

```
In[1]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\Theta"];
Once[<< Theta.m];
SetOptions[PolyPlot, ImageSize \[Rule] Tiny];
Clear[\[Theta]]
```

Loading KnotTheory` version of October 29, 2024, 10:29:52.1301.
Read more at <http://katlas.org/wiki/KnotTheory>.

```
In[2]:= RandomVK[n_]:= {
  Prepend[#, 2 RandomInteger[1]-1]& /@
  Partition[PermutationList[RandomPermutation[2 n], 2 n], 2],
  Table[RandomInteger[{-1, 1}], 2 n+1]
};
```

```
In[3]:= RandomVK[5]
Out[3]= {{ {1, 7, 3}, {-1, 6, 2}, {1, 4, 9}, {-1, 10, 1}, {1, 5, 8} },
{-1, -1, 0, 1, 0, -1, 1, 1, -1, 1, 0}}
```

```
In[4]:= CF[\[Epsilon]] := Expand@Collect[\[Epsilon], g_, F] /. F \[Rule] Factor@*PowerExpand;
```

```
In[5]:= Short[Options[\[Theta]] = {F1 \[Rule] (F1i = F1[{s0, i0, j0}]),
F2 \[Rule] (F2i = F2[{s0, i0, j0}, {s1, i1, j1}]), F3 \[Rule] (F3i = F3[\[Phi], k])}]
```

```
Out[5]//Short=
{F1 \[Rule]  $\frac{s\theta}{2} + s\theta T_2^{s\theta} g_{1,i0,i0} g_{2,j0,i0} + \text{Omit terms} - s\theta g_{2,i0,i0} g_{3,j0,j0} - s\theta T_2^{s\theta} g_{2,j0,i0} g_{3,j0,j0},$ 
F2 \[Rule] \text{Omit terms}, F3 \[Rule]  $-\frac{\varphi}{2} + \varphi g_{\text{Omit terms}}\}$ 
```

```
In[1]:= Θ[K_, opts___Rule] := Module[{X, ϕφ, n, A, Δ, G, ev, θ, kk, k0, k1, f1, f2, f3},
  f1 = F1 /. {opts} /. Options[θ];
  f2 = F2 /. {opts} /. Options[θ];
  f3 = F3 /. {opts} /. Options[θ];
  {X, ϕφ} = Rot[K];
  n = Length[X];
  A = IdentityMatrix[2 n + 1];
  Cases[X, {s_, i_, j_} :> (A[[{i, j}, {i + 1, j + 1}]] += {{-T^s T^s - 1}, {θ, -1}})];
  Δ = T^{(-Total[ϕφ] - Total[X[[All, 1]]])/2} Det[A];
  G = Inverse[A];
  ev[ε_] := Factor[
    ε /. {k_+ :> k + 1, $ → 2 n + 1} /. {g[α, β] :> (G[[α, β]] /. T → T), XTrue → 1, XFalse → 0}];
  θ = ev@Sum[f1 /. Thread[{s0, i0, j0} → X[[kk]]], {kk, n}];
  θ += ev@Sum[f2 /. Thread[{s0, i0, j0} → X[[k0]]] //.
    Thread[{s1, i1, j1} → X[[k1]]], {k0, n}, {k1, n}];
  θ += ev@Sum[f3 /. {φ → ϕφ[[kk]], k → kk}, {kk, Length@ϕφ}];
  Factor@{Δ, (Δ /. T → T1) (Δ /. T → T2) (Δ /. T → T3) θ}
];

```

```
In[2]:= Θ[Knot[7, 3]]
```

KnotTheory: Loading precomputed data in PD4Knots`.

```
Out[2]= 
$$\left\{ \frac{2 - 3 T + 3 T^2 - 3 T^3 + 2 T^4}{T^2}, \right.$$


$$\frac{1}{T_1^4 T_2^4} (17 - 25 T_1 + 25 T_1^2 - 25 T_1^3 + 17 T_1^4 - 25 T_2 + 12 T_1 T_2 + 12 T_1^4 T_2 - 25 T_1^5 T_2 + 25 T_2^2 -$$


$$T_1^2 T_2^2 - 7 T_1^3 T_2^2 - T_1^4 T_2^2 + 25 T_1^6 T_2^2 - 25 T_2^3 - 7 T_1^2 T_2^3 + 6 T_1^3 T_2^3 + 6 T_1^4 T_2^3 - 7 T_1^5 T_2^3 - 25 T_1^7 T_2^3 +$$

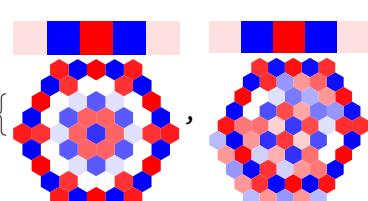
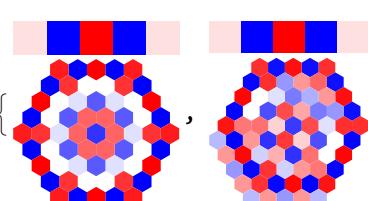

$$17 T_2^4 + 12 T_1 T_2^4 - T_1^2 T_2^4 + 6 T_1^3 T_2^4 - 12 T_1^4 T_2^4 + 6 T_1^5 T_2^4 - T_1^6 T_2^4 + 12 T_1^7 T_2^4 + 17 T_1^8 T_2^4 - 25 T_1 T_2^5 -$$


$$7 T_1^3 T_2^5 + 6 T_1^4 T_2^5 + 6 T_1^5 T_2^5 - 7 T_1^6 T_2^5 - 25 T_1^8 T_2^5 + 25 T_1^2 T_2^6 - T_1^4 T_2^6 - 7 T_1^5 T_2^6 - T_1^6 T_2^6 + 25 T_1^8 T_2^6 -$$


$$25 T_1^3 T_2^7 + 12 T_1^4 T_2^7 + 12 T_1^7 T_2^7 - 25 T_1^8 T_2^7 + 17 T_1^4 T_2^8 - 25 T_1^5 T_2^8 + 25 T_1^6 T_2^8 - 25 T_1^7 T_2^8 + 17 T_1^8 T_2^8 \Big\}$$


```

```
In[3]:= PolyPlot /@ {Θ[Knot[7, 3]], Θ[Knot[7, 3], F3 → 0]}
```

```
Out[3]= {, 
```

```
In[=]:= δi_,j_ := χi=j; λp̄h_p /; p > 1 ^:= χp̄h;
bRules[{s_, i_, j_}] := { (* b for "push indices backwards" *)
  gv_,j_,β ↪ gv_,j_,β - δj_,β, gv_,i_,β ↪ Tv-s gv_,i_,β + (1 - Tv-s) gv_,j_,β - Tv-s δi_,β - (1 - Tv-s) δj_,β,
  gv_,α_,i_ ↪ Tvs gv_,α_,i_ + δα_,i_, gv_,α_,j_ ↪ gv_,α_,j_ + (1 - Tvs) gv_,α_,i_ + δα_,j_
}};

bRules[X___List] := Union @@ Table[bRules[c], {c, {X}}]
```

In[=]:= {g_{v_},_{i_},_β, g_{v_},_{j_},_β} /. gRules[{s, i, j}]

Out[=]= {χ_{i=j} + T_v^s g_{v_},_{i_},_β + (1 - T_v^s) g_{v_},_{j_},_β, χ_{j=j} + g_{v_},_{j_},_β}

In[=]:= Expand[{g_{v_},_{i_},_β, g_{v_},_{j_},_β} /. gRules[{s, i, j}] /. bRules[{s, i, j}]]

Out[=]= {g_{v_},_{i_},_β, g_{v_},_{j_},_β}

```
In[=]:= D{s_, i_, j_}[E_] := CF[Expand[Plus[
  E /. {gv_,#,β_ ↪ gv_,i_,β, gv_,α_,# ↪ gv_,α_,i_},
  E /. {gv_,#,β_ ↪ gv_,j_,β, gv_,α_,# ↪ gv_,α_,j_},
  -E /. {gv_,#,β_ ↪ gv_,i_,β, gv_,α_,# ↪ gv_,α_,i_},
  -E /. {gv_,#,β_ ↪ gv_,j_,β, gv_,α_,# ↪ gv_,α_,j_}
] /. bRules[{s, i, j}] /. bRules[{s0, i0, j0}] /. XeqEqual ↪ XSort@eq // . {
  Xj0==j1 → Xj0==i1, Xj0+=j1+ → Xj0==j1,
  Xj0==j1 → 0, Xj0+=j1 → 0, Xj0+=j1+ → 0, Xj0+=i1 → Xj0==i1, Xj0+=j0+ → 0, Xj0+=j0+ → 0,
  Xj0==i1 A_ ↪ Xj0==i1 (A /. {s1 → s0, i1 → i0, j1 → j0})
} /. {Xj0==j0 → 0, Xj0+=j0+ → 0, XTrue → 1}
];
B[E_] := CF[
  (E /. {gv_,#,β_ ↪ gv_,$,β, gv_,α_,# ↪ gv_,α_,$}) - (E /. {gv_,#,β_ ↪ gv_,1,β, gv_,α_,# ↪ gv_,α_,1}) / . {
    gv_,$,β_ → X$==β, gv_,α_,$ → 1, gv_,α_,1 → Xα==1} / . {X$==i0 → 0, Xj0+=1 → 0, X$==j0 → 0, Xj0+=1 → 0}
];
```

In[=]:= D_{s1, i1, j1}[g_{1, #, i0}]

Out[=]= -T₁^{-s0} χ_{i0=i1} - T₁^{-s1} (-1 + T₁^{s1}) g_{1, i1, i0} + T₁^{-s1} (-1 + T₁^{s1}) g_{1, j1, i0}

In[=]:= D_{s1, i1, j1}[g_{3, j0, #}]

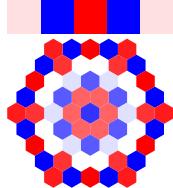
Out[=]= χ_{j0=i1+} + χ_{j0=j1+}

In[=]:= B[g_{3, j0, #}]

Out[=]= 1 - χ_{j0=1}

```
In[6]:= tw = g1, #, i0 ;
θ [Knot[7, 3], F1 → F1i - B[tw], F2 → F2i + D{s1, i1, j1} [tw]] // Echo // PolyPlot
» { $\frac{2 - 3T + 3T^2 - 3T^3 + 2T^4}{T^2}, \frac{1}{T_1^4 T_2^4}$ 
 $(17 - 25T_1 + 25T_1^2 - 25T_1^3 + 17T_1^4 - 25T_2 + 12T_1T_2 + 12T_1^4T_2 - 25T_1^5T_2 + 25T_2^2 - T_1^2T_2^2 - 7T_1^3T_2^2 - T_1^4T_2^2 + 25T_1^6T_2^2 - 25T_2^3 - 7T_2^2T_1^3 + 6T_1^3T_2^3 + 6T_1^4T_2^3 - 7T_1^5T_2^3 - 25T_1^7T_2^3 + 17T_1^4 + 12T_1T_2^4 - T_1^2T_2^4 + 6T_1^3T_2^4 - 12T_1^4T_2^4 + 6T_1^5T_2^4 - T_1^6T_2^4 + 12T_1^7T_2^4 + 17T_1^8T_2^4 - 25T_1T_2^5 - 7T_1^3T_2^5 + 6T_1^4T_2^5 + 6T_1^5T_2^5 - 7T_1^6T_2^5 - 25T_1^8T_2^5 + 25T_1^2T_2^6 - T_1^4T_2^6 - 7T_1^5T_2^6 - T_1^6T_2^6 + 25T_1^8T_2^6 - 25T_1^3T_2^7 + 12T_1^4T_2^7 + 12T_1^7T_2^7 - 25T_1^8T_2^7 + 17T_1^4T_2^8 - 25T_1^5T_2^8 + 25T_1^6T_2^8 - 25T_1^7T_2^8 + 17T_1^8T_2^8)$ }
```

Out[•]=



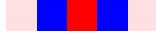
In[•]:= **tw = g_{1,j0,#}**;

```

Θ[Knot[7, 3], F1 → -B[tw], F2 → D{s1, i1, j1}[tw], F3 → 0] // Echo // PolyPlot
{
$$\frac{2 - 3 T + 3 T^2 - 3 T^3 + 2 T^4}{T^2}, \theta}$$


```

Out[•] =

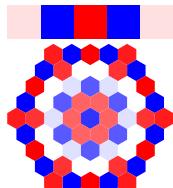


In[•]:= **tw = g_{1,j0,#}**;

```

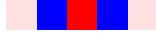
Θ[Knot[7, 3], F1 → F1i - B[tw], F2 → F2i + D{s1,i1,j1}[tw]] // Echo // PolyPlot
{ 2 - 3 T + 3 T2 - 3 T3 + 2 T4 , 1
   T2 , T14 T24
(17 - 25 T1 + 25 T12 - 25 T13 + 17 T14 - 25 T2 + 12 T1 T2 + 12 T14 T2 - 25 T15 T2 + 25 T22 - T12 T22 - 7 T13 T23 - 25 T23 - 7 T12 T23 + 6 T13 T23 + 6 T14 T23 - 7 T15 T23 - 25 T17 T23 + 17 T14 T24 + 12 T1 T24 - T12 T24 + 6 T13 T24 - 12 T16 T24 + 12 T17 T24 + 17 T18 T24 - 25 T1 T25 - 7 T13 T25 + 6 T14 T25 + 6 T15 T25 - 7 T16 T25 - 25 T18 T25 + 25 T26 T12 + 25 T16 T26 - 25 T3 T26 + 12 T14 T27 + 12 T17 T27 - 25 T18 T27 + 17 T14 T28 - 25 T15 T28 + 25 T16 T28 + 25 T17 T28 - 25 T18 T28 - 25 T29 T12 + 25 T16 T29 - 25 T3 T29 + 12 T14 T210 + 12 T17 T210 - 25 T18 T210 + 17 T14 T211 - 25 T15 T211 + 25 T16 T211 + 25 T17 T211 - 25 T18 T211 - 25 T212 T12 + 25 T16 T212 - 25 T3 T212 + 12 T14 T213 + 12 T17 T213 - 25 T18 T213 + 17 T14 T214 - 25 T15 T214 + 25 T16 T214 + 25 T17 T214 - 25 T18 T214 - 25 T215 T12 + 25 T16 T215 - 25 T3 T215 + 12 T14 T216 + 12 T17 T216 - 25 T18 T216 + 17 T14 T217 - 25 T15 T217 + 25 T16 T217 + 25 T17 T217 - 25 T18 T217 - 25 T218 T12 + 25 T16 T218 - 25 T3 T218 + 12 T14 T219 + 12 T17 T219 - 25 T18 T219 + 17 T14 T220 - 25 T15 T220 + 25 T16 T220 + 25 T17 T220 - 25 T18 T220 - 25 T221 T12 + 25 T16 T221 - 25 T3 T221 + 12 T14 T222 + 12 T17 T222 - 25 T18 T222 + 17 T14 T223 - 25 T15 T223 + 25 T16 T223 + 25 T17 T223 - 25 T18 T223 - 25 T224 T12 + 25 T16 T224 - 25 T3 T224 + 12 T14 T225 + 12 T17 T225 - 25 T18 T225 + 17 T14 T226 - 25 T15 T226 + 25 T16 T226 + 25 T17 T226 - 25 T18 T226 - 25 T227 T12 + 25 T16 T227 - 25 T3 T227 + 12 T14 T228 + 12 T17 T228 - 25 T18 T228 + 17 T14 T229 - 25 T15 T229 + 25 T16 T229 + 25 T17 T229 - 25 T18 T229 - 25 T230 T12 + 25 T16 T230 - 25 T3 T230 + 12 T14 T231 + 12 T17 T231 - 25 T18 T231 + 17 T14 T232 - 25 T15 T232 + 25 T16 T232 + 25 T17 T232 - 25 T18 T232 - 25 T233 T12 + 25 T16 T233 - 25 T3 T233 + 12 T14 T234 + 12 T17 T234 - 25 T18 T234 + 17 T14 T235 - 25 T15 T235 + 25 T16 T235 + 25 T17 T235 - 25 T18 T235 - 25 T236 T12 + 25 T16 T236 - 25 T3 T236 + 12 T14 T237 + 12 T17 T237 - 25 T18 T237 + 17 T14 T238 - 25 T15 T238 + 25 T16 T238 + 25 T17 T238 - 25 T18 T238 - 25 T239 T12 + 25 T16 T239 - 25 T3 T239 + 12 T14 T240 + 12 T17 T240 - 25 T18 T240 + 17 T14 T241 - 25 T15 T241 + 25 T16 T241 + 25 T17 T241 - 25 T18 T241 - 25 T242 T12 + 25 T16 T242 - 25 T3 T242 + 12 T14 T243 + 12 T17 T243 - 25 T18 T243 + 17 T14 T244 - 25 T15 T244 + 25 T16 T244 + 25 T17 T244 - 25 T18 T244 - 25 T245 T12 + 25 T16 T245 - 25 T3 T245 + 12 T14 T246 + 12 T17 T246 - 25 T18 T246 + 17 T14 T247 - 25 T15 T247 + 25 T16 T247 + 25 T17 T247 - 25 T18 T247 - 25 T248 T12 + 25 T16 T248 - 25 T3 T248 + 12 T14 T249 + 12 T17 T249 - 25 T18 T249 + 17 T14 T250 - 25 T15 T250 + 25 T16 T250 + 25 T17 T250 - 25 T18 T250 - 25 T251 T12 + 25 T16 T251 - 25 T3 T251 + 12 T14 T252 + 12 T17 T252 - 25 T18 T252 + 17 T14 T253 - 25 T15 T253 + 25 T16 T253 + 25 T17 T253 - 25 T18 T253 - 25 T254 T12 + 25 T16 T254 - 25 T3 T254 + 12 T14 T255 + 12 T17 T255 - 25 T18 T255 + 17 T14 T256 - 25 T15 T256 + 25 T16 T256 + 25 T17 T256 - 25 T18 T256 - 25 T257 T12 + 25 T16 T257 - 25 T3 T257 + 12 T14 T258 + 12 T17 T258 - 25 T18 T258 + 17 T14 T259 - 25 T15 T259 + 25 T16 T259 + 25 T17 T259 - 25 T18 T259 - 25 T260 T12 + 25 T16 T260 - 25 T3 T260 + 12 T14 T261 + 12 T17 T261 - 25 T18 T261 + 17 T14 T262 - 25 T15 T262 + 25 T16 T262 + 25 T17 T262 - 25 T18 T262 - 25 T263 T12 + 25 T16 T263 - 25 T3 T263 + 12 T14 T264 + 12 T17 T264 - 25 T18 T264 + 17 T14 T265 - 25 T15 T265 + 25 T16 T265 + 25 T17 T265 - 25 T18 T265 - 25 T266 T12 + 25 T16 T266 - 25 T3 T266 + 12 T14 T267 + 12 T17 T267 - 25 T18 T267 + 17 T14 T268 - 25 T15 T268 + 25 T16 T268 + 25 T17 T268 - 25 T18 T268 - 25 T269 T12 + 25 T16 T269 - 25 T3 T269 + 12 T14 T270 + 12 T17 T270 - 25 T18 T270 + 17 T14 T271 - 25 T15 T271 + 25 T16 T271 + 25 T17 T271 - 25 T18 T271 - 25 T272 T12 + 25 T16 T272 - 25 T3 T272 + 12 T14 T273 + 12 T17 T273 - 25 T18 T273 + 17 T14 T274 - 25 T15 T274 + 25 T16 T274 + 25 T17 T274 - 25 T18 T274 - 25 T275 T12 + 25 T16 T275 - 25 T3 T275 + 12 T14 T276 + 12 T17 T276 - 25 T18 T276 + 17 T14 T277 - 25 T15 T277 + 25 T16 T277 + 25 T17 T277 - 25 T18 T277 - 25 T278 T12 + 25 T16 T278 - 25 T3 T278 + 12 T14 T279 + 12 T17 T279 - 25 T18 T279 + 17 T14 T280 - 25 T15 T280 + 25 T16 T280 + 25 T17 T280 - 25 T18 T280 - 25 T281 T12 + 25 T16 T281 - 25 T3 T281 + 12 T14 T282 + 12 T17 T282 - 25 T18 T282 + 17 T14 T283 - 25 T15 T283 + 25 T16 T283 + 25 T17 T283 - 25 T18 T283 - 25 T284 T12 + 25 T16 T284 - 25 T3 T284 + 12 T14 T285 + 12 T17 T285 - 25 T18 T285 + 17 T14 T286 - 25 T15 T286 + 25 T16 T286 + 25 T17 T286 - 25 T18 T286 - 25 T287 T12 + 25 T16 T287 - 25 T3 T287 + 12 T14 T288 + 12 T17 T288 - 25 T18 T288 + 17 T14 T289 - 25 T15 T289 + 25 T16 T289 + 25 T17 T289 - 25 T18 T289 - 25 T290 T12 + 25 T16 T290 - 25 T3 T290 + 12 T14 T291 + 12 T17 T291 - 25 T18 T291 + 17 T14 T292 - 25 T15 T292 + 25 T16 T292 + 25 T17 T292 - 25 T18 T292 - 25 T293 T12 + 25 T16 T293 - 25 T3 T293 + 12 T14 T294 + 12 T17 T294 - 25 T18 T294 + 17 T14 T295 - 25 T15 T295 + 25 T16 T295 + 25 T17 T295 - 25 T18 T295 - 25 T296 T12 + 25 T16 T296 - 25 T3 T296 + 12 T14 T297 + 12 T17 T297 - 25 T18 T297 + 17 T14 T298 - 25 T15 T298 + 25 T16 T298 + 25 T17 T298 - 25 T18 T298 - 25 T299 T12 + 25 T16 T299 - 25 T3 T299 + 12 T14 T2100 + 12 T17 T2100 - 25 T18 T2100 + 17 T14 T2101 - 25 T15 T2101 + 25 T16 T2101 + 25 T17 T2101 - 25 T18 T2101 - 25 T2<sup
```

Out[•]=



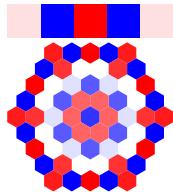
```
In[=]:= tw = g1, #1, j0 g2, #1, j0 g3, i0, #1;
θ[Knot[7, 3], F1 → -B[tw], F2 → D{s1, i1, j1}[tw], F3 → 0] // Echo // PolyPlot
θ[Knot[7, 3], F1 → F1i - B[tw], F2 → F2i + D{s1, i1, j1}[tw]] // Echo // PolyPlot
» { $\frac{2 - 3 T + 3 T^2 - 3 T^3 + 2 T^4}{T^2}, \theta$ }
```

Out[=]=



$$\gg \left\{ \frac{2 - 3 T + 3 T^2 - 3 T^3 + 2 T^4}{T^2}, \frac{1}{T_1^4 T_2^4} \right. \\ \left(17 - 25 T_1 + 25 T_1^2 - 25 T_1^3 + 17 T_1^4 - 25 T_2 + 12 T_1 T_2 + 12 T_1^4 T_2 - 25 T_1^5 T_2 + 25 T_2^2 - T_1^2 T_2^2 - 7 T_1^3 T_2^2 - T_1^4 T_2^2 + 25 T_1^6 T_2^2 - 25 T_2^3 - 7 T_1^2 T_2^3 + 6 T_1^3 T_2^3 + 6 T_1^4 T_2^3 - 7 T_1^5 T_2^3 - 25 T_1^7 T_2^3 + 17 T_1^8 T_2^3 + 12 T_1 T_2^4 - T_1^2 T_2^4 + 6 T_1^3 T_2^4 - 12 T_1^4 T_2^4 + 6 T_1^5 T_2^4 - T_1^6 T_2^4 + 12 T_1^7 T_2^4 + 17 T_1^8 T_2^4 - 25 T_1 T_2^5 - 7 T_1^3 T_2^5 + 6 T_1^4 T_2^5 + 6 T_1^5 T_2^5 - 7 T_1^6 T_2^5 - 25 T_1^8 T_2^5 + 25 T_1^2 T_2^6 - T_1^4 T_2^6 - 7 T_1^5 T_2^6 - T_1^6 T_2^6 + 25 T_1^8 T_2^6 - 25 T_1^3 T_2^7 + 12 T_1^4 T_2^7 + 12 T_1^7 T_2^7 - 25 T_1^8 T_2^7 + 17 T_1^4 T_2^8 - 25 T_1^5 T_2^8 + 25 T_1^6 T_2^8 - 25 T_1^7 T_2^8 + 17 T_1^8 T_2^8 \right) \}$$

Out[=]=

In[=]:= **bas = List @@ Expand[(g_{1, #, i0} + g_{1, #, j0}) (g_{2, #, i0} + g_{2, #, j0}) (g_{3, i0+, #} + g_{3, j0+, #})]**

Out[=]=

```
{g1, #1, i0 g2, #1, i0 g3, i0+, #1, g1, #1, j0 g2, #1, i0 g3, i0+, #1, g1, #1, i0 g2, #1, j0 g3, i0+, #1, g1, #1, j0 g2, #1, i0 g3, i0+, #1, g1, #1, i0 g2, #1, i0 g3, j0+, #1, g1, #1, j0 g2, #1, i0 g3, j0+, #1, g1, #1, i0 g2, #1, j0 g3, j0+, #1, g1, #1, j0 g2, #1, j0 g3, j0+, #1}
```

In[=]:= **Column[(# → θ[Knot[7, 3], F1 → -B[#], F2 → D_{s1, i1, j1}[#], F3 → 0] //., bas]**
Column[(# → {B[#], D_{s1, i1, j1}[#]})) & /@ bas]

Out[=]=

```
g1, #1, i0 g2, #1, i0 g3, i0+, #1 → 0
g1, #1, j0 g2, #1, i0 g3, i0+, #1 → 0
g1, #1, i0 g2, #1, j0 g3, i0+, #1 → 0
g1, #1, j0 g2, #1, j0 g3, i0+, #1 → 0
g1, #1, i0 g2, #1, i0 g3, j0+, #1 → 0
g1, #1, j0 g2, #1, i0 g3, j0+, #1 → 0
g1, #1, i0 g2, #1, j0 g3, j0+, #1 → 0
g1, #1, j0 g2, #1, j0 g3, j0+, #1 → 0
```

Out[=]=

```
g1, #1, i0 g2, #1, i0 g3, i0+, #1 →
{0, -T1-s0 T2-s0 (-2 + T1s1 + T2s1) g1, j1, i0 g2, j1, i0 g3, i0, i1 + χi0==i1 (T1-s0 T2-s0 g3, i0, i0 +
T1-s0 T2-s0 (-1 + T1s0 T2s0) g3, j0, i0 + g2, i0, i0 (-T1-s0 T2-s0 g3, i0, i0 - T1-s0 T2-s0 (-1 + T1s0 T2s0) g3, j0, i0) +
g1, i0, i0 (T1-s0 T2-s0 g2, i0, i0 - T1-s0 T2-s0 g3, i0, i0 - T1-s0 T2-s0 (-1 + T1s0 T2s0) g3, j0, i0) +
g1, j0, i0 (T1-s0 T2-s0 (-1 + T1s0 T2s0) g2, j0, i0 - T1-s0 (-1 + T1s0) T2-s0 g3, i0, i0 -
T1-s0 (-1 + T1s0) T2-s0 (-1 + T1s0 T2s0) g3, j0, i0) +
T1-s0 T2-s0 (-1 + T1s0 T2s0) (-2 + T1s1 + T2s1) g1, j1, i0 g2, j1, i0 g3, j0, i1 +
g2, i1, i0 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s1) T2-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g2, j1, i0 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, i0, i0 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, j0, i0 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, i0, i1 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, j0, i1 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, i1, i0 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, j1, i0 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, i0, j0 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, j0, j0 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, i1, j0 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, j1, j0 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, i0, i2 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, j0, i2 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, i1, i2 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, j1, i2 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, i0, j2 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, j0, j2 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, i1, j2 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, j1, j2 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, i0, i3 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, j0, i3 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, i1, i3 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, j1, i3 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, i0, j3 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, j0, j3 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, i1, j3 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, j1, j3 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, i0, i4 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, j0, i4 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, i1, i4 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, j1, i4 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, i0, j4 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, j0, j4 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, i1, j4 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, j1, j4 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, i0, i5 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, j0, i5 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, i1, i5 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, j1, i5 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, i0, j5 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, j0, j5 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, i1, j5 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, j1, j5 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, i0, i6 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, j0, i6 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, i1, i6 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, j1, i6 (T1-s0 (-1 + T1s1) T2-s0 g1, j1, i0 g3, i0, i1 + T1-s0 (-1 + T1s0 T2s0) g1, j1, i0 g3, j0, i1) +
g3, i0, j
```

$$\begin{aligned}
& g_{1,\#1,j0} g_{2,\#1,i0} g_{3,i0^+, \#1} \rightarrow \left\{ \theta, -T_1^{-s0} T_2^{-s0} (-2 + T_1^{s1} + T_2^{s1}) g_{1,j1,j0} g_{2,j1,i0} g_{3,i0,i1} - \right. \\
& \quad T_1^{-s0} T_2^{-s0} (-1 + T_1^{s0} T_2^{s0}) (-2 + T_1^{s1} + T_2^{s1}) g_{1,j1,j0} g_{2,j1,i0} g_{3,j0,i1} + \\
& \quad g_{2,i1,i0} (T_1^{-s0} (-1 + T_1^{s1}) T_2^{-s0} g_{1,j1,j0} g_{3,i0,i1} + T_1^{-s0} (-1 + T_1^{s1}) T_2^{-s0} (-1 + T_1^{s0} T_2^{s0}) g_{1,j1,j0} g_{3,j0,i1}) + \\
& \quad g_{1,i1,j0} (T_1^{-s0} T_2^{-s0} (-1 + T_2^{s1}) g_{2,j1,i0} g_{3,i0,i1} + T_1^{-s0} T_2^{-s0} (-1 + T_1^{s0} T_2^{s0}) (-1 + T_2^{s1}) g_{2,j1,i0} g_{3,j0,i1}) + \\
& \quad \chi_{i0=i1} (T_1^{-s0} (-1 + T_1^{s0}) T_2^{-s0} g_{3,i0,i0} + T_1^{-s0} (-1 + T_1^{s0}) T_2^{-s0} (-1 + T_1^{s0} T_2^{s0}) g_{3,j0,i0} + \\
& \quad g_{1,i0,j0} (T_1^{-s0} T_2^{-s0} g_{2,i0,i0} - T_1^{-s0} T_2^{-s0} (-1 + T_1^{s0} T_2^{s0}) g_{3,j0,i0}) + \\
& \quad g_{2,i0,i0} (-T_1^{-s0} (-1 + T_1^{s0}) T_2^{-s0} g_{3,i0,i0} - T_1^{-s0} (-1 + T_1^{s0}) T_2^{-s0} (-1 + T_1^{s0} T_2^{s0}) g_{3,j0,i0}) + \\
& \quad g_{1,j0,j0} (T_1^{-s0} T_2^{-s0} (-1 + T_1^{s0} T_2^{s0}) g_{2,j0,i0} - T_1^{-s0} (-1 + T_1^{s0}) T_2^{-s0} g_{3,i0,i0} - T_1^{-s0} (-1 + T_1^{s0}) T_2^{-s0} \\
& \quad (-1 + T_1^{s0} T_2^{s0}) g_{3,j0,i0}) + g_{2,j0,i0} (T_1^{-s0} T_2^{-s0} (-2 + T_1^{s0} + T_2^{s0}) g_{3,i0,i0} - T_1^{-s0} T_2^{-s0} g_{3,i0,j0} + \\
& \quad T_1^{-s0} T_2^{-s0} (-2 + T_1^{s0} + T_2^{s0}) (-1 + T_1^{s0} T_2^{s0}) g_{3,j0,i0} - T_1^{-s0} T_2^{-s0} (-1 + T_1^{s0} T_2^{s0}) g_{3,j0,j0}) \} \\
g_{1,\#1,i0} g_{2,\#1,j0} g_{3,i0^+, \#1} & \rightarrow \left\{ \theta, -T_1^{-s0} T_2^{-s0} (-2 + T_1^{s1} + T_2^{s1}) g_{1,j1,i0} g_{2,j1,j0} g_{3,i0,i1} - \right. \\
& \quad T_1^{-s0} T_2^{-s0} (-1 + T_1^{s0} T_2^{s0}) (-2 + T_1^{s1} + T_2^{s1}) g_{1,j1,i0} g_{2,j1,j0} g_{3,j0,i1} + \\
& \quad g_{2,i1,j0} (T_1^{-s0} (-1 + T_1^{s1}) T_2^{-s0} g_{1,j1,i0} g_{3,i0,i1} + T_1^{-s0} (-1 + T_1^{s1}) T_2^{-s0} (-1 + T_1^{s0} T_2^{s0}) g_{1,j1,i0} g_{3,j0,i1}) + \\
& \quad g_{1,i1,i0} (T_1^{-s0} T_2^{-s0} (-1 + T_2^{s1}) g_{2,j1,j0} g_{3,i0,i1} + T_1^{-s0} T_2^{-s0} (-1 + T_1^{s0} T_2^{s0}) (-1 + T_2^{s1}) g_{2,j1,j0} g_{3,j0,i1}) + \\
& \quad \chi_{i0=i1} (T_1^{-s0} T_2^{-s0} (-1 + T_2^{s0}) g_{3,i0,i0} + T_1^{-s0} T_2^{-s0} (-1 + T_2^{s0}) (-1 + T_1^{s0} T_2^{s0}) g_{3,j0,i0} + \\
& \quad g_{2,i0,j0} (-T_1^{-s0} T_2^{-s0} g_{3,i0,i0} - T_1^{-s0} T_2^{-s0} (-1 + T_1^{s0} T_2^{s0}) g_{3,j0,i0}) + \\
& \quad g_{2,j0,i0} (-T_1^{-s0} T_2^{-s0} (-1 + T_2^{s0}) g_{3,i0,i0} - T_1^{-s0} T_2^{-s0} (-1 + T_1^{s0} T_2^{s0}) (-1 + T_1^{s0} T_2^{s0}) g_{3,j0,i0}) + g_{1,i0,i0} \\
& \quad (T_1^{-s0} T_2^{-s0} g_{2,i0,j0} - T_1^{-s0} T_2^{-s0} (-1 + T_2^{s0}) g_{3,i0,i0} - T_1^{-s0} T_2^{-s0} (-1 + T_2^{s0}) (-1 + T_1^{s0} T_2^{s0}) g_{3,j0,i0}) + \\
& \quad g_{1,j0,i0} (T_1^{-s0} T_2^{-s0} (-1 + T_1^{s0} T_2^{s0}) g_{2,j0,j0} + T_1^{-s0} T_2^{-s0} (-2 + T_1^{s0} + T_2^{s0}) g_{3,i0,i0} - T_1^{-s0} T_2^{-s0} g_{3,i0,j0} + \\
& \quad T_1^{-s0} T_2^{-s0} (-2 + T_1^{s0} + T_2^{s0}) (-1 + T_1^{s0} T_2^{s0}) g_{3,j0,i0} - T_1^{-s0} T_2^{-s0} (-1 + T_1^{s0} T_2^{s0}) g_{3,j0,j0}) \} \\
g_{1,\#1,j0} g_{2,\#1,i0} g_{3,i0^+, \#1} & \rightarrow \left\{ \theta, -T_1^{-s0} T_2^{-s0} (-2 + T_1^{s1} + T_2^{s1}) g_{1,j1,j0} g_{2,j1,i0} g_{3,i0,i1} - \right. \\
& \quad T_1^{-s0} T_2^{-s0} (-1 + T_1^{s0} T_2^{s0}) (-2 + T_1^{s1} + T_2^{s1}) g_{1,j1,j0} g_{2,j1,i0} g_{3,j0,i1} + \\
& \quad g_{2,i1,j0} (T_1^{-s0} (-1 + T_1^{s1}) T_2^{-s0} g_{1,j1,j0} g_{3,i0,i1} + T_1^{-s0} (-1 + T_1^{s1}) T_2^{-s0} (-1 + T_1^{s0} T_2^{s0}) g_{1,j1,j0} g_{3,j0,i1}) + \\
& \quad g_{1,i1,j0} (T_1^{-s0} T_2^{-s0} (-1 + T_2^{s1}) g_{2,j1,j0} g_{3,i0,i1} + T_1^{-s0} T_2^{-s0} (-1 + T_1^{s0} T_2^{s0}) (-1 + T_2^{s1}) g_{2,j1,j0} g_{3,j0,i1}) + \\
& \quad \chi_{i0=i1} (-T_1^{-s0} T_2^{-s0} (-2 + T_1^{s0} + T_2^{s0}) g_{3,i0,i0} + \\
& \quad T_1^{-s0} T_2^{-s0} g_{3,i0,j0} - T_1^{-s0} T_2^{-s0} (-2 + T_1^{s0} + T_2^{s0}) (-1 + T_1^{s0} T_2^{s0}) g_{3,j0,i0} + \\
& \quad g_{2,i0,j0} (-T_1^{-s0} (-1 + T_1^{s0}) T_2^{-s0} g_{3,i0,i0} - T_1^{-s0} (-1 + T_1^{s0}) T_2^{-s0} (-1 + T_1^{s0} T_2^{s0}) g_{3,j0,i0}) + g_{1,i0,j0} \\
& \quad (T_1^{-s0} T_2^{-s0} g_{2,i0,j0} - T_1^{-s0} T_2^{-s0} (-1 + T_2^{s0}) g_{3,i0,i0} - T_1^{-s0} T_2^{-s0} (-1 + T_2^{s0}) (-1 + T_1^{s0} T_2^{s0}) g_{3,j0,i0}) + \\
& \quad T_1^{-s0} T_2^{-s0} (-1 + T_1^{s0} T_2^{s0}) g_{3,j0,j0} + g_{2,j0,j0} (T_1^{-s0} T_2^{-s0} (-2 + T_1^{s0} + T_2^{s0}) g_{3,i0,i0} - T_1^{-s0} T_2^{-s0} g_{3,i0,j0} + \\
& \quad T_1^{-s0} T_2^{-s0} (-2 + T_1^{s0} + T_2^{s0}) (-1 + T_1^{s0} T_2^{s0}) g_{3,j0,i0} - T_1^{-s0} T_2^{-s0} (-1 + T_1^{s0} T_2^{s0}) g_{3,j0,j0}) \} \\
g_{1,\#1,i0} g_{2,\#1,i0} g_{3,j0^+, \#1} & \rightarrow \left\{ \theta, \chi_{i0=i1} \right. \\
& \quad \left((1 - g_{1,i0,i0} - g_{2,i0,i0}) g_{3,j0,i0} + (1 - T_2^{s0}) g_{2,j0,i0} g_{3,j0,i0} + g_{1,j0,i0} (g_{2,j0,i0} + (1 - T_1^{s0}) g_{3,j0,i0}) \right) + \\
& \quad \left(-1 + T_1^{s1} \right) g_{1,j1,i0} g_{2,i1,i0} g_{3,j0,i1} + \left(-1 + T_2^{s1} \right) g_{1,i1,i0} g_{2,j1,i0} g_{3,j0,i1} + \\
& \quad \left(2 - T_1^{s1} - T_2^{s1} \right) g_{1,j1,i0} g_{2,j1,i0} g_{3,j0,i1} \} \\
g_{1,\#1,j0} g_{2,\#1,i0} g_{3,j0^+, \#1} & \rightarrow \left\{ \theta, \left(-1 + T_1^{s1} \right) g_{1,j1,j0} g_{2,i1,i0} g_{3,j0,i1} + \left(-1 + T_2^{s1} \right) g_{1,i1,j0} g_{2,j1,i0} g_{3,j0,i1} + \right. \\
& \quad \left(2 - T_1^{s1} - T_2^{s1} \right) g_{1,j1,j0} g_{2,j1,i0} g_{3,j0,i1} + \chi_{i0=i1} \left(\left(-1 + T_1^{s0} - g_{1,i0,j0} + (1 - T_1^{s0}) g_{2,i0,i0} \right) g_{3,j0,i0} \right. \\
& \quad \left. \left. + g_{1,j0,j0} (g_{2,j0,i0} + (1 - T_1^{s0}) g_{3,j0,i0}) + g_{2,j0,i0} \left(\left(-2 + T_1^{s0} + T_2^{s0} \right) g_{3,j0,i0} - g_{3,j0,j0} \right) \right) \right\}
\end{aligned}$$

$$\begin{aligned}
& g_{1,\#1,i0} g_{2,\#1,j0} g_{3,j0^+,\#1} \rightarrow \\
& \left\{ 0, (-1 + T_1^{s1}) g_{1,j1,i0} g_{2,i1,j0} g_{3,j0,i1} + (-1 + T_2^{s1}) g_{1,i1,i0} g_{2,j1,j0} g_{3,j0,i1} + (2 - T_1^{s1} - T_2^{s1}) g_{1,j1,i0} g_{2,j1,j0} \right. \\
& g_{3,j0,i1} + \chi_{i0=i1} \left((-1 + T_2^{s0}) + (1 - T_2^{s0}) g_{1,i0,i0} - g_{2,i0,j0} \right) g_{3,j0,i0} + (1 - T_2^{s0}) g_{2,j0,j0} g_{3,j0,i0} + \\
& \left. g_{1,j0,i0} (g_{2,j0,j0} + (-2 + T_1^{s0} + T_2^{s0}) g_{3,j0,i0} - g_{3,j0,j0}) \right\} \\
& g_{1,\#1,j0} g_{2,\#1,j0} g_{3,j0^+,\#1} \rightarrow \\
& \left\{ 0, (-1 + T_1^{s1}) g_{1,j1,j0} g_{2,i1,j0} g_{3,j0,i1} + (-1 + T_2^{s1}) g_{1,i1,j0} g_{2,j1,j0} g_{3,j0,i1} + (2 - T_1^{s1} - T_2^{s1}) g_{1,j1,j0} \right. \\
& g_{2,j1,j0} g_{3,j0,i1} + \chi_{i0=i1} \left((2 - T_1^{s0} - T_2^{s0}) + (1 - T_2^{s0}) g_{1,i0,j0} + (1 - T_1^{s0}) g_{2,i0,j0} \right) g_{3,j0,i0} + \\
& (1 - T_2^{s0}) g_{3,j0,i0} + \\
& \left. g_{1,j0,j0} (g_{2,j0,j0} + (-2 + T_1^{s0} + T_2^{s0}) g_{3,j0,i0} - g_{3,j0,j0}) \right\}
\end{aligned}$$

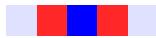
```

In[=]: tw = ((1 - T_1^{s0}) g_{1,\#1,i0} g_{2,\#1,i0} g_{3,j0^+,\#1} + (-1 + T_1^{s0}) g_{1,\#1,j0} g_{2,\#1,i0} g_{3,j0^+,\#1}) / (T_2 - 1)
{D_{\{s1,i1,j1\}}[tw], B[Tw]}
Theta[Knot[7, 6], F1 -> B[tw], F2 -> D_{\{s1,i1,j1\}}[tw], F3 -> 0] // Echo // PolyPlot
Theta[Knot[7, 6], F1 -> F1i - B[tw], F2 -> F2i + D_{\{s1,i1,j1\}}[tw]] // Echo // PolyPlot

```

$$\frac{(1 - T_1^{s0}) g_{1,\#1,i0} g_{2,\#1,i0} g_{3,j0^+,\#1} + (-1 + T_1^{s0}) g_{1,\#1,j0} g_{2,\#1,i0} g_{3,j0^+,\#1}}{-1 + T_2}$$

$$\begin{aligned}
Out[=]= & \left\{ -\frac{(-1 + T_1^{s0}) \chi_{i0=i1} g_{1,j0,i0} g_{2,j0,i0}}{-1 + T_2} + \frac{(-1 + T_1^{s0}) \chi_{i0=i1} g_{1,j0,j0} g_{2,j0,i0}}{-1 + T_2} + \right. \\
& \frac{(-2 + T_1^{s0}) (-1 + T_1^{s0}) \chi_{i0=i1} g_{3,j0,i0}}{-1 + T_2} + \frac{(-1 + T_1^{s0}) \chi_{i0=i1} g_{1,i0,i0} g_{3,j0,i0}}{-1 + T_2} - \\
& \frac{(-1 + T_1^{s0}) \chi_{i0=i1} g_{1,i0,j0} g_{3,j0,i0}}{-1 + T_2} + \frac{(-1 + T_1^{s0})^2 \chi_{i0=i1} g_{1,j0,i0} g_{3,j0,i0}}{-1 + T_2} - \\
& \frac{(-1 + T_1^{s0})^2 \chi_{i0=i1} g_{1,j0,j0} g_{3,j0,i0}}{-1 + T_2} - \frac{(-2 + T_1^{s0}) (-1 + T_1^{s0}) \chi_{i0=i1} g_{2,i0,i0} g_{3,j0,i0}}{-1 + T_2} + \\
& \frac{(-1 + T_1^{s0}) (-3 + T_1^{s0} + 2 T_2^{s0}) \chi_{i0=i1} g_{2,j0,i0} g_{3,j0,i0}}{-1 + T_2} - \frac{(-1 + T_1^{s0}) (-1 + T_1^{s1}) g_{1,j1,i0} g_{2,i1,i0} g_{3,j0,i1}}{-1 + T_2} + \\
& \frac{(-1 + T_1^{s0}) (-1 + T_1^{s1}) g_{1,j1,j0} g_{2,i1,i0} g_{3,j0,i1}}{-1 + T_2} - \frac{(-1 + T_1^{s0}) (-1 + T_2^{s1}) g_{1,i1,i0} g_{2,j1,i0} g_{3,j0,i1}}{-1 + T_2} + \\
& \frac{(-1 + T_1^{s0}) (-1 + T_2^{s1}) g_{1,i1,j0} g_{2,j1,i0} g_{3,j0,i1}}{-1 + T_2} + \frac{(-1 + T_1^{s0}) (-2 + T_1^{s1} + T_2^{s1}) g_{1,j1,i0} g_{2,j1,i0} g_{3,j0,i1}}{-1 + T_2} - \\
& \frac{(-1 + T_1^{s0}) (-2 + T_1^{s1} + T_2^{s1}) g_{1,j1,j0} g_{2,j1,i0} g_{3,j0,i1}}{-1 + T_2} - \frac{(-1 + T_1^{s0}) \chi_{i0=i1} g_{2,j0,i0} g_{3,j0,j0}}{-1 + T_2}, 0 \} \\
\gg & \left\{ -\frac{1 - 5 T + 7 T^2 - 5 T^3 + T^4}{T^2}, 0 \right\}
\end{aligned}$$

Out[*o*] =

$$\gg \left\{ -\frac{1 - 5 T + 7 T^2 - 5 T^3 + T^4}{T^2}, \right.$$

$$\frac{1}{T_1^4 T_2^4} (1 - 5 T_1 + 7 T_1^2 - 5 T_1^3 + T_1^4 - 5 T_2 + 20 T_1 T_2 - 10 T_1^2 T_2 - 10 T_1^3 T_2 + 20 T_1^4 T_2 - 5 T_1^5 T_2 + 7 T_2^2 -$$

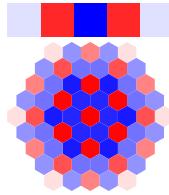
$$10 T_1 T_2^2 - 64 T_1^2 T_2^2 + 98 T_1^3 T_2^2 - 64 T_1^4 T_2^2 - 10 T_1^5 T_2^2 + 7 T_1^6 T_2^2 - 5 T_2^3 - 10 T_1 T_2^3 + 98 T_1^2 T_2^3 - 50 T_1^3 T_2^3 -$$

$$50 T_1^4 T_2^3 + 98 T_1^5 T_2^3 - 10 T_1^6 T_2^3 - 5 T_1^7 T_2^3 + T_2^4 + 20 T_1 T_2^4 - 64 T_1^2 T_2^4 - 50 T_1^3 T_2^4 + 108 T_1^4 T_2^4 - 50 T_1^5 T_2^4 -$$

$$64 T_1^6 T_2^4 + 20 T_1^7 T_2^4 + T_1^8 T_2^4 - 5 T_1 T_2^5 - 10 T_1^2 T_2^5 + 98 T_1^3 T_2^5 - 50 T_1^4 T_2^5 - 50 T_1^5 T_2^5 + 98 T_1^6 T_2^5 -$$

$$10 T_1^7 T_2^5 - 5 T_1^8 T_2^5 + 7 T_1^2 T_2^6 - 10 T_1^3 T_2^6 - 64 T_1^4 T_2^6 + 98 T_1^5 T_2^6 - 64 T_1^6 T_2^6 - 10 T_1^7 T_2^6 + 7 T_1^8 T_2^6 - 5 T_1^3 T_2^7 +$$

$$20 T_1^4 T_2^7 - 10 T_1^5 T_2^7 - 10 T_1^6 T_2^7 + 20 T_1^7 T_2^7 - 5 T_1^8 T_2^7 + T_1^4 T_2^8 - 5 T_1^5 T_2^8 + 7 T_1^6 T_2^8 - 5 T_1^7 T_2^8 + T_1^8 T_2^8) \Big\}$$

Out[*j*] =

$$\text{In[*o*]} := \text{CF}[\text{Residue}[\text{CF}[\chi_{i0==i1} \text{F1} i + \text{F2} i + \text{D}_{\{s1, i1, j1\}}[\text{tw}], \{T_2, 1\}] /.$$

$$\{\text{g3, } \alpha, \beta \rightarrow \text{g1, } \alpha, \beta, \text{g2, } \alpha, \beta \rightarrow \chi_{\alpha \leq \beta}\} /. \{\chi_{i0 \leq i0} \rightarrow 1, \chi_{j0 \leq j0} \rightarrow 1\}]$$

Out[*o*] =

$$(-1 + T_1^{s0}) \chi_{i0==i1} (-1 + \chi_{i0 \leq j0} - \chi_{j0 \leq i0} + T_1^{s0} \chi_{j0 \leq i0}) \text{g1, j0, i0} -$$

$$(-1 + T_1^{s0}) \chi_{i0==i1} \text{g1, i0, j0} \text{g1, j0, i0} - (-1 + T_1^{s0})^2 \chi_{i0==i1} \text{g1, j0, i0}^2 -$$

$$(-2 + T_1^{s0}) (-1 + T_1^{s0}) \chi_{i0==i1} \text{g1, j0, i0} \text{g1, j0, j0} - (-1 + T_1^{s0}) (-1 + T_1^{s1}) (\chi_{i1 \leq j0} - \chi_{j1 \leq j0}) \text{g1, j0, i1} \text{g1, j1, i0} +$$

$$(-1 + T_1^{s0}) (-1 + T_1^{s1}) (\chi_{i1 \leq i0} - \chi_{j1 \leq i0}) \text{g1, j0, i1} \text{g1, j1, j0}$$