

Pensieve header: The PBW multiplication tensor for \mathfrak{sl}_n using the λ -tangent formalism. Some material from pensieve://Projects/UEA.

```
In[*]:= SetDirectory@"C:\\drorbn\\AcademicPensieve\\Projects\\SolvablePBW";
<< KnotTheory`
```

Loading KnotTheory` version of October 29, 2024, 10:29:52.1301.
Read more at <http://katlas.org/wiki/KnotTheory>.

Prolog

```
In[*]:= BeginPackage["UEA`"];
Print["UEA` does computations in general universal enveloping
algebras and PBW algebras. It is in the public domain, available
at http://drorbn.net/AcademicPensieve/Projects/SolvablePBW/.
Dror Bar-Natan is committed to support it within
reason until June 1, 2027. This is version 260601."];
Print["UEA` implements / extends ",
Sort@{"**",  $\epsilon$ , $k, CF, SSQ, NilQ, B, m, SetAlgebra, U,
UB, UProducts, USimp, UU, $Basis, $PBWRule,  $\delta$ , adPower, adExp},
"."];
Begin["`Private`"];
```

UEA` does computations in general universal enveloping algebras and PBW algebras. It is in the public domain, available at <http://drorbn.net/AcademicPensieve/Projects/SolvablePBW/>. Dror Bar-Natan is committed to support it within reason until June 1, 2027. This is version 260601.

UEA` implements / extends {**, adExp, adPower, B, CF, m, NilQ, SetAlgebra, SSQ, U, UB, UProducts, USimp, UU, δ , ϵ , \$Basis, \$k, \$PBWRule}.

Utilities

```
In[*]:=  $\delta_{i,j} := \text{If}[i == j, 1, 0];$ 
```

```
In[*]:= SSQ[c_.*x_] /; MemberQ[$Basis, x] :=  $\neg$  NilQ[x]; (* Semi-Simple Q *)
NilQ[c_.*x_] /; MemberQ[$Basis, x] := NilQ[x];
SSQ[x_Plus] := And @@ (SSQ /@ (List @@ x));
NilQ[x_Plus] := And @@ (NilQ /@ (List @@ x));
```

```
In[*]:= $k = 1;
```

```
In[*]:= CF[ $\mathcal{E}_-$ ] := Expand[ $\mathcal{E} / . e^L \rightarrow e^{\text{Expand}[L]}$ ] /. { $e^{k \cdot} /; k > $k \rightarrow 0$ };
```

Implementing general universal enveloping algebras

```
In[*]:= B[0, _] = 0; B[_ , 0] = 0;
B[c_*x_, y_] /; MemberQ[$Basis, x] := CF[c B[x, y]];
B[y_, c_*x_] /; MemberQ[$Basis, x] := CF[c B[y, x]];
B[x_Plus, y_] := B[#, y] & /@ x;
B[x_, y_Plus] := B[x, #] & /@ y;
B[x_, x_] = 0;
B[y_, x_] := CF[-B[x, y]];
```

```
In[*]:= x_ ≤ y_ := OrderedQ[{x, y} /. $PBWRule]; x_ < y_ := ! OrderedQ[{y, x} /. $PBWRule];
UU_i_[1] := U_i[];
UU_i_[x^p_] := UU_i_@@Table[x, {p}];
UU_i_[ε_] := ε /. {
  U[xs__] => U_i[xs],
  x_ /; MemberQ[$Basis, x] => U_i[x]
};
UU_i_[x_, xs__] := UU_t1[x] UU_t2[xs] // Expand // m_{t1,t2->i};
USimp[ε_] := Collect[ε, Times[U_[] ..], Expand];
USimp[ε_] := Expand[ε];
```

```
In[*]:= m_s_[0] = 0;
m_s_[x_Plus] := m_s_ /@ x;
m_s_[sd_SeriesData] := MapAt[m_s, sd, {3, All}];
m_{i->j}[ε_] := ε /. U_i -> U_j;
```

```
In[*]:= m_{i,j->k}[c_. U_i[x__] U_j[]] := c U_k[x];
m_{i,j->k}[c_. U_i[] U_j[y__]] := c U_k[y];
m_{i,j->k}[c_. U_i[xx__, x_] U_j[y_, yy__]] := If[x ≤ y,
  c U_k[xx, x, y, yy],
  ((U_i[xx] (U_j[y, x] + UU_j[B[x, y]]) // Expand // m_{i,j->i}) U_j[yy] // Expand // m_{i,j->k})
  c // USimp
];
```

```
In[*]:= UProducts[{}, 0] = {1}; UProducts[{}, d_Integer] /; d > 0 = {};
UProducts[{i_, is__}, d_Integer] := Sort@
  Flatten@Table[(U_i@@@Subsets[$Basis, {j}]) u, {j, 0, d}, {u, UProducts[is, d-j]}];
```

```
In[*]:= Supp[ε_] := Union@Cases[ε, U_i[___] => i, ∞];
```

```
In[*]:= Unprotect[NonCommutativeMultiply];
NonCommutativeMultiply[x_] := x;
x_ ** y_ := Module[{is = Supp[x] ∩ Supp[y], σ, z},
  z = x; Do[z = mi→σi[z], {i, is}];
  z = Expand[y z]; Do[z = mσi→i[z], {i, is}]; z];
UB[x_, y_] := USimp[x ** y - y ** x];
```

Epilog

```
In[*]:= End[]; EndPackage[];
```

Predefined Algebras

sl(2)

```
In[*]:= Print["UEA`SetAlgebra knows \"sl2\"."];
```

UEA`SetAlgebra knows "sl2".

```
In[*]:= SetAlgebra["sl2"] := (
  Print["In sl2: ⟨e,h,f⟩ / ([h,e]=2e, [h,f]=-2f, [e,f]=h)."];
  B[h, e] = 2 e; B[h, f] = -2 f; B[e, f] = h;
  $Algebra = "sl2";
  $Basis = {e, h, f};
  $PBWRule = {e → 1, h → 2, f → 3};
  NilQ[e] = NilQ[f] = True; NilQ[h] = False;
);
```

$gl_{n,\epsilon}$

```
In[*]:= Print["UEA`SetAlgebra knows the ε-nilpotent algebra gln,ε."];
```

UEA`SetAlgebra knows the ϵ -nilpotent algebra $gl_{n,\epsilon}$.

```
In[ ]:= SetAlgebra[gln,ε] := (
  $Algebra = gln,ε;
  $Basis = Flatten@{
    Table[yα,β, {β, 2, n}, {α, 1, β - 1}],
    Table[xα,β, {β, 1, n}, {α, 1, β}]
  };
  NilQ[y_] = True; NilQ[xα,β] := (α != β); (* Nilpotent Q *)
  $PBWRule = Thread[$Basis → Range@Length@$Basis];
  B[xi,j, xk,l] := δj,k xi,l - δl,i xk,j;
  B[yi,j, yk,l] := CF[ε δj,k yi,l - ε δl,i yk,j];
  B[xi,j, yk,l] := CF[δj,k xi,l - δl,i xk,j / . xα,β → If[α ≤ β, ε xα,β, yβ,α]];
);
```

```
In[ ]:= SetAlgebra[gl2,ε];
$PBWRule
MatrixForm@Table[{b1, b2} → B[b1, b2], {b1, $Basis}, {b2, $Basis}]
```

Out[]:= {y_{1,2} → 1, x_{1,1} → 2, x_{1,2} → 3, x_{2,2} → 4}

Out[]//MatrixForm=

{y _{1,2} , y _{1,2} } → 0	{y _{1,2} , x _{1,1} } → y _{1,2}	{y _{1,2} , x _{1,2} } → -ε x _{1,1} + ε x _{2,2}	{y _{1,2} , x _{2,2} } → -y _{1,2}
{x _{1,1} , y _{1,2} } → -y _{1,2}	{x _{1,1} , x _{1,1} } → 0	{x _{1,1} , x _{1,2} } → x _{1,2}	{x _{1,1} , x _{2,2} } → 0
{x _{1,2} , y _{1,2} } → ε x _{1,1} - ε x _{2,2}	{x _{1,2} , x _{1,1} } → -x _{1,2}	{x _{1,2} , x _{1,2} } → 0	{x _{1,2} , x _{2,2} } → x _{1,2}
{x _{2,2} , y _{1,2} } → y _{1,2}	{x _{2,2} , x _{1,1} } → 0	{x _{2,2} , x _{1,2} } → -x _{1,2}	{x _{2,2} , x _{2,2} } → 0

```
In[ ]:= adPower[x_, k_, Env_] [c * y_] /; MemberQ[$Basis, y] := CF[c adPower[x, k, Env] [y]];
adPower[x_, k_, Env_] [y_Plus] := adPower[x, k, Env] /@ y;
adPower[_, 0, Env_] [y_] := y;
adPower[x_, k_, Env_] [0] = 0;
adPower[x_, k_, Env_] [y_] /; MemberQ[$Basis, y] :=
  adPower[x, k, Env] [y] = B[adPower[x, k - 1, Env] [y], x];
adPower[x_, k_] [y_] := adPower[x, k, {$Algebra, $k}] [y];
```

In[]:= x0 = (\$Basis / . {x → ξ0, y → η0}).\$Basis
 x1 = (\$Basis / . {x → ξ1, y → η1}).\$Basis

Out[]:= y_{1,2} η_{0,2} + x_{1,1} ξ_{0,1} + x_{1,2} ξ_{0,2} + x_{2,2} ξ_{0,2}

Out[]:= y_{1,2} η_{1,2} + x_{1,1} ξ_{1,1} + x_{1,2} ξ_{1,2} + x_{2,2} ξ_{1,2}

```
In[ ]:= adExp[x_] [y_] /; SSQ[x] := Expand@Module[{A, b, c},
  A = Table[Coefficient[B[b, x], c], {c, $Basis}, {b, $Basis}];
  CF[$Basis.Normal[Series[MatrixExp[A], {ε, 0, $k}]]].
  Table[Coefficient[y, b], {b, $Basis}]
]
```

In[*]:= **SSQ**[$\xi_{0,1}$ $x_{1,1}$]

Out[*]=
True

In[*]:= **adExp**[$\xi_{0,1}$ $x_{1,1}$] [**x1**]

Out[*]=
 $e^{\xi_{0,1}} y_{1,2} \eta_{1,2} + x_{1,1} \xi_{1,1} + e^{-\xi_{0,1}} x_{1,2} \xi_{1,2} + x_{2,2} \xi_{1,2}$

```
In[*]:= adExp[x_] [y_] /; NilQ[x] := Expand@Module [{k = 0, s = 0},
  While[adPower[x, k] [y] != 0,
    s += adPower[x, k] [y] / k!;
    ++k
  ];
  s
]
```

In[*]:= **adExp**[$\xi_{0,2}$ $x_{1,2}$] [**x1**]

Out[*]=
 $y_{1,2} \eta_{1,2} - \epsilon x_{1,1} \eta_{1,2} \xi_{0,2} + \epsilon x_{2,2} \eta_{1,2} \xi_{0,2} - \epsilon x_{1,2} \eta_{1,2} \xi_{0,2}^2 +$
 $x_{1,1} \xi_{1,1} + x_{1,2} \xi_{0,2} \xi_{1,1} + x_{1,2} \xi_{1,2} + x_{2,2} \xi_{1,2} - x_{1,2} \xi_{0,2} \xi_{1,2}$

```
In[*]:= λTangent[ ] = 0;
λTangent[xs____, x_] := adExp[x] [λTangent[xs]] +  $\partial_{\lambda}$  x;
λTangent[xs_List] := λTangent @@ xs
```

In[*]:= **\$Basis** (**\$Basis** /. {**x** → ξ , **y** → η })

Out[*]=
{ $y_{1,2} \eta_{1,2}$, $x_{1,1} \xi_{1,1}$, $x_{1,2} \xi_{1,2}$, $x_{2,2} \xi_{2,2}$ }

In[*]:= **λTangent**@(λ **\$Basis** (**\$Basis** /. {**x** → ξ , **y** → η }))

Out[*]=
 $e^{\lambda \xi_{1,1} - \lambda \xi_{2,2}} y_{1,2} \eta_{1,2} + x_{1,1} \xi_{1,1} + e^{\lambda \xi_{2,2}} x_{1,2} \xi_{1,2} - e^{\lambda \xi_{1,1}} \epsilon \lambda x_{1,1} \eta_{1,2} \xi_{1,2} +$
 $e^{\lambda \xi_{1,1}} \epsilon \lambda x_{2,2} \eta_{1,2} \xi_{1,2} + e^{\lambda \xi_{2,2}} \lambda x_{1,2} \xi_{1,1} \xi_{1,2} - e^{\lambda \xi_{1,1} + \lambda \xi_{2,2}} \epsilon \lambda^2 x_{1,2} \eta_{1,2} \xi_{1,2}^2 + x_{2,2} \xi_{2,2}$

In[*]:= **\$Basis**

Out[*]=
{ $y_{1,2}$, $x_{1,1}$, $x_{1,2}$, $x_{2,2}$ }

In[*]:= **Lhs = λ Tangent [**

Join [λ \$Basis (\$Basis /. {x \rightarrow ξ_1 , y \rightarrow η_1 }), λ \$Basis (\$Basis /. {x \rightarrow ξ_2 , y \rightarrow η_2 })]

Out[*]=

$$\begin{aligned} & e^{\lambda \xi_{1,1} - \lambda \xi_{1,2} + \lambda \xi_{2,1} - \lambda \xi_{2,2}} y_{1,2} \eta_{1,2} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} y_{1,2} \eta_{2,2} + x_{1,1} \xi_{1,1} - \\ & e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \lambda y_{1,2} \eta_{2,2} \xi_{1,1} + e^{\lambda \xi_{1,2} - \lambda \xi_{2,1} + \lambda \xi_{2,2}} x_{1,2} \xi_{1,2} - e^{\lambda \xi_{1,1}} \in \lambda x_{1,1} \eta_{1,2} \xi_{1,2} + \\ & e^{\lambda \xi_{1,1}} \in \lambda x_{2,2} \eta_{1,2} \xi_{1,2} + e^{\lambda \xi_{1,2}} \in \lambda x_{1,1} \eta_{2,2} \xi_{1,2} - e^{\lambda \xi_{1,2}} \in \lambda x_{2,2} \eta_{2,2} \xi_{1,2} + \\ & 2 e^{\lambda \xi_{1,1} + \lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^2 y_{1,2} \eta_{1,2} \eta_{2,2} \xi_{1,2} - e^{\lambda \xi_{1,2} + \lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^2 y_{1,2} \eta_{2,2}^2 \xi_{1,2} + \\ & e^{\lambda \xi_{1,2} - \lambda \xi_{2,1} + \lambda \xi_{2,2}} \lambda x_{1,2} \xi_{1,1} \xi_{1,2} + e^{\lambda \xi_{1,2}} \in \lambda^2 x_{1,1} \eta_{2,2} \xi_{1,1} \xi_{1,2} - e^{\lambda \xi_{1,2}} \in \lambda^2 x_{2,2} \eta_{2,2} \xi_{1,1} \xi_{1,2} - \\ & e^{\lambda \xi_{1,2} + \lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^3 y_{1,2} \eta_{2,2}^2 \xi_{1,1} \xi_{1,2} - e^{\lambda \xi_{1,1} + \lambda \xi_{1,2} - \lambda \xi_{2,1} + \lambda \xi_{2,2}} \in \lambda^2 x_{1,2} \eta_{1,2} \xi_{1,2}^2 + x_{2,2} \xi_{1,2} + \\ & e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \lambda y_{1,2} \eta_{2,2} \xi_{1,2} + x_{1,1} \xi_{2,1} + e^{\lambda \xi_{2,2}} x_{1,2} \xi_{2,2} - e^{\lambda \xi_{1,1} - \lambda \xi_{1,2} + \lambda \xi_{2,1}} \in \lambda x_{1,1} \eta_{1,2} \xi_{2,2} + \\ & e^{\lambda \xi_{1,1} - \lambda \xi_{1,2} + \lambda \xi_{2,1}} \in \lambda x_{2,2} \eta_{1,2} \xi_{2,2} - e^{\lambda \xi_{2,1}} \in \lambda x_{1,1} \eta_{2,2} \xi_{2,2} + e^{\lambda \xi_{2,1}} \in \lambda x_{2,2} \eta_{2,2} \xi_{2,2} + \\ & e^{\lambda \xi_{2,2}} \lambda x_{1,2} \xi_{1,1} \xi_{2,2} + e^{\lambda \xi_{2,1}} \in \lambda^2 x_{1,1} \eta_{2,2} \xi_{1,1} \xi_{2,2} - e^{\lambda \xi_{2,1}} \in \lambda^2 x_{2,2} \eta_{2,2} \xi_{1,1} \xi_{2,2} - \\ & 2 e^{\lambda \xi_{1,1} + \lambda \xi_{2,2}} \in \lambda^2 x_{1,2} \eta_{1,2} \xi_{1,2} \xi_{2,2} + 2 e^{\lambda \xi_{1,2} + \lambda \xi_{2,2}} \in \lambda^2 x_{1,2} \eta_{2,2} \xi_{1,2} \xi_{2,2} + \\ & 2 e^{\lambda \xi_{1,2} + \lambda \xi_{2,2}} \in \lambda^3 x_{1,2} \eta_{2,2} \xi_{1,1} \xi_{1,2} \xi_{2,2} - e^{\lambda \xi_{2,2}} \lambda x_{1,2} \xi_{1,2} \xi_{2,2} - \\ & e^{\lambda \xi_{2,1}} \in \lambda^2 x_{1,1} \eta_{2,2} \xi_{1,2} \xi_{2,2} + e^{\lambda \xi_{2,1}} \in \lambda^2 x_{2,2} \eta_{2,2} \xi_{1,2} \xi_{2,2} + e^{\lambda \xi_{2,2}} \lambda x_{1,2} \xi_{2,1} \xi_{2,2} - \\ & e^{\lambda \xi_{1,1} - \lambda \xi_{1,2} + \lambda \xi_{2,1} + \lambda \xi_{2,2}} \in \lambda^2 x_{1,2} \eta_{1,2} \xi_{2,2}^2 - e^{\lambda \xi_{2,1} + \lambda \xi_{2,2}} \in \lambda^2 x_{1,2} \eta_{2,2} \xi_{2,2}^2 + \\ & e^{\lambda \xi_{2,1} + \lambda \xi_{2,2}} \in \lambda^3 x_{1,2} \eta_{2,2} \xi_{1,1} \xi_{2,2}^2 - e^{\lambda \xi_{2,1} + \lambda \xi_{2,2}} \in \lambda^3 x_{1,2} \eta_{2,2} \xi_{1,2} \xi_{2,2}^2 + x_{2,2} \xi_{2,2} \end{aligned}$$

In[*]:= **rhs = λ Tangent@ (\$Basis (\$Basis /. {x _{α} , α \rightarrow CF[f _{α} , α][λ], x _{$\alpha\beta$} \rightarrow f _{$\alpha\beta$} [λ], y _{$\alpha\beta$} \rightarrow g _{$\alpha\beta$} [λ])}**

Out[*]=

$$\begin{aligned} & x_{1,1} f_{1,1}'[\lambda] + e^{f_{2,2}[\lambda]} x_{1,2} f_{1,2}[\lambda] f_{1,1}'[\lambda] + e^{f_{2,2}[\lambda]} x_{1,2} f_{1,2}'[\lambda] + \\ & x_{2,2} f_{2,2}'[\lambda] + e^{f_{1,1}[\lambda] - f_{2,2}[\lambda]} y_{1,2} g_{1,2}'[\lambda] - e^{f_{1,1}[\lambda]} \in x_{1,1} f_{1,2}[\lambda] g_{1,2}'[\lambda] + \\ & e^{f_{1,1}[\lambda]} \in x_{2,2} f_{1,2}[\lambda] g_{1,2}'[\lambda] - e^{f_{1,1}[\lambda] + f_{2,2}[\lambda]} \in x_{1,2} f_{1,2}[\lambda]^2 g_{1,2}'[\lambda] \end{aligned}$$

In[*]:= **SSBasis = Select[\$Basis, SSQ]**

Out[*]=

$$\{x_{1,1}, x_{2,2}\}$$

In[*]:= eqns = Simplify[(Coefficient[lhs - rhs, #] == 0) & /@ \$Basis]

Out[*]=

$$\begin{aligned} & \left\{ e^{\lambda (\xi_{1,1} - \xi_{1,2} + \xi_{2,1} - \xi_{2,2})} \eta_{1,2} \left(1 + 2 e^{\lambda \xi_{1,2}} \in \lambda^2 \eta_{2,2} \xi_{1,2} \right) = \right. \\ & \quad e^{\lambda (\xi_{1,2} + \xi_{2,1} - \xi_{2,2})} \in \lambda^2 \eta_{2,2}^2 \left(1 + \lambda \xi_{1,1} \right) \xi_{1,2} + \\ & \quad e^{\lambda (\xi_{2,1} - \xi_{2,2})} \eta_{2,2} \left(-1 + \lambda \xi_{1,1} - \lambda \xi_{1,2} \right) + e^{f_{1,1}[\lambda] - f_{2,2}[\lambda]} g_{1,2}'[\lambda], \\ & \quad \xi_{2,1} + \xi_{1,1} \left(1 + e^{\lambda^2 \eta_{2,2}} \left(e^{\lambda \xi_{1,2}} \xi_{1,2} + e^{\lambda \xi_{2,1}} \xi_{2,2} \right) \right) + e^{f_{1,1}[\lambda]} \in f_{1,2}[\lambda] g_{1,2}'[\lambda] = \\ & \quad e^{\lambda (\xi_{1,1} - \xi_{1,2})} \in \lambda \eta_{1,2} \left(e^{\lambda \xi_{1,2}} \xi_{1,2} + e^{\lambda \xi_{2,1}} \xi_{2,2} \right) + \\ & \quad \in \lambda \eta_{2,2} \left(-e^{\lambda \xi_{1,2}} \xi_{1,2} + e^{\lambda \xi_{2,1}} \left(1 + \lambda \xi_{1,2} \right) \xi_{2,2} \right) + f_{1,1}'[\lambda], \\ & \quad e^{-\lambda (\xi_{1,2} + \xi_{2,1})} \left(-e^{\lambda (\xi_{1,1} + 2 \xi_{1,2} + \xi_{2,2})} \in \lambda^2 \eta_{1,2} \xi_{1,2}^2 + e^{\lambda (\xi_{1,2} + \xi_{2,2})} \xi_{1,2} \left(-2 e^{\lambda (\xi_{1,1} + \xi_{2,1})} \in \lambda^2 \eta_{1,2} \right. \right. \\ & \quad \left. \left. \xi_{2,2} + e^{\lambda \xi_{1,2}} \left(1 + 2 e^{\lambda \xi_{2,1}} \in \lambda^2 \eta_{2,2} \xi_{2,2} \right) + e^{\lambda \xi_{1,2}} \lambda \xi_{1,1} \left(1 + 2 e^{\lambda \xi_{2,1}} \in \lambda^2 \eta_{2,2} \xi_{2,2} \right) \right) - \right. \\ & \quad \left. e^{\lambda \xi_{2,1}} \left(e^{\lambda (\xi_{1,2} + \xi_{2,2})} \left(-1 - \lambda \xi_{1,1} + \lambda \xi_{1,2} - \lambda \xi_{2,1} \right) \xi_{2,2} + \right. \right. \\ & \quad \left. \left. e^{\lambda (\xi_{2,1} + \xi_{2,2})} \in \lambda^2 \left(e^{\lambda \xi_{1,1}} \eta_{1,2} + e^{\lambda \xi_{1,2}} \eta_{2,2} \left(1 - \lambda \xi_{1,1} + \lambda \xi_{1,2} \right) \right) \xi_{2,2}^2 + \right. \right. \\ & \quad \left. \left. e^{\lambda \xi_{1,2} + f_{2,2}[\lambda]} \left(f_{1,2}[\lambda] f_{1,1}'[\lambda] + f_{1,2}'[\lambda] - e^{f_{1,1}[\lambda]} \in f_{1,2}[\lambda]^2 g_{1,2}'[\lambda] \right) \right) \right) = 0, \\ & \quad \xi_{1,2} + e^{\lambda \eta_{1,2}} \left(e^{\lambda \xi_{1,1}} \xi_{1,2} + e^{\lambda (\xi_{1,1} - \xi_{1,2} + \xi_{2,1})} \xi_{2,2} \right) + \xi_{2,2} = \\ & \quad \in \lambda \eta_{2,2} \left(e^{\lambda \xi_{1,2}} \left(1 + \lambda \xi_{1,1} \right) \xi_{1,2} + e^{\lambda \xi_{2,1}} \left(-1 + \lambda \xi_{1,1} - \lambda \xi_{1,2} \right) \xi_{2,2} \right) + \\ & \quad \left. f_{2,2}'[\lambda] + e^{f_{1,1}[\lambda]} \in f_{1,2}[\lambda] g_{1,2}'[\lambda] \right\} \end{aligned}$$

In[*]:= Length@eqns

Out[*]=

4

In[*]:= unknowns = \$Basis /. {x_{αβ} → f_{αβ}[λ], y_{αβ} → g_{αβ}[λ]}

Out[*]=

{g_{1,2}[λ], f_{1,1}[λ], f_{1,2}[λ], f_{2,2}[λ]}

In[*]:= \$Basis /. {x_{αβ} → f_{αβ}[0], y_{αβ} → g_{αβ}[0]}

Out[*]=

{g_{1,2}[0] == 0, f_{1,1}[0] == 0, f_{1,2}[0] == 0, f_{2,2}[0] == 0}

In[*]:= {sol0} = Block[{\$k = 0}, Simplify@DSolve[

lhs = λTangent[

Join[λ \$Basis (\$Basis /. {x → ξ₁, y → η₁}), λ \$Basis (\$Basis /. {x → ξ₂, y → η₂})]

];

rhs = λTangent@(\$Basis (\$Basis /. {x_α, y_α → CF[f_{α,α}[λ]], x_{αβ} → f_{αβ}[λ], y_{αβ} → g_{αβ}[λ]}));

Join[

(Coefficient[CF[lhs - rhs], #] == 0) & /@ \$Basis,

\$Basis /. {x_{αβ} → f_{αβ}[0], y_{αβ} → g_{αβ}[0]}]

],

\$Basis /. {x_{αβ} → f_{αβ}[λ], y_{αβ} → g_{αβ}[λ]},

λ

]]

Out[*]=

{ {f_{1,1}[λ] → λ (ξ_{1,1} + ξ_{2,1}), f_{2,2}[λ] → λ (ξ_{1,2} + ξ_{2,2}),
f_{1,2}[λ] → e^{-λ ξ_{2,1}} λ ξ_{1,2} + e^{-λ ξ_{1,2}} λ ξ_{2,2}, g_{1,2}[λ] → λ (η_{1,2} + e^{λ (-ξ_{1,1} + ξ_{1,2})} η_{2,2}) } }

```

In[*]:= {sol1} = Block[{k = 1}, Simplify@DSolve[
  lhs = λTangent[
    Join[λ $Basis ($Basis /. {x → ξ1, y → η1}), λ $Basis ($Basis /. {x → ξ2, y → η2})]
  ];
  rhs = λTangent[
    $Basis ($Basis /. {xα,α ⇒ CF[fα,α[λ]], xαβ ⇒ fαβ[λ], yαβ ⇒ gαβ[λ]}) /.
    (sol0 /. ((a → b) ⇒ (a → e a + b)))
  ];
  Join[
    (Coefficient[CF[lhs - rhs], #] == 0) & /@ $Basis,
    $Basis /. {xαβ ⇒ fαβ[0] == 0, yαβ ⇒ gαβ[0] == 0}
  ],
  $Basis /. {xαβ ⇒ fαβ[λ], yαβ ⇒ gαβ[λ]},
  λ
]]

```

```

Out[*]= { {f1,1[λ] → eλ ξ1,2 λ2 η2,2 ξ1,2, f2,2[λ] → -eλ ξ1,2 λ2 η2,2 ξ1,2,
  f1,2[λ] → λ3 η2,2 ξ1,2 ξ2,2, g1,2[λ] → -e-λ ξ1,1 + 2 λ ξ1,2 λ3 η2,22 ξ1,2 } }

```

```

In[*]:= TriangularDSolve[eqns_, funs_, iv_] := Module[{sol = {}, e, fun, der},
  MapThread[
    {e, fun} ↦ sol = Echo@Union[sol,
      MapAt[CF, First@DSolve[Echo@CF[e /. Derivative[1][f_][v_] ⇒ der[f[v], v] /. sol /.
        der[f_, v_] ⇒ ∂vf], fun, iv], {1, 2}]
    ],
  {eqns, funs}
];
sol
]

```

```

In[*]:= SetAlgebra[gl3,ε];
      $k = 1
      $PBWRule
      MatrixForm@Table[{b1, b2} → B[b1, b2], {b1, $Basis}, {b2, $Basis}]

Out[*]=
1

Out[*]=
{y1,2 → 1, y1,3 → 2, y2,3 → 3, x1,1 → 4, x1,2 → 5, x2,2 → 6, x1,3 → 7, x2,3 → 8, x3,3 → 9}

Out[*]//MatrixForm=

$$\left( \begin{array}{cccc}
\{y_{1,2}, y_{1,2}\} \rightarrow 0 & \{y_{1,2}, y_{1,3}\} \rightarrow 0 & \{y_{1,2}, y_{2,3}\} \rightarrow \in y_{1,3} & \{y_{1,2}, x_{1,1}\} \rightarrow y_{1,2} \\
\{y_{1,3}, y_{1,2}\} \rightarrow 0 & \{y_{1,3}, y_{1,3}\} \rightarrow 0 & \{y_{1,3}, y_{2,3}\} \rightarrow 0 & \{y_{1,3}, x_{1,1}\} \rightarrow y_{1,3} \\
\{y_{2,3}, y_{1,2}\} \rightarrow -\in y_{1,3} & \{y_{2,3}, y_{1,3}\} \rightarrow 0 & \{y_{2,3}, y_{2,3}\} \rightarrow 0 & \{y_{2,3}, x_{1,1}\} \rightarrow y_{2,3} \\
\{x_{1,1}, y_{1,2}\} \rightarrow -y_{1,2} & \{x_{1,1}, y_{1,3}\} \rightarrow -y_{1,3} & \{x_{1,1}, y_{2,3}\} \rightarrow 0 & \{x_{1,1}, x_{1,1}\} \rightarrow x_{1,1} \\
\{x_{1,2}, y_{1,2}\} \rightarrow \in x_{1,1} - \in x_{2,2} & \{x_{1,2}, y_{1,3}\} \rightarrow -y_{2,3} & \{x_{1,2}, y_{2,3}\} \rightarrow 0 & \{x_{1,2}, x_{1,1}\} \rightarrow -x_{1,2} \\
\{x_{2,2}, y_{1,2}\} \rightarrow y_{1,2} & \{x_{2,2}, y_{1,3}\} \rightarrow 0 & \{x_{2,2}, y_{2,3}\} \rightarrow -y_{2,3} & \{x_{2,2}, x_{1,1}\} \rightarrow -x_{2,2} \\
\{x_{1,3}, y_{1,2}\} \rightarrow -\in x_{2,3} & \{x_{1,3}, y_{1,3}\} \rightarrow \in x_{1,1} - \in x_{3,3} & \{x_{1,3}, y_{2,3}\} \rightarrow \in x_{1,2} & \{x_{1,3}, x_{1,1}\} \rightarrow -x_{1,3} \\
\{x_{2,3}, y_{1,2}\} \rightarrow 0 & \{x_{2,3}, y_{1,3}\} \rightarrow y_{1,2} & \{x_{2,3}, y_{2,3}\} \rightarrow \in x_{2,2} - \in x_{3,3} & \{x_{2,3}, x_{1,1}\} \rightarrow -x_{2,3} \\
\{x_{3,3}, y_{1,2}\} \rightarrow 0 & \{x_{3,3}, y_{1,3}\} \rightarrow y_{1,3} & \{x_{3,3}, y_{2,3}\} \rightarrow y_{2,3} & \{x_{3,3}, x_{1,1}\} \rightarrow -x_{3,3}
\end{array} \right)$$


In[*]:= $Basis
Out[*]=
{y1,2, x1,1, x1,2, x2,2}

In[*]:= SortBy[$Basis, If[#[[1]] === x, {0, #[[3]] - #[[2]]}, {1, #[[2]] - #[[3]]}] &]
Out[*]=
{x1,1, x2,2, x1,2, y1,2}

```

```

In[*]:= SBasis = SortBy[$Basis, If[#[[1]] === x, {0, #[[3]] - #[[2]], {1, #[[2]] - #[[3]]}] &];
sol[-1] = 0 SBasis;
Do[
  Block[{$k = k},
    s0 = Thread[SBasis → sol[k - 1]];
    lhs = λTangent[
      Join[λ SBasis (SBasis /. {x → ξ1, y → η1}), λ SBasis (SBasis /. {x → ξ2, y → η2})]
    ];
    rhs = λTangent[
      SBasis (SBasis /. {xαβ → (xαβ /. s0) + εk fαβ[λ], yαβ → (yαβ /. s0) + εk gαβ[λ]})
    ];
    Echo@"computed lhs and rhs";
    eqns = EchoLabel["eqns"]@Transpose@{
      (Coefficient[CF[lhs - rhs], #] == 0) & /@ SBasis,
      SBasis /. {xαβ → fαβ[0] == 0, yαβ → gαβ[0] == 0}
    };
    unknowns = SBasis /. {xαβ → fαβ[λ], yαβ → gαβ[λ]};
    s = TriangularDSolve[eqns, unknowns, λ];
    sol[k] = CF[sol[k - 1] + εk (SBasis /. {xαβ → fαβ[λ], yαβ → gαβ[λ]} /. s)]
  ],
  {k, 0, $k}
];
sol[$k].SBasis
» computed lhs and rhs

```

- » eqns $\{ \{ \xi_{1,1} + \xi_{2,1} - f_{1,1}'[\lambda] = 0, f_{1,1}[0] = 0 \},$
 $\{ \xi_{1,2} + \xi_{2,2} - f_{2,2}'[\lambda] = 0, f_{2,2}[0] = 0 \}, \{ \xi_{1,3} + \xi_{2,3} - f_{3,3}'[\lambda] = 0, f_{3,3}[0] = 0 \},$
 $\{ e^{-\lambda \xi_{2,1} + \lambda \xi_{2,2}} \xi_{1,2} + e^{-\lambda \xi_{2,1} + \lambda \xi_{2,2}} \lambda \xi_{1,1} \xi_{1,2} - e^{-\lambda \xi_{2,1} + \lambda \xi_{2,2}} \lambda \xi_{1,2} \xi_{1,2} + \xi_{2,2} + \lambda \xi_{1,1} \xi_{2,2} - \lambda \xi_{1,2} \xi_{2,2} + \lambda \xi_{2,1} \xi_{2,2} - \lambda \xi_{2,2} \xi_{2,2} - f_{1,2}[\lambda] f_{1,1}'[\lambda] - f_{1,2}'[\lambda] + f_{1,2}[\lambda] f_{2,2}'[\lambda] = 0, f_{1,2}[0] = 0 \},$
 $\{ e^{-\lambda \xi_{2,2} + \lambda \xi_{2,3}} \xi_{1,2,3} + e^{-\lambda \xi_{2,2} + \lambda \xi_{2,3}} \lambda \xi_{1,2,2} \xi_{1,2,3} - e^{-\lambda \xi_{2,2} + \lambda \xi_{2,3}} \lambda \xi_{1,2,3} \xi_{1,3,3} + \xi_{2,2,3} + \lambda \xi_{1,2,2} \xi_{2,2,3} - \lambda \xi_{1,3,3} \xi_{2,2,3} + \lambda \xi_{2,2,2} \xi_{2,2,3} - \lambda \xi_{2,2,3} \xi_{2,3,3} - f_{2,3}[\lambda] f_{2,2}'[\lambda] - f_{2,3}'[\lambda] + f_{2,3}[\lambda] f_{3,3}'[\lambda] = 0,$
 $f_{2,3}[0] = 0 \}, \{ e^{-\lambda \xi_{2,1} + \lambda \xi_{2,3}} \xi_{1,1,3} + e^{-\lambda \xi_{2,1} + \lambda \xi_{2,3}} \lambda \xi_{1,1,1} \xi_{1,1,3} + e^{-\lambda \xi_{2,1} + \lambda \xi_{2,3}} \lambda \xi_{1,1,2} \xi_{1,2,3} + e^{-\lambda \xi_{2,1} + \lambda \xi_{2,3}} \lambda^2 \xi_{1,1,1} \xi_{1,1,2} \xi_{1,2,3} - e^{-\lambda \xi_{2,1} + \lambda \xi_{2,3}} \lambda^2 \xi_{1,1,2} \xi_{1,2,2} \xi_{1,2,3} - e^{-\lambda \xi_{2,1} + \lambda \xi_{2,3}} \lambda \xi_{1,1,3} \xi_{1,3,3} - e^{-\lambda \xi_{2,2} + \lambda \xi_{2,3}} \lambda \xi_{1,2,3} \xi_{2,1,2} - e^{-\lambda \xi_{2,2} + \lambda \xi_{2,3}} \lambda^2 \xi_{1,2,2} \xi_{1,2,3} \xi_{2,1,2} + e^{-\lambda \xi_{2,2} + \lambda \xi_{2,3}} \lambda^2 \xi_{1,2,3} \xi_{1,3,3} \xi_{2,1,2} + \xi_{2,1,3} + \lambda \xi_{1,1,1} \xi_{2,1,3} - \lambda \xi_{1,3,3} \xi_{2,1,3} + \lambda \xi_{2,1,1} \xi_{2,1,3} + e^{-\lambda \xi_{2,1} + \lambda \xi_{2,2}} \lambda \xi_{1,1,2} \xi_{2,2,3} + e^{-\lambda \xi_{2,1} + \lambda \xi_{2,2}} \lambda^2 \xi_{1,1,1} \xi_{1,1,2} \xi_{2,2,3} - e^{-\lambda \xi_{2,1} + \lambda \xi_{2,2}} \lambda^2 \xi_{1,1,2} \xi_{1,2,2} \xi_{2,2,3} + \lambda \xi_{2,1,2} \xi_{2,2,3} + \lambda^2 \xi_{1,1,1} \xi_{2,1,2} \xi_{2,2,3} - \lambda^2 \xi_{1,2,2} \xi_{2,1,2} \xi_{2,2,3} + \lambda^2 \xi_{2,1,1} \xi_{2,1,2} \xi_{2,2,3} - \lambda^2 \xi_{2,1,2} \xi_{2,2,2} \xi_{2,2,3} - \lambda \xi_{2,1,3} \xi_{2,3,3} - f_{1,3}[\lambda] f_{1,1}'[\lambda] - f_{1,2}[\lambda] f_{2,3}[\lambda] f_{1,1}'[\lambda] - f_{2,3}[\lambda] f_{1,2}'[\lambda] - f_{1,3}'[\lambda] + f_{1,2}[\lambda] f_{2,3}[\lambda] f_{2,2}'[\lambda] + f_{1,3}[\lambda] f_{3,3}'[\lambda] = 0, f_{1,3}[0] = 0 \},$
 $\{ e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \eta_{1,3} + \eta_{2,1,3} - e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda \eta_{1,3} \xi_{1,1,1} - \lambda \eta_{2,1,3} \xi_{1,1,1} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda \eta_{1,3} \xi_{1,3,3} + \lambda \eta_{2,1,3} \xi_{1,3,3} - \lambda \eta_{2,1,3} \xi_{2,1,1} + \lambda \eta_{2,1,3} \xi_{2,3,3} + g_{1,3}[\lambda] f_{1,1}'[\lambda] - g_{1,3}[\lambda] f_{3,3}'[\lambda] - g_{1,3}'[\lambda] = 0, g_{1,3}[0] = 0 \},$
 $\{ e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \eta_{1,2} + \eta_{2,1,2} - e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \lambda \eta_{1,2} \xi_{1,1,1} - \lambda \eta_{2,1,2} \xi_{1,1,1} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \lambda \eta_{1,2} \xi_{1,2,2} + \lambda \eta_{2,1,2} \xi_{1,2,2} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \lambda \eta_{1,3} \xi_{1,2,3} + e^{-\lambda \xi_{2,2} + \lambda \xi_{2,3}} \lambda \eta_{2,1,3} \xi_{1,2,3} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \lambda^2 \eta_{1,3} \xi_{1,2,2} \xi_{1,2,3} + e^{-\lambda \xi_{2,2} + \lambda \xi_{2,3}} \lambda^2 \eta_{2,1,3} \xi_{1,2,3} \xi_{1,3,3} - e^{-\lambda \xi_{2,2} + \lambda \xi_{2,3}} \lambda^2 \eta_{2,1,3} \xi_{1,2,3} \xi_{1,3,3} - \lambda \eta_{2,1,2} \xi_{2,1,1} + \lambda \eta_{2,1,2} \xi_{2,2,2} - e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda \eta_{1,3} \xi_{2,2,3} + \lambda \eta_{2,1,3} \xi_{2,2,3} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda^2 \eta_{1,3} \xi_{1,1,1} \xi_{2,2,3} + \lambda^2 \eta_{2,1,3} \xi_{1,2,2} \xi_{2,2,3} - e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda^2 \eta_{1,3} \xi_{1,3,3} \xi_{2,2,3} - \lambda^2 \eta_{2,1,3} \xi_{1,3,3} \xi_{2,2,3} + \lambda^2 \eta_{2,1,3} \xi_{2,2,2} \xi_{2,2,3} - \lambda^2 \eta_{2,1,3} \xi_{2,3,3} + g_{1,2}[\lambda] f_{1,1}'[\lambda] - g_{1,2}[\lambda] f_{2,2}'[\lambda] - f_{2,3}[\lambda] g_{1,3}[\lambda] f_{2,2}'[\lambda] - g_{1,3}[\lambda] f_{2,3}'[\lambda] + f_{2,3}[\lambda] g_{1,3}[\lambda] f_{3,3}'[\lambda] - g_{1,2}'[\lambda] = 0, g_{1,2}[0] = 0 \},$
 $\{ e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \eta_{1,2,3} + \eta_{2,2,3} - e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda \eta_{1,2,3} \xi_{1,1,2} - e^{-\lambda \xi_{2,1} + \lambda \xi_{2,2}} \lambda \eta_{2,1,3} \xi_{1,1,2} - e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda \eta_{1,2,3} \xi_{1,2,2} - \lambda \eta_{2,2,3} \xi_{1,2,2} + e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^2 \eta_{1,3} \xi_{1,1,2} \xi_{1,2,2} + e^{-\lambda \xi_{2,1} + \lambda \xi_{2,2}} \lambda^2 \eta_{2,1,3} \xi_{1,1,2} \xi_{1,2,2} + e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda \eta_{1,2,3} \xi_{1,3,3} + \lambda \eta_{2,2,3} \xi_{1,3,3} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda \eta_{1,3} \xi_{2,1,2} - \lambda \eta_{2,1,3} \xi_{2,1,2} - e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda^2 \eta_{1,3} \xi_{1,1,1} \xi_{2,1,2} - \lambda^2 \eta_{2,1,3} \xi_{1,1,1} \xi_{2,1,2} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda^2 \eta_{1,3} \xi_{1,3,3} \xi_{2,1,2} - \lambda^2 \eta_{2,1,3} \xi_{2,1,1} \xi_{2,1,2} - \lambda \eta_{2,2,3} \xi_{2,2,2} + \lambda^2 \eta_{2,1,3} \xi_{2,1,2} \xi_{2,2,2} + \lambda \eta_{2,2,3} \xi_{2,3,3} + f_{1,2}[\lambda] g_{1,3}[\lambda] f_{1,1}'[\lambda] + g_{1,3}[\lambda] f_{1,2}'[\lambda] - f_{1,2}[\lambda] g_{1,3}[\lambda] f_{2,2}'[\lambda] + g_{2,3}[\lambda] f_{2,2}'[\lambda] - g_{2,3}[\lambda] f_{3,3}'[\lambda] - g_{2,3}'[\lambda] = 0, g_{2,3}[0] = 0 \}$
- » $\{ \xi_{1,1} + \xi_{2,1} - f_{1,1}'[\lambda] = 0, f_{1,1}[0] = 0 \}$
- » $\{ f_{1,1}[\lambda] \rightarrow \lambda \xi_{1,1} + \lambda \xi_{2,1} \}$
- » $\{ \xi_{1,2} + \xi_{2,2} - f_{2,2}'[\lambda] = 0, f_{2,2}[0] = 0 \}$
- » $\{ f_{1,1}[\lambda] \rightarrow \lambda \xi_{1,1} + \lambda \xi_{2,1}, f_{2,2}[\lambda] \rightarrow \lambda \xi_{1,2} + \lambda \xi_{2,2} \}$
- » $\{ \xi_{1,3} + \xi_{2,3} - f_{3,3}'[\lambda] = 0, f_{3,3}[0] = 0 \}$
- » $\{ f_{1,1}[\lambda] \rightarrow \lambda \xi_{1,1} + \lambda \xi_{2,1}, f_{2,2}[\lambda] \rightarrow \lambda \xi_{1,2} + \lambda \xi_{2,2}, f_{3,3}[\lambda] \rightarrow \lambda \xi_{1,3} + \lambda \xi_{2,3} \}$
- » $\{ e^{-\lambda \xi_{2,1} + \lambda \xi_{2,2}} \xi_{1,2} + e^{-\lambda \xi_{2,1} + \lambda \xi_{2,2}} \lambda \xi_{1,1} \xi_{1,2} - e^{-\lambda \xi_{2,1} + \lambda \xi_{2,2}} \lambda \xi_{1,2} \xi_{1,2} + \xi_{2,2} + \lambda \xi_{1,1} \xi_{2,2} - \lambda \xi_{1,2} \xi_{2,2} - \xi_{1,1} f_{1,2}[\lambda] + \xi_{1,2} f_{1,2}[\lambda] - \xi_{2,1} f_{1,2}[\lambda] + \xi_{2,2} f_{1,2}[\lambda] - f_{1,2}'[\lambda] = 0, f_{1,2}[0] = 0 \}$
- » $\{ f_{1,1}[\lambda] \rightarrow \lambda \xi_{1,1} + \lambda \xi_{2,1}, f_{1,2}[\lambda] \rightarrow e^{-\lambda \xi_{2,1} + \lambda \xi_{2,2}} \lambda \xi_{1,2} + \lambda \xi_{2,1,2}, f_{2,2}[\lambda] \rightarrow \lambda \xi_{1,2} + \lambda \xi_{2,2,2}, f_{3,3}[\lambda] \rightarrow \lambda \xi_{1,3} + \lambda \xi_{2,3,3} \}$
- » $\{ e^{-\lambda \xi_{2,2} + \lambda \xi_{2,3}} \xi_{1,2,3} + e^{-\lambda \xi_{2,2} + \lambda \xi_{2,3}} \lambda \xi_{1,2,2} \xi_{1,2,3} - e^{-\lambda \xi_{2,2} + \lambda \xi_{2,3}} \lambda \xi_{1,2,3} \xi_{1,3,3} + \xi_{2,2,3} + \lambda \xi_{1,2,2} \xi_{2,2,3} - \lambda \xi_{1,3,3} \xi_{2,2,3} - \lambda \xi_{2,2,2} \xi_{2,2,3} - \lambda \xi_{2,2,3} \xi_{2,3,3} - \xi_{1,2,2} f_{2,3}[\lambda] + \xi_{1,3,3} f_{2,3}[\lambda] - \xi_{2,2,2} f_{2,3}[\lambda] + \xi_{2,3,3} f_{2,3}[\lambda] - f_{2,3}'[\lambda] = 0, f_{2,3}[0] = 0 \}$

- $\gg \left\{ \begin{aligned} &2 e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \in \lambda \eta_{1,3} \xi_{2,1,3} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \in \lambda^2 \eta_{1,3} \xi_{2,1,1} \xi_{2,1,3} + 2 e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda \eta_{1,2,3} \xi_{2,2,3} + \\ &3 e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \in \lambda^2 \eta_{1,3} \xi_{2,1,2} \xi_{2,2,3} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,3} \xi_{2,1,1} \xi_{2,1,2} \xi_{2,2,3} + \\ &e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^2 \eta_{1,2,3} \xi_{2,2,2} \xi_{2,2,3} - e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \in \lambda^2 \eta_{1,3} \xi_{2,1,3} \xi_{2,3,3} - \\ &e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^2 \eta_{1,2,3} \xi_{2,2,3} \xi_{2,3,3} - e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,3} \xi_{2,1,2} \xi_{2,2,3} \xi_{2,3,3} - \in f_{3,3}'[\lambda] = \theta, f_{3,3}[\theta] = \theta \end{aligned} \right\}$
- $\gg \left\{ \begin{aligned} f_{1,1}[\lambda] &\rightarrow -e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \lambda^2 \eta_{1,2} \xi_{2,1,2} - e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda^2 \eta_{1,3} \xi_{2,1,3}, \\ f_{2,2}[\lambda] &\rightarrow e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \lambda^2 \eta_{1,2} \xi_{2,1,2} - e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^2 \eta_{1,2,3} \xi_{2,2,3} - e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda^3 \eta_{1,3} \xi_{2,1,2} \xi_{2,2,3}, \\ f_{3,3}[\lambda] &\rightarrow e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda^2 \eta_{1,3} \xi_{2,1,3} + e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^2 \eta_{1,2,3} \xi_{2,2,3} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda^3 \eta_{1,3} \xi_{2,1,2} \xi_{2,2,3} \end{aligned} \right\}$
- $\gg \left\{ \begin{aligned} &6 \in \lambda^2 \eta_{1,2} \xi_{1,1,2} \xi_{2,1,2} + 2 \in \lambda^3 \eta_{1,2} \xi_{1,1,1} \xi_{1,2} \xi_{2,1,2} - 2 \in \lambda^3 \eta_{1,2} \xi_{1,1,2} \xi_{1,2,2} \xi_{2,1,2} + \\ &2 \in \lambda^3 \eta_{1,2} \xi_{1,1,2} \xi_{2,1,1} \xi_{2,1,2} + 3 \in e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^2 \eta_{1,2} \xi_{2,1,2}^2 + e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^3 \eta_{1,2} \xi_{1,1,1} \xi_{2,1,2}^2 - \\ &e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^3 \eta_{1,2} \xi_{1,1,2} \xi_{2,1,2}^2 + 2 \in e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^3 \eta_{1,2} \xi_{2,1,1} \xi_{2,1,2}^2 - \\ &2 \in e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda \eta_{1,2,3} \xi_{2,1,3} - e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^2 \eta_{1,2,3} \xi_{1,1,1} \xi_{2,1,3} + 3 \in e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^2 \eta_{1,3} \xi_{1,1,2} \xi_{2,1,3} + \\ &e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,3} \xi_{1,1,1} \xi_{1,1,2} \xi_{2,1,3} + e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^2 \eta_{1,2,3} \xi_{1,1,2} \xi_{2,1,3} - \\ &e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,3} \xi_{1,1,2} \xi_{1,2,2} \xi_{2,1,3} - e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^2 \eta_{1,2,3} \xi_{2,1,1} \xi_{2,1,3} + \\ &e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,3} \xi_{1,1,2} \xi_{2,1,1} \xi_{2,1,3} - 2 \in \lambda^3 \eta_{1,2} \xi_{1,1,2} \xi_{2,1,2} \xi_{2,2,2} - \\ &2 \in e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^3 \eta_{1,2} \xi_{2,1,2}^2 \xi_{2,2,2} - 3 \in e^{-\lambda \xi_{2,1} + 2 \lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^2 \eta_{1,2,3} \xi_{1,1,2} \xi_{2,2,3} - \\ &e^{-\lambda \xi_{2,1} + 2 \lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2,3} \xi_{1,1,1} \xi_{1,1,2} \xi_{2,2,3} + e^{-\lambda \xi_{2,1} + 2 \lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2,3} \xi_{1,1,2} \xi_{1,2,2} \xi_{2,2,3} - \\ &3 \in e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^2 \eta_{1,2,3} \xi_{2,1,2} \xi_{2,2,3} - e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2,3} \xi_{1,1,1} \xi_{2,1,2} \xi_{2,2,3} - \\ &4 \in e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,3} \xi_{1,1,2} \xi_{2,1,2} \xi_{2,2,3} - e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,3} \xi_{1,1,1} \xi_{1,1,2} \xi_{2,1,2} \xi_{2,2,3} + \\ &e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2,3} \xi_{1,2,2} \xi_{2,1,2} \xi_{2,2,3} + e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,3} \xi_{1,1,2} \xi_{1,2,2} \xi_{2,1,2} \xi_{2,2,3} - \\ &e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2,3} \xi_{2,1,1} \xi_{2,1,2} \xi_{2,2,3} - e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,3} \xi_{1,1,2} \xi_{2,1,1} \xi_{2,1,2} \xi_{2,2,3} - \\ &4 \in e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,3} \xi_{2,1,2}^2 \xi_{2,2,3} - e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,3} \xi_{1,1,1} \xi_{2,1,2}^2 \xi_{2,2,3} + \\ &e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,3} \xi_{1,2,2} \xi_{2,1,2}^2 \xi_{2,2,3} - 2 \in e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,3} \xi_{2,1,1} \xi_{2,1,2}^2 \xi_{2,2,3} - \\ &e^{-\lambda \xi_{2,1} + 2 \lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2,3} \xi_{1,1,2} \xi_{2,2,2} \xi_{2,2,3} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,3} \xi_{2,1,2}^2 \xi_{2,2,2} \xi_{2,2,3} + \\ &e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^2 \eta_{1,2,3} \xi_{2,1,3} \xi_{2,3,3} - e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,3} \xi_{1,1,2} \xi_{2,1,3} \xi_{2,3,3} + \\ &e^{-\lambda \xi_{2,1} + 2 \lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2,3} \xi_{1,1,2} \xi_{2,2,3} \xi_{2,3,3} + e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2,3} \xi_{2,1,2} \xi_{2,2,3} \xi_{2,3,3} + \\ &e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,3} \xi_{1,1,2} \xi_{2,1,2} \xi_{2,2,3} \xi_{2,3,3} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,3} \xi_{2,1,2}^2 \xi_{2,2,3} \xi_{2,3,3} - \\ &\in \xi_{1,1,1} f_{1,2}[\lambda] + \in \xi_{1,2,2} f_{1,2}[\lambda] - \in \xi_{2,1,1} f_{1,2}[\lambda] + \in \xi_{2,2,2} f_{1,2}[\lambda] - \in f_{1,2}'[\lambda] = \theta, f_{1,2}[\theta] = \theta \end{aligned} \right\}$
- $\gg \left\{ \begin{aligned} f_{1,1}[\lambda] &\rightarrow -e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \lambda^2 \eta_{1,2} \xi_{2,1,2} - e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda^2 \eta_{1,3} \xi_{2,1,3}, \\ f_{1,2}[\lambda] &\rightarrow 2 \lambda^3 \eta_{1,2} \xi_{1,1,2} \xi_{2,1,2} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \lambda^3 \eta_{1,2} \xi_{2,1,2}^2 - e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^2 \eta_{1,2,3} \xi_{2,1,3} + \\ &e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^3 \eta_{1,3} \xi_{1,1,2} \xi_{2,1,3} - e^{-\lambda \xi_{2,1} + 2 \lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^3 \eta_{1,2,3} \xi_{1,1,2} \xi_{2,2,3} - \\ &e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^3 \eta_{1,2,3} \xi_{2,1,2} \xi_{2,2,3} - e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^4 \eta_{1,3} \xi_{1,1,2} \xi_{2,1,2} \xi_{2,2,3} - e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda^4 \eta_{1,3} \xi_{2,1,2}^2 \xi_{2,2,3}, \\ f_{2,2}[\lambda] &\rightarrow e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \lambda^2 \eta_{1,2} \xi_{2,1,2} - e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^2 \eta_{1,2,3} \xi_{2,2,3} - e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda^3 \eta_{1,3} \xi_{2,1,2} \xi_{2,2,3}, \\ f_{3,3}[\lambda] &\rightarrow e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda^2 \eta_{1,3} \xi_{2,1,3} + e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^2 \eta_{1,2,3} \xi_{2,2,3} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda^3 \eta_{1,3} \xi_{2,1,2} \xi_{2,2,3} \end{aligned} \right\}$

$$\begin{aligned}
 & \gg \left\{ -3 e^{\lambda \xi_{2,1} - 2 \lambda \xi_{2,2} + \lambda \xi_{2,3}} \in \lambda^2 \eta_{1,2} \xi_{1,2,3} \xi_{2,1,2} - e^{\lambda \xi_{2,1} - 2 \lambda \xi_{2,2} + \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2} \xi_{1,2,3} \xi_{2,1,2} + \right. \\
 & \quad e^{\lambda \xi_{2,1} - 2 \lambda \xi_{2,2} + \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2} \xi_{1,2,3} \xi_{1,3,3} \xi_{2,1,2} - e^{\lambda \xi_{2,1} - 2 \lambda \xi_{2,2} + \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2} \xi_{1,2,3} \xi_{2,1,1} \xi_{2,1,2} + \\
 & \quad 2 e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda \eta_{1,2} \xi_{2,1,3} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^2 \eta_{1,2} \xi_{1,2,2} \xi_{2,1,3} + 3 e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^2 \eta_{1,3} \xi_{1,2,3} \xi_{2,1,3} + \\
 & \quad e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^3 \eta_{1,3} \xi_{1,2,2} \xi_{1,2,3} \xi_{2,1,3} - e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^2 \eta_{1,2} \xi_{1,3,3} \xi_{2,1,3} - \\
 & \quad e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^3 \eta_{1,3} \xi_{1,2,3} \xi_{1,3,3} \xi_{2,1,3} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^2 \eta_{1,2} \xi_{2,1,1} \xi_{2,1,3} + \\
 & \quad e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^3 \eta_{1,3} \xi_{1,2,3} \xi_{2,1,1} \xi_{2,1,3} + e^{\lambda \xi_{2,1} - 2 \lambda \xi_{2,2} + \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2} \xi_{1,2,3} \xi_{2,1,2} \xi_{2,2,2} + \\
 & \quad 6 \in \lambda^2 \eta_{1,2,3} \xi_{1,2,3} \xi_{2,2,3} + 2 \in \lambda^3 \eta_{1,2,3} \xi_{1,2,2} \xi_{1,2,3} \xi_{2,2,3} - 2 \in \lambda^3 \eta_{1,2,3} \xi_{1,2,3} \xi_{1,3,3} \xi_{2,2,3} + \\
 & \quad 8 e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^3 \eta_{1,3} \xi_{1,2,3} \xi_{2,1,2} \xi_{2,2,3} + 2 e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^4 \eta_{1,3} \xi_{1,2,2} \xi_{1,2,3} \xi_{2,1,2} \xi_{2,2,3} - \\
 & \quad 2 e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^4 \eta_{1,3} \xi_{1,2,3} \xi_{1,3,3} \xi_{2,1,2} \xi_{2,2,3} + 2 e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^4 \eta_{1,3} \xi_{1,2,3} \xi_{2,1,1} \xi_{2,1,2} \xi_{2,2,3} + \\
 & \quad 2 \in \lambda^3 \eta_{1,2,3} \xi_{1,2,3} \xi_{2,2,2} \xi_{2,2,3} + 3 e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^2 \eta_{1,2,3} \xi_{2,2,3}^2 + \\
 & \quad e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2,3} \xi_{1,2,2} \xi_{2,2,3}^2 - e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2,3} \xi_{1,3,3} \xi_{2,2,3}^2 + \\
 & \quad 4 e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,3} \xi_{2,1,2} \xi_{2,2,3}^2 + e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,3} \xi_{1,2,2} \xi_{2,1,2} \xi_{2,2,3}^2 - \\
 & \quad e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,3} \xi_{1,3,3} \xi_{2,1,2} \xi_{2,2,3}^2 + e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,3} \xi_{2,1,1} \xi_{2,1,2} \xi_{2,2,3}^2 + \\
 & \quad 2 e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2,3} \xi_{2,2,2} \xi_{2,2,3}^2 + e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,3} \xi_{2,1,2} \xi_{2,2,2} \xi_{2,2,3}^2 - \\
 & \quad e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^2 \eta_{1,2} \xi_{2,1,3} \xi_{2,3,3} - e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^3 \eta_{1,3} \xi_{1,2,3} \xi_{2,1,3} \xi_{2,3,3} - \\
 & \quad 2 \in \lambda^3 \eta_{1,2,3} \xi_{1,2,3} \xi_{2,2,3} \xi_{2,3,3} - 2 e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \in \lambda^4 \eta_{1,3} \xi_{1,2,3} \xi_{2,1,2} \xi_{2,2,3} \xi_{2,3,3} - \\
 & \quad 2 e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2,3} \xi_{2,2,3}^2 \xi_{2,3,3} - 2 e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,3} \xi_{2,1,2} \xi_{2,2,3}^2 \xi_{2,3,3} - \in \xi_{1,2,2} f_{2,3}[\lambda] + \\
 & \quad \in \xi_{1,3,3} f_{2,3}[\lambda] - \in \xi_{2,2,2} f_{2,3}[\lambda] + \in \xi_{2,3,3} f_{2,3}[\lambda] - \in f_{2,3}'[\lambda] = \theta, f_{2,3}[\theta] = \theta \} \\
 & \gg \left\{ f_{1,1}[\lambda] \rightarrow -e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \lambda^2 \eta_{1,2} \xi_{2,1,2} - e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda^2 \eta_{1,3} \xi_{2,1,3}, \right. \\
 & \quad f_{1,2}[\lambda] \rightarrow 2 \lambda^3 \eta_{1,2} \xi_{1,1,2} \xi_{2,1,2} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \lambda^3 \eta_{1,2} \xi_{2,1,2}^2 - e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^2 \eta_{1,2,3} \xi_{2,1,3} + \\
 & \quad e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^3 \eta_{1,3} \xi_{1,1,2} \xi_{2,1,3} - e^{-\lambda \xi_{2,1} + 2 \lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^3 \eta_{1,2,3} \xi_{1,1,2} \xi_{2,2,3} - \\
 & \quad e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^3 \eta_{1,2,3} \xi_{2,1,2} \xi_{2,2,3} - e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^4 \eta_{1,3} \xi_{1,1,2} \xi_{2,1,2} \xi_{2,2,3} - e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda^4 \eta_{1,3} \xi_{2,1,2}^2 \xi_{2,2,3}, \\
 & \quad f_{2,2}[\lambda] \rightarrow e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \lambda^2 \eta_{1,2} \xi_{2,1,2} - e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^2 \eta_{1,2,3} \xi_{2,2,3} - e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda^3 \eta_{1,3} \xi_{2,1,2} \xi_{2,2,3}, \\
 & \quad f_{2,3}[\lambda] \rightarrow -e^{\lambda \xi_{2,1} - 2 \lambda \xi_{2,2} + \lambda \xi_{2,3}} \lambda^3 \eta_{1,2} \xi_{1,2,3} \xi_{2,1,2} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \lambda^2 \eta_{1,2} \xi_{2,1,3} + \\
 & \quad e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \lambda^3 \eta_{1,3} \xi_{1,2,3} \xi_{2,1,3} + 2 \lambda^3 \eta_{1,2,3} \xi_{1,2,3} \xi_{2,2,3} + 2 e^{\lambda \xi_{2,1} - \lambda \xi_{2,2}} \lambda^4 \eta_{1,3} \xi_{1,2,3} \xi_{2,1,2} \xi_{2,2,3} + \\
 & \quad e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^3 \eta_{1,2,3} \xi_{2,2,3}^2 + e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda^4 \eta_{1,3} \xi_{2,1,2} \xi_{2,2,3}^2, \\
 & \quad f_{3,3}[\lambda] \rightarrow e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda^2 \eta_{1,3} \xi_{2,1,3} + e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \lambda^2 \eta_{1,2,3} \xi_{2,2,3} + e^{\lambda \xi_{2,1} - \lambda \xi_{2,3}} \lambda^3 \eta_{1,3} \xi_{2,1,2} \xi_{2,2,3} \}
 \end{aligned}$$

$$\begin{aligned}
& \gg \{ 3 e^{-\lambda \xi_{2,2} + \lambda \xi_{2,3}} \in \lambda^2 \eta_{1,2} \xi_{1,3} \xi_{2,1,2} + e^{-\lambda \xi_{2,2} + \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2} \xi_{1,1,1} \xi_{1,3} \xi_{2,1,2} - \\
& e^{-\lambda \xi_{2,2} + \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2} \xi_{1,1,3} \xi_{1,3,3} \xi_{2,1,2} + e^{-\lambda \xi_{2,2} + \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2} \xi_{1,1,3} \xi_{2,1,1} \xi_{2,1,2} + \\
& 6 \in \lambda^2 \eta_{1,3} \xi_{1,1,3} \xi_{2,1,3} + 2 \in \lambda^3 \eta_{1,1,3} \xi_{1,1,1} \xi_{1,3} \xi_{2,1,3} + 3 \in \lambda^2 \eta_{1,2,3} \xi_{1,2,3} \xi_{2,1,3} + \\
& \in \lambda^3 \eta_{1,2,3} \xi_{1,1,1} \xi_{1,2,3} \xi_{2,1,3} - 2 \in \lambda^3 \eta_{1,1,3} \xi_{1,1,3} \xi_{1,3,3} \xi_{2,1,3} - \in \lambda^3 \eta_{1,2,3} \xi_{1,2,3} \xi_{1,3,3} \xi_{2,1,3} + \\
& 2 \in \lambda^3 \eta_{1,1,3} \xi_{1,1,3} \xi_{2,1,1} \xi_{2,1,3} + \in \lambda^3 \eta_{1,2,3} \xi_{1,2,3} \xi_{2,1,1} \xi_{2,1,3} - 4 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2,2}} \in \lambda^3 \eta_{1,1,3} \xi_{1,2,3} \xi_{2,1,2} \xi_{2,1,3} - \\
& e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2,2}} \in \lambda^4 \eta_{1,1,3} \xi_{1,1,1} \xi_{1,2,3} \xi_{2,1,2} \xi_{2,1,3} + e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2,2}} \in \lambda^4 \eta_{1,1,3} \xi_{1,2,3} \xi_{1,3,3} \xi_{2,1,2} \xi_{2,1,3} - \\
& 2 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2,2}} \in \lambda^4 \eta_{1,1,3} \xi_{1,2,3} \xi_{2,1,1} \xi_{2,1,3} + 3 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2,2}} \in \lambda^2 \eta_{1,1,3} \xi_{2,1,3}^2 + \\
& e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2,2}} \in \lambda^3 \eta_{1,1,3} \xi_{1,1,1} \xi_{2,1,3}^2 - e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2,2}} \in \lambda^3 \eta_{1,1,3} \xi_{1,1,3} \xi_{2,1,3}^2 + 2 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2,2}} \in \lambda^3 \eta_{1,1,3} \xi_{2,1,1} \xi_{2,1,3}^2 - \\
& e^{-\lambda \xi_{2,2} + \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2} \xi_{1,1,3} \xi_{2,1,2} \xi_{2,2,2} + e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2,2}} \in \lambda^4 \eta_{1,1,3} \xi_{1,2,3} \xi_{2,1,2} \xi_{2,1,3} \xi_{2,2,2} + \\
& 3 e^{-\lambda \xi_{2,1,1} + \lambda \xi_{2,2,2}} \in \lambda^2 \eta_{1,2,3} \xi_{1,1,3} \xi_{2,2,3} + e^{-\lambda \xi_{2,1,1} + \lambda \xi_{2,2,2}} \in \lambda^3 \eta_{1,2,3} \xi_{1,1,1} \xi_{1,3} \xi_{2,2,3} - \\
& e^{-\lambda \xi_{2,1,1} + \lambda \xi_{2,2,2}} \in \lambda^3 \eta_{1,2,3} \xi_{1,1,3} \xi_{1,3,3} \xi_{2,2,3} + 4 \in \lambda^3 \eta_{1,1,3} \xi_{1,1,3} \xi_{2,1,2} \xi_{2,2,3} + \in \lambda^4 \eta_{1,1,3} \xi_{1,1,1} \xi_{1,3} \xi_{2,1,2} \xi_{2,2,3} - \\
& 4 \in \lambda^3 \eta_{1,2,3} \xi_{1,2,3} \xi_{2,1,2} \xi_{2,2,3} - \in \lambda^4 \eta_{1,2,3} \xi_{1,1,1} \xi_{1,2,3} \xi_{2,1,2} \xi_{2,2,3} - \in \lambda^4 \eta_{1,1,3} \xi_{1,1,3} \xi_{1,3,3} \xi_{2,1,2} \xi_{2,2,3} + \\
& \in \lambda^4 \eta_{1,2,3} \xi_{1,2,3} \xi_{1,3,3} \xi_{2,1,2} \xi_{2,2,3} + \in \lambda^4 \eta_{1,1,3} \xi_{1,1,3} \xi_{2,1,1} \xi_{2,1,2} \xi_{2,2,3} - \in \lambda^4 \eta_{1,2,3} \xi_{1,2,3} \xi_{2,1,1} \xi_{2,1,2} \xi_{2,2,3} - \\
& 5 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2,2}} \in \lambda^4 \eta_{1,1,3} \xi_{1,2,3} \xi_{2,1,2}^2 \xi_{2,2,3} - e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2,2}} \in \lambda^5 \eta_{1,1,3} \xi_{1,1,1} \xi_{1,2,3} \xi_{2,1,2}^2 \xi_{2,2,3} + \\
& e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2,2}} \in \lambda^5 \eta_{1,1,3} \xi_{1,2,3} \xi_{1,3,3} \xi_{2,1,2}^2 \xi_{2,2,3} - 2 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2,2}} \in \lambda^5 \eta_{1,1,3} \xi_{1,2,3} \xi_{2,1,1} \xi_{2,1,2}^2 \xi_{2,2,3} + \\
& 3 e^{\lambda \xi_{2,2,2} - \lambda \xi_{2,3,3}} \in \lambda^2 \eta_{1,2,3} \xi_{2,1,3} \xi_{2,2,3} + e^{\lambda \xi_{2,2,2} - \lambda \xi_{2,3,3}} \in \lambda^3 \eta_{1,2,3} \xi_{1,1,1} \xi_{2,1,3} \xi_{2,2,3} - \\
& e^{\lambda \xi_{2,2,2} - \lambda \xi_{2,3,3}} \in \lambda^3 \eta_{1,2,3} \xi_{1,3,3} \xi_{2,1,3} \xi_{2,2,3} + e^{\lambda \xi_{2,2,2} - \lambda \xi_{2,3,3}} \in \lambda^3 \eta_{1,2,3} \xi_{2,1,1} \xi_{2,1,3} \xi_{2,2,3} + \\
& 4 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,3,3}} \in \lambda^3 \eta_{1,1,3} \xi_{2,1,2} \xi_{2,1,3} \xi_{2,2,3} + e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,3,3}} \in \lambda^4 \eta_{1,1,3} \xi_{1,1,1} \xi_{2,1,2} \xi_{2,1,3} \xi_{2,2,3} - \\
& e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,3,3}} \in \lambda^4 \eta_{1,1,3} \xi_{1,1,3} \xi_{2,1,2} \xi_{2,1,3} \xi_{2,2,3} + 2 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,3,3}} \in \lambda^4 \eta_{1,1,3} \xi_{2,1,1} \xi_{2,1,2} \xi_{2,1,3} \xi_{2,2,3} + \\
& e^{-\lambda \xi_{2,1,1} + \lambda \xi_{2,2,2}} \in \lambda^3 \eta_{1,2,3} \xi_{1,1,3} \xi_{2,2,2} \xi_{2,2,3} + e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2,2}} \in \lambda^5 \eta_{1,1,3} \xi_{1,2,3} \xi_{2,1,2}^2 \xi_{2,2,2} \xi_{2,2,3} + \\
& e^{\lambda \xi_{2,2,2} - \lambda \xi_{2,3,3}} \in \lambda^3 \eta_{1,2,3} \xi_{2,1,3} \xi_{2,2,2} \xi_{2,2,3} - 2 \in \lambda^3 \eta_{1,1,3} \xi_{1,1,3} \xi_{2,1,3} \xi_{2,3,3} - \in \lambda^3 \eta_{1,2,3} \xi_{1,2,3} \xi_{2,1,3} \xi_{2,3,3} + \\
& e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2,2}} \in \lambda^4 \eta_{1,1,3} \xi_{1,2,3} \xi_{2,1,2} \xi_{2,1,3} \xi_{2,3,3} - 2 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2,2}} \in \lambda^3 \eta_{1,1,3} \xi_{2,1,3}^2 \xi_{2,3,3} - \\
& e^{-\lambda \xi_{2,1,1} + \lambda \xi_{2,2,2}} \in \lambda^3 \eta_{1,2,3} \xi_{1,1,3} \xi_{2,2,3} \xi_{2,3,3} - \in \lambda^4 \eta_{1,1,3} \xi_{1,1,3} \xi_{2,1,2} \xi_{2,2,3} \xi_{2,3,3} + \\
& \in \lambda^4 \eta_{1,2,3} \xi_{1,2,3} \xi_{2,1,2} \xi_{2,2,3} \xi_{2,3,3} + e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2,2}} \in \lambda^5 \eta_{1,1,3} \xi_{1,2,3} \xi_{2,1,2}^2 \xi_{2,2,3} \xi_{2,3,3} - \\
& 2 e^{\lambda \xi_{2,2,2} - \lambda \xi_{2,3,3}} \in \lambda^3 \eta_{1,2,3} \xi_{2,1,3} \xi_{2,2,3} \xi_{2,3,3} - 2 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2,2}} \in \lambda^4 \eta_{1,1,3} \xi_{2,1,2} \xi_{2,1,3} \xi_{2,2,3} \xi_{2,3,3} - \\
& \in \xi_{1,1,1} f_{1,3}[\lambda] + \in \xi_{1,3,3} f_{1,3}[\lambda] - \in \xi_{2,1,1} f_{1,3}[\lambda] + \in \xi_{2,3,3} f_{1,3}[\lambda] - \in f_{1,3}'[\lambda] = \theta, f_{1,3}[\theta] = \theta \} \\
& \gg \{ f_{1,1}[\lambda] \rightarrow -e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2,2}} \lambda^2 \eta_{1,2} \xi_{2,1,2} - e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,3,3}} \lambda^2 \eta_{1,1,3} \xi_{2,1,3}, \\
& f_{1,2}[\lambda] \rightarrow 2 \lambda^3 \eta_{1,2} \xi_{1,1,2} \xi_{2,1,2} + e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2,2}} \lambda^3 \eta_{1,1,2} \xi_{2,1,2}^2 - e^{\lambda \xi_{2,2,2} - \lambda \xi_{2,3,3}} \lambda^2 \eta_{1,2,3} \xi_{2,1,3} + \\
& e^{\lambda \xi_{2,2,2} - \lambda \xi_{2,3,3}} \lambda^3 \eta_{1,1,3} \xi_{1,1,2} \xi_{2,1,3} - e^{-\lambda \xi_{2,1,1} + 2 \lambda \xi_{2,2,2} - \lambda \xi_{2,3,3}} \lambda^3 \eta_{1,2,3} \xi_{1,1,2} \xi_{2,2,3} - \\
& e^{\lambda \xi_{2,2,2} - \lambda \xi_{2,3,3}} \lambda^3 \eta_{1,2,3} \xi_{2,1,2} \xi_{2,2,3} - e^{\lambda \xi_{2,2,2} - \lambda \xi_{2,3,3}} \lambda^4 \eta_{1,1,3} \xi_{1,1,2} \xi_{2,1,2} \xi_{2,2,3} - e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,3,3}} \lambda^4 \eta_{1,1,3} \xi_{2,1,2}^2 \xi_{2,2,3}, \\
& f_{1,3}[\lambda] \rightarrow e^{-\lambda \xi_{2,2,2} + \lambda \xi_{2,3,3}} \lambda^3 \eta_{1,1,2} \xi_{1,1,3} \xi_{2,1,2} + 2 \lambda^3 \eta_{1,1,3} \xi_{1,1,3} \xi_{2,1,3} + \lambda^3 \eta_{1,2,3} \xi_{1,2,3} \xi_{2,1,3} - \\
& e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2,2}} \lambda^4 \eta_{1,1,3} \xi_{1,2,3} \xi_{2,1,2} \xi_{2,1,3} + e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,3,3}} \lambda^3 \eta_{1,1,3} \xi_{2,1,3}^2 + e^{-\lambda \xi_{2,1,1} + \lambda \xi_{2,2,2}} \lambda^3 \eta_{1,2,3} \xi_{1,1,3} \xi_{2,2,3} + \\
& \lambda^4 \eta_{1,1,3} \xi_{1,1,3} \xi_{2,1,2} \xi_{2,2,3} - \lambda^4 \eta_{1,2,3} \xi_{1,2,3} \xi_{2,1,2} \xi_{2,2,3} - e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2,2}} \lambda^5 \eta_{1,1,3} \xi_{1,2,3} \xi_{2,1,2}^2 \xi_{2,2,3} + \\
& e^{\lambda \xi_{2,2,2} - \lambda \xi_{2,3,3}} \lambda^3 \eta_{1,2,3} \xi_{2,1,3} \xi_{2,2,3} + e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,3,3}} \lambda^4 \eta_{1,1,3} \xi_{2,1,2} \xi_{2,1,3} \xi_{2,2,3}, \\
& f_{2,2}[\lambda] \rightarrow e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2,2}} \lambda^2 \eta_{1,1,2} \xi_{2,1,2} - e^{\lambda \xi_{2,2,2} - \lambda \xi_{2,3,3}} \lambda^2 \eta_{1,2,3} \xi_{2,2,3} - e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,3,3}} \lambda^3 \eta_{1,1,3} \xi_{2,1,2} \xi_{2,2,3}, \\
& f_{2,3}[\lambda] \rightarrow -e^{\lambda \xi_{2,1,1} - 2 \lambda \xi_{2,2,2} + \lambda \xi_{2,3,3}} \lambda^3 \eta_{1,1,2} \xi_{1,2,3} \xi_{2,1,2} + e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2,2}} \lambda^2 \eta_{1,1,2} \xi_{2,1,3} + \\
& e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2,2}} \lambda^3 \eta_{1,1,3} \xi_{1,2,3} \xi_{2,1,3} + 2 \lambda^3 \eta_{1,2,3} \xi_{1,2,3} \xi_{2,2,3} + 2 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2,2}} \lambda^4 \eta_{1,1,3} \xi_{1,2,3} \xi_{2,1,2} \xi_{2,2,3} + \\
& e^{\lambda \xi_{2,2,2} - \lambda \xi_{2,3,3}} \lambda^3 \eta_{1,2,3} \xi_{2,2,3}^2 + e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,3,3}} \lambda^4 \eta_{1,1,3} \xi_{2,1,2} \xi_{2,2,3}, \\
& f_{3,3}[\lambda] \rightarrow e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,3,3}} \lambda^2 \eta_{1,1,3} \xi_{2,1,3} + e^{\lambda \xi_{2,2,2} - \lambda \xi_{2,3,3}} \lambda^2 \eta_{1,2,3} \xi_{2,2,3} + e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,3,3}} \lambda^3 \eta_{1,1,3} \xi_{2,1,2} \xi_{2,2,3} \}
\end{aligned}$$

$$\gg \left\{ -2 e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda \eta_{1,2,3} \eta_{2,1,2} + e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^2 \eta_{1,2,3} \eta_{2,1,2} \xi_{1,1,1} - 2 \in \lambda^2 \eta_{1,1,2} \eta_{2,1,3} \xi_{1,1,2} - \right. \\
 2 \in \lambda^3 \eta_{1,1,2} \eta_{2,1,3} \xi_{1,1,1} \xi_{1,1,2} + 2 \in \lambda^3 \eta_{1,1,2} \eta_{2,1,3} \xi_{1,1,2} \xi_{1,2,2} - 2 \in \lambda^2 \eta_{1,2,3} \eta_{2,1,3} \xi_{1,2,3} - \\
 2 \in \lambda^3 \eta_{1,2,3} \eta_{2,1,3} \xi_{1,2,2} \xi_{1,2,3} - e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^2 \eta_{1,2,3} \eta_{2,1,2} \xi_{1,3,3} + 2 \in \lambda^3 \eta_{1,2,3} \eta_{2,1,3} \xi_{1,2,3} \xi_{1,3,3} + \\
 e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^2 \eta_{1,2,3} \eta_{2,1,2} \xi_{2,1,1} - 4 e^{2 \lambda \xi_{2,1,1} - \lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^2 \eta_{1,1,2} \eta_{1,1,3} \xi_{2,1,2} - \\
 3 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,3}} \in \lambda^2 \eta_{1,1,3} \eta_{2,1,2} \xi_{2,1,2} - 2 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \in \lambda^2 \eta_{1,1,2} \eta_{2,1,3} \xi_{2,1,2} + \\
 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,1,3} \eta_{2,1,2} \xi_{1,1,1} \xi_{2,1,2} - 2 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \in \lambda^3 \eta_{1,1,2} \eta_{2,1,3} \xi_{1,1,1} \xi_{2,1,2} + \\
 2 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \in \lambda^3 \eta_{1,1,2} \eta_{2,1,3} \xi_{1,2,2} \xi_{2,1,2} - 2 e^{2 \lambda \xi_{2,1,1} - \lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,1,3}^2 \xi_{1,2,3} \xi_{2,1,2} - \\
 2 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \in \lambda^3 \eta_{1,1,3} \eta_{2,1,3} \xi_{1,2,3} \xi_{2,1,2} - 2 e^{2 \lambda \xi_{2,1,1} - \lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,1,3}^2 \xi_{1,2,2} \xi_{1,2,3} \xi_{2,1,2} - \\
 2 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \in \lambda^4 \eta_{1,1,3} \eta_{2,1,3} \xi_{1,2,2} \xi_{1,2,3} \xi_{2,1,2} - e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,1,3} \eta_{2,1,2} \xi_{1,3,3} \xi_{2,1,2} + \\
 2 e^{2 \lambda \xi_{2,1,1} - \lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,1,3}^2 \xi_{1,2,3} \xi_{1,3,3} \xi_{2,1,2} + 2 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \in \lambda^4 \eta_{1,1,3} \eta_{2,1,3} \xi_{1,2,3} \xi_{1,3,3} \xi_{2,1,2} - \\
 2 e^{2 \lambda \xi_{2,1,1} - \lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,1,2} \eta_{1,1,3} \xi_{2,1,1} \xi_{2,1,2} - 2 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \in \lambda^3 \eta_{1,1,2} \eta_{2,1,3} \xi_{2,1,1} \xi_{2,1,2} - \\
 3 e^{2 \lambda \xi_{2,1,1} - 2 \lambda \xi_{2,3}} \in \lambda^2 \eta_{1,1,3}^2 \xi_{2,1,3} + e^{2 \lambda \xi_{2,1,1} - 2 \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,1,3}^2 \xi_{1,1,1} \xi_{2,1,3} - \\
 e^{2 \lambda \xi_{2,1,1} - 2 \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,1,3}^2 \xi_{1,3,3} \xi_{2,1,3} - e^{2 \lambda \xi_{2,1,1} - 2 \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,1,3}^2 \xi_{2,1,1} \xi_{2,1,3} - \\
 e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^2 \eta_{1,2,3} \eta_{2,1,2} \xi_{2,2,2} + 2 e^{2 \lambda \xi_{2,1,1} - \lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,1,2} \eta_{1,1,3} \xi_{2,1,2} \xi_{2,2,2} + \\
 2 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \in \lambda^3 \eta_{1,1,2} \eta_{2,1,3} \xi_{2,1,2} \xi_{2,2,2} - e^{\lambda \xi_{2,1,1} + \lambda \xi_{2,2} - 2 \lambda \xi_{2,3}} \in \lambda^2 \eta_{1,1,3} \eta_{1,2,3} \xi_{2,2,3} - \\
 2 e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^2 \eta_{1,2,3} \eta_{2,1,3} \xi_{2,2,3} - e^{\lambda \xi_{2,1,1} + \lambda \xi_{2,2} - 2 \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,1,3} \eta_{1,2,3} \xi_{1,1,1} \xi_{2,2,3} + \\
 2 e^{\lambda \xi_{2,1,1} + \lambda \xi_{2,2} - 2 \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,1,3}^2 \xi_{1,1,2} \xi_{2,2,3} + 2 e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,1,3} \eta_{2,1,3} \xi_{1,1,2} \xi_{2,2,3} + \\
 2 e^{\lambda \xi_{2,1,1} + \lambda \xi_{2,2} - 2 \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,1,3}^2 \xi_{1,1,1} \xi_{1,1,2} \xi_{2,2,3} + 2 e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,1,3} \eta_{2,1,3} \xi_{1,1,1} \xi_{1,1,2} \xi_{2,2,3} - \\
 2 e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2,3} \eta_{2,1,3} \xi_{1,2,2} \xi_{2,2,3} - 2 e^{\lambda \xi_{2,1,1} + \lambda \xi_{2,2} - 2 \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,1,3}^2 \xi_{1,1,2} \xi_{1,2,2} \xi_{2,2,3} - \\
 2 e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,1,3} \eta_{2,1,3} \xi_{1,1,2} \xi_{1,2,2} \xi_{2,2,3} + e^{\lambda \xi_{2,1,1} + \lambda \xi_{2,2} - 2 \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,1,3} \eta_{1,2,3} \xi_{1,3,3} \xi_{2,2,3} + \\
 2 e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2,3} \eta_{2,1,3} \xi_{1,3,3} \xi_{2,2,3} + e^{2 \lambda \xi_{2,1,1} - 2 \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,1,3}^2 \xi_{1,1,1} \xi_{2,1,2} \xi_{2,2,3} + \\
 2 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,1,3} \eta_{2,1,3} \xi_{1,1,1} \xi_{2,1,2} \xi_{2,2,3} - 2 e^{2 \lambda \xi_{2,1,1} - 2 \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,1,3}^2 \xi_{1,1,2} \xi_{1,2,2} \xi_{2,2,3} - \\
 4 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,1,3} \eta_{2,1,3} \xi_{1,2,2} \xi_{2,1,2} \xi_{2,2,3} + e^{2 \lambda \xi_{2,1,1} - 2 \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,1,3}^2 \xi_{1,3,3} \xi_{2,1,2} \xi_{2,2,3} + \\
 2 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,1,3} \eta_{2,1,3} \xi_{1,3,3} \xi_{2,1,2} \xi_{2,2,3} + e^{2 \lambda \xi_{2,1,1} - 2 \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,1,3}^2 \xi_{2,1,1} \xi_{2,1,2} \xi_{2,2,3} + \\
 2 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,1,3} \eta_{2,1,3} \xi_{2,1,1} \xi_{2,1,2} \xi_{2,2,3} - e^{\lambda \xi_{2,1,1} + \lambda \xi_{2,2} - 2 \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,1,3} \eta_{1,2,3} \xi_{2,2,2} \xi_{2,2,3} - \\
 2 e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2,3} \eta_{2,1,3} \xi_{2,2,2} \xi_{2,2,3} - 2 e^{2 \lambda \xi_{2,1,1} - 2 \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,1,3}^2 \xi_{2,1,2} \xi_{2,2,2} \xi_{2,2,3} - \\
 4 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,1,3} \eta_{2,1,3} \xi_{2,1,2} \xi_{2,2,2} \xi_{2,2,3} + e^{2 \lambda \xi_{2,1,1} - 2 \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,1,3}^2 \xi_{2,1,3} \xi_{2,1,3} \xi_{2,3,3} + \\
 e^{\lambda \xi_{2,1,1} + \lambda \xi_{2,2} - 2 \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,1,3} \eta_{1,2,3} \xi_{2,2,3} \xi_{2,3,3} + 2 e^{\lambda \xi_{2,2} - \lambda \xi_{2,3}} \in \lambda^3 \eta_{1,2,3} \eta_{2,1,3} \xi_{2,2,3} \xi_{2,3,3} + \\
 e^{2 \lambda \xi_{2,1,1} - 2 \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,1,3}^2 \xi_{2,1,2} \xi_{2,2,3} \xi_{2,3,3} + 2 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,3}} \in \lambda^4 \eta_{1,1,3} \eta_{2,1,3} \xi_{2,1,2} \xi_{2,2,3} \xi_{2,3,3} + \\
 \left. \in \xi_{1,1,1} \mathbf{g}_{1,3}[\lambda] - \in \xi_{1,3,3} \mathbf{g}_{1,3}[\lambda] + \in \xi_{2,1,1} \mathbf{g}_{1,3}[\lambda] - \in \xi_{2,3,3} \mathbf{g}_{1,3}[\lambda] - \in \mathbf{g}_{1,3}'[\lambda] = \mathbf{0}, \mathbf{g}_{1,3}[\mathbf{0}] = \mathbf{0} \right\}$$

Out[6]=

\$Aborted

Dot: Tensors

$$\{ \lambda \xi_{1,1} + \lambda \xi_{2,1} - e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \in \lambda^2 \eta_{1,1,2} \xi_{2,1,2}, \lambda \xi_{1,2,2} + e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \in \lambda^2 \eta_{1,1,2} \xi_{2,1,2} + \lambda \xi_{2,2,2}, e^{-\lambda \xi_{2,1,1} + \lambda \xi_{2,2}} \lambda \xi_{1,1,2} + \lambda \xi_{2,1,2} + 2 \in \lambda^3 \eta_{1,1,2} \\
 \xi_{1,1,2} \xi_{2,1,2} + e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \in \lambda^3 \eta_{1,1,2}^2 \xi_{2,1,2}, e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \lambda \eta_{1,1,2} + \lambda \eta_{2,1,2} - e^{2 \lambda \xi_{2,1,1} - 2 \lambda \xi_{2,2}} \in \lambda^3 \eta_{1,1,2}^2 \xi_{2,1,2} \} \text{ and} \\
 \{ x_{1,1}, x_{2,2}, x_{3,3}, x_{1,2}, x_{2,3}, x_{1,3}, y_{1,3}, y_{1,2}, y_{2,3} \} \text{ have incompatible shapes.}$$

Out[6]=

$$\{ \lambda \xi_{1,1,1} + \lambda \xi_{2,1,1} - e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \in \lambda^2 \eta_{1,1,2} \xi_{2,1,2}, \lambda \xi_{1,2,2} + e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \in \lambda^2 \eta_{1,1,2} \xi_{2,1,2} + \lambda \xi_{2,2,2}, \\
 e^{-\lambda \xi_{2,1,1} + \lambda \xi_{2,2}} \lambda \xi_{1,1,2} + \lambda \xi_{2,1,2} + 2 \in \lambda^3 \eta_{1,1,2} \xi_{1,1,2} \xi_{2,1,2} + e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \in \lambda^3 \eta_{1,1,2}^2 \xi_{2,1,2}, \\
 e^{\lambda \xi_{2,1,1} - \lambda \xi_{2,2}} \lambda \eta_{1,1,2} + \lambda \eta_{2,1,2} - e^{2 \lambda \xi_{2,1,1} - 2 \lambda \xi_{2,2}} \in \lambda^3 \eta_{1,1,2}^2 \xi_{2,1,2} \}. \\
 \{ x_{1,1}, x_{2,2}, x_{3,3}, x_{1,2}, x_{2,3}, x_{1,3}, y_{1,3}, y_{1,2}, y_{2,3} \}$$

$$\text{osol} = \mathbb{E}_{\{i,j\} \rightarrow \{k\}} \left[y_{1,2}[k] \eta_{1,2}[i] + e^{-\xi_{1,1}[i] + \xi_{2,2}[i]} y_{1,2}[k] \eta_{1,2}[j] + y_{1,3}[k] \eta_{1,3}[i] + \right. \\
 e^{-\xi_{1,1}[i] + \xi_{3,3}[i]} y_{1,3}[k] \eta_{1,3}[j] + y_{2,3}[k] \eta_{2,3}[i] + e^{-\xi_{2,2}[i] + \xi_{3,3}[i]} y_{2,3}[k] \eta_{2,3}[j] + \\
 \left. x_{1,1}[k] \xi_{1,1}[i] + x_{1,1}[k] \xi_{1,1}[j] + e^{-\xi_{1,1}[j]} x_{1,2}[k] \xi_{1,2}[i] - e^{\xi_{3,3}[i]} y_{2,3}[k] \eta_{1,3}[j] \xi_{1,2}[i] + \right.$$

$$\begin{aligned}
 & e^{-\epsilon_{2,2}[i]} x_{1,2}[k] \xi_{1,2}[j] + e^{-\epsilon_{1,1}[j]} x_{1,3}[k] \xi_{1,3}[i] + e^{-\epsilon_{3,3}[i]} x_{1,3}[k] \xi_{1,3}[j] + x_{2,2}[k] \xi_{2,2}[i] + \\
 & x_{2,2}[k] \xi_{2,2}[j] + e^{-\epsilon_{2,2}[j]} x_{2,3}[k] \xi_{2,3}[i] + e^{-\epsilon_{1,1}[i] + \epsilon_{2,2}[i] + \epsilon_{3,3}[i]} y_{1,2}[k] \eta_{1,3}[j] \xi_{2,3}[i] - \\
 & x_{1,3}[k] \xi_{1,2}[j] \xi_{2,3}[i] + e^{-\epsilon_{3,3}[i]} x_{2,3}[k] \xi_{2,3}[j] + x_{3,3}[k] \xi_{3,3}[i] + x_{3,3}[k] \xi_{3,3}[j], \\
 & - e^{-\epsilon_{2,2}[i] + \epsilon_{3,3}[i]} y_{1,3}[k] \eta_{1,2}[i] \eta_{2,3}[j] + e^{\epsilon_{2,2}[i]} x_{1,1}[k] \eta_{1,2}[j] \xi_{1,2}[i] - \\
 & e^{\epsilon_{2,2}[i]} x_{2,2}[k] \eta_{1,2}[j] \xi_{1,2}[i] - e^{-\epsilon_{1,1}[i] + 2\epsilon_{2,2}[i]} y_{1,2}[k] \eta_{1,2}[j]^2 \xi_{1,2}[i] + \\
 & e^{\epsilon_{3,3}[i]} y_{1,3}[k] \eta_{1,2}[i] \eta_{1,3}[j] \xi_{1,2}[i] + x_{1,2}[k] \eta_{1,2}[j] \xi_{1,2}[i] \xi_{1,2}[j] - \\
 & e^{-\epsilon_{2,2}[j]} x_{2,3}[k] \eta_{1,2}[j] \xi_{1,3}[i] + e^{\epsilon_{3,3}[i]} x_{1,1}[k] \eta_{1,3}[j] \xi_{1,3}[i] - e^{\epsilon_{3,3}[i]} x_{3,3}[k] \eta_{1,3}[j] \xi_{1,3}[i] - \\
 & e^{-\epsilon_{1,1}[i] + \epsilon_{2,2}[i] + \epsilon_{3,3}[i]} y_{1,2}[k] \eta_{1,2}[j] \eta_{1,3}[j] \xi_{1,3}[i] - e^{-\epsilon_{1,1}[i] + 2\epsilon_{3,3}[i]} y_{1,3}[k] \eta_{1,3}[j]^2 \xi_{1,3}[i] + \\
 & e^{-\epsilon_{1,1}[j] - \epsilon_{2,2}[i] + \epsilon_{3,3}[i]} x_{1,2}[k] \eta_{2,3}[j] \xi_{1,3}[i] - e^{-\epsilon_{2,2}[i] + 2\epsilon_{3,3}[i]} y_{2,3}[k] \eta_{1,3}[j] \eta_{2,3}[j] \xi_{1,3}[i] - \\
 & e^{-\epsilon_{1,1}[j] + \epsilon_{3,3}[i]} x_{1,2}[k] \eta_{1,3}[j] \xi_{1,2}[i] \xi_{1,3}[i] + e^{2\epsilon_{3,3}[i]} y_{2,3}[k] \eta_{1,3}[j]^2 \xi_{1,2}[i] \xi_{1,3}[i] + \\
 & x_{1,3}[k] \eta_{1,2}[j] \xi_{1,2}[j] \xi_{1,3}[i] + x_{1,3}[k] \eta_{1,3}[j] \xi_{1,3}[i] \xi_{1,3}[j] + e^{\epsilon_{3,3}[i]} x_{2,2}[k] \eta_{2,3}[j] \xi_{2,3}[i] - \\
 & e^{\epsilon_{3,3}[i]} x_{3,3}[k] \eta_{2,3}[j] \xi_{2,3}[i] + e^{-\epsilon_{1,1}[i] + \epsilon_{2,2}[i] + \epsilon_{3,3}[i]} y_{1,2}[k] \eta_{1,2}[j] \eta_{2,3}[j] \xi_{2,3}[i] - \\
 & e^{-\epsilon_{1,1}[i] + 2\epsilon_{3,3}[i]} y_{1,3}[k] \eta_{1,3}[j] \eta_{2,3}[j] \xi_{2,3}[i] - e^{-\epsilon_{2,2}[i] + \epsilon_{3,3}[i]} x_{2,2}[k] \eta_{1,3}[j] \xi_{1,2}[i] \xi_{2,3}[i] - \\
 & 2 e^{-\epsilon_{1,1}[i] + 2\epsilon_{2,2}[i] + \epsilon_{3,3}[i]} y_{1,2}[k] \eta_{1,2}[j] \eta_{1,3}[j] \xi_{1,2}[i] \xi_{2,3}[i] + \\
 & e^{2\epsilon_{3,3}[i]} y_{2,3}[k] \eta_{1,3}[j] \eta_{2,3}[j] \xi_{1,2}[i] \xi_{2,3}[i] - \\
 & e^{-\epsilon_{2,2}[i] + \epsilon_{3,3}[i]} x_{1,2}[k] \eta_{2,3}[j] \xi_{1,2}[j] \xi_{2,3}[i] + e^{\epsilon_{3,3}[i]} x_{1,2}[k] \eta_{1,3}[j] \xi_{1,2}[i] \xi_{1,2}[j] \xi_{2,3}[i] - \\
 & e^{-\epsilon_{1,1}[i] + \epsilon_{2,2}[i] + 2\epsilon_{3,3}[i]} y_{1,2}[k] \eta_{1,3}[j]^2 \xi_{1,3}[i] \xi_{2,3}[i] + x_{1,3}[k] \eta_{2,3}[j] \xi_{1,3}[j] \xi_{2,3}[i] - \\
 & e^{-\epsilon_{1,1}[i] + 2\epsilon_{2,2}[i] + 2\epsilon_{3,3}[i]} y_{1,2}[k] \eta_{1,3}[j]^2 \xi_{1,2}[i] \xi_{2,3}[i]^2 + \\
 & x_{2,3}[k] \eta_{1,3}[j] \xi_{1,3}[i] \xi_{2,3}[j] + x_{2,3}[k] \eta_{2,3}[j] \xi_{2,3}[i] \xi_{2,3}[j], \\
 & - \frac{1}{2} e^{2\epsilon_{2,2}[i]} x_{1,1}[k] \eta_{1,2}[j]^2 \xi_{1,2}[i]^2 + \frac{1}{2} e^{2\epsilon_{2,2}[i]} x_{2,2}[k] \eta_{1,2}[j]^2 \xi_{1,2}[i]^2 + \\
 & e^{-\epsilon_{1,1}[i] + 3\epsilon_{2,2}[i]} y_{1,2}[k] \eta_{1,2}[j]^3 \xi_{1,2}[i]^2 + e^{\epsilon_{3,3}[i]} x_{1,1}[k] \eta_{1,2}[j] \eta_{2,3}[j] \xi_{1,3}[i] - \\
 & e^{\epsilon_{3,3}[i]} x_{2,2}[k] \eta_{1,2}[j] \eta_{2,3}[j] \xi_{1,3}[i] - e^{-\epsilon_{1,1}[i] + \epsilon_{2,2}[i] + \epsilon_{3,3}[i]} y_{1,2}[k] \eta_{1,2}[j]^2 \eta_{2,3}[j] \xi_{1,3}[i] + \\
 & e^{-\epsilon_{2,2}[i] + 2\epsilon_{3,3}[i]} y_{1,3}[k] \eta_{1,2}[i] \eta_{1,3}[j] \eta_{2,3}[j] \xi_{1,3}[i] - e^{\epsilon_{2,2}[i] + \epsilon_{3,3}[i]} x_{1,1}[k] \eta_{1,2}[j] \\
 & \eta_{1,3}[j] \xi_{1,2}[i] \xi_{1,3}[i] + e^{\epsilon_{2,2}[i] + \epsilon_{3,3}[i]} x_{2,2}[k] \eta_{1,2}[j] \eta_{1,3}[j] \xi_{1,2}[i] \xi_{1,3}[i] + \\
 & 2 e^{-\epsilon_{1,1}[i] + 2\epsilon_{2,2}[i] + \epsilon_{3,3}[i]} y_{1,2}[k] \eta_{1,2}[j]^2 \eta_{1,3}[j] \xi_{1,2}[i] \xi_{1,3}[i] - e^{2\epsilon_{3,3}[i]} y_{1,3}[k] \\
 & \eta_{1,2}[i] \eta_{1,3}[j]^2 \xi_{1,2}[i] \xi_{1,3}[i] + e^{-\epsilon_{2,2}[i] + \epsilon_{3,3}[i]} x_{1,2}[k] \eta_{1,2}[j] \eta_{2,3}[j] \xi_{1,2}[j] \xi_{1,3}[i] - \\
 & e^{\epsilon_{3,3}[i]} x_{1,2}[k] \eta_{1,2}[j] \eta_{1,3}[j] \xi_{1,2}[i] \xi_{1,2}[j] \xi_{1,3}[i] - \frac{1}{2} e^{2\epsilon_{3,3}[i]} x_{1,1}[k] \eta_{1,3}[j]^2 \xi_{1,3}[i]^2 + \\
 & \frac{1}{2} e^{2\epsilon_{3,3}[i]} x_{3,3}[k] \eta_{1,3}[j]^2 \xi_{1,3}[i]^2 + e^{-\epsilon_{1,1}[i] + \epsilon_{2,2}[i] + 2\epsilon_{3,3}[i]} y_{1,2}[k] \eta_{1,2}[j] \eta_{1,3}[j]^2 \xi_{1,3}[i]^2 + \\
 & e^{-\epsilon_{1,1}[i] + 3\epsilon_{3,3}[i]} y_{1,3}[k] \eta_{1,3}[j]^3 \xi_{1,3}[i]^2 - e^{-\epsilon_{1,1}[j] - \epsilon_{2,2}[i] + 2\epsilon_{3,3}[i]} x_{1,2}[k] \eta_{1,3}[j] \eta_{2,3}[j] \xi_{1,3}[i]^2 + \\
 & e^{-\epsilon_{2,2}[i] + 3\epsilon_{3,3}[i]} y_{2,3}[k] \eta_{1,3}[j]^2 \eta_{2,3}[j] \xi_{1,3}[i]^2 + \\
 & e^{-\epsilon_{1,1}[j] + 2\epsilon_{3,3}[i]} x_{1,2}[k] \eta_{1,3}[j]^2 \xi_{1,2}[i] \xi_{1,3}[i]^2 - e^{3\epsilon_{3,3}[i]} y_{2,3}[k] \eta_{1,3}[j]^3 \xi_{1,2}[i] \xi_{1,3}[i]^2 + \\
 & e^{-\epsilon_{2,2}[i] + 2\epsilon_{3,3}[i]} y_{1,3}[k] \eta_{1,2}[i] \eta_{2,3}[j]^2 \xi_{2,3}[i] + e^{\epsilon_{2,2}[i] + \epsilon_{3,3}[i]} x_{1,1}[k] \eta_{1,2}[j] \\
 & \eta_{2,3}[j] \xi_{1,2}[i] \xi_{2,3}[i] - e^{\epsilon_{2,2}[i] + \epsilon_{3,3}[i]} x_{2,2}[k] \eta_{1,2}[j] \eta_{2,3}[j] \xi_{1,2}[i] \xi_{2,3}[i] - \\
 & 2 e^{-\epsilon_{1,1}[i] + 2\epsilon_{2,2}[i] + \epsilon_{3,3}[i]} y_{1,2}[k] \eta_{1,2}[j]^2 \eta_{2,3}[j] \xi_{1,2}[i] \xi_{2,3}[i] - \\
 & e^{2\epsilon_{3,3}[i]} y_{1,3}[k] \eta_{1,2}[i] \eta_{1,3}[j] \eta_{2,3}[j] \xi_{1,2}[i] \xi_{2,3}[i] - e^{2\epsilon_{2,2}[i] + \epsilon_{3,3}[i]} x_{1,1}[k] \eta_{1,2}[j] \\
 & \eta_{1,3}[j] \xi_{1,2}[i]^2 \xi_{2,3}[i] + e^{2\epsilon_{2,2}[i] + \epsilon_{3,3}[i]} x_{2,2}[k] \eta_{1,2}[j] \eta_{1,3}[j] \xi_{1,2}[i]^2 \xi_{2,3}[i] + \\
 & 3 e^{-\epsilon_{1,1}[i] + 3\epsilon_{2,2}[i] + \epsilon_{3,3}[i]} y_{1,2}[k] \eta_{1,2}[j]^2 \eta_{1,3}[j] \xi_{1,2}[i]^2 \xi_{2,3}[i] - \\
 & e^{2\epsilon_{3,3}[i]} x_{2,2}[k] \eta_{1,3}[j] \eta_{2,3}[j] \xi_{1,3}[i] \xi_{2,3}[i] + e^{2\epsilon_{3,3}[i]} x_{3,3}[k] \eta_{1,3}[j] \eta_{2,3}[j] \xi_{1,3}[i] \xi_{2,3}[i] - \\
 & 2 e^{-\epsilon_{1,1}[i] + \epsilon_{2,2}[i] + 2\epsilon_{3,3}[i]} y_{1,2}[k] \eta_{1,2}[j] \eta_{1,3}[j] \eta_{2,3}[j] \xi_{1,3}[i] \xi_{2,3}[i] + \\
 & 2 e^{-\epsilon_{1,1}[i] + 3\epsilon_{3,3}[i]} y_{1,3}[k] \eta_{1,3}[j]^2 \eta_{2,3}[j] \xi_{1,3}[i] \xi_{2,3}[i] -
 \end{aligned}$$

$$\begin{aligned}
& e^{-\epsilon_{1,1}[j]-\epsilon_{2,2}[i]+2\epsilon_{3,3}[i]} x_{1,2}[k] \eta_{2,3}[j]^2 \epsilon_{1,3}[i] \epsilon_{2,3}[i] + \\
& 2 e^{-\epsilon_{2,2}[i]+3\epsilon_{3,3}[i]} y_{2,3}[k] \eta_{1,3}[j] \eta_{2,3}[j]^2 \epsilon_{1,3}[i] \epsilon_{2,3}[i] - e^{\epsilon_{2,2}[i]+2\epsilon_{3,3}[i]} x_{1,1}[k] \eta_{1,3}[j]^2 \\
& \quad \epsilon_{1,2}[i] \epsilon_{1,3}[i] \epsilon_{2,3}[i] + e^{\epsilon_{2,2}[i]+2\epsilon_{3,3}[i]} x_{2,2}[k] \eta_{1,3}[j]^2 \epsilon_{1,2}[i] \epsilon_{1,3}[i] \epsilon_{2,3}[i] + \\
& 4 e^{-\epsilon_{1,1}[i]+2\epsilon_{2,2}[i]+2\epsilon_{3,3}[i]} y_{1,2}[k] \eta_{1,2}[j] \eta_{1,3}[j]^2 \epsilon_{1,2}[i] \epsilon_{1,3}[i] \epsilon_{2,3}[i] + \\
& e^{-\epsilon_{1,1}[j]+2\epsilon_{3,3}[i]} x_{1,2}[k] \eta_{1,3}[j] \eta_{2,3}[j] \epsilon_{1,2}[i] \epsilon_{1,3}[i] \epsilon_{2,3}[i] - \\
& 2 e^{3\epsilon_{3,3}[i]} y_{2,3}[k] \eta_{1,3}[j]^2 \eta_{2,3}[j] \epsilon_{1,2}[i] \epsilon_{1,3}[i] \epsilon_{2,3}[i] + \\
& e^{-\epsilon_{2,2}[i]+2\epsilon_{3,3}[i]} x_{1,2}[k] \eta_{1,3}[j] \eta_{2,3}[j] \epsilon_{1,2}[j] \epsilon_{1,3}[i] \epsilon_{2,3}[i] - \\
& e^{2\epsilon_{3,3}[i]} x_{1,2}[k] \eta_{1,3}[j]^2 \epsilon_{1,2}[i] \epsilon_{1,2}[j] \epsilon_{1,3}[i] \epsilon_{2,3}[i] + \\
& e^{-\epsilon_{1,1}[i]+2\epsilon_{2,2}[i]+3\epsilon_{3,3}[i]} y_{1,2}[k] \eta_{1,3}[j]^3 \epsilon_{1,3}[i]^2 \epsilon_{2,3}[i] - \\
& \frac{1}{2} e^{2\epsilon_{3,3}[i]} x_{2,2}[k] \eta_{2,3}[j]^2 \epsilon_{2,3}[i]^2 + \frac{1}{2} e^{2\epsilon_{3,3}[i]} x_{3,3}[k] \eta_{2,3}[j]^2 \epsilon_{2,3}[i]^2 + \\
& e^{-\epsilon_{1,1}[i]+3\epsilon_{3,3}[i]} y_{1,3}[k] \eta_{1,3}[j] \eta_{2,3}[j]^2 \epsilon_{2,3}[i]^2 + e^{-\epsilon_{2,2}[i]+3\epsilon_{3,3}[i]} y_{2,3}[k] \eta_{2,3}[j]^3 \epsilon_{2,3}[i]^2 - \\
& 2 e^{-\epsilon_{1,1}[i]+2\epsilon_{2,2}[i]+2\epsilon_{3,3}[i]} y_{1,2}[k] \eta_{1,2}[j] \eta_{1,3}[j] \eta_{2,3}[j] \epsilon_{1,2}[i] \epsilon_{2,3}[i]^2 - \\
& e^{3\epsilon_{3,3}[i]} y_{2,3}[k] \eta_{1,3}[j] \eta_{2,3}[j]^2 \epsilon_{1,2}[i] \epsilon_{2,3}[i]^2 - \frac{1}{2} e^{2\epsilon_{2,2}[i]+2\epsilon_{3,3}[i]} x_{1,1}[k] \\
& \quad \eta_{1,3}[j]^2 \epsilon_{1,2}[i]^2 \epsilon_{2,3}[i]^2 + \frac{1}{2} e^{2\epsilon_{2,2}[i]+2\epsilon_{3,3}[i]} x_{2,2}[k] \eta_{1,3}[j]^2 \epsilon_{1,2}[i]^2 \epsilon_{2,3}[i]^2 + \\
& 3 e^{-\epsilon_{1,1}[i]+3\epsilon_{2,2}[i]+2\epsilon_{3,3}[i]} y_{1,2}[k] \eta_{1,2}[j] \eta_{1,3}[j]^2 \epsilon_{1,2}[i]^2 \epsilon_{2,3}[i]^2 + \\
& e^{-\epsilon_{2,2}[i]+2\epsilon_{3,3}[i]} x_{1,2}[k] \eta_{2,3}[j]^2 \epsilon_{1,2}[j] \epsilon_{2,3}[i]^2 - e^{2\epsilon_{3,3}[i]} x_{1,2}[k] \eta_{1,3}[j] \eta_{2,3}[j] \epsilon_{1,2}[i] \\
& \quad \epsilon_{1,2}[j] \epsilon_{2,3}[i]^2 + 2 e^{-\epsilon_{1,1}[i]+2\epsilon_{2,2}[i]+3\epsilon_{3,3}[i]} y_{1,2}[k] \eta_{1,3}[j]^3 \epsilon_{1,2}[i] \epsilon_{1,3}[i] \epsilon_{2,3}[i]^2 + \\
& e^{-\epsilon_{1,1}[i]+3\epsilon_{2,2}[i]+3\epsilon_{3,3}[i]} y_{1,2}[k] \eta_{1,3}[j]^3 \epsilon_{1,2}[i]^2 \epsilon_{2,3}[i]^3];
\end{aligned}$$

`osol = (List@@osol).Table[e^k , {k, 0, Length[osol] - 1}]`

`Out[*]=`

$$\begin{aligned}
& y_{1,2}[k] \eta_{1,2}[i] + e^{-\epsilon_{1,1}[i]+2\epsilon_{2,2}[i]} y_{1,2}[k] \eta_{1,2}[j] + \\
& x_{1,1}[k] \epsilon_{1,1}[i] + x_{1,1}[k] \epsilon_{1,1}[j] + e^{-\epsilon_{1,1}[j]} x_{1,2}[k] \epsilon_{1,2}[i] + \\
& \epsilon^2 \left(-\frac{1}{2} e^{2\epsilon_{2,2}[i]} x_{1,1}[k] \eta_{1,2}[j]^2 \epsilon_{1,2}[i]^2 + \frac{1}{2} e^{2\epsilon_{2,2}[i]} x_{2,2}[k] \eta_{1,2}[j]^2 \epsilon_{1,2}[i]^2 + \right. \\
& \quad \left. e^{-\epsilon_{1,1}[i]+3\epsilon_{2,2}[i]} y_{1,2}[k] \eta_{1,2}[j]^3 \epsilon_{1,2}[i]^2 \right) + e^{-\epsilon_{2,2}[i]} x_{1,2}[k] \epsilon_{1,2}[j] + \\
& \in \left(e^{\epsilon_{2,2}[i]} x_{1,1}[k] \eta_{1,2}[j] \epsilon_{1,2}[i] - e^{\epsilon_{2,2}[i]} x_{2,2}[k] \eta_{1,2}[j] \epsilon_{1,2}[i] - e^{-\epsilon_{1,1}[i]+2\epsilon_{2,2}[i]} y_{1,2}[k] \right. \\
& \quad \left. \eta_{1,2}[j]^2 \epsilon_{1,2}[i] + x_{1,2}[k] \eta_{1,2}[j] \epsilon_{1,2}[i] \epsilon_{1,2}[j] \right) + x_{2,2}[k] \epsilon_{2,2}[i] + x_{2,2}[k] \epsilon_{2,2}[j]
\end{aligned}$$

`In[*]:= CF[(Expand@osol[$k].$Basis /. { $\xi_{1\alpha\beta} \Rightarrow \xi_{\alpha\beta}[i]$, $\eta_{1\alpha\beta} \Rightarrow \eta_{\alpha\beta}[i]$,
 $\xi_{2\alpha\beta} \Rightarrow \xi_{\alpha\beta}[j]$, $\eta_{2\alpha\beta} \Rightarrow \eta_{\alpha\beta}[j]$, $\lambda \rightarrow 1$ }) - (osol /. $v_-[k] \Rightarrow v$)]`

`Out[*]=`

0