```
In[•]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\Theta"];
        Once[<< Theta.m]
```

```
Loading KnotTheory` version of October 29, 2024, 10:29:52.1301.
Read more at http://katlas.org/wiki/KnotTheory.
```

```
In[•]:= RandomVK[n_] := {
          Prepend[#, 2 RandomInteger[1] - 1] & /@
           Partition[PermutationList[RandomPermutation[2 n], 2 n], 2],
          Table[RandomInteger[{-1, 1}], 2 n + 1]
        };
```

```
In[•]:= RandomVK[5]
```

```
Out[•]=
{{{-1, 9, 4}, {1, 7, 10}, {1, 1, 3}, {1, 2, 6}, {-1, 5, 8}},
 {1, 0, 1, -1, 0, 0, 0, 1, 0, -1, -1}}
```

```
In[•]:= CF[ℰ_] := Collect[
          Expand@Collect[ℰ, g__ | δ__ | χ__, F] /. F → Factor@*PowerExpand,
          χ_{i0==i1}
        ];
```

```
In[•]:= Θ_{F1_,F2_}[K_] := Module[{X, φ, n, A, Δ, G, ev, θ, k, k1, k2},
          {X, φ} = Rot[K]; n = Length[X]; A = IdentityMatrix[2 n + 1];
          Cases[X, {s_, i_, j_} :> (A[[{i, j}, {i + 1, j + 1}]] += ((-T^s  T^s - 1)
                                                                    (  0     -1  )))];
          Δ = T^((-Total[φ] - Total[X[[All, 1]]])/2) Det[A];
          G = Inverse[A];
          ev[ℰ_] :=
           Factor[ℰ /. k_^+ :> k + 1 /. {g_{ν_,α_,β_} :> (G[[α, β]] /. T → T_ν), χ_True → 1, χ_False → 0}];
          θ = ev@Sum[F1 //. Thread[{s, i, j} → X[[k]]], {k, n}];
          θ += ev@Sum[F2 //. Thread[{s0, i0, j0} → X[[k1]]] //.
                Thread[{s1, i1, j1} → X[[k2]]], {k1, n}, {k2, n}];
          θ += ev@Sum[F_3[φ[[k]], k], {k, Length@φ}];
          Factor@{Δ, (Δ /. T → T_1) (Δ /. T → T_2) (Δ /. T → T_3) θ}
        ];
```

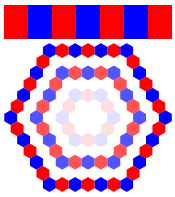
In[•]:=
```
FO = CF[χ_{i0==i1} F_1[{s0, i0, j0}] + F_2[{s0, i0, j0}, {s1, i1, j1}]]
```

Out[•]=

$$\frac{s1\left(-1+T_1^{s0}\right)T_2^{s0}\left(-1+T_1^{s1}T_2^{s1}\right)g_{1,j1,i0}\,g_{2,i1,i0}\,g_{3,j0,i1}}{-1+T_2^{s1}}-$$
$$\frac{s1\left(-1+T_1^{s0}\right)\left(-1+T_1^{s1}T_2^{s1}\right)g_{1,j1,i0}\,g_{2,i1,j0}\,g_{3,j0,i1}}{-1+T_2^{s1}}-$$
$$\frac{s1\left(-1+T_1^{s0}\right)T_2^{s0}\left(-1+T_1^{s1}T_2^{s1}\right)g_{1,j1,i0}\,g_{2,j1,i0}\,g_{3,j0,i1}}{-1+T_2^{s1}}+$$
$$\frac{s1\left(-1+T_1^{s0}\right)\left(-1+T_1^{s1}T_2^{s1}\right)g_{1,j1,i0}\,g_{2,j1,j0}\,g_{3,j0,i1}}{-1+T_2^{s1}}+$$
$$\chi_{i0==i1}\left(\frac{s0}{2}+s0\,T_2^{s0}\,g_{1,i0,i0}\,g_{2,j0,i0}+\frac{s0\left(-1+T_1^{s0}\right)T_2^{2\,s0}\,g_{1,j0,i0}\,g_{2,j0,i0}}{-1+T_2^{s0}}-s0\,g_{1,i0,i0}\,g_{2,j0,j0}-\right.$$
$$\frac{s0\left(-1+T_1^{s0}\right)T_2^{s0}\,g_{1,j0,i0}\,g_{2,j0,j0}}{-1+T_2^{s0}}-s0\,g_{3,i0,i0}-s0\left(-1+T_2^{s0}\right)g_{2,j0,i0}\,g_{3,i0,i0}+$$
$$2\,s0\,g_{2,j0,j0}\,g_{3,i0,i0}+\frac{s0\left(-1+T_1^{s0}T_2^{s0}\right)g_{3,j0,i0}}{-1+T_2^{s0}}-\frac{s0\,T_2^{s0}\left(-1+T_1^{s0}T_2^{s0}\right)g_{1,i0,i0}\,g_{3,j0,i0}}{-1+T_2^{s0}}-$$
$$\frac{s0\left(-1+T_1^{s0}\right)\left(1+T_2^{s0}\right)\left(-1+T_1^{s0}T_2^{s0}\right)g_{1,j0,i0}\,g_{3,j0,i0}}{-1+T_2^{s0}}+\frac{s0\left(-1+T_1^{s0}T_2^{s0}\right)g_{2,i0,j0}\,g_{3,j0,i0}}{-1+T_2^{s0}}+$$
$$s0\left(-1+T_1^{s0}T_2^{s0}\right)g_{2,j0,i0}\,g_{3,j0,i0}+\frac{s0\left(-2+T_2^{s0}\right)\left(-1+T_1^{s0}T_2^{s0}\right)g_{2,j0,j0}\,g_{3,j0,i0}}{-1+T_2^{s0}}+$$
$$\left.s0\,g_{1,i0,i0}\,g_{3,j0,j0}+\frac{s0\left(-1+T_1^{s0}\right)T_2^{s0}\,g_{1,j0,i0}\,g_{3,j0,j0}}{-1+T_2^{s0}}-s0\,g_{2,i0,i0}\,g_{3,j0,j0}-s0\,T_2^{s0}\,g_{2,j0,i0}\,g_{3,j0,j0}\right)$$

In[•]:=
```
PolyPlot[Θ_{0,FO}[Knot[7, 1]], ImageSize → Tiny]
```

Out[•]=



In[•]:=
```
δ_{i_,j_} := χ_{i==j};
bRules[{s_, i_, j_}] := { (* b for "push indices backwards" *)
    g_{ν_,j^+,β_} :→ g_{ν,j,β} - δ_{j,β}, g_{ν_,i^+,β_} :→ T_ν^{-s} g_{ν,i,β} + (1 - T_ν^{-s}) g_{ν,j,β} - T_ν^{-s} δ_{i,β} - (1 - T_ν^{-s}) δ_{j,β},
    g_{ν_,α_,i^+} :→ T_ν^s g_{ν,α,i} + δ_{α,i^+}, g_{ν_,α_,j^+} :→ g_{ν,α,j} + (1 - T_ν^s) g_{ν,α,i} + δ_{α,j^+}
  };
bRules[X___List] := Union @@ Table[bRules[c], {c, {X}}]
```

In[•]:=
```
{g_{ν,i,β}, g_{ν,j,β}} /. gRules[{s, i, j}]
```
Out[•]=
$$\{\chi_{i==\beta} + T^{s}_{\nu}\, g_{\nu,i^{+},\beta} + (1 - T^{s}_{\nu})\, g_{\nu,j^{+},\beta},\ \chi_{j==\beta} + g_{\nu,j^{+},\beta}\}$$

In[•]:=
```
Expand[{g_{ν,i,β}, g_{ν,j,β}} /. gRules[{s, i, j}] /. bRules[{s, i, j}]]
```
Out[•]=
$$\{g_{\nu,i,\beta},\ g_{\nu,j,\beta}\}$$

In[•]:=
```
D_{s_,i_,j_}[ℰ_] := CF[
  CF[
        ((ℰ /. ⌗ → i^+) + (ℰ /. ⌗ → j^+) - (ℰ /. ⌗ → i) - (ℰ /. ⌗ → j)) /.
          {g_{ν_,α_,β_:(i|j)^+} :↦ g_{ν,α,β} - δ_{α,β}} /. bRules[{s, i, j}]
      ] /. χ_{j1==i0} | χ_{i1==j0} → 0 /. χ_{j1==j0} | χ_{i1==i0} → χ_{i0==i1} /. χ^2_{eq_Equal} → χ_{eq} //.
   {χ_{i0==i1} g_{ν_,i1,β_} :↦ χ_{i0==i1} g_{ν,i0,β}, χ_{i0==i1} g_{ν_,j1,β_} :↦ χ_{i0==i1} g_{ν,j0,β},
    χ_{i0==i1} g_{ν_,α_,i1} :↦ χ_{i0==i1} g_{ν,α,i0}, χ_{i0==i1} g_{ν_,α_,j1} :↦ χ_{i0==i1} g_{ν,α,j0}}
 ]
```

In[•]:=
```
D_{s1,i1,j1}[g_{1,⌗,i0} g_{2,⌗,j0} g_{3,j0,⌗}]
```
Out[•]=
$$\left(-1 + T^{s1}_{1}\right) g_{1,j1,i0}\, g_{2,i1,j0}\, g_{3,j0,i1} + \left(-1 + T^{s1}_{2}\right) g_{1,i1,i0}\, g_{2,j1,j0}\, g_{3,j0,i1} + \left(2 - T^{s1}_{1} - T^{s1}_{2}\right) g_{1,j1,i0}\, g_{2,j1,j0}\, g_{3,j0,i1} + \chi_{i0==i1}\left(\left(-1 + T^{s1}_{2}\right) g_{3,j0,i0} + \left(1 - T^{s1}_{2}\right) g_{1,i0,i0}\, g_{3,j0,i0} + \left(-2 + T^{s1}_{1} + T^{s1}_{2}\right) g_{1,j0,i0}\, g_{3,j0,i0} - g_{2,i0,j0}\, g_{3,j0,i0} + \left(1 - T^{s1}_{2}\right) g_{2,j0,j0}\, g_{3,j0,i0} - g_{1,j0,i0}\, g_{3,j0,j0}\right)$$

In[•]:=
```
D_{s_,i_,j_}[ℰ_] := CF[Plus[
      ℰ /. {g_{ν_,⌗,β_} :↦ g_{ν,i^+,β}, g_{ν_,α_,⌗} :↦ g_{ν,α,i^+}},
      ℰ /. {g_{ν_,⌗,β_} :↦ g_{ν,j^+,β}, g_{ν_,α_,⌗} :↦ g_{ν,α,j^+}},
      -ℰ /. {g_{ν_,⌗,β_} :↦ g_{ν,i,β}, g_{ν_,α_,⌗} :↦ g_{ν,α,i}},
      -ℰ /. {g_{ν_,⌗,β_} :↦ g_{ν,j,β}, g_{ν_,α_,⌗} :↦ g_{ν,α,j}}
    ] /. bRules[{s, i, j}]
  ];
B_L_[ℰ_] := CF[
    (ℰ /. {g_{ν_,⌗,β_} :↦ g_{ν,L,β}, g_{ν_,α_,⌗} :↦ g_{ν,α,L}}) - (ℰ /. {g_{ν_,⌗,β_} :↦ g_{ν,1,β}, g_{ν_,α_,⌗} :↦ g_{ν,α,1}}) /. {
      g_{_,L,β_} → χ_{L==β}, g_{_,_,L} → 1, g_{_,α_,1} → χ_{α==1}}
  ];
```

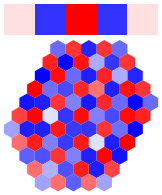
In[•]:=
```
D_{s1,i1,j1}[g_{1,⌗,i0}]
```
Out[•]=
$$-T^{-s1}_{1}\,\chi_{i1==i0} - T^{-s1}_{1}\left(-1 + 2\,T^{s1}_{1}\right)\chi_{j1==i0} - T^{-s1}_{1}\left(-1 + T^{s1}_{1}\right) g_{1,i1,i0} + T^{-s1}_{1}\left(-1 + T^{s1}_{1}\right) g_{1,j1,i0}$$

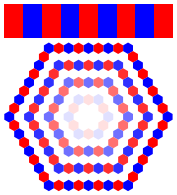
In[•]:=
```
D_{s1,i1,j1}[g_{3,j0,⌗}]
```
Out[•]=
$$\chi_{j0==i1^+} + \chi_{j0==j1^+}$$

In[•]:=
```
B_{2 n+1}[g_{3,j0,#}]
```
Out[•]=
$$1 - \chi_{j0==1}$$

In[•]:=
```
Θ_{-B19[g_{3,j,#}],D_{s1,i1,j1}[g_{3,j0,#}]}[Knot[9, 1]]
```
Out[•]=
$$\left\{\frac{\left(1 - T + T^2\right)\ \left(1 - T^3 + T^6\right)}{T^4},\ 0\right\}$$

In[•]:=
```
Θ_{-B19[g_{1,j,#}],D_{s1,i1,j1}[g_{1,j0,#}]}[Knot[9, 7]]
```
Out[•]=
$$\left\{\frac{3 - 7\,T + 9\,T^2 - 7\,T^3 + 3\,T^4}{T^2},\ 0\right\}$$

In[•]:=
```
PolyPlot[Θ_{-B19[g_{1,j,#}],D_{s1,i1,j1}[g_{1,j0,#}]}[Knot[9, 7]], ImageSize → Tiny]
```
Out[•]=


In[•]:=
```
PolyPlot[Θ_{-B19[g_{1,#,i} g_{2,#,j} g_{3,j,#}],FO+D_{s1,i1,j1}[g_{1,#,i0} g_{2,#,j0} g_{3,j0,#}]}[Knot[9, 1]], ImageSize → Tiny]
```
Out[•]=


In[•]:=
```
List @@ Expand[(g_{1,#,i0} + g_{1,#,j0}) (g_{2,#,i0} + g_{2,#,j0}) (g_{3,i0,#} + g_{3,j0,#})]
```
Out[•]=
$$\{g_{1,\#1,i0}\,g_{2,\#1,i0}\,g_{3,i0,\#1},\ g_{1,\#1,j0}\,g_{2,\#1,i0}\,g_{3,i0,\#1},\ g_{1,\#1,i0}\,g_{2,\#1,j0}\,g_{3,i0,\#1},\ g_{1,\#1,j0}\,g_{2,\#1,j0}\,g_{3,i0,\#1},$$
$$g_{1,\#1,i0}\,g_{2,\#1,i0}\,g_{3,j0,\#1},\ g_{1,\#1,j0}\,g_{2,\#1,i0}\,g_{3,j0,\#1},\ g_{1,\#1,i0}\,g_{2,\#1,j0}\,g_{3,j0,\#1},\ g_{1,\#1,j0}\,g_{2,\#1,j0}\,g_{3,j0,\#1}\}$$

In[•]:=
```
Column[
 (# → D_{s1,i1,j1}[#]) & /@ List @@ Expand[(g_{1,#,i0} + g_{1,#,j0}) (g_{2,#,i0} + g_{2,#,j0}) (g_{3,i0,#} + g_{3,j0,#})]]
```
Out[•]=
$$g_{1,\#1,i0}\,g_{2,\#1,i0}\,g_{3,i0,\#1} \to T_1^{-s1}\,T_2^{-s1}\,\chi_{i0==i1^+}\,\chi_{i1==i0}^2 + T_1^{-s1}\,T_2^{-s1}\left(-2 + T_1^{s1} + T_2^{s1}\right)\chi_{i0==i1^+}\,\chi_{i1==i0}\,\chi_{j1==i0} +$$
$$T_1^{-s1}\left(-1 + T_1^{s1}\right)T_2^{-s1}\left(-1 + T_2^{s1}\right)\chi_{i0==i1^+}\,\chi_{j1==i0}^2 + \chi_{i0==j1^+}\,\chi_{j1==i0}^2 - T_1^{-s1}\,T_2^{-s1}\,\chi_{i0==i1^+}\,\chi_{i1==i0}\,g_{1,i1,i0} -$$
$$T_1^{-s1}\,T_2^{-s1}\left(-1 + T_2^{s1}\right)\chi_{i0==i1^+}\,\chi_{j1==i0}\,g_{1,i1,i0} - T_1^{-s1}\left(-1 + T_1^{s1}\right)T_2^{-s1}\,\chi_{i0==i1^+}\,\chi_{i1==i0}\,g_{1,j1,i0} -$$
$$T_1^{-s1}\left(-1 + T_1^{s1}\right)T_2^{-s1}\left(-1 + T_2^{s1}\right)\chi_{i0==i1^+}\,\chi_{j1==i0}\,g_{1,j1,i0} - \chi_{i0==j1^+}\,\chi_{j1==i0}\,g_{1,j1,i0} -$$
$$T_1^{-s1}\,T_2^{-s1}\,\chi_{i0==i1^+}\,\chi_{i1==i0}\,g_{2,i1,i0} - T_1^{-s1}\left(-1 + T_1^{s1}\right)T_2^{-s1}\,\chi_{i0==i1^+}\,\chi_{j1==i0}\,g_{2,i1,i0} +$$
$$T_1^{-s1}\,T_2^{-s1}\,\chi_{i0==i1^+}\,g_{1,i1,i0}\,g_{2,i1,i0} + T_1^{-s1}\left(-1 + T_1^{s1}\right)T_2^{-s1}\,\chi_{i0==i1^+}\,g_{1,j1,i0}\,g_{2,i1,i0} -$$
$$T_1^{-s1}\,T_2^{-s1}\left(-1 + T_2^{s1}\right)\chi_{i0==i1^+}\,\chi_{i1==i0}\,g_{2,j1,i0} - T_1^{-s1}\left(-1 + T_1^{s1}\right)T_2^{-s1}\left(-1 + T_2^{s1}\right)\chi_{i0==i1^+}\,\chi_{j1==i0}\,g_{2,j1,i0} -$$
$$\chi_{i0==j1^+}\,\chi_{j1==i0}\,g_{2,j1,i0} + T_1^{-s1}\,T_2^{-s1}\left(-1 + T_2^{s1}\right)\chi_{i0==i1^+}\,g_{1,i1,i0}\,g_{2,j1,i0} +$$
$$T_1^{-s1}\left(-1 + T_1^{s1}\right)T_2^{-s1}\left(-1 + T_2^{s1}\right)\chi_{i0==i1^+}\,g_{1,j1,i0}\,g_{2,j1,i0} + \chi_{i0==j1^+}\,g_{1,j1,i0}\,g_{2,j1,i0} +$$
$$\chi_{i1==i0}^2\,g_{3,i0,i1} + \left(-2 + T_1^{s1} + T_2^{s1}\right)\chi_{i1==i0}\,\chi_{j1==i0}\,g_{3,i0,i1} + \left(2 - T_1^{s1} - T_2^{s1}\right)\chi_{j1==i0}^2\,g_{3,i0,i1} -$$
$$\chi_{i1==i0}\,g_{1,i1,i0}\,g_{3,i0,i1} + \left(1 - T_2^{s1}\right)\chi_{j1==i0}\,g_{1,i1,i0}\,g_{3,i0,i1} + \left(1 - T_1^{s1}\right)\chi_{i1==i0}\,g_{1,j1,i0}\,g_{3,i0,i1} +$$
$$\left(-2 + T_1^{s1} + T_2^{s1}\right)\chi_{j1==i0}\,g_{1,j1,i0}\,g_{3,i0,i1} + \left(1 - T_1^{s1}\right)\chi_{i1==i0}\,g_{2,i1,i0}\,g_{3,i0,i1} + \left(1 - T_2^{s1}\right)\chi_{j1==i0}\,g_{2,i1,i0}\,g_{3,i0,i1} +$$
$$\left(-1 + T_1^{s1}\right)g_{1,j1,i0}\,g_{2,i1,i0}\,g_{3,i0,i1} + \left(1 - T_2^{s1}\right)\chi_{i1==i0}\,g_{2,j1,i0}\,g_{3,i0,i1} +$$
$$\left(-2 + T_1^{s1} + T_2^{s1}\right)\chi_{j1==i0}\,g_{2,j1,i0}\,g_{3,i0,i1} + \left(-1 + T_2^{s1}\right)g_{1,i1,i0}\,g_{2,j1,i0}\,g_{3,i0,i1} +$$

$$g_{1,\#1,j0}\,g_{2,\#1,i0}\,g_{3,i0,\#1} \to T_1^{-s1}\,T_2^{-s1}\,\chi_{i0==i1^+}\,\chi_{i1==i0}\,\chi_{i1==j0} + T_1^{-s1}\,T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{i0==i1^+}\,\chi_{i1==j0}\,\chi_{j1==i0} + T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\,\chi_{i0==i1^+}\,\chi_{i1==i0}\,\chi_{j1==j0} + T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{i0==i1^+}\,\chi_{j1==i0}\,\chi_{j1==j0} + \chi_{i0==j1^+}\,\chi_{j1==i0}\,\chi_{j1==j0} - T_1^{-s1}\,T_2^{-s1}\,\chi_{i0==i1^+}\,\chi_{i1==i0}\,g_{1,i1,j0} - T_1^{-s1}\,T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{i0==i1^+}\,\chi_{j1==i0}\,g_{1,i1,j0} - T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\,\chi_{i0==i1^+}\,\chi_{i1==i0}\,g_{1,j1,j0} - T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{i0==i1^+}\,\chi_{j1==i0}\,g_{1,j1,j0} - \chi_{i0==j1^+}\,\chi_{j1==i0}\,g_{1,j1,j0} - T_1^{-s1}\,T_2^{-s1}\,\chi_{i0==i1^+}\,\chi_{i1==j0}\,g_{2,i1,i0} - T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\,\chi_{i0==i1^+}\,\chi_{j1==j0}\,g_{2,i1,i0} + T_1^{-s1}\,T_2^{-s1}\,\chi_{i0==i1^+}\,g_{1,i1,j0}\,g_{2,i1,i0} + T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\,\chi_{i0==i1^+}\,g_{1,j1,j0}\,g_{2,i1,i0} - T_1^{-s1}\,T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{i0==i1^+}\,\chi_{i1==j0}\,g_{2,j1,i0} - T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{i0==i1^+}\,\chi_{j1==j0}\,g_{2,j1,i0} - \chi_{i0==j1^+}\,\chi_{j1==j0}\,g_{2,j1,i0} + T_1^{-s1}\,T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{i0==i1^+}\,g_{1,i1,j0}\,g_{2,j1,i0} + T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{i0==i1^+}\,g_{1,j1,j0}\,g_{2,j1,i0} + \chi_{i0==j1^+}\,g_{1,j1,j0}\,g_{2,j1,i0} + \chi_{i1==i0}\,\chi_{i1==j0}\,g_{3,i0,i1} + \left(-1+T_2^{s1}\right)\chi_{i1==j0}\,\chi_{j1==i0}\,g_{3,i0,i1} + \left(-1+T_1^{s1}\right)\chi_{i1==i0}\,\chi_{j1==j0}\,g_{3,i0,i1} + \left(2-T_1^{s1}-T_2^{s1}\right)\chi_{j1==i0}\,\chi_{j1==j0}\,g_{3,i0,i1} - \chi_{i1==i0}\,g_{1,i1,j0}\,g_{3,i0,i1} + \left(1-T_2^{s1}\right)\chi_{j1==i0}\,g_{1,i1,j0}\,g_{3,i0,i1} + \left(1-T_1^{s1}\right)\chi_{i1==i0}\,g_{1,j1,j0}\,g_{3,i0,i1} + \left(-2+T_1^{s1}+T_2^{s1}\right)\chi_{j1==i0}\,g_{1,j1,j0}\,g_{3,i0,i1} - \chi_{i1==j0}\,g_{2,i1,i0}\,g_{3,i0,i1} + \left(1-T_1^{s1}\right)\chi_{j1==j0}\,g_{2,i1,i0}\,g_{3,i0,i1} + \left(-1+T_1^{s1}\right)g_{1,j1,j0}\,g_{2,i1,i0}\,g_{3,i0,i1} + \left(1-T_2^{s1}\right)\chi_{i1==j0}\,g_{2,j1,i0}\,g_{3,i0,i1} + \left(-2+T_1^{s1}+T_2^{s1}\right)\chi_{j1==j0}\,g_{2,j1,i0}\,g_{3,i0,i1} + \left(-1+T_2^{s1}\right)g_{1,i1,j0}\,g_{2,j1,i0}\,g_{3,i0,i1} + \left(2-T_1^{s1}-T_2^{s1}\right)g_{1,j1,j0}\,g_{2,j1,i0}\,g_{3,i0,i1} + \chi_{j1==i0}\,\chi_{j1==j0}\,g_{3,i0,j1} - \chi_{j1==i0}\,g_{1,j1,j0}\,g_{3,i0,j1} - \chi_{j1==j0}\,g_{2,j1,i0}\,g_{3,i0,j1}$$

$$g_{1,\#1,i0}\,g_{2,\#1,j0}\,g_{3,i0,\#1} \to T_1^{-s1}\,T_2^{-s1}\,\chi_{i0==i1^+}\,\chi_{i1==i0}\,\chi_{i1==j0} + T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\,\chi_{i0==i1^+}\,\chi_{i1==j0}\,\chi_{j1==i0} + T_1^{-s1}\,T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{i0==i1^+}\,\chi_{i1==i0}\,\chi_{j1==j0} + \chi_{i0==j1^+}\,\chi_{j1==i0}\,\chi_{j1==j0} - T_1^{-s1}\,T_2^{-s1}\,\chi_{i0==i1^+}\,\chi_{i1==j0}\,g_{1,i1,i0} - T_1^{-s1}\,T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{i0==i1^+}\,\chi_{j1==j0}\,g_{1,i1,i0} - T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\,\chi_{i0==i1^+}\,\chi_{i1==j0}\,g_{1,j1,i0} - T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{i0==i1^+}\,\chi_{j1==j0}\,g_{1,j1,i0} - \chi_{i0==j1^+}\,\chi_{j1==j0}\,g_{1,j1,i0} - T_1^{-s1}\,T_2^{-s1}\,\chi_{i0==i1^+}\,\chi_{i1==i0}\,g_{2,i1,j0} - T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\,\chi_{i0==i1^+}\,\chi_{j1==i0}\,g_{2,i1,j0} + T_1^{-s1}\,T_2^{-s1}\,\chi_{i0==i1^+}\,g_{1,i1,i0}\,g_{2,i1,j0} + T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\,\chi_{i0==i1^+}\,g_{1,j1,i0}\,g_{2,i1,j0} - T_1^{-s1}\,T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{i0==i1^+}\,\chi_{i1==i0}\,g_{2,j1,j0} - T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{i0==i1^+}\,\chi_{j1==i0}\,g_{2,j1,j0} + \chi_{i0==j1^+}\,\chi_{j1==i0}\,g_{2,j1,j0} + T_1^{-s1}\,T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{i0==i1^+}\,g_{1,i1,i0}\,g_{2,j1,j0} + T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{i0==i1^+}\,g_{1,j1,i0}\,g_{2,j1,j0} + \chi_{i0==j1^+}\,g_{1,j1,i0}\,g_{2,j1,j0} + \chi_{i1==i0}\,\chi_{i1==j0}\,g_{3,i0,i1} + \left(-1+T_2^{s1}\right)\chi_{i1==j0}\,\chi_{j1==i0}\,g_{3,i0,i1} + \left(-1+T_1^{s1}\right)\chi_{i1==i0}\,\chi_{j1==j0}\,g_{3,i0,i1} + \left(2-T_1^{s1}-T_2^{s1}\right)\chi_{j1==i0}\,\chi_{j1==j0}\,g_{3,i0,i1} - \chi_{i1==j0}\,g_{1,i1,i0}\,g_{3,i0,i1} + \left(1-T_2^{s1}\right)\chi_{j1==j0}\,g_{1,i1,i0}\,g_{3,i0,i1} + \left(1-T_1^{s1}\right)\chi_{i1==j0}\,g_{1,j1,i0}\,g_{3,i0,i1} + \left(-2+T_1^{s1}+T_2^{s1}\right)\chi_{j1==j0}\,g_{1,j1,i0}\,g_{3,i0,i1} - \chi_{i1==i0}\,g_{2,i1,j0}\,g_{3,i0,i1} + \left(1-T_1^{s1}\right)\chi_{j1==i0}\,g_{2,i1,j0}\,g_{3,i0,i1} + \left(-1+T_1^{s1}\right)g_{1,j1,i0}\,g_{2,i1,j0}\,g_{3,i0,i1} + \left(1-T_2^{s1}\right)\chi_{i1==i0}\,g_{2,j1,j0}\,g_{3,i0,i1} + \left(-2+T_1^{s1}+T_2^{s1}\right)\chi_{j1==i0}\,g_{2,j1,j0}\,g_{3,i0,i1} + \left(-1+T_2^{s1}\right)g_{1,i1,i0}\,g_{2,j1,j0}\,g_{3,i0,i1} + \left(2-T_1^{s1}-T_2^{s1}\right)g_{1,j1,i0}\,g_{2,j1,j0}\,g_{3,i0,i1} + \chi_{j1==i0}\,\chi_{j1==j0}\,g_{3,i0,j1} - \chi_{j1==j0}\,g_{1,j1,i0}\,g_{3,i0,j1} - \chi_{j1==i0}\,g_{2,j1,j0}\,g_{3,i0,j1}$$

$$g_{1,\sharp 1,j0}\,g_{2,\sharp 1,j0}\,g_{3,i0,\sharp 1}\to T_1^{-s1}\,T_2^{-s1}\,\chi_{i0==i1^+}\,\chi^2_{i1==j0}+T_1^{-s1}\,T_2^{-s1}\left(-2+T_1^{s1}+T_2^{s1}\right)\chi_{i0==i1^+}\,\chi_{i1==j0}\,\chi_{j1==j0}+T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{i0==i1^+}\,\chi^2_{j1==j0}+\chi_{i0==j1^+}\,\chi^2_{j1==j0}-T_1^{-s1}\,T_2^{-s1}\,\chi_{i0==i1^+}\,\chi_{i1==j0}\,g_{1,i1,j0}-T_1^{-s1}\,T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{i0==i1^+}\,\chi_{j1==j0}\,g_{1,i1,j0}-T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\,\chi_{i0==i1^+}\,\chi_{i1==j0}\,g_{1,j1,j0}-T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{i0==i1^+}\,\chi_{j1==j0}\,g_{1,j1,j0}-\chi_{i0==j1^+}\,\chi_{j1==j0}\,g_{1,j1,j0}-T_1^{-s1}\,T_2^{-s1}\,\chi_{i0==i1^+}\,\chi_{i1==j0}\,g_{2,i1,j0}-T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\,\chi_{i0==i1^+}\,\chi_{j1==j0}\,g_{2,i1,j0}+T_1^{-s1}\,T_2^{-s1}\,\chi_{i0==i1^+}\,g_{1,i1,j0}\,g_{2,i1,j0}+T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\,\chi_{i0==i1^+}\,g_{1,j1,j0}\,g_{2,i1,j0}-T_1^{-s1}\,T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{i0==i1^+}\,\chi_{i1==j0}\,g_{2,j1,j0}-T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{i0==i1^+}\,\chi_{j1==j0}\,g_{2,j1,j0}-\chi_{i0==j1^+}\,\chi_{j1==j0}\,g_{2,j1,j0}+T_1^{-s1}\,T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{i0==i1^+}\,g_{1,i1,j0}\,g_{2,j1,j0}+T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{i0==i1^+}\,g_{1,j1,j0}\,g_{2,j1,j0}+\chi_{i0==j1^+}\,g_{1,j1,j0}\,g_{2,j1,j0}+\chi^2_{i1==j0}\,g_{3,i0,i1}+\left(-2+T_1^{s1}+T_2^{s1}\right)\chi_{i1==j0}\,\chi_{j1==j0}\,g_{3,i0,i1}+\left(2-T_1^{s1}-T_2^{s1}\right)\chi^2_{j1==j0}\,g_{3,i0,i1}-\chi_{i1==j0}\,g_{1,i1,j0}\,g_{3,i0,i1}+\left(1-T_2^{s1}\right)\chi_{j1==j0}\,g_{1,i1,j0}\,g_{3,i0,i1}+\left(1-T_1^{s1}\right)\chi_{i1==j0}\,g_{1,j1,j0}\,g_{3,i0,i1}+\left(-2+T_1^{s1}+T_2^{s1}\right)\chi_{j1==j0}\,g_{1,j1,j0}\,g_{3,i0,i1}-\chi_{i1==j0}\,g_{2,i1,j0}\,g_{3,i0,i1}+\left(1-T_1^{s1}\right)\chi_{j1==j0}\,g_{2,i1,j0}\,g_{3,i0,i1}+\left(-1+T_1^{s1}\right)g_{1,j1,j0}\,g_{2,i1,j0}\,g_{3,i0,i1}+\left(1-T_2^{s1}\right)\chi_{i1==j0}\,g_{2,j1,j0}\,g_{3,i0,i1}+\left(-2+T_1^{s1}+T_2^{s1}\right)\chi_{j1==j0}\,g_{2,j1,j0}\,g_{3,i0,i1}+\left(-1+T_2^{s1}\right)g_{1,i1,j0}\,g_{2,j1,j0}\,g_{3,i0,i1}+\left(2-T_1^{s1}-T_2^{s1}\right)g_{1,j1,j0}\,g_{2,j1,j0}\,g_{3,i0,i1}+\chi^2_{j1==j0}\,g_{3,i0,j1}-\chi_{j1==j0}\,g_{1,j1,j0}\,g_{3,i0,j1}-\chi_{j1==j0}\,g_{2,j1,j0}\,g_{3,i0,j1}$$

$$g_{1,\sharp 1,i0}\,g_{2,\sharp 1,i0}\,g_{3,j0,\sharp 1}\to T_1^{-s1}\,T_2^{-s1}\,\chi^2_{i1==i0}\,\chi_{j0==i1^+}+T_1^{-s1}\,T_2^{-s1}\left(-2+T_1^{s1}+T_2^{s1}\right)\chi_{i1==i0}\,\chi_{j0==i1^+}\,\chi_{j1==i0}+T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{j0==i1^+}\,\chi^2_{j1==i0}+\chi_{j0==j1^+}\,\chi^2_{j1==i0}-T_1^{-s1}\,T_2^{-s1}\,\chi_{i1==i0}\,\chi_{j0==i1^+}\,g_{1,i1,i0}-T_1^{-s1}\,T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{j0==i1^+}\,\chi_{j1==i0}\,g_{1,i1,i0}-T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\,\chi_{i1==i0}\,\chi_{j0==i1^+}\,g_{1,j1,i0}-T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{j0==i1^+}\,\chi_{j1==i0}\,g_{1,j1,i0}-\chi_{j0==j1^+}\,\chi_{j1==i0}\,g_{1,j1,i0}-T_1^{-s1}\,T_2^{-s1}\,\chi_{i1==i0}\,\chi_{j0==i1^+}\,g_{2,i1,i0}-T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\,\chi_{j0==i1^+}\,\chi_{j1==i0}\,g_{2,i1,i0}+T_1^{-s1}\,T_2^{-s1}\,\chi_{j0==i1^+}\,g_{1,i1,i0}\,g_{2,i1,i0}+T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\,\chi_{j0==i1^+}\,g_{1,j1,i0}\,g_{2,i1,i0}-T_1^{-s1}\,T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{i1==i0}\,\chi_{j0==i1^+}\,g_{2,j1,i0}-T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{j0==i1^+}\,\chi_{j1==i0}\,g_{2,j1,i0}-\chi_{j0==j1^+}\,\chi_{j1==i0}\,g_{2,j1,i0}+T_1^{-s1}\,T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{j0==i1^+}\,g_{1,i1,i0}\,g_{2,j1,i0}+T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{j0==i1^+}\,g_{1,j1,i0}\,g_{2,j1,i0}+\chi_{j0==j1^+}\,g_{1,j1,i0}\,g_{2,j1,i0}+\chi^2_{i1==i0}\,g_{3,j0,i1}+\left(-2+T_1^{s1}+T_2^{s1}\right)\chi_{i1==i0}\,\chi_{j1==i0}\,g_{3,j0,i1}+\left(2-T_1^{s1}-T_2^{s1}\right)\chi^2_{j1==i0}\,g_{3,j0,i1}-\chi_{i1==i0}\,g_{1,i1,i0}\,g_{3,j0,i1}+\left(1-T_2^{s1}\right)\chi_{j1==i0}\,g_{1,i1,i0}\,g_{3,j0,i1}+\left(1-T_1^{s1}\right)\chi_{i1==i0}\,g_{1,j1,i0}\,g_{3,j0,i1}+\left(-2+T_1^{s1}+T_2^{s1}\right)\chi_{j1==i0}\,g_{1,j1,i0}\,g_{3,j0,i1}-\chi_{i1==i0}\,g_{2,i1,i0}\,g_{3,j0,i1}+\left(1-T_1^{s1}\right)\chi_{j1==i0}\,g_{2,i1,i0}\,g_{3,j0,i1}+\left(-1+T_1^{s1}\right)g_{1,j1,i0}\,g_{2,i1,i0}\,g_{3,j0,i1}+\left(1-T_2^{s1}\right)\chi_{i1==i0}\,g_{2,j1,i0}\,g_{3,j0,i1}+\left(-2+T_1^{s1}+T_2^{s1}\right)\chi_{j1==i0}\,g_{2,j1,i0}\,g_{3,j0,i1}+\left(-1+T_2^{s1}\right)g_{1,i1,i0}\,g_{2,j1,i0}\,g_{3,j0,i1}+\left(2-T_1^{s1}-T_2^{s1}\right)g_{1,j1,i0}\,g_{2,j1,i0}\,g_{3,j0,i1}+\chi^2_{j1==i0}\,g_{3,j0,j1}-\chi_{j1==i0}\,g_{1,j1,i0}\,g_{3,j0,j1}-\chi_{j1==i0}\,g_{2,j1,i0}\,g_{3,j0,j1}$$

$$g_{1,\#1,j0}\,g_{2,\#1,i0}\,g_{3,j0,\#1} \to T_1^{-s1}\,T_2^{-s1}\,\chi_{i1==i0}\,\chi_{i1==j0}\,\chi_{j0==i1^+} + T_1^{-s1}\,T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{i1==j0}\,\chi_{j0==i1^+}\,\chi_{j1==i0} + T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\,\chi_{i1==i0}\,\chi_{j0==i1^+}\,\chi_{j1==j0} + T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{j0==i1^+}\,\chi_{j1==i0}\,\chi_{j1==j0} + \chi_{j0==j1^+}\,\chi_{j1==i0}\,\chi_{j1==j0} - T_1^{-s1}\,T_2^{-s1}\,\chi_{i1==i0}\,\chi_{j0==i1^+}\,g_{1,i1,j0} - T_1^{-s1}\,T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{j0==i1^+}\,\chi_{j1==i0}\,g_{1,i1,j0} - T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\,\chi_{i1==i0}\,\chi_{j0==i1^+}\,g_{1,j1,j0} - T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{j0==i1^+}\,\chi_{j1==i0}\,g_{1,j1,j0} - \chi_{j0==j1^+}\,\chi_{j1==i0}\,g_{1,j1,j0} - T_1^{-s1}\,T_2^{-s1}\,\chi_{i1==j0}\,\chi_{j0==i1^+}\,g_{2,i1,i0} - T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\,\chi_{j0==i1^+}\,\chi_{j1==j0}\,g_{2,i1,i0} + T_1^{-s1}\,T_2^{-s1}\,\chi_{j0==i1^+}\,g_{1,i1,j0}\,g_{2,i1,i0} + T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\,\chi_{j0==i1^+}\,g_{1,j1,j0}\,g_{2,i1,i0} - T_1^{-s1}\,T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{i1==j0}\,\chi_{j0==i1^+}\,g_{2,j1,i0} - T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{j0==i1^+}\,\chi_{j1==j0}\,g_{2,j1,i0} - \chi_{j0==j1^+}\,\chi_{j1==j0}\,g_{2,j1,i0} + T_1^{-s1}\,T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{j0==i1^+}\,g_{1,i1,j0}\,g_{2,j1,i0} + T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{j0==i1^+}\,g_{1,j1,j0}\,g_{2,j1,i0} + \chi_{j0==j1^+}\,g_{1,j1,j0}\,g_{2,j1,i0} + \chi_{i1==i0}\,\chi_{i1==j0}\,g_{3,j0,i1} + \left(-1+T_2^{s1}\right)\chi_{i1==j0}\,\chi_{j1==i0}\,g_{3,j0,i1} + \left(-1+T_1^{s1}\right)\chi_{i1==i0}\,\chi_{j1==j0}\,g_{3,j0,i1} + \left(2-T_1^{s1}-T_2^{s1}\right)\chi_{j1==i0}\,\chi_{j1==j0}\,g_{3,j0,i1} - \chi_{i1==i0}\,g_{1,i1,j0}\,g_{3,j0,i1} + \left(1-T_2^{s1}\right)\chi_{j1==i0}\,g_{1,i1,j0}\,g_{3,j0,i1} + \left(1-T_1^{s1}\right)\chi_{i1==i0}\,g_{1,j1,j0}\,g_{3,j0,i1} + \left(-2+T_1^{s1}+T_2^{s1}\right)\chi_{j1==i0}\,g_{1,j1,j0}\,g_{3,j0,i1} - \chi_{i1==j0}\,g_{2,i1,i0}\,g_{3,j0,i1} + \left(1-T_1^{s1}\right)\chi_{j1==j0}\,g_{2,i1,i0}\,g_{3,j0,i1} + \left(-1+T_1^{s1}\right)g_{1,j1,j0}\,g_{2,i1,i0}\,g_{3,j0,i1} + \left(1-T_2^{s1}\right)\chi_{i1==j0}\,g_{2,j1,i0}\,g_{3,j0,i1} + \left(-2+T_1^{s1}+T_2^{s1}\right)\chi_{j1==j0}\,g_{2,j1,i0}\,g_{3,j0,i1} + \left(-1+T_2^{s1}\right)g_{1,i1,j0}\,g_{2,j1,i0}\,g_{3,j0,i1} + \left(2-T_1^{s1}-T_2^{s1}\right)g_{1,j1,j0}\,g_{2,j1,i0}\,g_{3,j0,i1} + \chi_{j1==i0}\,\chi_{j1==j0}\,g_{3,j0,j1} - \chi_{j1==i0}\,g_{1,j1,j0}\,g_{3,j0,j1} - \chi_{j1==j0}\,g_{2,j1,i0}\,g_{3,j0,j1}$$

$$g_{1,\#1,i0}\,g_{2,\#1,j0}\,g_{3,j0,\#1} \to T_1^{-s1}\,T_2^{-s1}\,\chi_{i1==i0}\,\chi_{i1==j0}\,\chi_{j0==i1^+} + T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\,\chi_{i1==j0}\,\chi_{j0==i1^+}\,\chi_{j1==i0} + T_1^{-s1}\,T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{i1==i0}\,\chi_{j0==i1^+}\,\chi_{j1==j0} + T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{j0==i1^+}\,\chi_{j1==i0}\,\chi_{j1==j0} + \chi_{j0==j1^+}\,\chi_{j1==i0}\,\chi_{j1==j0} - T_1^{-s1}\,T_2^{-s1}\,\chi_{i1==j0}\,\chi_{j0==i1^+}\,g_{1,i1,i0} - T_1^{-s1}\,T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{j0==i1^+}\,\chi_{j1==j0}\,g_{1,i1,i0} - T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\,\chi_{i1==j0}\,\chi_{j0==i1^+}\,g_{1,j1,i0} - T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{j0==i1^+}\,\chi_{j1==j0}\,g_{1,j1,i0} - \chi_{j0==j1^+}\,\chi_{j1==j0}\,g_{1,j1,i0} - T_1^{-s1}\,T_2^{-s1}\,\chi_{i1==i0}\,\chi_{j0==i1^+}\,g_{2,i1,j0} - T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\,\chi_{j0==i1^+}\,\chi_{j1==i0}\,g_{2,i1,j0} + T_1^{-s1}\,T_2^{-s1}\,\chi_{j0==i1^+}\,g_{1,i1,i0}\,g_{2,i1,j0} + T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{i1==i0}\,\chi_{j0==i1^+}\,g_{2,j1,j0} - T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{j0==i1^+}\,\chi_{j1==i0}\,g_{2,j1,j0} - \chi_{j0==j1^+}\,\chi_{j1==i0}\,g_{2,j1,j0} + T_1^{-s1}\,T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{j0==i1^+}\,g_{1,i1,i0}\,g_{2,j1,j0} + T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{j0==i1^+}\,g_{1,j1,i0}\,g_{2,j1,j0} + \chi_{j0==j1^+}\,g_{1,j1,i0}\,g_{2,j1,j0} + \chi_{i1==i0}\,\chi_{i1==j0}\,g_{3,j0,i1} + \left(-1+T_1^{s1}\right)\chi_{i1==j0}\,\chi_{j1==i0}\,g_{3,j0,i1} + \left(-1+T_2^{s1}\right)\chi_{i1==i0}\,\chi_{j1==j0}\,g_{3,j0,i1} + \left(2-T_1^{s1}-T_2^{s1}\right)\chi_{j1==i0}\,\chi_{j1==j0}\,g_{3,j0,i1} - \chi_{i1==j0}\,g_{1,i1,i0}\,g_{3,j0,i1} + \left(1-T_2^{s1}\right)\chi_{j1==j0}\,g_{1,i1,i0}\,g_{3,j0,i1} + \left(1-T_1^{s1}\right)\chi_{i1==j0}\,g_{1,j1,i0}\,g_{3,j0,i1} + \left(-2+T_1^{s1}+T_2^{s1}\right)\chi_{j1==j0}\,g_{1,j1,i0}\,g_{3,j0,i1} - \chi_{i1==i0}\,g_{2,i1,j0}\,g_{3,j0,i1} + \left(1-T_1^{s1}\right)\chi_{j1==i0}\,g_{2,i1,j0}\,g_{3,j0,i1} + \left(-1+T_1^{s1}\right)g_{1,j1,i0}\,g_{2,i1,j0}\,g_{3,j0,i1} + \left(1-T_2^{s1}\right)\chi_{i1==i0}\,g_{2,j1,j0}\,g_{3,j0,i1} + \left(-2+T_1^{s1}+T_2^{s1}\right)\chi_{j1==i0}\,g_{2,j1,j0}\,g_{3,j0,i1} + \left(-1+T_2^{s1}\right)g_{1,i1,i0}\,g_{2,j1,j0}\,g_{3,j0,i1} + \left(2-T_1^{s1}-T_2^{s1}\right)g_{1,j1,i0}\,g_{2,j1,j0}\,g_{3,j0,i1} + \chi_{j1==i0}\,\chi_{j1==j0}\,g_{3,j0,j1} - \chi_{j1==j0}\,g_{1,j1,i0}\,g_{3,j0,j1} - \chi_{j1==i0}\,g_{2,j1,j0}\,g_{3,j0,j1}$$

$$g_{1,\#1,j0}\,g_{2,\#1,j0}\,g_{3,j0,\#1} \to T_1^{-s1}\,T_2^{-s1}\,\chi^2_{i1==j0}\,\chi_{j0==i1^+} + T_1^{-s1}\,T_2^{-s1}\left(-2+T_1^{s1}+T_2^{s1}\right)\chi_{i1==j0}\,\chi_{j0==i1^+}\,\chi_{j1==j0} +$$
$$T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{j0==i1^+}\,\chi^2_{j1==j0} + \chi_{j0==j1^+}\,\chi^2_{j1==j0} - T_1^{-s1}\,T_2^{-s1}\,\chi_{i1==j0}\,\chi_{j0==i1^+}\,g_{1,i1,j0} -$$
$$T_1^{-s1}\,T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{j0==i1^+}\,\chi_{j1==j0}\,g_{1,i1,j0} - T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\,\chi_{i1==j0}\,\chi_{j0==i1^+}\,g_{1,j1,j0} -$$
$$T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{j0==i1^+}\,\chi_{j1==j0}\,g_{1,j1,j0} - \chi_{j0==j1^+}\,\chi_{j1==j0}\,g_{1,j1,j0} -$$
$$T_1^{-s1}\,T_2^{-s1}\,\chi_{i1==j0}\,\chi_{j0==i1^+}\,g_{2,i1,j0} - T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\,\chi_{j0==i1^+}\,\chi_{j1==j0}\,g_{2,i1,j0} +$$
$$T_1^{-s1}\,T_2^{-s1}\,\chi_{j0==i1^+}\,g_{1,i1,j0}\,g_{2,i1,j0} + T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\,\chi_{j0==i1^+}\,g_{1,j1,j0}\,g_{2,i1,j0} -$$
$$T_1^{-s1}\,T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{i1==j0}\,\chi_{j0==i1^+}\,g_{2,j1,j0} - T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{j0==i1^+}\,\chi_{j1==j0}\,g_{2,j1,j0} -$$
$$\chi_{j0==j1^+}\,\chi_{j1==j0}\,g_{2,j1,j0} + T_1^{-s1}\,T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{j0==i1^+}\,g_{1,i1,j0}\,g_{2,j1,j0} +$$
$$T_1^{-s1}\left(-1+T_1^{s1}\right)T_2^{-s1}\left(-1+T_2^{s1}\right)\chi_{j0==i1^+}\,g_{1,j1,j0}\,g_{2,j1,j0} + \chi_{j0==j1^+}\,g_{1,j1,j0}\,g_{2,j1,j0} +$$
$$\chi^2_{i1==j0}\,g_{3,j0,i1} + \left(-2+T_1^{s1}+T_2^{s1}\right)\chi_{i1==j0}\,\chi_{j1==j0}\,g_{3,j0,i1} + \left(2-T_1^{s1}-T_2^{s1}\right)\chi^2_{j1==j0}\,g_{3,j0,i1} -$$
$$\chi_{i1==j0}\,g_{1,i1,j0}\,g_{3,j0,i1} + \left(1-T_2^{s1}\right)\chi_{j1==j0}\,g_{1,i1,j0}\,g_{3,j0,i1} + \left(1-T_1^{s1}\right)\chi_{i1==j0}\,g_{1,j1,j0}\,g_{3,j0,i1} +$$
$$\left(-2+T_1^{s1}+T_2^{s1}\right)\chi_{j1==j0}\,g_{1,j1,j0}\,g_{3,j0,i1} - \chi_{i1==j0}\,g_{2,i1,j0}\,g_{3,j0,i1} + \left(1-T_1^{s1}\right)\chi_{j1==j0}\,g_{2,i1,j0}\,g_{3,j0,i1} +$$
$$\left(-1+T_1^{s1}\right)g_{1,j1,j0}\,g_{2,i1,j0}\,g_{3,j0,i1} + \left(1-T_2^{s1}\right)\chi_{i1==j0}\,g_{2,j1,j0}\,g_{3,j0,i1} +$$
$$\left(-2+T_1^{s1}+T_2^{s1}\right)\chi_{j1==j0}\,g_{2,j1,j0}\,g_{3,j0,i1} + \left(-1+T_2^{s1}\right)g_{1,i1,j0}\,g_{2,j1,j0}\,g_{3,j0,i1} +$$
$$\left(2-T_1^{s1}-T_2^{s1}\right)g_{1,j1,j0}\,g_{2,j1,j0}\,g_{3,j0,i1} + \chi^2_{j1==j0}\,g_{3,j0,j1} - \chi_{j1==j0}\,g_{1,j1,j0}\,g_{3,j0,j1} - \chi_{j1==j0}\,g_{2,j1,j0}\,g_{3,j0,j1}$$