i0,i0 +
 ((-2 + s0 - s0 T2 + 3 T2^s0 - T2^(2 s0) + T1^s0 (2 - (3 + s0) T2^s0 + T2^(2 s0) + s0 T2^(1+s0))) g3,j0,i0)/((-1 + T2) (-1 + T2^s0)) -
 ((-2 + (3 + s0) T2^s0 - T2^(2 s0) - s0 T2^(1+s0) + T1^s0 (2 - 3 T2^s0 - (-1 + s0) T2^(2 s0) + s0 T2^(1+2 s0))) g1,i0,i0 g3,j0,i0)/((-1 + T2) (-1 + T2^s0)) -
 1/((-1 + T2) (-1 + T2^s0)) (-1 + T1^s0) (-3 + s0 - s0 T2 + (4 + s0) T2^s0 - T2^(2 s0) - s0 T2^(1+s0) +
    T1^s0 (2 - (2 + s0) T2^s0 - s0 T2^(2 s0) + s0 T2^(1+s0) + s0 T2^(1+2 s0))) g1,j0,i0 g3,j0,i0 + ((-1 + T1^s0) g2,i0,i0 g3,j0,i0)/(-1 + T2) +
 ((-1 + s0 - s0 T2 + T2^s0 + T1^s0 (1 - (1 + s0) T2^s0 + s0 T2^(1+s0))) g2,i0,j0 g3,j0,i0)/((-1 + T2) (-1 + T2^s0)) +
 ((1 + s0 - s0 T2 - T2^s0 + T1^s0 (-1 - (-1 + s0) T2^s0 + s0 T2^(1+s0))) g2,j0,i0 g3,j0,i0)/(-1 + T2) +
 1/((-1 + T2) (-1 + T2^s0)) (1 - 2 s0 + 2 s0 T2 + (-2 + s0) T2^s0 + T2^(2 s0) - s0 T2^(1+s0) +
    T1^s0 (-1 + 2 (1 + s0) T2^s0 - (1 + s0) T2^(2 s0) - 2 s0 T2^(1+s0) + s0 T2^(1+2 s0))) g2,j0,j0 g3,j0,i0 +
 s0 g1,i0,i0 g3,j0,j0 + ((-1 + T1^s0) (1 - (1 + s0) T2^s0 + s0 T2^(1+s0)) g1,j0,i0 g3,j0,j0)/((-1 + T2) (-1 + T2^s0)) -
 s0 g2,i0,i0 g3,j0,j0 - s0 T2^s0 g2,j0,i0 g3,j0,j0 /. s0 -> 1]
```

Out[•]=
$$\frac{1}{2} + T_2\, g_{1,i0,i0}\, g_{2,j0,i0} + (-1+T_1)(1+T_2)\, g_{1,j0,i0}\, g_{2,j0,i0} - g_{1,i0,i0}\, g_{2,j0,j0} + (1-T_1)\, g_{1,j0,i0}\, g_{2,j0,j0} - g_{3,i0,i0} + (1-T_2)\, g_{2,j0,i0}\, g_{3,i0,i0} + 2\, g_{2,j0,j0}\, g_{3,i0,i0} + (-1+2T_1)\, g_{3,j0,i0} + (2-2T_1-T_1T_2)\, g_{1,i0,i0}\, g_{3,j0,i0} - (-1+T_1)(-2+2T_1+T_1T_2)\, g_{1,j0,i0}\, g_{3,j0,i0} + \frac{(-1+T_1)\, g_{2,i0,i0}\, g_{3,j0,i0}}{-1+T_2} + T_1\, g_{2,i0,j0}\, g_{3,j0,i0} + (-2+T_1+T_1T_2)\, g_{2,j0,i0}\, g_{3,j0,i0} + \frac{(1+T_1-3T_1T_2+T_1T_2^2)\, g_{2,j0,j0}\, g_{3,j0,i0}}{-1+T_2} + g_{1,i0,i0}\, g_{3,j0,j0} + (-1+T_1)\, g_{1,j0,i0}\, g_{3,j0,j0} - g_{2,i0,i0}\, g_{3,j0,j0} - T_2\, g_{2,j0,i0}\, g_{3,j0,j0}$$

In[•]:=
```
Factor[(-1 + T1) + (1 + T1 - 3 T1 T2 + T1 T2^2)]
```

Out[•]=
$$T_1\,(-2+T_2)\,(-1+T_2)$$

In[•]:=
```
CF[s0/2 + s0 T2^s0 g1,i0,i0 g2,j0,i0 + ((-1 + T1^s0) (1 - T2^s0 - s0 T2^(2 s0) + s0 T2^(1+2 s0)) g1,j0,i0 g2,j0,i0)/((-1 + T2) (-1 + T2^s0)) -
 s0 g1,i0,i0 g2,j0,j0 - ((-1 + T1^s0) (1 - (1 + s0) T2^s0 + s0 T2^(1+s0)) g1,j0,i0 g2,j0,j0)/((-1 + T2) (-1 + T2^s0)) -
 s0 g3,i0,i0 - s0 (-1 + T2^s0) g2,j0,i0 g3,i0,i0 + 2 s0 g2,j0,j0 g3,i0,i0 +
 ((-2 + s0 - s0 T2 + 3 T2^s0 - T2^(2 s0) + T1^s0 (2 - (3 + s0) T2^s0 + T2^(2 s0) + s0 T2^(1+s0))) g3,j0,i0)/((-1 + T2) (-1 + T2^s0)) -
 ((-2 + (3 + s0) T2^s0 - T2^(2 s0) - s0 T2^(1+s0) + T1^s0 (2 - 3 T2^s0 - (-1 + s0) T2^(2 s0) + s0 T2^(1+2 s0))) g1,i0,i0 g3,j0,i0)/((-1 + T2) (-1 + T2^s0)) -
 1/((-1 + T2) (-1 + T2^s0)) (-1 + T1^s0) (-3 + s0 - s0 T2 + (4 + s0) T2^s0 - T2^(2 s0) - s0 T2^(1+s0) +
    T1^s0 (2 - (2 + s0) T2^s0 - s0 T2^(2 s0) + s0 T2^(1+s0) + s0 T2^(1+2 s0))) g1,j0,i0 g3,j0,i0 + ((-1 + T1^s0) g2,i0,i0 g3,j0,i0)/(-1 + T2) +
 ((-1 + s0 - s0 T2 + T2^s0 + T1^s0 (1 - (1 + s0) T2^s0 + s0 T2^(1+s0))) g2,i0,j0 g3,j0,i0)/((-1 + T2) (-1 + T2^s0)) +
 ((1 + s0 - s0 T2 - T2^s0 + T1^s0 (-1 - (-1 + s0) T2^s0 + s0 T2^(1+s0))) g2,j0,i0 g3,j0,i0)/(-1 + T2) +
 1/((-1 + T2) (-1 + T2^s0)) (1 - 2 s0 + 2 s0 T2 + (-2 + s0) T2^s0 + T2^(2 s0) - s0 T2^(1+s0) +
    T1^s0 (-1 + 2 (1 + s0) T2^s0 - (1 + s0) T2^(2 s0) - 2 s0 T2^(1+s0) + s0 T2^(1+2 s0))) g2,j0,j0 g3,j0,i0 +
 s0 g1,i0,i0 g3,j0,j0 + ((-1 + T1^s0) (1 - (1 + s0) T2^s0 + s0 T2^(1+s0)) g1,j0,i0 g3,j0,j0)/((-1 + T2) (-1 + T2^s0)) -
 s0 g2,i0,i0 g3,j0,j0 - s0 T2^s0 g2,j0,i0 g3,j0,j0 /. s0 → -1]
```

Out[•]=
$$-\frac{1}{2}-\frac{g_{1,i0,i0}\,g_{2,j0,i0}}{T_2}+\frac{(-1+T_1)\,g_{1,j0,i0}\,g_{2,j0,i0}}{T_1\,T_2}+g_{1,i0,i0}\,g_{2,j0,j0}+g_{3,i0,i0}-\frac{(-1+T_2)\,g_{2,j0,i0}\,g_{3,i0,i0}}{T_2}-2\,g_{2,j0,j0}\,g_{3,i0,i0}-\frac{(1-T_1+T_1\,T_2)\,g_{3,j0,i0}}{T_1\,T_2}-\frac{(-2+T_1)\,g_{1,i0,i0}\,g_{3,j0,i0}}{T_1\,T_2}-\frac{(-1+T_1)\,(1-T_1+T_1\,T_2)\,g_{1,j0,i0}\,g_{3,j0,i0}}{T_1^2\,T_2}-\frac{(-1+T_1)\,g_{2,i0,i0}\,g_{3,j0,i0}}{T_1\,(-1+T_2)}-g_{2,i0,j0}\,g_{3,j0,i0}+\frac{(-2+T_1+T_1\,T_2)\,g_{2,j0,i0}\,g_{3,j0,i0}}{T_1\,T_2}+\frac{\left(T_1-T_2-2\,T_1\,T_2+2\,T_1\,T_2^2\right)g_{2,j0,j0}\,g_{3,j0,i0}}{T_1\,(-1+T_2)\,T_2}-g_{1,i0,i0}\,g_{3,j0,j0}+g_{2,i0,i0}\,g_{3,j0,j0}+\frac{g_{2,j0,i0}\,g_{3,j0,j0}}{T_2}$$

In[•]:=
```
Factor[-T2 (T1 - 1) + (T1 - T2 - 2 T1 T2 + 2 T1 T2^2)]
```

Out[•]=
$$T_1\,(-1+T_2)\,(-1+2\,T_2)$$