

Pensieve header: An isolated attempt to compute the multiplication tensor of QU.

The implementation of QU is modified from pensieve://Projects/SL2Portfolio/SL2PortfolioProgram.nb.

## Initialization / Utilities

```
In[*]:=
$k = 1; $U = QU; $E := {$k};
$trim := {e^{k-} /; k > $k -> 0};
SetAttributes[{SS, SST}, HoldAll];
q_{h-} := e^{Y \epsilon^h};
(* Upper to lower and lower to Upper: *)
U2l = {B_{i-}^{p-} -> e^{-p h Y} b_i, B_{i-}^{p-} -> e^{-p h Y} b, T_{i-}^{p-} -> e^{p h t_i}, T_{i-}^{p-} -> e^{p h t}, A_{i-}^{p-} -> e^{p Y \alpha_i}, A_{i-}^{p-} -> e^{p Y \alpha}};
l2U = {e_{-}^{c-} b_{i-} + d_{-} -> B_{i-}^{-c/(h Y)} e^d, e_{-}^{c-} b + d_{-} -> B^{-c/(h Y)} e^d,
e_{-}^{c-} t_{i-} + d_{-} -> T_{i-}^{c/h} e^d, e_{-}^{c-} t + d_{-} -> T^{c/h} e^d,
e_{-}^{c-} \alpha_{i-} + d_{-} -> A_{i-}^{c/Y} e^d, e_{-}^{c-} \alpha + d_{-} -> A^{c/Y} e^d,
e_{-}^{\epsilon} -> e^{Expand@E}};
SS[E_, op_] := Collect[Normal@Series[E, {e, 0, $k}], e, op];
SS[E_] := SS[E, Together];
SST[E_, op___] := SS[E /. U2l, op];
Simp[E_, op_] := Collect[E, _CU | _QU, op];
Simp[E_] := Simp[E, SS[#, Expand] &];
SimpT[E_] := Collect[E, _CU | _QU, SST[#, Expand] &];
K\delta /: K\delta_{i-, j-} := If[i == j, 1, 0];
c_Integer_{k_Integer} := c + O[e]^{k+1};
```

```
In[*]:=
CF[E_] := ExpandDenominator@
ExpandNumerator@Together[Expand[E] /. e^{x-} e^{y-} -> e^{x+y} /. e^{x-} -> e^{CF[x]}];
```

```
In[*]:=
Unprotect[SeriesData];
SeriesData /: CF[sd_SeriesData] := MapAt[CF, sd, 3];
SeriesData /: Expand[sd_SeriesData] := MapAt[Expand, sd, 3];
SeriesData /: Simplify[sd_SeriesData] := MapAt[Simplify, sd, 3];
SeriesData /: Together[sd_SeriesData] := MapAt[Together, sd, 3];
SeriesData /: Collect[sd_SeriesData, specs___] := MapAt[Collect[#, specs] &, sd, 3];
Protect[SeriesData];
```

Self-Pair (SP):

```
In[*]:=
SP_{ }[P_] := P; SP_{E- -> x-, ps___}[P_] := Expand[P // SP_{ps}] /. f_{-} \cdot \epsilon^{d-} -> \partial_{\{x, d\}} f
```

## DeclareAlgebra

```
In[*]:= Unprotect[NonCommutativeMultiply]; Attributes[NonCommutativeMultiply] = {};
(NCM = NonCommutativeMultiply)[x_] := x;
NCM[x_, y_, z_] := (x ⓧ y) ⓧ z;
0 ⓧ _ = _ ⓧ 0 = 0;
(x_Plus) ⓧ y_ := (# ⓧ y) & /@ x; x_ ⓧ (y_Plus) := (x ⓧ #) & /@ y;
B[x_, x_] = 0; B[x_, y_] := x ⓧ y - y ⓧ x;
B[x_, y_, e_] := B[x, y, e] = B[x, y];
```

```
In[*]:= DeclareAlgebra[U_Symbol, opts__Rule] := Module[
{gp, sr, g, cp, M, pow, k = 0,
gs = Generators /. {opts},
cs = Centrals /. {opts} /. Centrals -> {}
},
U[Generators] = gs;
(#_ = U@#) & /@ gs;
gp = Alternatives @@ gs; gp = gp | gp_ (* gens *)
sr = Flatten@Table[{g -> ++k, g_i -> {i, k}}, {g, gs}]; (* sorting -> *)
cp = Alternatives @@ cs; (* cents *)
SetAttributes[M, HoldRest]; M[0, _] = 0; M[a_, x_] := a x;
CE[ε_] := Collect[ε, _U, Expand] /. e^x_ -> e^Expand[x] /. $trim;
U_i_[ε_] := ε /. {t : cp -> t_i, u_U -> (#_ &) /@ u};
U_i_[NCM[]] = pow[ε_, 0] = U@{} = 1_U = U[];
B[U@(x_)_i_, U@(y_)_i_] := U_i_@B[U@x, U@y];
B[U@(x_)_i_, U@(y_)_j_] /; i != j := 0;
B[U@y_, U@x_] := CE[-B[U@x, U@y]];
x_ ⓧ (c_. 1_U) := CE[c x]; (c_. 1_U) ⓧ x_ := CE[c x];
(a_. U[xx_, x_]) ⓧ (b_. U[y_, yy_]) := If[OrderedQ[{x, y} /. sr],
CE@M[a b /. $trim, U[xx, x, y, yy]],
U@xx ⓧ CE@M[a b /. $trim, U@y ⓧ U@x + B[U@x, U@y, $E]] ⓧ U@yy];
U@{c_. * (l : gp)^n_, r___} /; FreeQ[c, gp] := CE[c U@Table[l, {n}] ⓧ U@{r}];
U@{c_. * l : gp, r___} := CE[c U[l] ⓧ U@{r}];
U@{c_, r___} /; FreeQ[c, gp] := CE[c U@{r}];
U@{l_Plus, r___} := CE[U@{#, r} & /@ l];
U@{l_, r___} := U@{Expand[l], r};
U[ε_NonCommutativeMultiply] := U /@ ε;
0_U[specs___, poly_] := Module[{sp, null, vs, us},
sp = Replace[{specs}, l_List -> l_null, {1}];
vs = Join @@ (First /@ sp);
us = Join @@ (sp /. l_s_ -> (l /. x_i_ -> x_s));
CE[Total[
CoefficientRules[poly, vs] /. (p_ -> c_) -> c U@{us^p}
]] /. x_null -> x];
```

```

 $\mathcal{O}_U[\text{specs\_}, \mathbb{E}[L\_ , Q\_ , P\_]] := \mathcal{O}_U[\text{specs}, \text{SS@Normal}[P e^{L+Q}]];$ 
 $\text{pow}[\mathcal{E}_ , n\_ ] := \text{pow}[\mathcal{E}_ , n - 1] \otimes \mathcal{E}_ ;$ 
 $\text{S}_U[\mathcal{E}_ , \text{ss\_Rule}] := \text{CE@Total}[\text{CoefficientRules}[\mathcal{E}_ , \text{First} / @ \{\text{ss}\}] /. \text{ }]$ 
 $(p\_ \rightarrow c\_ ) \Rightarrow c \text{ NCM} @ @ \text{MapThread}[\text{pow}, \{\text{Last} / @ \{\text{ss}\}, p\}]]];$ 
 $\sigma_{rs\_} [c\_ . * u\_ U] := (c / . (t : cp)_{j\_} \Rightarrow t_{j / . \{rs\}}) U[\text{List} @ @ (u / . v_{-j\_} \Rightarrow v_{j / . \{rs\}})];$ 
 $m_{j \rightarrow k\_} [c\_ . * u\_ U] := \text{CE}[(c / . (t : cp)_{j\_} \rightarrow t_k) \text{DeleteCases}[u, \_j | k] \otimes U @ @ \text{Cases}[u, w_{-j} \Rightarrow w_k] \otimes U @ @ \text{Cases}[u, \_k]];]$ 
 $U / : c\_ . * u\_ U * v\_ U := \text{CE}[c u \otimes v];$ 
 $S_{i\_} [c\_ . * u\_ U] := \text{CE}[(c / . S_i[U, \text{Centrals}]) \text{DeleteCases}[u, \_i] \otimes U[i[\text{NCM} @ @ \text{Reverse} @ \text{Cases}[u, x_{-i} \Rightarrow S @ U @ x]]]];]$ 
 $\Delta_{i \rightarrow j\_ , k\_} [c\_ . * u\_ U] := \text{CE}[(c / . \Delta_{i \rightarrow j, k}[U, \text{Centrals}]) \text{DeleteCases}[u, \_i] \otimes (\text{NCM} @ @ \text{Cases}[u, x_{-i} \Rightarrow \sigma_{1 \rightarrow j, 2 \rightarrow k} @ \Delta @ U @ x] / . \text{NCM}[] \rightarrow U[])]];]$ 

```

$\text{In}[*]:= \text{S}_{\text{QU}}[(a^2 + a) b, a \rightarrow \text{QU}[y, x], b \rightarrow 7 \text{QU}[]]$

$\text{Out}[*]=$

$(14 - 7 T) \text{QU}[y, x] + 14 T \in \text{QU}[y, a, x] + (7 + 7 \in) \text{QU}[y, y, x, x]$

## Meta-Operations

$\text{In}[*]:=$

```

 $\sigma_{rs\_} [\mathcal{E}_{\text{Plus}}] := \sigma_{rs} / @ \mathcal{E};$ 
 $m_{j \rightarrow j\_} = \text{Identity}; m_{j \rightarrow k\_} [0] = 0;$ 
 $m_{j \rightarrow k\_} [\mathcal{E}_{\text{Plus}}] := \text{Simp}[m_{j \rightarrow k} / @ \mathcal{E};]$ 
 $m_{is\_ , i\_ , j \rightarrow k\_} [\mathcal{E}_] := m_{j \rightarrow k} @ m_{is, i \rightarrow j} @ \mathcal{E};$ 
 $S_{i\_} [\mathcal{E}_{\text{Plus}}] := \text{Simp}[S_i / @ \mathcal{E};]$ 
 $\Delta_{is\_} [\mathcal{E}_{\text{Plus}}] := \text{Simp}[\Delta_{is} / @ \mathcal{E};]$ 

```

## Implementing $\text{CU} = \mathcal{U}(\text{sl}_2^{\vee \epsilon})$

Verify  $\sigma$  and  $\Delta$ ! Also Generalize  $\Delta$  to  $\Delta_{i, j_1, j_2, \dots}$ .

$\text{In}[*]:=$

```

 $\text{DeclareAlgebra}[\text{CU}, \text{Generators} \rightarrow \{y, a, x\}, \text{Centrals} \rightarrow \{t\}];$ 
 $B[a_{\text{CU}}, y_{\text{CU}}] = -y_{\text{CU}}; B[x_{\text{CU}}, a_{\text{CU}}] = -x_{\text{CU}};$ 
 $B[x_{\text{CU}}, y_{\text{CU}}] = 2 \in a_{\text{CU}} - t 1_{\text{CU}};$ 
 $(S @ y_{\text{CU}} = -y_{\text{CU}}; S @ a_{\text{CU}} = -a_{\text{CU}}; S @ x_{\text{CU}} = -x_{\text{CU}};)$ 
 $S_{i\_} [\text{CU}, \text{Centrals}] = \{t_i \rightarrow -t_i\};$ 
 $\Delta @ y_{\text{CU}} = \text{CU} @ y_1 + \text{CU} @ y_2; \Delta @ a_{\text{CU}} = \text{CU} @ a_1 + \text{CU} @ a_2; \Delta @ x_{\text{CU}} = \text{CU} @ x_1 + \text{CU} @ x_2;$ 
 $\Delta_{i \rightarrow j\_ , k\_} [\text{CU}, \text{Centrals}] = \{t_i \rightarrow t_j + t_k\};$ 

```

## Implementing $QU = \mathcal{U}_q(\mathfrak{sl}_3^{\vee \epsilon})$

```
In[*]:=  $\hbar = \gamma = 1;$ 
DeclareAlgebra[QU, Generators  $\rightarrow \{X, Y, Z, b, a, z, y, x\}$ , Centrals  $\rightarrow \{s, t\}$ ];
B[aQU, xQU] = -2 QU@x; B[bQU, xQU] = QU@x;
B[bQU, yQU] = -2 QU@y; B[aQU, yQU] = QU@y;
B[aQU, zQU] = -QU@z; B[bQU, zQU] = -QU@z;
B[yQU, xQU] = -zQU +  $\epsilon \hbar$  QU[y, x];
B[zQU, xQU] = - $\epsilon \hbar$  QU[z, x];
B[bQU, aQU] = 0;
B[zQU, yQU] =  $\epsilon \hbar$  QU[z, y];
B[aQU, xQU] = 2 xQU; B[bQU, xQU] = -xQU;
B[aQU, yQU] = -yQU; B[bQU, yQU] = 2 yQU;
B[aQU, zQU] = zQU; B[bQU, zQU] = zQU;
B[xQU, xQU] = -2  $\epsilon \hbar$  QU@{X, x} +  $\frac{(1-s) QU[]}{\hbar}$  + 2 s  $\epsilon$  QU[a];
B[yQU, yQU] = -2  $\epsilon \hbar$  QU@{Y, y} +  $\frac{(1-t) QU[]}{\hbar}$  + 2 t  $\epsilon$  QU[b];
B[zQU, zQU] = -2  $\epsilon \hbar$  QU@{Z, z} +  $\frac{(1-s t) QU[]}{\hbar}$  + 2 s t  $\epsilon$  QU[a] + 2 s t  $\epsilon$  QU[b];
B[yQU, zQU] = xQU -  $\epsilon \hbar$  QU@{Z, y};
B[xQU, zQU] = - $\epsilon \hbar$  QU@{Z, x} - (s + s  $\epsilon \hbar$ ) QU[Y] + 2 s  $\epsilon \hbar$  QU[Y, a];
B[zQU, xQU] = 2  $\epsilon$  yQU -  $\epsilon \hbar$  QU@{X, z};
B[zQU, yQU] = -2 t  $\epsilon$  QU[x] -  $\epsilon \hbar$  QU[Y, z]; B[yQU, xQU] =  $\epsilon \hbar$  QU[X, y];
B[xQU, yQU] =  $\epsilon \hbar$  QU[Y, x];
B[xQU, yQU] = 2  $\epsilon$  zQU -  $\epsilon \hbar$  QU[X, Y];
B[xQU, zQU] =  $\epsilon \hbar$  QU[X, Z];
B[zQU, yQU] =  $\epsilon \hbar$  QU[Y, Z];
```

```
In[*]:= B[xQU, yQU]
```

```
Out[*]= QU[z] -  $\epsilon$  QU[y, x]
```

```
In[*]:= QU[x]  $\otimes$  QU[y, Z]
```

```
Out[*]= (1 - s t) QU[] + (-s + s t) QU[] + 2 s t  $\epsilon$  QU[a] + (2 s  $\epsilon$  - 2 s t  $\epsilon$ ) QU[a] + (1 - 2  $\epsilon$ ) QU[X, x] -
2 s  $\epsilon$  QU[Y, y] + (-s + 2 s  $\epsilon$ ) QU[Y, y] + (1 - 2  $\epsilon$ ) QU[Z, z] + 2 s  $\epsilon$  QU[Y, a, y] + (1 - 3  $\epsilon$ ) QU[Z, y, x]
```

## tSW

Logoi from Pensieve://Talks/Toulouse-1705/DogmaDemo.nb and from Pensieve://Talks/Sydney-1708/ExtraDetails@@.nb.

Goal. In either  $U$ , compute  $F = e^{-\eta Y} e^{\xi X} e^{\eta Y} e^{-\xi X}$ . First compute  $G = e^{\xi X} y e^{-\xi X}$ , a finite sum. Now  $F$  satisfies the ODE  $\partial_\eta F = \partial_\eta (e^{-\eta Y} e^{\eta G}) = -YF + FG$  with initial conditions  $F(\eta = 0) = 1$ . So we set it up and solve:

```

In[*]:= SWxy[U_, kk_] :=
  SWxy[U, kk] = Block[{ $U = U, $k = kk, $p = kk }, Module[{ G, F, fs, f, bs, e, b, es },
    G = Simp[Table[ξk/k!, {k, 0, $k + 1}].NestList[Simp[B[xU, #]] &, yU, $k + 1]];
    fs = Flatten@Table[f1,i,j,k[η], {1, 0, $k}, {i, 0, 1}, {j, 0, 1}, {k, 0, 1}];
    F = fs.(bs = fs /. fl,i,j,k[η] => el U @ {yi, aj, xk});
    es = Flatten[
      Table[Coefficient[e, b] == 0, {e, {F - 1U /. η → 0, F ⊗ G - yU ⊗ F - ∂η F}}, {b, bs}]];
    F = F /. DSolve[es, fs, η][[1]];
    E[0,
      ξ x + η y + (U /. {CU → -t η ξ, QU → η ξ (1 - T) / ħ}),
      F + 0$k /. {e → 1, U → Times}
    ] /. (v : η | ξ | t | T | y | a | x) → v1
  ]];
  tSWxy,i,j→k := SWxy[$U, $k] /. {ξ1 → ξi, η1 → ηj, (v : t | T | y | a | x)1 → vk};
  tSWxa,i,j→k := E[αj ak, e-γ αj ξi xk, 1];
  tSWay,i,j→k := E[αi ak, e-γ αi ηj yk, 1];

```

## Exponentials as needed.

Task. Define  $\text{Exp}_{U_i,k}[\xi, P]$  which computes  $e^{\xi \mathcal{O}(P)}$  to  $e^k$  in the algebra  $U_i$ , where  $\xi$  is a scalar,  $X$  is  $x_i$  or  $y_i$ , and  $P$  is an  $\epsilon$ -dependent near-docile element, giving the answer in  $\mathbb{E}$ -form. Should satisfy

$$U @ \text{Exp}_{U_i,k}[\xi, P] = \mathbb{S}_U[e^{\xi X}, x \rightarrow \mathcal{O}(P)].$$

Methodology. If  $P_0 := P_{\epsilon=0}$  and  $e^{\xi \mathcal{O}(P)} = \mathcal{O}(e^{\xi P_0} F(\xi))$ , then  $F(\xi = 0) = 1$  and we have:

$$\mathcal{O}(e^{\xi P_0} (P_0 F(\xi) + \partial_\xi F)) = \mathcal{O}(\partial_\xi e^{\xi P_0} F(\xi)) = \partial_\xi \mathcal{O}(e^{\xi P_0} F(\xi)) = \partial_\xi e^{\xi \mathcal{O}(P)} = e^{\xi \mathcal{O}(P)} \mathcal{O}(P) = \mathcal{O}(e^{\xi P_0} F(\xi)) \mathcal{O}(P).$$

This is an ODE for  $F$ . Setting inductively  $F_k = F_{k-1} + \epsilon^k \varphi$  we find that  $F_0 = 1$  and solve for  $\varphi$ .

```

In[*]:= (* Bug: The first line is valid only if 0(eP0) == e0(P0). *)
(* Bug: ξ must be a symbol. *)
ExpUi,0[$_, P_] := Module[{LQ = Normal@P /. ε → 0},
  E[ξ LQ /. (x | y)i → 0, ξ LQ /. (t | a)i → 0, 1]];
ExpUi,k[$_, P_] := Block[{ $U = U, $k = k },
  Module[{P0, φ, φs, F, j, rhs, at0, atξ},
    P0 = Normal@P /. ε → 0;
    φs =
      Flatten@Table[φj1,j2,j3[ξ], {j2, 0, k}, {j1, 0, 2k + 1 - j2}, {j3, 0, 2k + 1 - j2 - j1}];
    F = Normal@Last@ExpUi,k-1[$_, P] + eε k φs.(φs /. φjs[[ξ]] => Times @@ {yi, ai, xi}{js});
    rhs = Normal@
      Last@mi,j→i[E[ξ P0 /. (x | y)i → 0, ξ P0 /. (t | a)i → 0, F + 0k] mi→j@E[0, 0, P + 0k]];
    at0 = (# == 0) & /@ Flatten@CoefficientList[F - 1 /. ξ → 0, {yi, ai, xi}];
    atξ = (# == 0) & /@ Flatten@CoefficientList[(∂ξ F) + P0 F - rhs, {yi, ai, xi}];
    E[ξ P0 /. (x | y)i → 0, ξ P0 /. (t | a)i → 0, F + 0k] /.
    DSolve[And @@ (at0 ∪ atξ), φs, ξ][[1]]]

```

```

In[*]:= $k = 1
Out[*]=
1

In[*]:= $Basis = {aQU, xQU, yQU};
Module[{a, b}, Table[{a, b} → CE[B[a, b]], {a, $Basis}, {b, $Basis}]] // MatrixForm
Out[*]//MatrixForm=

$$\begin{pmatrix} \{QU[a], QU[a]\} \rightarrow 0 & \{QU[a], QU[x]\} \rightarrow \gamma QU[x] & \{QU[x], QU[a]\} \rightarrow -\gamma QU[x] & \{QU[x], QU[x]\} \rightarrow 0 & \{QU[x], QU[y]\} \rightarrow \gamma QU[y] \\ \{QU[y], QU[a]\} \rightarrow \gamma QU[y] & \{QU[y], QU[x]\} \rightarrow \left(-\frac{1}{\hbar} + \frac{T}{\hbar}\right) QU[] - 2 T \in QU[a] - \gamma \in \hbar QU[y, x] & \{QU[x], QU[y]\} \rightarrow \gamma QU[y] & \{QU[x], QU[x]\} \rightarrow 0 & \{QU[x], QU[y]\} \rightarrow \gamma QU[y] \end{pmatrix}$$


In[*]:= $Basis = {aQU, xQU, yQU};
Module[{a, b},
DeleteCases[
Flatten[Table[{a, b, c} → CE[B[a, b] + B[b, a]], {a, $Basis}, {b, $Basis}]], _ → 0]
]
Module[{a, b, c},
DeleteCases[Flatten[Table[{a, b, c} → CE[B[a, B[b, c]] + B[b, B[c, a]] + B[c, B[a, b]]],
{a, $Basis}, {b, $Basis}, {c, $Basis}]], _ → 0]
]
$ABasis = {aQU, xQU, yQU, QU[y, y], QU[y, a], QU[y, x], QU[a, a], QU[a, x], QU[x, x]};
Module[{a, b, c},
DeleteCases[Flatten[Table[{a, b, c} → CE[a ⊗ CE[b ⊗ c] - CE[a ⊗ b] ⊗ c],
{a, $ABasis}, {b, $ABasis}, {c, $ABasis}]], _ → 0]
]
Out[*]=
{}

Out[*]=
{}

Out[*]=
{}

In[*]:= Module[{a, b, c},
Table[{a, b, c} → CE[a ⊗ CE[b ⊗ c]], {a, $ABasis}, {b, $ABasis}, {c, $ABasis}]
];

In[*]:= B[xQU, yQU] // CE
Out[*]=

$$\left(\frac{1}{\hbar} - \frac{T}{\hbar}\right) QU[] + 2 T \in QU[a] + \gamma \in \hbar QU[y, x]$$


In[*]:= B[xQU, B[xQU, yQU]] // CE
Out[*]=

$$(\gamma \in - 3 T \gamma \in) QU[x]$$


In[*]:= B[xQU, B[xQU, B[xQU, yQU]] // CE
Out[*]=
0

```

```
In[*]:= B[xQU, B[xQU, B[xQU, B[xQU, yQU]]]] // CE
```

```
Out[*]=
```

```
0
```

```
In[*]:= B[xQU, B[xQU, B[xQU, B[xQU, B[xQU, yQU]]]] // Expand
```

```
Out[*]=
```

```
0
```

```
In[*]:= B[xQU, B[xQU, B[xQU, B[xQU, B[xQU, B[xQU, yQU]]]] // Expand
```

```
Out[*]=
```

```
0
```

```
In[*]:= adPower[x_, k_, Env_] [c_ * y_QU] := CE[c adPower[x, k, Env] [y]];
adPower[x_, k_, Env_] [y_Plus] := adPower[x, k, Env] /@ y;
adPower[_ , 0, Env_] [y_] := y;
adPower[x_, k_, Env_] [0] = 0;
adPower[x_, k_, Env_] [y_QU] :=
  adPower[x, k, Env] [y] = CE@B[adPower[x, k - 1, Env] [y], x];
adPower[x_, k_] [y_] := adPower[x, k, $E] [y];
```

```
In[*]:= adPower[xQU, 1] [yQU]
```

```
Out[*]=
```

```
-QU[z] + ∈ QU[y, x]
```

```
In[*]:= adPower[xQU, 2] [yQU]
```

```
Out[*]=
```

```
0
```

```
In[*]:= adPower[xQU, 3] [yQU]
```

```
Out[*]=
```

```
0
```

```
In[*]:= adPower[xQU, 4] [yQU]
```

```
Out[*]=
```

```
0
```

```
In[*]:= adPower[xQU, 5] [yQU]
```

```
Out[*]=
```

```
0
```

```
In[*]:= NilQ[zQU] = NilQ[ZQU] = NilQ[XQU] = NilQ[YQU] = NilQ[xQU] = NilQ[yQU] = True; NilQ[aQU] = False;
NilQ[bQU] = False;
PotQ[c_ * x_QU] := ¬ NilQ[x]; (* Potent Q *)
NilQ[c_ * x_QU] := NilQ[x];
PotQ[x_Plus] := And @@ (PotQ /@ (List @@ x));
NilQ[x_Plus] := And @@ (NilQ /@ (List @@ x));
```

```
In[*]:= Ad[0] = Identity;
Ad[c_. aQu][z_] := z /. w_Qu => ec Y (Sum[ $\frac{B[gQu, aQu]}{gQu}$  Count[w, g], {g, QU@Generators}]] w;
Ad[c_. bQu][z_] := z /. w_Qu => ec Y (Sum[ $\frac{B[gQu, bQu]}{gQu}$  Count[w, g], {g, QU@Generators}]] w;
Ad[c_. QU[]][z_] := z;
```

```
In[*]:= Sum[ $\frac{B[gQu, aQu]}{gQu}$  Cnt[w, g], {g, QU@Generators}]
```

```
Out[*]=
2 Cnt[w, x] - 2 Cnt[w, X] - Cnt[w, y] + Cnt[w, Y] + Cnt[w, z] - Cnt[w, Z]
```

```
In[*]:= Table[SeriesCoefficient[Ad[c aQu][gQu], {c, 0, 1}] -
  B[gQu, aQu], {g, Generators // QU}]
Table[SeriesCoefficient[Ad[c bQu][gQu], {c, 0, 1}] -
  B[gQu, bQu], {g, Generators // QU}]
```

```
Out[*]=
{0, 0, 0, 0, 0, 0, 0, 0}
```

```
Out[*]=
{0, 0, 0, 0, 0, 0, 0, 0}
```

```
In[*]:= B[XQu, aQu]
B[YQu, aQu]
B[ZQu, aQu]
```

```
Out[*]=
-2 QU[X]
```

```
Out[*]=
QU[Y]
```

```
Out[*]=
-QU[Z]
```

```
In[*]:= Ad[x_][y_] /; NilQ[x] := Expand@Module[{k = 0, s = 0},
  While[adPower[x, k][y] != 0,
    s += adPower[x, k][y] / k!;
    ++k
  ];
  s
]
```

```
In[*]:= Ad[XQu][YQu]
```

```
Out[*]=
QU[y] - QU[z] + e QU[y, x]
```

```
In[*]:= $k
```

```
Out[*]=
1
```

In[\*]:= **Ad**[**c** **a<sub>QU</sub>**] /@ {**QU**[], **x<sub>QU</sub>**, **y<sub>QU</sub>**, **QU**[**x**, **y**]}

Out[\*]=  
 $\{QU[], e^{-c \gamma} QU[x], e^{c \gamma} QU[y], QU[x, y]\}$

In[\*]:= **Ad**[**( $\alpha 1 + \alpha 2 + \epsilon fa[\lambda]$ )** **a<sub>QU</sub>**] /@ {**QU**[], **x<sub>QU</sub>**, **y<sub>QU</sub>**, **QU**[**x**, **y**]}

Out[\*]=  
 $\{QU[], e^{-\gamma (\alpha 1 + \alpha 2 + \epsilon fa[\lambda])} QU[x], e^{\gamma (\alpha 1 + \alpha 2 + \epsilon fa[\lambda])} QU[y], QU[x, y]\}$

In[\*]:= **Normal@Series**[**e** <sup>$-\gamma (\alpha 1 + \alpha 2 + \epsilon fa[\lambda])$</sup> , { **$\epsilon$** , **0**,  **$\$k$** }]

Out[\*]=  
 $e^{-(\alpha 1 + \alpha 2) \gamma} - e^{-(\alpha 1 + \alpha 2) \gamma} \gamma \epsilon fa[\lambda]$

In[\*]:= **CE**[  
**Ad**[**( $\alpha 1 + \alpha 2 + \epsilon fa[\lambda]$ )** **a<sub>QU</sub>**] /@ {**QU**[], **x<sub>QU</sub>**, **y<sub>QU</sub>**, **QU**[**x**, **y**]}/. **e** <sup>$\epsilon$</sup>  :> **Normal@Series**[**e** <sup>$\epsilon$</sup> , { **$\epsilon$** , **0**,  **$\$k$** }]]

Out[\*]=  
 $\{QU[], (e^{-\alpha 1 \gamma - \alpha 2 \gamma} - e^{-\alpha 1 \gamma - \alpha 2 \gamma} \gamma \epsilon fa[\lambda]) QU[x], (e^{\alpha 1 \gamma + \alpha 2 \gamma} + e^{\alpha 1 \gamma + \alpha 2 \gamma} \gamma \epsilon fa[\lambda]) QU[y], QU[x, y]\}$

In[\*]:= **Ad**[**a<sub>QU</sub>**] **Ad**[**x<sub>QU</sub>**] **y<sub>QU</sub>**]

Out[\*]=  
 $\epsilon QU[y] - QU[z] + \epsilon QU[y, x]$

```
In[*]:= TT_λ[] = 0; (* Translated Tangent *)
TT_λ[xs___, x_] := CE[Ad[x][TT_λ[xs]] + ∂_λ x];
TT_λ[xs_List] := TT_λ @@ xs
```

In[\*]:= **TT<sub>λ</sub>**[**λ** **y<sub>QU</sub>**, **λ** **a<sub>QU</sub>**, **λ** **x<sub>QU</sub>**]

Out[\*]=  
 $(-e^{\lambda} \lambda + e^{\lambda} T \lambda) QU[] + (1 - 2 e^{\lambda} T \epsilon \lambda) QU[a] +$   
 $\left(1 + \lambda + \frac{1}{2} e^{\lambda} \epsilon \lambda^2 - \frac{3}{2} e^{\lambda} T \epsilon \lambda^2\right) QU[x] + e^{\lambda} QU[y] - e^{\lambda} \epsilon \lambda QU[y, x]$

```
In[*]:= Seepage_U[{ }, c_] = c;
Seepage_U[ξs_List, P_] := Module[{gs = U[Generators], lx},
  lx = gs[[Length@ξs]];
  Collect[P, lx, (Ad[Last[ξs] lx_U][O_U[gs, Seepage_U[Most@ξs, #]]] /. U → Times) &]
]
```

In[\*]:= **B**[**y<sub>QU</sub>**, **α** **a<sub>QU</sub>**]

Out[\*]=  
 $\alpha QU[y]$

In[\*]:= **Seepage<sub>QU</sub>**[{**η**, **α**, **ξ**}, **y**]

Out[\*]=  
 $e^{\alpha} y - e^{\alpha} \xi + e^{\alpha} T \xi - 2 a e^{\alpha} T \epsilon \xi - e^{\alpha} x y \epsilon \xi + \frac{1}{2} e^{\alpha} x \epsilon \xi^2 - \frac{3}{2} e^{\alpha} T x \epsilon \xi^2$

$$\begin{aligned}
L_\lambda &= \mathbb{C}E(\lambda_{\mathbb{Z}_W} x) \mathbb{C}E(\lambda_{\mathbb{Z}_W} x) & \text{Seepage is defined by: } \mathbb{C}E(f_i x_i) \mathbb{O}(\text{Seepage}(\{f_i\}, A)) &= \mathbb{O}(e^{f_i x_i} A) \\
\text{TT}(L_\lambda) &= L'_\lambda \alpha_\lambda L_\lambda & S(B) &= \text{Seepage}(\{f_i\}, B). \\
R_\lambda &= \mathbb{C}E(f+g), \quad f = f_i x_i, \quad g = \sum_{i \in W} g_i w & e^f = F, \quad e^g = G, \quad R_\lambda = \mathbb{O}(FG), \quad \alpha_\lambda F = F \alpha_\lambda f \\
\text{TT}(R_\lambda) &= R'_\lambda \alpha_\lambda R_\lambda = \mathbb{O}(FG)^{-1} \mathbb{O}(\alpha_\lambda FG) = S(G)^{-1} \mathbb{O}(F) \mathbb{O}(F(\alpha_\lambda FG + \alpha_\lambda G)) = S(G)^{-1} S(\alpha_\lambda FG + \alpha_\lambda G)
\end{aligned}$$

In[\*]:= **TTL** = **TT**<sub>λ</sub>[**λ** **Ξ1** **X**<sub>QU</sub>, **λ** **Υ1** **Y**<sub>QU</sub>, **λ** **Ω1** **Z**<sub>QU</sub>, **β1** **b**<sub>QU</sub>, **α1** **a**<sub>QU</sub>, **λ** **ξ1** **z**<sub>QU</sub>, **λ** **η1** **y**<sub>QU</sub>,  
**λ** **ξ1** **x**<sub>QU</sub>, **λ** **Ξ2** **X**<sub>QU</sub>, **λ** **Υ2** **Y**<sub>QU</sub>, **λ** **Ω2** **Z**<sub>QU</sub>, **β2** **b**<sub>QU</sub>, **α2** **a**<sub>QU</sub>, **λ** **ξ2** **z**<sub>QU</sub>, **λ** **η2** **y**<sub>QU</sub>, **λ** **ξ2** **x**<sub>QU</sub>]

Out[\*]=

In[\*]:= **Total**[  
**f1**<sub>#</sub>[**λ**] **#** & /@ (**Times** @@ **#** & /@ {**QU**[**X**, **x**], **QU**[**X**, **X**], **QU**[**X**, **y**], **QU**[**X**, **Y**], **QU**[**X**, **z**], **QU**[**X**, **Z**],  
**QU**[**y**, **x**], **QU**[**Y**, **a**], **QU**[**Y**, **x**], **QU**[**Y**, **y**], **QU**[**Y**, **Y**], **QU**[**Y**, **z**], **QU**[**Y**, **Z**],  
**QU**[**z**, **x**], **QU**[**z**, **y**], **QU**[**z**, **z**], **QU**[**Z**, **x**], **QU**[**Z**, **y**], **QU**[**Z**, **Z**]})]

Out[\*]=

$$\begin{aligned}
& x X f_{1_{XX}}[\lambda] + X^2 f_{1_{X^2}}[\lambda] + x y f_{1_{xy}}[\lambda] + X y f_{1_{Xy}}[\lambda] + a Y f_{1_{aY}}[\lambda] + x Y f_{1_{xY}}[\lambda] + \\
& X Y f_{1_{XY}}[\lambda] + y Y f_{1_{yY}}[\lambda] + Y^2 f_{1_{Y^2}}[\lambda] + x z f_{1_{xz}}[\lambda] + X z f_{1_{Xz}}[\lambda] + y z f_{1_{yz}}[\lambda] + \\
& Y z f_{1_{Yz}}[\lambda] + z^2 f_{1_{z^2}}[\lambda] + x Z f_{1_{xZ}}[\lambda] + X Z f_{1_{XZ}}[\lambda] + y Z f_{1_{yZ}}[\lambda] + Y Z f_{1_{YZ}}[\lambda] + z Z f_{1_{zZ}}[\lambda]
\end{aligned}$$

In[\*]:= **f** = {**fx0**[**λ**] + **e** **fx1**[**λ**], **fy0**[**λ**] + **e** **fy1**[**λ**], **fz0**[**λ**] + **e** **fz1**[**λ**], **β1** + **β2** + **e** **fb1**[**λ**],  
**α1** + **α2** + **e** **fa1**[**λ**], **fz0**[**λ**] + **e** **fz1**[**λ**], **fy0**[**λ**] + **e** **fy1**[**λ**], **fx0**[**λ**] + **e** **fx1**[**λ**]}  
**g** = (**ft0**[**λ**] + **e** **ft1**[**λ**]) +  
**e** **Total**[**f1**<sub>#</sub>[**λ**] **#** & /@ (**Times** @@ **#** & /@ {**QU**[**X**, **x**], **QU**[**X**, **X**], **QU**[**X**, **y**], **QU**[**X**, **Y**], **QU**[**X**, **z**],  
**QU**[**X**, **Z**], **QU**[**y**, **x**], **QU**[**Y**, **a**], **QU**[**Y**, **x**], **QU**[**Y**, **y**], **QU**[**Y**, **Y**], **QU**[**Y**, **z**],  
**QU**[**Y**, **Z**], **QU**[**z**, **x**], **QU**[**z**, **y**], **QU**[**z**, **z**], **QU**[**Z**, **x**], **QU**[**Z**, **y**], **QU**[**Z**, **Z**]})]  
**G** = **SS**[**e**<sup>**g**</sup>]

Out[\*]=

$$\begin{aligned}
& \{fx0[\lambda] + e fx1[\lambda], fy0[\lambda] + e fy1[\lambda], fz0[\lambda] + e fz1[\lambda], \beta1 + \beta2 + e fb1[\lambda], \\
& \alpha1 + \alpha2 + e fa1[\lambda], fz0[\lambda] + e fz1[\lambda], fy0[\lambda] + e fy1[\lambda], fx0[\lambda] + e fx1[\lambda]\}
\end{aligned}$$

Out[\*]=

$$\begin{aligned}
& ft0[\lambda] + e ft1[\lambda] + \\
& e (x X f_{1_{XX}}[\lambda] + X^2 f_{1_{X^2}}[\lambda] + x y f_{1_{xy}}[\lambda] + X y f_{1_{Xy}}[\lambda] + a Y f_{1_{aY}}[\lambda] + x Y f_{1_{xY}}[\lambda] + X Y f_{1_{XY}}[\lambda] + \\
& y Y f_{1_{yY}}[\lambda] + Y^2 f_{1_{Y^2}}[\lambda] + x z f_{1_{xz}}[\lambda] + X z f_{1_{Xz}}[\lambda] + y z f_{1_{yz}}[\lambda] + Y z f_{1_{Yz}}[\lambda] + \\
& z^2 f_{1_{z^2}}[\lambda] + x Z f_{1_{xZ}}[\lambda] + X Z f_{1_{XZ}}[\lambda] + y Z f_{1_{yZ}}[\lambda] + Y Z f_{1_{YZ}}[\lambda] + z Z f_{1_{zZ}}[\lambda])
\end{aligned}$$

Out[\*]=

$$\begin{aligned}
& e^{ft0[\lambda]} + \\
& e^{ft0[\lambda]} e (ft1[\lambda] + x X f_{1_{XX}}[\lambda] + X^2 f_{1_{X^2}}[\lambda] + x y f_{1_{xy}}[\lambda] + X y f_{1_{Xy}}[\lambda] + a Y f_{1_{aY}}[\lambda] + x Y f_{1_{xY}}[\lambda] + \\
& X Y f_{1_{XY}}[\lambda] + y Y f_{1_{yY}}[\lambda] + Y^2 f_{1_{Y^2}}[\lambda] + x z f_{1_{xz}}[\lambda] + X z f_{1_{Xz}}[\lambda] + y z f_{1_{yz}}[\lambda] + \\
& Y z f_{1_{Yz}}[\lambda] + z^2 f_{1_{z^2}}[\lambda] + x Z f_{1_{xZ}}[\lambda] + X Z f_{1_{XZ}}[\lambda] + y Z f_{1_{yZ}}[\lambda] + Y Z f_{1_{YZ}}[\lambda] + z Z f_{1_{zZ}}[\lambda])
\end{aligned}$$

In[\*]:= **S**[**B**<sub>-</sub>] := **CE**[**SS**@**QU**[{**X**, **Y**, **Z**, **b**, **a**, **z**, **y**, **x**}, **Seepage**<sub>QU</sub>[**f**, **SS**@**B**]]];  
**S**[**G**]

Out[\*]=

$$\begin{aligned}
& e^{ft0[\lambda]} e \in QU[y, x] f_{1_{xy}}[\lambda] + e^{\alpha1 + \alpha2 - 2\beta1 - 2\beta2 + ft0[\lambda]} e \in QU[Y, a] f_{1_{aY}}[\lambda] + \\
& QU[a] (-e^{\alpha1 + \alpha2 - 2\beta1 - 2\beta2 + ft0[\lambda]} e \in fy0[\lambda] f_{1_{aY}}[\lambda] + e^{\alpha1 + \alpha2 - 2\beta1 - 2\beta2 + ft0[\lambda]} t \in fy0[\lambda] f_{1_{aY}}[\lambda]) +
\end{aligned}$$

$$\begin{aligned}
& QU[z, x] \left( -e^{ft_0[\lambda]} \in fx_0[\lambda] f1_{xy}[\lambda] + e^{ft_0[\lambda]} \in f1_{xz}[\lambda] \right) + \\
& e^{ft_0[\lambda]} \in QU[z, y] f1_{yz}[\lambda] + QU[z, z] \left( -e^{ft_0[\lambda]} \in fx_0[\lambda] f1_{yz}[\lambda] + e^{ft_0[\lambda]} \in f1_{z^2}[\lambda] \right) + \\
& e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2 + ft_0[\lambda]} \in QU[z, x] f1_{xz}[\lambda] + QU[y, x] \left( -2 e^{\alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2 + ft_0[\lambda]} \in fx_0[\lambda] f1_{ay}[\lambda] + \right. \\
& \quad \left. e^{\alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2 + ft_0[\lambda]} \in f1_{xy}[\lambda] + e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2 + ft_0[\lambda]} s \in fx_0[\lambda] f1_{xz}[\lambda] \right) + \\
& QU[x, x] \left( e^{-2\alpha_1 - 2\alpha_2 + \beta_1 + \beta_2 + ft_0[\lambda]} \in f1_{xx}[\lambda] - e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2 + ft_0[\lambda]} \in fy_0[\lambda] f1_{xz}[\lambda] \right) + \\
& QU[x] \left( -e^{-2\alpha_1 - 2\alpha_2 + \beta_1 + \beta_2 + ft_0[\lambda]} \in fx_0[\lambda] f1_{xx}[\lambda] + \right. \\
& \quad e^{-2\alpha_1 - 2\alpha_2 + \beta_1 + \beta_2 + ft_0[\lambda]} s \in fx_0[\lambda] f1_{xx}[\lambda] + 2 e^{\alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2 + ft_0[\lambda]} \in fx_0[\lambda] fy_0[\lambda] f1_{ay}[\lambda] - \\
& \quad 2 e^{\alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2 + ft_0[\lambda]} t \in fx_0[\lambda] fy_0[\lambda] f1_{ay}[\lambda] - e^{\alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2 + ft_0[\lambda]} \in fy_0[\lambda] f1_{xy}[\lambda] + \\
& \quad e^{\alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2 + ft_0[\lambda]} t \in fy_0[\lambda] f1_{xy}[\lambda] + e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2 + ft_0[\lambda]} \in fx_0[\lambda] fy_0[\lambda] f1_{xz}[\lambda] - \\
& \quad e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2 + ft_0[\lambda]} s \in fx_0[\lambda] fy_0[\lambda] f1_{xz}[\lambda] - e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2 + ft_0[\lambda]} \in fz_0[\lambda] f1_{xz}[\lambda] + \\
& \quad \left. e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2 + ft_0[\lambda]} s t \in fz_0[\lambda] f1_{xz}[\lambda] \right) + e^{-3\alpha_1 - 3\alpha_2 + ft_0[\lambda]} \in QU[X, Z] f1_{xz}[\lambda] + \\
& QU[X, X] \left( e^{-4\alpha_1 - 4\alpha_2 + 2\beta_1 + 2\beta_2 + ft_0[\lambda]} \in f1_{x^2}[\lambda] - e^{-3\alpha_1 - 3\alpha_2 + ft_0[\lambda]} \in fy_0[\lambda] f1_{xz}[\lambda] \right) + \\
& e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2 + ft_0[\lambda]} \in QU[z, y] f1_{yz}[\lambda] + \\
& QU[y, y] \left( e^{\alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2 + ft_0[\lambda]} \in fy_0[\lambda] f1_{ay}[\lambda] + \right. \\
& \quad \left. e^{\alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2 + ft_0[\lambda]} \in f1_{yy}[\lambda] + e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2 + ft_0[\lambda]} s \in fx_0[\lambda] f1_{yz}[\lambda] \right) + \\
& QU[X, y] \left( e^{-2\alpha_1 - 2\alpha_2 + \beta_1 + \beta_2 + ft_0[\lambda]} \in f1_{xy}[\lambda] - e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2 + ft_0[\lambda]} \in fy_0[\lambda] f1_{yz}[\lambda] \right) + \\
& QU[y] \left( -e^{-2\alpha_1 - 2\alpha_2 + \beta_1 + \beta_2 + ft_0[\lambda]} \in fx_0[\lambda] f1_{xy}[\lambda] + e^{-2\alpha_1 - 2\alpha_2 + \beta_1 + \beta_2 + ft_0[\lambda]} s \in fx_0[\lambda] f1_{xy}[\lambda] - \right. \\
& \quad e^{\alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2 + ft_0[\lambda]} \in fy_0[\lambda]^2 f1_{ay}[\lambda] + e^{\alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2 + ft_0[\lambda]} t \in fy_0[\lambda]^2 f1_{ay}[\lambda] - \\
& \quad e^{\alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2 + ft_0[\lambda]} \in fy_0[\lambda] f1_{yy}[\lambda] + e^{\alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2 + ft_0[\lambda]} t \in fy_0[\lambda] f1_{yy}[\lambda] + \\
& \quad e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2 + ft_0[\lambda]} \in fx_0[\lambda] fy_0[\lambda] f1_{yz}[\lambda] - e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2 + ft_0[\lambda]} s \in fx_0[\lambda] fy_0[\lambda] f1_{yz}[\lambda] - \\
& \quad \left. e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2 + ft_0[\lambda]} \in fz_0[\lambda] f1_{yz}[\lambda] + e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2 + ft_0[\lambda]} s t \in fz_0[\lambda] f1_{yz}[\lambda] \right) + \\
& e^{-3\beta_1 - 3\beta_2 + ft_0[\lambda]} \in QU[Y, Z] f1_{yz}[\lambda] + QU[Y, Y] \\
& \quad \left( e^{2\alpha_1 + 2\alpha_2 - 4\beta_1 - 4\beta_2 + ft_0[\lambda]} \in f1_{y^2}[\lambda] + e^{-3\beta_1 - 3\beta_2 + ft_0[\lambda]} s \in fx_0[\lambda] f1_{yz}[\lambda] \right) + \\
& QU[X, Y] \left( e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2 + ft_0[\lambda]} \in f1_{xy}[\lambda] + e^{-3\alpha_1 - 3\alpha_2 + ft_0[\lambda]} s \in fx_0[\lambda] f1_{xz}[\lambda] - \right. \\
& \quad \left. e^{-3\beta_1 - 3\beta_2 + ft_0[\lambda]} \in fy_0[\lambda] f1_{yz}[\lambda] \right) + \\
& QU[Z] \left( -e^{-3\alpha_1 - 3\alpha_2 + ft_0[\lambda]} \in fx_0[\lambda] f1_{xz}[\lambda] + e^{-3\alpha_1 - 3\alpha_2 + ft_0[\lambda]} s \in fx_0[\lambda] f1_{xz}[\lambda] - \right. \\
& \quad \left. e^{-3\beta_1 - 3\beta_2 + ft_0[\lambda]} \in fy_0[\lambda] f1_{yz}[\lambda] + e^{-3\beta_1 - 3\beta_2 + ft_0[\lambda]} t \in fy_0[\lambda] f1_{yz}[\lambda] \right) + \\
& QU[X] \left( -2 e^{-4\alpha_1 - 4\alpha_2 + 2\beta_1 + 2\beta_2 + ft_0[\lambda]} \in fx_0[\lambda] f1_{x^2}[\lambda] + 2 e^{-4\alpha_1 - 4\alpha_2 + 2\beta_1 + 2\beta_2 + ft_0[\lambda]} s \in fx_0[\lambda] f1_{x^2}[\lambda] - \right. \\
& \quad e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2 + ft_0[\lambda]} \in fy_0[\lambda] f1_{xy}[\lambda] + e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2 + ft_0[\lambda]} t \in fy_0[\lambda] f1_{xy}[\lambda] + \\
& \quad 2 e^{-3\alpha_1 - 3\alpha_2 + ft_0[\lambda]} \in fx_0[\lambda] fy_0[\lambda] f1_{xz}[\lambda] - 2 e^{-3\alpha_1 - 3\alpha_2 + ft_0[\lambda]} s \in fx_0[\lambda] fy_0[\lambda] f1_{xz}[\lambda] - \\
& \quad e^{-3\alpha_1 - 3\alpha_2 + ft_0[\lambda]} \in fz_0[\lambda] f1_{xz}[\lambda] + e^{-3\alpha_1 - 3\alpha_2 + ft_0[\lambda]} s t \in fz_0[\lambda] f1_{xz}[\lambda] + \\
& \quad \left. e^{-3\beta_1 - 3\beta_2 + ft_0[\lambda]} \in fy_0[\lambda]^2 f1_{yz}[\lambda] - e^{-3\beta_1 - 3\beta_2 + ft_0[\lambda]} t \in fy_0[\lambda]^2 f1_{yz}[\lambda] \right) + \\
& QU[Y] \left( -e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2 + ft_0[\lambda]} \in fx_0[\lambda] f1_{xy}[\lambda] + e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2 + ft_0[\lambda]} s \in fx_0[\lambda] f1_{xy}[\lambda] - \right. \\
& \quad 2 e^{2\alpha_1 + 2\alpha_2 - 4\beta_1 - 4\beta_2 + ft_0[\lambda]} \in fy_0[\lambda] f1_{y^2}[\lambda] + 2 e^{2\alpha_1 + 2\alpha_2 - 4\beta_1 - 4\beta_2 + ft_0[\lambda]} t \in fy_0[\lambda] f1_{y^2}[\lambda] - \\
& \quad e^{-3\alpha_1 - 3\alpha_2 + ft_0[\lambda]} s \in fx_0[\lambda]^2 f1_{xz}[\lambda] + e^{-3\alpha_1 - 3\alpha_2 + ft_0[\lambda]} s^2 \in fx_0[\lambda]^2 f1_{xz}[\lambda] + \\
& \quad e^{-3\beta_1 - 3\beta_2 + ft_0[\lambda]} \in fx_0[\lambda] fy_0[\lambda] f1_{yz}[\lambda] - 2 e^{-3\beta_1 - 3\beta_2 + ft_0[\lambda]} s \in fx_0[\lambda] fy_0[\lambda] f1_{yz}[\lambda] + \\
& \quad e^{-3\beta_1 - 3\beta_2 + ft_0[\lambda]} s t \in fx_0[\lambda] fy_0[\lambda] f1_{yz}[\lambda] - \\
& \quad \left. e^{-3\beta_1 - 3\beta_2 + ft_0[\lambda]} \in fz_0[\lambda] f1_{yz}[\lambda] + e^{-3\beta_1 - 3\beta_2 + ft_0[\lambda]} s t \in fz_0[\lambda] f1_{yz}[\lambda] \right) + \\
& QU[] \left( e^{ft_0[\lambda]} + e^{ft_0[\lambda]} \in ft_1[\lambda] + e^{-4\alpha_1 - 4\alpha_2 + 2\beta_1 + 2\beta_2 + ft_0[\lambda]} \in fx_0[\lambda]^2 f1_{x^2}[\lambda] - \right. \\
& \quad 2 e^{-4\alpha_1 - 4\alpha_2 + 2\beta_1 + 2\beta_2 + ft_0[\lambda]} s \in fx_0[\lambda]^2 f1_{x^2}[\lambda] + e^{-4\alpha_1 - 4\alpha_2 + 2\beta_1 + 2\beta_2 + ft_0[\lambda]} s^2 \in fx_0[\lambda]^2 f1_{x^2}[\lambda] + \\
& \quad e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2 + ft_0[\lambda]} \in fx_0[\lambda] fy_0[\lambda] f1_{xy}[\lambda] - e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2 + ft_0[\lambda]} s \in fx_0[\lambda] fy_0[\lambda] f1_{xy}[\lambda] - \\
& \quad e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2 + ft_0[\lambda]} t \in fx_0[\lambda] fy_0[\lambda] f1_{xy}[\lambda] + e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2 + ft_0[\lambda]} s t \in fx_0[\lambda] fy_0[\lambda] f1_{xy}[\lambda] + \\
& \quad \left. e^{2\alpha_1 + 2\alpha_2 - 4\beta_1 - 4\beta_2 + ft_0[\lambda]} \in fy_0[\lambda]^2 f1_{y^2}[\lambda] - 2 e^{2\alpha_1 + 2\alpha_2 - 4\beta_1 - 4\beta_2 + ft_0[\lambda]} t \in fy_0[\lambda]^2 f1_{y^2}[\lambda] + \right)
\end{aligned}$$

$$\begin{aligned}
& e^{2\alpha_1+2\alpha_2-4\beta_1-4\beta_2+ft_0[\lambda]} t^2 \in fy_0[\lambda]^2 f_{1_{YZ}}[\lambda] - e^{-3\alpha_1-3\alpha_2+ft_0[\lambda]} \in fx_0[\lambda]^2 fy_0[\lambda] f_{1_{XZ}}[\lambda] + \\
& 2 e^{-3\alpha_1-3\alpha_2+ft_0[\lambda]} s \in fx_0[\lambda]^2 fy_0[\lambda] f_{1_{XZ}}[\lambda] - e^{-3\alpha_1-3\alpha_2+ft_0[\lambda]} s^2 \in fx_0[\lambda]^2 fy_0[\lambda] f_{1_{XZ}}[\lambda] + \\
& e^{-3\alpha_1-3\alpha_2+ft_0[\lambda]} \in fx_0[\lambda] fz_0[\lambda] f_{1_{XZ}}[\lambda] - e^{-3\alpha_1-3\alpha_2+ft_0[\lambda]} s \in fx_0[\lambda] fz_0[\lambda] f_{1_{XZ}}[\lambda] - \\
& e^{-3\alpha_1-3\alpha_2+ft_0[\lambda]} s t \in fx_0[\lambda] fz_0[\lambda] f_{1_{XZ}}[\lambda] + e^{-3\alpha_1-3\alpha_2+ft_0[\lambda]} s^2 t \in fx_0[\lambda] fz_0[\lambda] f_{1_{XZ}}[\lambda] - \\
& e^{-3\beta_1-3\beta_2+ft_0[\lambda]} \in fx_0[\lambda] fy_0[\lambda]^2 f_{1_{YZ}}[\lambda] + e^{-3\beta_1-3\beta_2+ft_0[\lambda]} s \in fx_0[\lambda] fy_0[\lambda]^2 f_{1_{YZ}}[\lambda] + \\
& e^{-3\beta_1-3\beta_2+ft_0[\lambda]} t \in fx_0[\lambda] fy_0[\lambda]^2 f_{1_{YZ}}[\lambda] - e^{-3\beta_1-3\beta_2+ft_0[\lambda]} s t \in fx_0[\lambda] fy_0[\lambda]^2 f_{1_{YZ}}[\lambda] + \\
& e^{-3\beta_1-3\beta_2+ft_0[\lambda]} \in fy_0[\lambda] fz_0[\lambda] f_{1_{YZ}}[\lambda] - e^{-3\beta_1-3\beta_2+ft_0[\lambda]} t \in fy_0[\lambda] fz_0[\lambda] f_{1_{YZ}}[\lambda] - \\
& e^{-3\beta_1-3\beta_2+ft_0[\lambda]} s t \in fy_0[\lambda] fz_0[\lambda] f_{1_{YZ}}[\lambda] + e^{-3\beta_1-3\beta_2+ft_0[\lambda]} s t^2 \in fy_0[\lambda] fz_0[\lambda] f_{1_{YZ}}[\lambda] \Big) + \\
& QU[Z, z] \Big( -e^{-\alpha_1-\alpha_2-\beta_1-\beta_2+ft_0[\lambda]} \in fx_0[\lambda] f_{1_{YZ}}[\lambda] + e^{-\alpha_1-\alpha_2-\beta_1-\beta_2+ft_0[\lambda]} \in f_{1_{ZZ}}[\lambda] \Big) + \\
& QU[Y, z] \Big( -e^{\alpha_1+\alpha_2-2\beta_1-2\beta_2+ft_0[\lambda]} \in fx_0[\lambda] fy_0[\lambda] f_{1_{aY}}[\lambda] - e^{\alpha_1+\alpha_2-2\beta_1-2\beta_2+ft_0[\lambda]} \in fz_0[\lambda] f_{1_{aY}}[\lambda] - \\
& e^{\alpha_1+\alpha_2-2\beta_1-2\beta_2+ft_0[\lambda]} \in fx_0[\lambda] f_{1_{YV}}[\lambda] + e^{\alpha_1+\alpha_2-2\beta_1-2\beta_2+ft_0[\lambda]} \in f_{1_{YZ}}[\lambda] - \\
& e^{-\alpha_1-\alpha_2-\beta_1-\beta_2+ft_0[\lambda]} s \in fx_0[\lambda]^2 f_{1_{YZ}}[\lambda] + e^{-\alpha_1-\alpha_2-\beta_1-\beta_2+ft_0[\lambda]} s \in fx_0[\lambda] f_{1_{ZZ}}[\lambda] \Big) + \\
& QU[X, z] \Big( -e^{-2\alpha_1-2\alpha_2+\beta_1+\beta_2+ft_0[\lambda]} \in fx_0[\lambda] f_{1_{XY}}[\lambda] + e^{-2\alpha_1-2\alpha_2+\beta_1+\beta_2+ft_0[\lambda]} \in f_{1_{XZ}}[\lambda] + \\
& e^{-\alpha_1-\alpha_2-\beta_1-\beta_2+ft_0[\lambda]} \in fx_0[\lambda] fy_0[\lambda] f_{1_{YZ}}[\lambda] - e^{-\alpha_1-\alpha_2-\beta_1-\beta_2+ft_0[\lambda]} \in fy_0[\lambda] f_{1_{ZZ}}[\lambda] \Big) + \\
& QU[z] \Big( e^{-2\alpha_1-2\alpha_2+\beta_1+\beta_2+ft_0[\lambda]} \in fx_0[\lambda]^2 f_{1_{XY}}[\lambda] - e^{-2\alpha_1-2\alpha_2+\beta_1+\beta_2+ft_0[\lambda]} s \in fx_0[\lambda]^2 f_{1_{XY}}[\lambda] + \\
& e^{\alpha_1+\alpha_2-2\beta_1-2\beta_2+ft_0[\lambda]} \in fx_0[\lambda] fy_0[\lambda]^2 f_{1_{aY}}[\lambda] - e^{\alpha_1+\alpha_2-2\beta_1-2\beta_2+ft_0[\lambda]} t \in fx_0[\lambda] fy_0[\lambda]^2 f_{1_{aY}}[\lambda] + \\
& e^{\alpha_1+\alpha_2-2\beta_1-2\beta_2+ft_0[\lambda]} \in fy_0[\lambda] fz_0[\lambda] f_{1_{aY}}[\lambda] - e^{\alpha_1+\alpha_2-2\beta_1-2\beta_2+ft_0[\lambda]} t \in fy_0[\lambda] fz_0[\lambda] f_{1_{aY}}[\lambda] + \\
& e^{\alpha_1+\alpha_2-2\beta_1-2\beta_2+ft_0[\lambda]} \in fx_0[\lambda] fy_0[\lambda] f_{1_{YV}}[\lambda] - e^{\alpha_1+\alpha_2-2\beta_1-2\beta_2+ft_0[\lambda]} t \in fx_0[\lambda] fy_0[\lambda] f_{1_{YV}}[\lambda] - \\
& e^{-2\alpha_1-2\alpha_2+\beta_1+\beta_2+ft_0[\lambda]} \in fx_0[\lambda] f_{1_{XZ}}[\lambda] + e^{-2\alpha_1-2\alpha_2+\beta_1+\beta_2+ft_0[\lambda]} s \in fx_0[\lambda] f_{1_{XZ}}[\lambda] - \\
& e^{\alpha_1+\alpha_2-2\beta_1-2\beta_2+ft_0[\lambda]} \in fy_0[\lambda] f_{1_{YZ}}[\lambda] + e^{\alpha_1+\alpha_2-2\beta_1-2\beta_2+ft_0[\lambda]} t \in fy_0[\lambda] f_{1_{YZ}}[\lambda] - \\
& e^{-\alpha_1-\alpha_2-\beta_1-\beta_2+ft_0[\lambda]} \in fx_0[\lambda]^2 fy_0[\lambda] f_{1_{YZ}}[\lambda] + e^{-\alpha_1-\alpha_2-\beta_1-\beta_2+ft_0[\lambda]} s \in fx_0[\lambda]^2 fy_0[\lambda] f_{1_{YZ}}[\lambda] + \\
& e^{-\alpha_1-\alpha_2-\beta_1-\beta_2+ft_0[\lambda]} \in fx_0[\lambda] fz_0[\lambda] f_{1_{YZ}}[\lambda] - e^{-\alpha_1-\alpha_2-\beta_1-\beta_2+ft_0[\lambda]} s t \in fx_0[\lambda] fz_0[\lambda] f_{1_{YZ}}[\lambda] + \\
& e^{-\alpha_1-\alpha_2-\beta_1-\beta_2+ft_0[\lambda]} \in fx_0[\lambda] fy_0[\lambda] f_{1_{ZZ}}[\lambda] - e^{-\alpha_1-\alpha_2-\beta_1-\beta_2+ft_0[\lambda]} s \in fx_0[\lambda] fy_0[\lambda] f_{1_{ZZ}}[\lambda] - \\
& e^{-\alpha_1-\alpha_2-\beta_1-\beta_2+ft_0[\lambda]} \in fz_0[\lambda] f_{1_{ZZ}}[\lambda] + e^{-\alpha_1-\alpha_2-\beta_1-\beta_2+ft_0[\lambda]} s t \in fz_0[\lambda] f_{1_{ZZ}}[\lambda] \Big)
\end{aligned}$$

```

In[*]:= InvertU[_[E_]] := Module[{A, B, β}, (* E=A U[] + E B *)
  A = Coefficient[E, 1U] /. E -> 0;
  B = CE[E^-1 (E - A 1U)];
  SU[SS[(A + E β)^-1], β -> B]
]

```

```

In[*]:= InvertQU[S[G]]

```

```

Out[*]= (e^-ft0[λ] - e^-ft0[λ] ∈ ft1[λ]) QU[] - e^-ft0[λ] ∈ fyx1[λ] QU[y, x] + e^-ft0[λ] ∈ fx0[λ] fyx1[λ] QU[z, x]

```

```

In[*]:= CE[InvertQU[S[G]]] ⊗ S[G]

```

```

Out[*]= QU[]

```

$$\begin{aligned}
L_\lambda &= \bigoplus_{i,j} (\lambda_{\lambda_{ij}} x) \bigoplus_{i,j} (\lambda_{\lambda_{ij}} x) & \text{Seepage is defined by: } \bigoplus_{\text{off}} (f_i x_i) \bigoplus (\text{Seepage}(\{f_i\}, A)) &= \bigoplus (e^{f_i x_i} A) \\
\text{TTR}(L_\lambda) &= L'_\lambda \partial_\lambda L_\lambda & S(B) &= \text{Seepage}(\{f_i\}, B). \\
R_\lambda &= \bigoplus (f + g), \quad f = f_i x_i, \quad g = \sum_{i,j} g_{ij} w & e^f = F, \quad e^g = G, \quad R_\lambda = \bigoplus (FG), \quad \partial_\lambda F = F \partial_\lambda f \\
\text{TTR}(R_\lambda) &= R'_\lambda (\partial_\lambda R_\lambda) = \bigoplus (FG)' \bigoplus (\partial_\lambda FG) = S(G)' \bigoplus (F)' \bigoplus (F(\partial_\lambda FG + \partial_\lambda G)) = S(G)' S(\partial_\lambda FG + \partial_\lambda G).
\end{aligned}$$

In[\*]:= **G**

Out[\*]=

$$e^{ft_0[\lambda]} + e^{ft_0[\lambda]} \in (ft_1[\lambda] + x y fyx_1[\lambda])$$

In[\*]:= **( $\partial_\lambda$  (f.QU@Generators) G +  $\partial_\lambda$  G)**

Out[\*]=

$$\begin{aligned}
&e^{ft_0[\lambda]} ft_0'[\lambda] + e^{ft_0[\lambda]} \in (ft_1[\lambda] + x y fyx_1[\lambda]) ft_0'[\lambda] + (e^{ft_0[\lambda]} + e^{ft_0[\lambda]} \in (ft_1[\lambda] + x y fyx_1[\lambda])) \\
&\quad (a \in fa_1'[\lambda] + x (fx_0'[\lambda] + \epsilon fx_1'[\lambda]) + y (fy_0'[\lambda] + \epsilon fy_1'[\lambda])) + \\
&\quad e^{ft_0[\lambda]} \in (ft_1'[\lambda] + x y fyx_1'[\lambda])
\end{aligned}$$

In[\*]:= **TTR = CE[Invert<sub>QU</sub>[S[G]]]  $\otimes$  S[( $\partial_\lambda$  (f.QU@Generators) G +  $\partial_\lambda$  G)]**

Out[\*]=

In[\*]:= **Cases[TTR, x\_QU, Infinity] // Sort**

**Cases[TTL, x\_QU, Infinity] // Sort**

Out[\*]=

$$\{QU[], QU[a], QU[b], QU[x], QU[X], QU[y], QU[Y], QU[z], QU[Z], QU[X, x], QU[X, X], \\
QU[X, y], QU[X, Y], QU[X, z], QU[X, Z], QU[y, x], QU[Y, a], QU[Y, x], QU[Y, y], QU[Y, Y], \\
QU[Y, z], QU[Y, Z], QU[z, x], QU[z, y], QU[z, z], QU[Z, x], QU[Z, y], QU[Z, z]\}$$

Out[\*]=

$$\{QU[], QU[a], QU[b], QU[x], QU[X], QU[y], QU[Y], QU[z], QU[Z], QU[X, x], QU[X, X], \\
QU[X, y], QU[X, Y], QU[X, z], QU[X, Z], QU[y, x], QU[Y, a], QU[Y, x], QU[Y, y], QU[Y, Y], \\
QU[Y, z], QU[Y, Z], QU[z, x], QU[z, y], QU[z, z], QU[Z, x], QU[Z, y], QU[Z, z]\}$$

In[\*]:= **diff = CE[TTL - TTR]**

Out[\*]=

In[\*]:= **diff** /. **ε** → 0

Out[\*]=

$$\begin{aligned}
& \text{QU}[x] \left( e^{2\alpha 2 - \beta 2} \xi 1 + \xi 2 - \text{fx0}'[\lambda] \right) + \text{QU}[y] \left( e^{-\alpha 2 + 2\beta 2} \eta 1 + \eta 2 - \text{fy0}'[\lambda] \right) + \\
& \text{QU}[z] \left( e^{\alpha 2 + \beta 2} \zeta 1 + \zeta 2 - e^{\alpha 2 + \beta 2} \eta 1 \lambda \xi 1 + e^{2\alpha 2 - \beta 2} \eta 2 \lambda \xi 1 - e^{-\alpha 2 + 2\beta 2} \eta 1 \lambda \xi 2 - \eta 2 \lambda \xi 2 + \right. \\
& \quad \left. \text{fx0}[\lambda] \text{fy0}'[\lambda] - \text{fz0}'[\lambda] \right) + \text{QU}[Z] \left( e^{-\alpha 1 - \alpha 2 - \beta 1 - \beta 2} \Omega 1 + e^{-\alpha 2 - \beta 2} \Omega 2 - e^{-\alpha 1 - \alpha 2 - \beta 1 - \beta 2} \text{fz0}'[\lambda] \right) + \\
& \text{QU}[Y] \left( e^{\alpha 1 + \alpha 2 - 2\beta 1 - 2\beta 2} \Upsilon 1 + e^{\alpha 2 - 2\beta 2} \Upsilon 2 + e^{-\alpha 1 + \alpha 2 - \beta 1 - 2\beta 2} s \lambda \xi 1 \Omega 1 + e^{-\alpha 1 - \alpha 2 - \beta 1 - \beta 2} s \lambda \xi 2 \Omega 1 - \right. \\
& \quad \left. e^{\alpha 2 - 2\beta 2} s \lambda \xi 1 \Omega 2 + e^{-\alpha 2 - \beta 2} s \lambda \xi 2 \Omega 2 - e^{\alpha 1 + \alpha 2 - 2\beta 1 - 2\beta 2} \text{fy0}'[\lambda] - e^{-\alpha 1 - \alpha 2 - \beta 1 - \beta 2} s \text{fx0}[\lambda] \text{fz0}'[\lambda] \right) + \\
& \text{QU}[X] \left( e^{-2\alpha 1 - 2\alpha 2 + \beta 1 + \beta 2} \Xi 1 + e^{-2\alpha 2 + \beta 2} \Xi 2 - e^{-\alpha 1 - 2\alpha 2 - \beta 1 + \beta 2} \eta 1 \lambda \Omega 1 - e^{-\alpha 1 - \alpha 2 - \beta 1 - \beta 2} \eta 2 \lambda \Omega 1 + \right. \\
& \quad \left. e^{-2\alpha 2 + \beta 2} \eta 1 \lambda \Omega 2 - e^{-\alpha 2 - \beta 2} \eta 2 \lambda \Omega 2 - e^{-2\alpha 1 - 2\alpha 2 + \beta 1 + \beta 2} \text{fx0}'[\lambda] + e^{-\alpha 1 - \alpha 2 - \beta 1 - \beta 2} \text{fy0}[\lambda] \text{fz0}'[\lambda] \right) + \\
& \text{QU}[] \left( -e^{-2\alpha 1 + \beta 1} \lambda \xi 1 \Xi 1 + e^{-2\alpha 1 + \beta 1} s \lambda \xi 1 \Xi 1 - e^{-2\alpha 1 - 2\alpha 2 + \beta 1 + \beta 2} \lambda \Xi 1 \xi 2 + e^{-2\alpha 1 - 2\alpha 2 + \beta 1 + \beta 2} s \lambda \Xi 1 \xi 2 + \right. \\
& \quad \lambda \xi 1 \Xi 2 - s \lambda \xi 1 \Xi 2 - e^{-2\alpha 2 + \beta 2} \lambda \xi 2 \Xi 2 + e^{-2\alpha 2 + \beta 2} s \lambda \xi 2 \Xi 2 - e^{\alpha 1 - 2\beta 1} \eta 1 \lambda \Upsilon 1 + \\
& \quad e^{\alpha 1 - 2\beta 1} t \eta 1 \lambda \Upsilon 1 - e^{\alpha 1 + \alpha 2 - 2\beta 1 - 2\beta 2} \eta 2 \lambda \Upsilon 1 + e^{\alpha 1 + \alpha 2 - 2\beta 1 - 2\beta 2} t \eta 2 \lambda \Upsilon 1 + \eta 1 \lambda \Upsilon 2 - \\
& \quad t \eta 1 \lambda \Upsilon 2 - e^{\alpha 2 - 2\beta 2} \eta 2 \lambda \Upsilon 2 + e^{\alpha 2 - 2\beta 2} t \eta 2 \lambda \Upsilon 2 - e^{-\alpha 1 - \beta 1} \zeta 1 \lambda \Omega 1 + e^{-\alpha 1 - \beta 1} s t \zeta 1 \lambda \Omega 1 - \\
& \quad e^{-\alpha 1 - \alpha 2 - \beta 1 - \beta 2} \zeta 2 \lambda \Omega 1 + e^{-\alpha 1 - \alpha 2 - \beta 1 - \beta 2} s t \zeta 2 \lambda \Omega 1 + e^{-\alpha 1 - \beta 1} \eta 1 \lambda^2 \xi 1 \Omega 1 - e^{-\alpha 1 - \beta 1} s \eta 1 \lambda^2 \xi 1 \Omega 1 - \\
& \quad e^{-\alpha 1 + \alpha 2 - \beta 1 - 2\beta 2} s \eta 2 \lambda^2 \xi 1 \Omega 1 + e^{-\alpha 1 + \alpha 2 - \beta 1 - 2\beta 2} s t \eta 2 \lambda^2 \xi 1 \Omega 1 + e^{-\alpha 1 - 2\alpha 2 - \beta 1 + \beta 2} \eta 1 \lambda^2 \xi 2 \Omega 1 - \\
& \quad e^{-\alpha 1 - 2\alpha 2 - \beta 1 + \beta 2} s \eta 1 \lambda^2 \xi 2 \Omega 1 + e^{-\alpha 1 - \alpha 2 - \beta 1 - \beta 2} \eta 2 \lambda^2 \xi 2 \Omega 1 - e^{-\alpha 1 - \alpha 2 - \beta 1 - \beta 2} s \eta 2 \lambda^2 \xi 2 \Omega 1 + \\
& \quad \zeta 1 \lambda \Omega 2 - s t \zeta 1 \lambda \Omega 2 - e^{-\alpha 2 - \beta 2} \zeta 2 \lambda \Omega 2 + e^{-\alpha 2 - \beta 2} s t \zeta 2 \lambda \Omega 2 - \eta 1 \lambda^2 \xi 1 \Omega 2 + s t \eta 1 \lambda^2 \xi 1 \Omega 2 + \\
& \quad e^{\alpha 2 - 2\beta 2} s \eta 2 \lambda^2 \xi 1 \Omega 2 - e^{\alpha 2 - 2\beta 2} s t \eta 2 \lambda^2 \xi 1 \Omega 2 - e^{-2\alpha 2 + \beta 2} \eta 1 \lambda^2 \xi 2 \Omega 2 + e^{-2\alpha 2 + \beta 2} s \eta 1 \lambda^2 \xi 2 \Omega 2 + \\
& \quad e^{-\alpha 2 - \beta 2} \eta 2 \lambda^2 \xi 2 \Omega 2 - e^{-\alpha 2 - \beta 2} s \eta 2 \lambda^2 \xi 2 \Omega 2 - \text{ft0}'[\lambda] + e^{-2\alpha 1 - 2\alpha 2 + \beta 1 + \beta 2} \text{fx0}[\lambda] \text{fx0}'[\lambda] - \\
& \quad e^{-2\alpha 1 - 2\alpha 2 + \beta 1 + \beta 2} s \text{fx0}[\lambda] \text{fx0}'[\lambda] + e^{\alpha 1 + \alpha 2 - 2\beta 1 - 2\beta 2} \text{fy0}[\lambda] \text{fy0}'[\lambda] - e^{\alpha 1 + \alpha 2 - 2\beta 1 - 2\beta 2} t \text{fy0}[\lambda] \text{fy0}'[\lambda] - \\
& \quad e^{-\alpha 1 - \alpha 2 - \beta 1 - \beta 2} \text{fx0}[\lambda] \text{fy0}[\lambda] \text{fz0}'[\lambda] + e^{-\alpha 1 - \alpha 2 - \beta 1 - \beta 2} s \text{fx0}[\lambda] \text{fy0}[\lambda] \text{fz0}'[\lambda] + \\
& \quad e^{-\alpha 1 - \alpha 2 - \beta 1 - \beta 2} \text{fz0}[\lambda] \text{fz0}'[\lambda] - e^{-\alpha 1 - \alpha 2 - \beta 1 - \beta 2} s t \text{fz0}[\lambda] \text{fz0}'[\lambda] \Big)
\end{aligned}$$

In[\*]:= **Generators** // **QU**

Out[\*]=

{X, Y, Z, b, a, z, y, x}

In[\*]:= **{sol1} = DSolve[Coefficient[diff /. ε → 0, x<sub>QU</sub>] == 0 ∧ fx0[0] == 0, fx0, λ]**

Out[\*]=

{{fx0 → Function[{λ], e<sup>-β<sup>2</sup></sup> λ (e<sup>2α<sup>2</sup></sup> ξ<sub>1</sub> + e<sup>β<sup>2</sup></sup> ξ<sub>2</sub>) ]}}

In[\*]:= **CE[diff /. sol1 /.  $\epsilon \rightarrow 0$ ]**

Out[\*]=

$$\begin{aligned}
& \text{QU}[y] \left( e^{-\alpha 2 + 2 \beta 2} \eta 1 + \eta 2 - \text{fy}0'[\lambda] \right) + \\
& \text{QU}[z] \left( e^{\alpha 2 + \beta 2} \zeta 1 + \zeta 2 - e^{\alpha 2 + \beta 2} \eta 1 \lambda \xi 1 + e^{2 \alpha 2 - \beta 2} \eta 2 \lambda \xi 1 - e^{-\alpha 2 + 2 \beta 2} \eta 1 \lambda \xi 2 - \right. \\
& \quad \left. \eta 2 \lambda \xi 2 + e^{2 \alpha 2 - \beta 2} \lambda \xi 1 \text{fy}0'[\lambda] + \lambda \xi 2 \text{fy}0'[\lambda] - \text{fz}0'[\lambda] \right) + \\
& \text{QU}[Z] \left( e^{-\alpha 1 - \alpha 2 - \beta 1 - \beta 2} \Omega 1 + e^{-\alpha 2 - \beta 2} \Omega 2 - e^{-\alpha 1 - \alpha 2 - \beta 1 - \beta 2} \text{fZ}0'[\lambda] \right) + \\
& \text{QU}[Y] \left( e^{\alpha 1 + \alpha 2 - 2 \beta 1 - 2 \beta 2} \Upsilon 1 + e^{\alpha 2 - 2 \beta 2} \Upsilon 2 + e^{-\alpha 1 + \alpha 2 - \beta 1 - 2 \beta 2} s \lambda \xi 1 \Omega 1 + \right. \\
& \quad e^{-\alpha 1 - \alpha 2 - \beta 1 - \beta 2} s \lambda \xi 2 \Omega 1 - e^{\alpha 2 - 2 \beta 2} s \lambda \xi 1 \Omega 2 + e^{-\alpha 2 - \beta 2} s \lambda \xi 2 \Omega 2 - e^{\alpha 1 + \alpha 2 - 2 \beta 1 - 2 \beta 2} \text{fY}0'[\lambda] - \\
& \quad \left. e^{-\alpha 1 + \alpha 2 - \beta 1 - 2 \beta 2} s \lambda \xi 1 \text{fZ}0'[\lambda] - e^{-\alpha 1 - \alpha 2 - \beta 1 - \beta 2} s \lambda \xi 2 \text{fZ}0'[\lambda] \right) + \\
& \text{QU}[X] \left( e^{-2 \alpha 1 - 2 \alpha 2 + \beta 1 + \beta 2} \Xi 1 + e^{-2 \alpha 2 + \beta 2} \Xi 2 - e^{-\alpha 1 - 2 \alpha 2 - \beta 1 + \beta 2} \eta 1 \lambda \Omega 1 - e^{-\alpha 1 - \alpha 2 - \beta 1 - \beta 2} \eta 2 \lambda \Omega 1 + \right. \\
& \quad \left. e^{-2 \alpha 2 + \beta 2} \eta 1 \lambda \Omega 2 - e^{-\alpha 2 - \beta 2} \eta 2 \lambda \Omega 2 - e^{-2 \alpha 1 - 2 \alpha 2 + \beta 1 + \beta 2} \text{fX}0'[\lambda] + e^{-\alpha 1 - \alpha 2 - \beta 1 - \beta 2} \text{fy}0[\lambda] \text{fZ}0'[\lambda] \right) + \\
& \text{QU}[] \left( -e^{-2 \alpha 1 + \beta 1} \lambda \xi 1 \Xi 1 + e^{-2 \alpha 1 + \beta 1} s \lambda \xi 1 \Xi 1 - e^{-2 \alpha 1 - 2 \alpha 2 + \beta 1 + \beta 2} \lambda \Xi 1 \xi 2 + e^{-2 \alpha 1 - 2 \alpha 2 + \beta 1 + \beta 2} s \lambda \Xi 1 \xi 2 + \right. \\
& \quad \lambda \xi 1 \Xi 2 - s \lambda \xi 1 \Xi 2 - e^{-2 \alpha 2 + \beta 2} \lambda \xi 2 \Xi 2 + e^{-2 \alpha 2 + \beta 2} s \lambda \xi 2 \Xi 2 - e^{\alpha 1 - 2 \beta 1} \eta 1 \lambda \Upsilon 1 + e^{\alpha 1 - 2 \beta 1} t \eta 1 \lambda \Upsilon 1 - \\
& \quad e^{\alpha 1 + \alpha 2 - 2 \beta 1 - 2 \beta 2} \eta 2 \lambda \Upsilon 1 + e^{\alpha 1 + \alpha 2 - 2 \beta 1 - 2 \beta 2} t \eta 2 \lambda \Upsilon 1 + \eta 1 \lambda \Upsilon 2 - t \eta 1 \lambda \Upsilon 2 - e^{\alpha 2 - 2 \beta 2} \eta 2 \lambda \Upsilon 2 + \\
& \quad e^{\alpha 2 - 2 \beta 2} t \eta 2 \lambda \Upsilon 2 - e^{-\alpha 1 - \beta 1} \zeta 1 \lambda \Omega 1 + e^{-\alpha 1 - \beta 1} s t \zeta 1 \lambda \Omega 1 - e^{-\alpha 1 - \alpha 2 - \beta 1 - \beta 2} \zeta 2 \lambda \Omega 1 + \\
& \quad e^{-\alpha 1 - \alpha 2 - \beta 1 - \beta 2} s t \zeta 2 \lambda \Omega 1 + e^{-\alpha 1 - \beta 1} \eta 1 \lambda^2 \xi 1 \Omega 1 - e^{-\alpha 1 - \beta 1} s \eta 1 \lambda^2 \xi 1 \Omega 1 - e^{-\alpha 1 + \alpha 2 - \beta 1 - 2 \beta 2} s \eta 2 \lambda^2 \xi 1 \Omega 1 + \\
& \quad e^{-\alpha 1 + \alpha 2 - \beta 1 - 2 \beta 2} s t \eta 2 \lambda^2 \xi 1 \Omega 1 + e^{-\alpha 1 - 2 \alpha 2 - \beta 1 + \beta 2} \eta 1 \lambda^2 \xi 2 \Omega 1 - e^{-\alpha 1 - 2 \alpha 2 - \beta 1 + \beta 2} s \eta 1 \lambda^2 \xi 2 \Omega 1 + \\
& \quad e^{-\alpha 1 - \alpha 2 - \beta 1 - \beta 2} \eta 2 \lambda^2 \xi 2 \Omega 1 - e^{-\alpha 1 - \alpha 2 - \beta 1 - \beta 2} s \eta 2 \lambda^2 \xi 2 \Omega 1 + \zeta 1 \lambda \Omega 2 - s t \zeta 1 \lambda \Omega 2 - \\
& \quad e^{-\alpha 2 - \beta 2} \zeta 2 \lambda \Omega 2 + e^{-\alpha 2 - \beta 2} s t \zeta 2 \lambda \Omega 2 - \eta 1 \lambda^2 \xi 1 \Omega 2 + s t \eta 1 \lambda^2 \xi 1 \Omega 2 + e^{\alpha 2 - 2 \beta 2} s \eta 2 \lambda^2 \xi 1 \Omega 2 - \\
& \quad e^{\alpha 2 - 2 \beta 2} s t \eta 2 \lambda^2 \xi 1 \Omega 2 - e^{-2 \alpha 2 + \beta 2} \eta 1 \lambda^2 \xi 2 \Omega 2 + e^{-2 \alpha 2 + \beta 2} s \eta 1 \lambda^2 \xi 2 \Omega 2 + e^{-\alpha 2 - \beta 2} \eta 2 \lambda^2 \xi 2 \Omega 2 - \\
& \quad e^{-\alpha 2 - \beta 2} s \eta 2 \lambda^2 \xi 2 \Omega 2 - \text{ft}0'[\lambda] + e^{-2 \alpha 1 + \beta 1} \lambda \xi 1 \text{fX}0'[\lambda] - e^{-2 \alpha 1 + \beta 1} s \lambda \xi 1 \text{fX}0'[\lambda] + \\
& \quad e^{-2 \alpha 1 - 2 \alpha 2 + \beta 1 + \beta 2} \lambda \xi 2 \text{fX}0'[\lambda] - e^{-2 \alpha 1 - 2 \alpha 2 + \beta 1 + \beta 2} s \lambda \xi 2 \text{fX}0'[\lambda] + e^{\alpha 1 + \alpha 2 - 2 \beta 1 - 2 \beta 2} \text{fy}0[\lambda] \text{fY}0'[\lambda] - \\
& \quad e^{\alpha 1 + \alpha 2 - 2 \beta 1 - 2 \beta 2} t \text{fy}0[\lambda] \text{fY}0'[\lambda] - e^{-\alpha 1 + \alpha 2 - \beta 1 - 2 \beta 2} \lambda \xi 1 \text{fy}0[\lambda] \text{fZ}0'[\lambda] + \\
& \quad e^{-\alpha 1 + \alpha 2 - \beta 1 - 2 \beta 2} s \lambda \xi 1 \text{fy}0[\lambda] \text{fZ}0'[\lambda] - e^{-\alpha 1 - \alpha 2 - \beta 1 - \beta 2} \lambda \xi 2 \text{fy}0[\lambda] \text{fZ}0'[\lambda] + \\
& \quad e^{-\alpha 1 - \alpha 2 - \beta 1 - \beta 2} s \lambda \xi 2 \text{fy}0[\lambda] \text{fZ}0'[\lambda] + e^{-\alpha 1 - \alpha 2 - \beta 1 - \beta 2} \text{fz}0[\lambda] \text{fZ}0'[\lambda] - e^{-\alpha 1 - \alpha 2 - \beta 1 - \beta 2} s t \text{fz}0[\lambda] \text{fZ}0'[\lambda] \Big)
\end{aligned}$$

In[\*]:= **{sol2} = DSolve[Coefficient[CE[diff /. sol1 /.  $\epsilon \rightarrow 0$ ],  $y_{qu}$ ] == 0  $\wedge$  fy0[0] == 0, fy0,  $\lambda$ ]**

Out[\*]=

$$\left\{ \left\{ \text{fy}0 \rightarrow \text{Function}[\{\lambda\}, e^{-\alpha 2} (e^{2 \beta 2} \eta 1 + e^{\alpha 2} \eta 2) \lambda] \right\} \right\}$$

In[\*]:= **CE[diff /. sol1 /. sol2 /.  $\epsilon \rightarrow 0$ ]**

Out[\*]=

$$\begin{aligned}
& \text{QU}[z] \left( e^{\alpha_2 + \beta_2} \zeta_1 + \zeta_2 + 2 e^{2\alpha_2 - \beta_2} \eta_2 \lambda \xi_1 - f_{z0}'[\lambda] \right) + \\
& \text{QU}[z] \left( e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2} \Omega_1 + e^{-\alpha_2 - \beta_2} \Omega_2 - e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2} f_{z0}'[\lambda] \right) + \\
& \text{QU}[x] \left( e^{-2\alpha_1 - 2\alpha_2 + \beta_1 + \beta_2} \Xi_1 + e^{-2\alpha_2 + \beta_2} \Xi_2 - e^{-\alpha_1 - 2\alpha_2 - \beta_1 + \beta_2} \eta_1 \lambda \Omega_1 - e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2} \eta_2 \lambda \Omega_1 + e^{-2\alpha_2 + \beta_2} \eta_1 \lambda \Omega_2 - \right. \\
& \quad \left. e^{-\alpha_2 - \beta_2} \eta_2 \lambda \Omega_2 - e^{-2\alpha_1 - 2\alpha_2 + \beta_1 + \beta_2} f_{x0}'[\lambda] + e^{-\alpha_1 - 2\alpha_2 - \beta_1 + \beta_2} \eta_1 \lambda f_{z0}'[\lambda] + e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2} \eta_2 \lambda f_{z0}'[\lambda] \right) + \\
& \text{QU}[y] \left( e^{\alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2} \Upsilon_1 + e^{\alpha_2 - 2\beta_2} \Upsilon_2 + e^{-\alpha_1 + \alpha_2 - \beta_1 - 2\beta_2} s \lambda \xi_1 \Omega_1 + e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2} s \lambda \xi_2 \Omega_1 - \right. \\
& \quad \left. e^{\alpha_2 - 2\beta_2} s \lambda \xi_1 \Omega_2 + e^{-\alpha_2 - \beta_2} s \lambda \xi_2 \Omega_2 - e^{\alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2} f_{y0}'[\lambda] - \right. \\
& \quad \left. e^{-\alpha_1 + \alpha_2 - \beta_1 - 2\beta_2} s \lambda \xi_1 f_{z0}'[\lambda] - e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2} s \lambda \xi_2 f_{z0}'[\lambda] \right) + \\
& \text{QU}[] \left( -e^{-2\alpha_1 + \beta_1} \lambda \xi_1 \Xi_1 + e^{-2\alpha_1 + \beta_1} s \lambda \xi_1 \Xi_1 - e^{-2\alpha_1 - 2\alpha_2 + \beta_1 + \beta_2} \lambda \Xi_1 \xi_2 + e^{-2\alpha_1 - 2\alpha_2 + \beta_1 + \beta_2} s \lambda \Xi_1 \xi_2 + \right. \\
& \quad \lambda \xi_1 \Xi_2 - s \lambda \xi_1 \Xi_2 - e^{-2\alpha_2 + \beta_2} \lambda \xi_2 \Xi_2 + e^{-2\alpha_2 + \beta_2} s \lambda \xi_2 \Xi_2 - e^{\alpha_1 - 2\beta_1} \eta_1 \lambda \Upsilon_1 + e^{\alpha_1 - 2\beta_1} t \eta_1 \lambda \Upsilon_1 - \\
& \quad e^{\alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2} \eta_2 \lambda \Upsilon_1 + e^{\alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2} t \eta_2 \lambda \Upsilon_1 + \eta_1 \lambda \Upsilon_2 - t \eta_1 \lambda \Upsilon_2 - e^{\alpha_2 - 2\beta_2} \eta_2 \lambda \Upsilon_2 + \\
& \quad e^{\alpha_2 - 2\beta_2} t \eta_2 \lambda \Upsilon_2 - e^{-\alpha_1 - \beta_1} \zeta_1 \lambda \Omega_1 + e^{-\alpha_1 - \beta_1} s t \zeta_1 \lambda \Omega_1 - e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2} \zeta_2 \lambda \Omega_1 + \\
& \quad e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2} s t \zeta_2 \lambda \Omega_1 + e^{-\alpha_1 - \beta_1} \eta_1 \lambda^2 \xi_1 \Omega_1 - e^{-\alpha_1 - \beta_1} s \eta_1 \lambda^2 \xi_1 \Omega_1 - e^{-\alpha_1 + \alpha_2 - \beta_1 - 2\beta_2} s \eta_2 \lambda^2 \xi_1 \Omega_1 + \\
& \quad e^{-\alpha_1 + \alpha_2 - \beta_1 - 2\beta_2} s t \eta_2 \lambda^2 \xi_1 \Omega_1 + e^{-\alpha_1 - 2\alpha_2 - \beta_1 + \beta_2} \eta_1 \lambda^2 \xi_2 \Omega_1 - e^{-\alpha_1 - 2\alpha_2 - \beta_1 + \beta_2} s \eta_1 \lambda^2 \xi_2 \Omega_1 + \\
& \quad e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2} \eta_2 \lambda^2 \xi_2 \Omega_1 - e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2} s \eta_2 \lambda^2 \xi_2 \Omega_1 + \zeta_1 \lambda \Omega_2 - s t \zeta_1 \lambda \Omega_2 - e^{-\alpha_2 - \beta_2} \zeta_2 \lambda \Omega_2 + \\
& \quad e^{-\alpha_2 - \beta_2} s t \zeta_2 \lambda \Omega_2 - \eta_1 \lambda^2 \xi_1 \Omega_2 + s t \eta_1 \lambda^2 \xi_1 \Omega_2 + e^{\alpha_2 - 2\beta_2} s \eta_2 \lambda^2 \xi_1 \Omega_2 - e^{\alpha_2 - 2\beta_2} s t \eta_2 \lambda^2 \xi_1 \Omega_2 - \\
& \quad e^{-2\alpha_2 + \beta_2} \eta_1 \lambda^2 \xi_2 \Omega_2 + e^{-2\alpha_2 + \beta_2} s \eta_1 \lambda^2 \xi_2 \Omega_2 + e^{-\alpha_2 - \beta_2} \eta_2 \lambda^2 \xi_2 \Omega_2 - e^{-\alpha_2 - \beta_2} s \eta_2 \lambda^2 \xi_2 \Omega_2 - \\
& \quad f_{t0}'[\lambda] + e^{-2\alpha_1 + \beta_1} \lambda \xi_1 f_{x0}'[\lambda] - e^{-2\alpha_1 + \beta_1} s \lambda \xi_1 f_{x0}'[\lambda] + e^{-2\alpha_1 - 2\alpha_2 + \beta_1 + \beta_2} \lambda \xi_2 f_{x0}'[\lambda] - \\
& \quad e^{-2\alpha_1 - 2\alpha_2 + \beta_1 + \beta_2} s \lambda \xi_2 f_{x0}'[\lambda] + e^{\alpha_1 - 2\beta_1} \eta_1 \lambda f_{y0}'[\lambda] - e^{\alpha_1 - 2\beta_1} t \eta_1 \lambda f_{y0}'[\lambda] + \\
& \quad e^{\alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2} \eta_2 \lambda f_{y0}'[\lambda] - e^{\alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2} t \eta_2 \lambda f_{y0}'[\lambda] - e^{-\alpha_1 - \beta_1} \eta_1 \lambda^2 \xi_1 f_{z0}'[\lambda] + \\
& \quad e^{-\alpha_1 - \beta_1} s \eta_1 \lambda^2 \xi_1 f_{z0}'[\lambda] - e^{-\alpha_1 + \alpha_2 - \beta_1 - 2\beta_2} \eta_2 \lambda^2 \xi_1 f_{z0}'[\lambda] + e^{-\alpha_1 + \alpha_2 - \beta_1 - 2\beta_2} s \eta_2 \lambda^2 \xi_1 f_{z0}'[\lambda] - \\
& \quad e^{-\alpha_1 - 2\alpha_2 - \beta_1 + \beta_2} \eta_1 \lambda^2 \xi_2 f_{z0}'[\lambda] + e^{-\alpha_1 - 2\alpha_2 - \beta_1 + \beta_2} s \eta_1 \lambda^2 \xi_2 f_{z0}'[\lambda] - e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2} \eta_2 \lambda^2 \xi_2 f_{z0}'[\lambda] + \\
& \quad e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2} s \eta_2 \lambda^2 \xi_2 f_{z0}'[\lambda] + e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2} f_{z0}[\lambda] f_{z0}'[\lambda] - e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2} s t f_{z0}[\lambda] f_{z0}'[\lambda] \Big)
\end{aligned}$$

In[\*]:= **{sol3} = DSolve[Coefficient[CE[diff /. sol1 /. sol2 /.  $\epsilon \rightarrow 0$ ], QU[z]] == 0  $\wedge$  fz0[0] == 0, fz0,  $\lambda$ ]**

Out[\*]=

$$\left\{ \left\{ f_{z0} \rightarrow \text{Function} \left[ \{ \lambda \}, e^{-\beta_2} \lambda \left( e^{\alpha_2 + 2\beta_2} \zeta_1 + e^{\beta_2} \zeta_2 + e^{2\alpha_2} \eta_2 \lambda \xi_1 \right) \right] \right\} \right\}$$

In[\*]:= **CE**[diff /. sol1 /. sol2 /. sol3] /.  $\epsilon \rightarrow 0$

Out[\*]=

$$\begin{aligned} & \text{QU}[Z] \left( e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2} \Omega_1 + e^{-\alpha_2 - \beta_2} \Omega_2 - e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2} fZ0'[\lambda] \right) + \\ & \text{QU}[X] \left( e^{-2\alpha_1 - 2\alpha_2 + \beta_1 + \beta_2} \Xi_1 + e^{-2\alpha_2 + \beta_2} \Xi_2 - e^{-\alpha_1 - 2\alpha_2 - \beta_1 + \beta_2} \eta_1 \lambda \Omega_1 - e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2} \eta_2 \lambda \Omega_1 + e^{-2\alpha_2 + \beta_2} \eta_1 \lambda \Omega_2 - \right. \\ & \quad \left. e^{-\alpha_2 - \beta_2} \eta_2 \lambda \Omega_2 - e^{-2\alpha_1 - 2\alpha_2 + \beta_1 + \beta_2} fX0'[\lambda] + e^{-\alpha_1 - 2\alpha_2 - \beta_1 + \beta_2} \eta_1 \lambda fZ0'[\lambda] + e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2} \eta_2 \lambda fZ0'[\lambda] \right) + \\ & \text{QU}[Y] \left( e^{\alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2} \Upsilon_1 + e^{\alpha_2 - 2\beta_2} \Upsilon_2 + e^{-\alpha_1 + \alpha_2 - \beta_1 - 2\beta_2} s \lambda \xi_1 \Omega_1 + e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2} s \lambda \xi_2 \Omega_1 - \right. \\ & \quad \left. e^{\alpha_2 - 2\beta_2} s \lambda \xi_1 \Omega_2 + e^{-\alpha_2 - \beta_2} s \lambda \xi_2 \Omega_2 - e^{\alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2} fY0'[\lambda] - \right. \\ & \quad \left. e^{-\alpha_1 + \alpha_2 - \beta_1 - 2\beta_2} s \lambda \xi_1 fZ0'[\lambda] - e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2} s \lambda \xi_2 fZ0'[\lambda] \right) + \\ & \text{QU}[] \left( -e^{-2\alpha_1 + \beta_1} \lambda \xi_1 \Xi_1 + e^{-2\alpha_1 + \beta_1} s \lambda \xi_1 \Xi_1 - e^{-2\alpha_1 - 2\alpha_2 + \beta_1 + \beta_2} \lambda \Xi_1 \xi_2 + e^{-2\alpha_1 - 2\alpha_2 + \beta_1 + \beta_2} s \lambda \Xi_1 \xi_2 + \right. \\ & \quad \lambda \xi_1 \Xi_2 - s \lambda \xi_1 \Xi_2 - e^{-2\alpha_2 + \beta_2} \lambda \xi_2 \Xi_2 + e^{-2\alpha_2 + \beta_2} s \lambda \xi_2 \Xi_2 - e^{\alpha_1 - 2\beta_1} \eta_1 \lambda \Upsilon_1 + e^{\alpha_1 - 2\beta_1} t \eta_1 \lambda \Upsilon_1 - \\ & \quad e^{\alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2} \eta_2 \lambda \Upsilon_1 + e^{\alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2} t \eta_2 \lambda \Upsilon_1 + \eta_1 \lambda \Upsilon_2 - t \eta_1 \lambda \Upsilon_2 - e^{\alpha_2 - 2\beta_2} \eta_2 \lambda \Upsilon_2 + \\ & \quad e^{\alpha_2 - 2\beta_2} t \eta_2 \lambda \Upsilon_2 - e^{-\alpha_1 - \beta_1} \xi_1 \lambda \Omega_1 + e^{-\alpha_1 - \beta_1} s t \xi_1 \lambda \Omega_1 - e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2} \xi_2 \lambda \Omega_1 + \\ & \quad e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2} s t \xi_2 \lambda \Omega_1 + e^{-\alpha_1 - \beta_1} \eta_1 \lambda^2 \xi_1 \Omega_1 - e^{-\alpha_1 - \beta_1} s \eta_1 \lambda^2 \xi_1 \Omega_1 - e^{-\alpha_1 + \alpha_2 - \beta_1 - 2\beta_2} s \eta_2 \lambda^2 \xi_1 \Omega_1 + \\ & \quad e^{-\alpha_1 + \alpha_2 - \beta_1 - 2\beta_2} s t \eta_2 \lambda^2 \xi_1 \Omega_1 + e^{-\alpha_1 - 2\alpha_2 - \beta_1 + \beta_2} \eta_1 \lambda^2 \xi_2 \Omega_1 - e^{-\alpha_1 - 2\alpha_2 - \beta_1 + \beta_2} s \eta_1 \lambda^2 \xi_2 \Omega_1 + \\ & \quad e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2} \eta_2 \lambda^2 \xi_2 \Omega_1 - e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2} s \eta_2 \lambda^2 \xi_2 \Omega_1 + \xi_1 \lambda \Omega_2 - s t \xi_1 \lambda \Omega_2 - e^{-\alpha_2 - \beta_2} \xi_2 \lambda \Omega_2 + \\ & \quad e^{-\alpha_2 - \beta_2} s t \xi_2 \lambda \Omega_2 - \eta_1 \lambda^2 \xi_1 \Omega_2 + s t \eta_1 \lambda^2 \xi_1 \Omega_2 + e^{\alpha_2 - 2\beta_2} s \eta_2 \lambda^2 \xi_1 \Omega_2 - e^{\alpha_2 - 2\beta_2} s t \eta_2 \lambda^2 \xi_1 \Omega_2 - \\ & \quad e^{-2\alpha_2 + \beta_2} \eta_1 \lambda^2 \xi_2 \Omega_2 + e^{-2\alpha_2 + \beta_2} s \eta_1 \lambda^2 \xi_2 \Omega_2 + e^{-\alpha_2 - \beta_2} \eta_2 \lambda^2 \xi_2 \Omega_2 - e^{-\alpha_2 - \beta_2} s \eta_2 \lambda^2 \xi_2 \Omega_2 - \\ & \quad f t0'[\lambda] + e^{-2\alpha_1 + \beta_1} \lambda \xi_1 fX0'[\lambda] - e^{-2\alpha_1 + \beta_1} s \lambda \xi_1 fX0'[\lambda] + e^{-2\alpha_1 - 2\alpha_2 + \beta_1 + \beta_2} \lambda \xi_2 fX0'[\lambda] - \\ & \quad e^{-2\alpha_1 - 2\alpha_2 + \beta_1 + \beta_2} s \lambda \xi_2 fX0'[\lambda] + e^{\alpha_1 - 2\beta_1} \eta_1 \lambda fY0'[\lambda] - e^{\alpha_1 - 2\beta_1} t \eta_1 \lambda fY0'[\lambda] + \\ & \quad e^{\alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2} \eta_2 \lambda fY0'[\lambda] - e^{\alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2} t \eta_2 \lambda fY0'[\lambda] + e^{-\alpha_1 - \beta_1} \xi_1 \lambda fZ0'[\lambda] - \\ & \quad e^{-\alpha_1 - \beta_1} s t \xi_1 \lambda fZ0'[\lambda] + e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2} \xi_2 \lambda fZ0'[\lambda] - e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2} s t \xi_2 \lambda fZ0'[\lambda] - \\ & \quad e^{-\alpha_1 - \beta_1} \eta_1 \lambda^2 \xi_1 fZ0'[\lambda] + e^{-\alpha_1 - \beta_1} s \eta_1 \lambda^2 \xi_1 fZ0'[\lambda] + e^{-\alpha_1 + \alpha_2 - \beta_1 - 2\beta_2} s \eta_2 \lambda^2 \xi_1 fZ0'[\lambda] - \\ & \quad e^{-\alpha_1 + \alpha_2 - \beta_1 - 2\beta_2} s t \eta_2 \lambda^2 \xi_1 fZ0'[\lambda] - e^{-\alpha_1 - 2\alpha_2 - \beta_1 + \beta_2} \eta_1 \lambda^2 \xi_2 fZ0'[\lambda] + \\ & \quad e^{-\alpha_1 - 2\alpha_2 - \beta_1 + \beta_2} s \eta_1 \lambda^2 \xi_2 fZ0'[\lambda] - e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2} \eta_2 \lambda^2 \xi_2 fZ0'[\lambda] + e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2} s \eta_2 \lambda^2 \xi_2 fZ0'[\lambda] \end{aligned}$$

In[\*]:= {sol4} =

**DSolve**[Coefficient[CE[diff /. sol1 /. sol2 /. sol3] /.  $\epsilon \rightarrow 0$ ], QU[Z]] == 0  $\wedge$  fZ0[0] == 0, fZ0,  $\lambda$

Out[\*]=

{ {fZ0  $\rightarrow$  Function[ $\{\lambda\}$ ,  $\lambda (\Omega_1 + e^{\alpha_1 + \beta_1} \Omega_2)$ ] } }

In[\*]:= **CE**[diff /. sol1 /. sol2 /. sol3 /. sol4] /.  $\epsilon \rightarrow 0$

Out[\*]=

$$\begin{aligned} & \text{QU}[X] \left( e^{-2\alpha_1 - 2\alpha_2 + \beta_1 + \beta_2} \Xi_1 + e^{-2\alpha_2 + \beta_2} \Xi_2 + 2 e^{-2\alpha_2 + \beta_2} \eta_1 \lambda \Omega_2 - e^{-2\alpha_1 - 2\alpha_2 + \beta_1 + \beta_2} fX0'[\lambda] \right) + \\ & \text{QU}[Y] \left( e^{\alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2} \Upsilon_1 + e^{\alpha_2 - 2\beta_2} \Upsilon_2 - 2 e^{\alpha_2 - 2\beta_2} s \lambda \xi_1 \Omega_2 - e^{\alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2} fY0'[\lambda] \right) + \\ & \text{QU}[] \left( -e^{-2\alpha_1 + \beta_1} \lambda \xi_1 \Xi_1 + e^{-2\alpha_1 + \beta_1} s \lambda \xi_1 \Xi_1 - e^{-2\alpha_1 - 2\alpha_2 + \beta_1 + \beta_2} \lambda \Xi_1 \xi_2 + e^{-2\alpha_1 - 2\alpha_2 + \beta_1 + \beta_2} s \lambda \Xi_1 \xi_2 + \right. \\ & \quad \lambda \xi_1 \Xi_2 - s \lambda \xi_1 \Xi_2 - e^{-2\alpha_2 + \beta_2} \lambda \xi_2 \Xi_2 + e^{-2\alpha_2 + \beta_2} s \lambda \xi_2 \Xi_2 - e^{\alpha_1 - 2\beta_1} \eta_1 \lambda \Upsilon_1 + \\ & \quad e^{\alpha_1 - 2\beta_1} t \eta_1 \lambda \Upsilon_1 - e^{\alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2} \eta_2 \lambda \Upsilon_1 + e^{\alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2} t \eta_2 \lambda \Upsilon_1 + \eta_1 \lambda \Upsilon_2 - t \eta_1 \lambda \Upsilon_2 - \\ & \quad e^{\alpha_2 - 2\beta_2} \eta_2 \lambda \Upsilon_2 + e^{\alpha_2 - 2\beta_2} t \eta_2 \lambda \Upsilon_2 + 2 \xi_1 \lambda \Omega_2 - 2 s t \xi_1 \lambda \Omega_2 - 2 \eta_1 \lambda^2 \xi_1 \Omega_2 + s \eta_1 \lambda^2 \xi_1 \Omega_2 + \\ & \quad s t \eta_1 \lambda^2 \xi_1 \Omega_2 + 2 e^{\alpha_2 - 2\beta_2} s \eta_2 \lambda^2 \xi_1 \Omega_2 - 2 e^{\alpha_2 - 2\beta_2} s t \eta_2 \lambda^2 \xi_1 \Omega_2 - 2 e^{-2\alpha_2 + \beta_2} \eta_1 \lambda^2 \xi_2 \Omega_2 + \\ & \quad 2 e^{-2\alpha_2 + \beta_2} s \eta_1 \lambda^2 \xi_2 \Omega_2 - f t0'[\lambda] + e^{-2\alpha_1 + \beta_1} \lambda \xi_1 fX0'[\lambda] - e^{-2\alpha_1 + \beta_1} s \lambda \xi_1 fX0'[\lambda] + \\ & \quad e^{-2\alpha_1 - 2\alpha_2 + \beta_1 + \beta_2} \lambda \xi_2 fX0'[\lambda] - e^{-2\alpha_1 - 2\alpha_2 + \beta_1 + \beta_2} s \lambda \xi_2 fX0'[\lambda] + e^{\alpha_1 - 2\beta_1} \eta_1 \lambda fY0'[\lambda] - \\ & \quad e^{\alpha_1 - 2\beta_1} t \eta_1 \lambda fY0'[\lambda] + e^{\alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2} \eta_2 \lambda fY0'[\lambda] - e^{\alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2} t \eta_2 \lambda fY0'[\lambda] \end{aligned}$$

In[\*]:= {sol5} = **DSolve**[

Coefficient[CE[diff /. sol1 /. sol2 /. sol3 /. sol4] /.  $\epsilon \rightarrow 0$ ], QU[Y]] == 0  $\wedge$  fY0[0] == 0, fY0,  $\lambda$

Out[\*]=

{ {fY0  $\rightarrow$  Function[ $\{\lambda\}$ ,  $e^{-\alpha_1} \lambda (e^{\alpha_1} \Upsilon_1 + e^{2\beta_1} \Upsilon_2 - e^{2\beta_1} s \lambda \xi_1 \Omega_2)$ ] } }

In[\*]:= **CE**[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5] /.  $\epsilon \rightarrow 0$

Out[\*]=

$$\begin{aligned} & \text{QU}[X] \left( e^{-2\alpha_1-2\alpha_2+\beta_1+\beta_2} \Xi_1 + e^{-2\alpha_2+\beta_2} \Xi_2 + 2 e^{-2\alpha_2+\beta_2} \eta_1 \lambda \Omega_2 - e^{-2\alpha_1-2\alpha_2+\beta_1+\beta_2} fX\theta'[\lambda] \right) + \\ & \text{QU}[] \left( -e^{-2\alpha_1+\beta_1} \lambda \xi_1 \Xi_1 + e^{-2\alpha_1+\beta_1} s \lambda \xi_1 \Xi_1 - e^{-2\alpha_1-2\alpha_2+\beta_1+\beta_2} \lambda \Xi_1 \xi_2 + e^{-2\alpha_1-2\alpha_2+\beta_1+\beta_2} s \lambda \Xi_1 \xi_2 + \right. \\ & \quad \lambda \xi_1 \Xi_2 - s \lambda \xi_1 \Xi_2 - e^{-2\alpha_2+\beta_2} \lambda \xi_2 \Xi_2 + e^{-2\alpha_2+\beta_2} s \lambda \xi_2 \Xi_2 + 2 \eta_1 \lambda \Upsilon_2 - 2 t \eta_1 \lambda \Upsilon_2 + \\ & \quad 2 \xi_1 \lambda \Omega_2 - 2 s t \xi_1 \lambda \Omega_2 - 2 \eta_1 \lambda^2 \xi_1 \Omega_2 - s \eta_1 \lambda^2 \xi_1 \Omega_2 + 3 s t \eta_1 \lambda^2 \xi_1 \Omega_2 - \\ & \quad 2 e^{-2\alpha_2+\beta_2} \eta_1 \lambda^2 \xi_2 \Omega_2 + 2 e^{-2\alpha_2+\beta_2} s \eta_1 \lambda^2 \xi_2 \Omega_2 - ft\theta'[\lambda] + e^{-2\alpha_1+\beta_1} \lambda \xi_1 fX\theta'[\lambda] - \\ & \quad e^{-2\alpha_1+\beta_1} s \lambda \xi_1 fX\theta'[\lambda] + e^{-2\alpha_1-2\alpha_2+\beta_1+\beta_2} \lambda \xi_2 fX\theta'[\lambda] - e^{-2\alpha_1-2\alpha_2+\beta_1+\beta_2} s \lambda \xi_2 fX\theta'[\lambda] \left. \right) \end{aligned}$$

In[\*]:= {sol6} =

**DSolve**[Coefficient[CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /.  $\epsilon \rightarrow 0$ ], QU[X]] == 0  $\wedge$  fX0[0] == 0, fX0,  $\lambda$ ]

Out[\*]=

$$\left\{ \left\{ fX0 \rightarrow \text{Function}[\{\lambda\}, e^{-\beta_1} \lambda (e^{\beta_1} \Xi_1 + e^{2\alpha_1} \Xi_2 + e^{2\alpha_1} \eta_1 \lambda \Omega_2)] \right\} \right\}$$

In[\*]:= **CE**[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6] /.  $\epsilon \rightarrow 0$

Out[\*]=

$$\begin{aligned} & \text{QU}[] \left( 2 \lambda \xi_1 \Xi_2 - 2 s \lambda \xi_1 \Xi_2 + 2 \eta_1 \lambda \Upsilon_2 - 2 t \eta_1 \lambda \Upsilon_2 + \right. \\ & \quad \left. 2 \xi_1 \lambda \Omega_2 - 2 s t \xi_1 \lambda \Omega_2 - 3 s \eta_1 \lambda^2 \xi_1 \Omega_2 + 3 s t \eta_1 \lambda^2 \xi_1 \Omega_2 - ft\theta'[\lambda] \right) \end{aligned}$$

In[\*]:= {sol7} = **DSolve**[

Coefficient[CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /.  $\epsilon \rightarrow 0$ ], QU[]] == 0  $\wedge$  ft0[0] == 0, ft0,  $\lambda$ ]

Out[\*]=

$$\left\{ \left\{ ft0 \rightarrow \text{Function}[\{\lambda\}, \lambda^2 \xi_1 \Xi_2 - s \lambda^2 \xi_1 \Xi_2 + \eta_1 \lambda^2 \Upsilon_2 - t \eta_1 \lambda^2 \Upsilon_2 + \xi_1 \lambda^2 \Omega_2 - s t \xi_1 \lambda^2 \Omega_2 - s \eta_1 \lambda^3 \xi_1 \Omega_2 + s t \eta_1 \lambda^3 \xi_1 \Omega_2] \right\} \right\}$$

In[\*]:= **CE**[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7] /.  $\epsilon \rightarrow 0$

Out[\*]=

0

In[\*]:= **CE**[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7]

Out[\*]=

In[\*]:= **Coefficient**[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7, QU[y, x]]

Out[\*]=

$$\begin{aligned} & e^{\alpha_2+\beta_2} \epsilon \eta_1 \lambda \xi_1 - e^{2\alpha_2-\beta_2} \epsilon \eta_2 \lambda \xi_1 + e^{-\alpha_2+2\beta_2} \epsilon \eta_1 \lambda \xi_2 + \\ & \epsilon \eta_2 \lambda \xi_2 - e^{-\alpha_2-\beta_2} \epsilon (e^{2\beta_2} \eta_1 + e^{\alpha_2} \eta_2) \lambda (e^{2\alpha_2} \xi_1 + e^{\beta_2} \xi_2) - \epsilon f1_{xy}'[\lambda] \end{aligned}$$

In[\*]:= {sol8} = **DSolve**[

Coefficient[CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7], QU[y, x]] == 0  $\wedge$  f1xy[0] == 0, f1xy,  $\lambda$ ]

Out[\*]=

$$\left\{ \left\{ f1_{xy} \rightarrow \text{Function}[\{\lambda\}, -e^{2\alpha_2-\beta_2} \eta_2 \lambda^2 \xi_1] \right\} \right\}$$

In[\*]:= **CE**[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8]

Out[\*]=

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In[*]:= Coefficient[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8, QU[z, x]]
Out[*]=

$$-e^{3\alpha^2} \in \zeta^1 \lambda \xi^1 + e^{2\alpha^2-\beta^2} \in \zeta^2 \lambda \xi^1 - e^{\alpha^2+\beta^2} \in \zeta^1 \lambda \xi^2 - e \zeta^2 \lambda \xi^2 - 2 e^{2\alpha^2-2\beta^2} \in \eta^2 \lambda^2 \xi^1 (e^{2\alpha^2} \xi^1 + e^{\beta^2} \xi^2) +$$


$$e^{-\beta^2} \in \lambda (e^{2\alpha^2-\beta^2} \eta^2 \lambda \xi^1 + e^{-\beta^2} (e^{\alpha^2+2\beta^2} \zeta^1 + e^{\beta^2} \zeta^2 + e^{2\alpha^2} \eta^2 \lambda \xi^1)) (e^{2\alpha^2} \xi^1 + e^{\beta^2} \xi^2) - e f_{1xz}'[\lambda]$$


In[*]:= {sol9} =
DSolve[Coefficient[CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8],
QU[z, x]] == 0 ∧ f1xz[0] == 0, f1xz, λ]
Out[*]=
{{f1xz → Function[{λ}, e^{2α2-β2} ζ2 λ^2 ξ1]}}

In[*]:= CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9]
Out[*]=

In[*]:= {sol10} = DSolve[
Coefficient[CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9],
QU[z, y]] == 0 ∧ f1yz[0] == 0, f1yz, λ]
Out[*]=
{{f1yz → Function[{λ}, -e^{-α2-β2} λ^2 (e^{3β2} ζ2 η1 + e^{2α2+2β2} η1 η2 λ ξ1 + e^{3α2} η2^2 λ ξ1)]}}

In[*]:= CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /. sol10]
Out[*]=

In[*]:= Cases[CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /. sol10],
_QU, Infinity] // Union
Out[*]=
{QU[], QU[a], QU[b], QU[x], QU[X], QU[y], QU[Y], QU[z], QU[Z],
QU[X, x], QU[X, X], QU[X, y], QU[X, Y], QU[X, z], QU[X, Z], QU[Y, a], QU[Y, x],
QU[Y, y], QU[Y, Y], QU[Y, z], QU[Y, Z], QU[z, z], QU[Z, x], QU[Z, y], QU[Z, z]}

In[*]:= Coefficient[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /. sol10,
QU[z, z]] // Simplify
Out[*]=
3 e^{2α2-β2} ∈ ζ2 η2 λ^2 ξ1 - e f1z^2'[\lambda]

In[*]:= {sol11} = DSolve[
Coefficient[CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /.
sol10], QU[z, z]] == 0 ∧ f1zz[0] == 0, f1zz, λ]
Out[*]=
{{f1zz → Function[{λ}, e^{2α2-β2} ζ2 η2 λ^3 ξ1]}}

In[*]:= Cases[CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /. sol10 /.
sol11], _QU, Infinity] // Union
Out[*]=
{QU[], QU[a], QU[b], QU[x], QU[X], QU[y], QU[Y], QU[z], QU[Z],
QU[X, x], QU[X, X], QU[X, y], QU[X, Y], QU[X, z], QU[X, Z], QU[Y, a], QU[Y, x],
QU[Y, y], QU[Y, Y], QU[Y, z], QU[Y, Z], QU[Z, x], QU[Z, y], QU[Z, z]}

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In[*]:= Coefficient[
  CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /. sol10 /. sol11],
  QU[Z, x]]
Out[*]=

$$-2 e^{\alpha^2 - 2 \beta^2} \in \lambda \xi^1 \Omega^2 - e^{-\alpha^1 - \alpha^2 - \beta^1 - \beta^2} \in f_{1xz}'[\lambda]$$


In[*]:= {sol12} = DSolve[
  Coefficient[CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /.
    sol10 /. sol11], QU[Z, x]] == 0 ∧ f1zx[0] == 0, f1zx, λ]
Out[*]=

$$\left\{ \left\{ f_{1xz} \rightarrow \text{Function}[\{\lambda\}, -e^{\alpha^1 + 2\alpha^2 + \beta^1 - \beta^2} \lambda^2 \xi^1 \Omega^2] \right\} \right\}$$


In[*]:= {sol13} = DSolve[
  Coefficient[CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /.
    sol10 /. sol11 /. sol12], QU[Z, y]] == 0 ∧ f1zy[0] == 0, f1zy, λ]
Out[*]=

$$\left\{ \left\{ f_{1yz} \rightarrow \text{Function}[\{\lambda\}, -e^{\alpha^1 - \alpha^2 + \beta^1 + 2\beta^2} \eta^1 \lambda^2 \Omega^2] \right\} \right\}$$


In[*]:= Cases[CE[
  diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /. sol10 /. sol11 /.
  sol12 /. sol13], _QU, Infinity] // Union
Out[*]=
{QU[], QU[a], QU[b], QU[x], QU[X], QU[y], QU[Y], QU[z],
  QU[Z], QU[X, x], QU[X, X], QU[X, y], QU[X, Y], QU[X, z], QU[X, Z],
  QU[Y, a], QU[Y, x], QU[Y, y], QU[Y, Y], QU[Y, z], QU[Y, Z], QU[Z, z]}

In[*]:= Coefficient[CE[
  diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /. sol10 /. sol11 /.
  sol12 /. sol13], QU[X, x]]
Out[*]=

$$-4 \in \lambda \xi^1 \Xi^2 - 3 \in \eta^1 \lambda^2 \xi^1 \Omega^2 - e^{-2\alpha^1 - 2\alpha^2 + \beta^1 + \beta^2} \in f_{1xx}'[\lambda]$$


In[*]:= {sol14} = DSolve[
  Coefficient[CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /.
    sol10 /. sol11 /. sol12 /. sol13], QU[X, x]] == 0 ∧ f1xx[0] == 0, f1xx, λ]
Out[*]=

$$\left\{ \left\{ f_{1xx} \rightarrow \text{Function}[\{\lambda\}, -e^{2\alpha^1 + 2\alpha^2 - \beta^1 - \beta^2} \lambda^2 \xi^1 (2\Xi^2 + \eta^1 \lambda \Omega^2)] \right\} \right\}$$


In[*]:= Coefficient[CE[
  diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /. sol10 /. sol11 /.
  sol12 /. sol13 /. sol14], QU[Y, a]]
Out[*]=

$$4 e^{\alpha^2 - 2\beta^2} s \in \lambda \xi^1 \Omega^2 - e^{\alpha^1 + \alpha^2 - 2\beta^1 - 2\beta^2} \in f_{1ay}'[\lambda]$$


In[*]:= {sol15} = DSolve[
  Coefficient[CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /.
    sol10 /. sol11 /. sol12 /. sol13 /. sol14], QU[Y, a]] == 0 ∧ f1ya[0] == 0, f1ya, λ]
Out[*]=

$$\left\{ \left\{ f_{1ay} \rightarrow \text{Function}[\{\lambda\}, 2 e^{-\alpha^1 + 2\beta^1} s \lambda^2 \xi^1 \Omega^2] \right\} \right\}$$


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In[*]:= Cases[CE[
    diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /. sol10 /. sol11 /.
    sol12 /. sol13 /. sol14 /. sol15], _QU, Infinity] // Union
Out[*]=
{QU[], QU[a], QU[b], QU[x], QU[X], QU[y], QU[Y], QU[z], QU[Z], QU[X, X], QU[X, y], QU[X, Y],
  QU[X, z], QU[X, Z], QU[Y, x], QU[Y, y], QU[Y, Y], QU[Y, z], QU[Y, Z], QU[Z, z]}

In[*]:= Coefficient[CE[
    diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /. sol10 /. sol11 /.
    sol12 /. sol13 /. sol14 /. sol15], QU[Z, z]]
Out[*]=

$$-4 \in \zeta_1 \lambda \Omega_2 - 3 e^{\alpha_2 - 2 \beta_2} \in \eta_2 \lambda^2 \xi_1 \Omega_2 - e^{-\alpha_1 - \alpha_2 - \beta_1 - \beta_2} \in f_{1_{zz}'}[\lambda]$$


In[*]:= {sol16} = DSolve[Coefficient[
    CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /. sol10 /.
    sol11 /. sol12 /. sol13 /. sol14 /. sol15], QU[Z, z]] == 0 & f_{1_{zz}}[0] == 0, f_{1_{zz}}, \lambda]
Out[*]=
{\{f_{1_{zz}} \rightarrow \text{Function}[\{\lambda\}, -e^{\alpha_1 + \alpha_2 + \beta_1 - \beta_2} \lambda^2 (2 e^{2 \beta_2} \zeta_1 + e^{\alpha_2} \eta_2 \lambda \xi_1) \Omega_2]\}\}}

In[*]:= Coefficient[CE[
    diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /. sol10 /. sol11 /.
    sol12 /. sol13 /. sol14 /. sol15 /. sol16], QU[Y, y]]
Out[*]=

$$-4 \in \eta_1 \lambda \Upsilon_2 - e^{\alpha_1 + \alpha_2 - 2 \beta_1 - 2 \beta_2} \in f_{1_{yy}'}[\lambda]$$


In[*]:= {sol17} = DSolve[
    Coefficient[CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /.
    sol10 /. sol11 /. sol12 /. sol13 /. sol14 /.
    sol15 /. sol16], QU[Y, y]] == 0 & f_{1_{yy}}[0] == 0, f_{1_{yy}}, \lambda]
Out[*]=
{\{f_{1_{yy}} \rightarrow \text{Function}[\{\lambda\}, -2 e^{-\alpha_1 - \alpha_2 + 2 \beta_1 + 2 \beta_2} \eta_1 \lambda^2 \Upsilon_2]\}\}}

In[*]:= Coefficient[CE[
    diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /. sol10 /. sol11 /.
    sol12 /. sol13 /. sol14 /. sol15 /. sol16 /. sol17], QU[Y, x]]
Out[*]=

$$2 e^{3 \alpha_2 - 3 \beta_2} \in \lambda \xi_1 \Upsilon_2 + 6 e^{3 \alpha_2 - 3 \beta_2} s \in \lambda^2 \xi_1^2 \Omega_2 - e^{\alpha_1 + \alpha_2 - 2 \beta_1 - 2 \beta_2} \in f_{1_{xy}'}[\lambda]$$


In[*]:= {sol18} = DSolve[
    Coefficient[CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /.
    sol10 /. sol11 /. sol12 /. sol13 /. sol14 /.
    sol15 /. sol16 /. sol17], QU[Y, x]] == 0 & f_{1_{yx}}[0] == 0, f_{1_{yx}}, \lambda]
Out[*]=
{\{f_{1_{yx}} \rightarrow \text{Function}[\{\lambda\}, e^{-\alpha_1 + 2 \alpha_2 + 2 \beta_1 - \beta_2} \lambda^2 \xi_1 (\Upsilon_2 + 2 s \lambda \xi_1 \Omega_2)]\}\}}

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In[*]:= Cases[CE[
    diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /. sol10 /. sol11 /.
    sol12 /. sol13 /. sol14 /. sol15 /. sol16 /. sol17 /. sol18], _QU, Infinity] // Union

Out[*]=
{QU[], QU[a], QU[b], QU[x], QU[X], QU[y], QU[Y], QU[z], QU[Z],
  QU[X, X], QU[X, y], QU[X, Y], QU[X, z], QU[X, Z], QU[Y, Y], QU[Y, z], QU[Y, Z]}

In[*]:= Coefficient[CE[
    diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /. sol10 /. sol11 /.
    sol12 /. sol13 /. sol14 /. sol15 /. sol16 /. sol17 /. sol18], QU[X, y]]

Out[*]=
 $2 e^{-3 \alpha 2 + 3 \beta 2} \in \eta 1 \lambda \Xi 2 - e^{-2 \alpha 1 - 2 \alpha 2 + \beta 1 + \beta 2} \in f_{1_{xy}}'[\lambda]$ 

In[*]:= {sol19} =
DSolve[Coefficient[CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /.
    sol9 /. sol10 /. sol11 /. sol12 /. sol13 /. sol14 /. sol15 /.
    sol16 /. sol17 /. sol18], QU[X, y]] == 0 & f_{1_{xy}}[0] == 0, f_{1_{xy}}, \lambda]

Out[*]=
{{f_{1_{xy}} \to Function[\{\lambda\}, e^{2 \alpha 1 - \alpha 2 - \beta 1 + 2 \beta 2} \eta 1 \lambda^2 \Xi 2]}}

In[*]:= Cases[CE[
    diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /. sol10 /. sol11 /.
    sol12 /. sol13 /. sol14 /. sol15 /.
    sol16 /. sol17 /. sol18 /. sol19], _QU, Infinity] // Union

Out[*]=
{QU[], QU[a], QU[b], QU[x], QU[X], QU[y], QU[Y], QU[z], QU[Z],
  QU[X, X], QU[X, Y], QU[X, z], QU[X, Z], QU[Y, Y], QU[Y, z], QU[Y, Z]}

In[*]:= {sol20} = DSolve[
    Coefficient[CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /.
    sol10 /. sol11 /. sol12 /. sol13 /. sol14 /. sol15 /. sol16 /.
    sol17 /. sol18 /. sol19], QU[X, z]] == 0 & f_{1_{xz}}[0] == 0, f_{1_{xz}}, \lambda]

Out[*]=
{{f_{1_{xz}} \to Function[\{\lambda\},
    -e^{2 \alpha 1 + \alpha 2 - \beta 1 - \beta 2} \lambda^2 (e^{2 \beta 2} \zeta 1 \Xi 2 + 2 e^{\alpha 2} \eta 2 \lambda \xi 1 \Xi 2 + e^{2 \beta 2} \zeta 1 \eta 1 \lambda \Omega 2 + e^{\alpha 2} \eta 1 \eta 2 \lambda^2 \xi 1 \Omega 2)]}}

In[*]:= Cases[CE[
    diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /. sol10 /. sol11 /.
    sol12 /. sol13 /. sol14 /. sol15 /. sol16 /.
    sol17 /. sol18 /. sol19 /. sol20], _QU, Infinity] // Union

Out[*]=
{QU[], QU[a], QU[b], QU[x], QU[X], QU[y], QU[Y], QU[z],
  QU[Z], QU[X, X], QU[X, Y], QU[X, Z], QU[Y, Y], QU[Y, z], QU[Y, Z]}

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In[*]:= Coefficient[
  CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /. sol10 /.
    sol11 /. sol12 /. sol13 /. sol14 /. sol15 /.
    sol16 /. sol17 /. sol18 /. sol19 /. sol20], QU[Y, Z]]
Out[*]=

$$2 e^{-\alpha 1 - \beta 1 - 3 \beta 2} \in \lambda \Upsilon 2 \Omega 1 - 3 e^{-\alpha 1 - \beta 1 - 3 \beta 2} s \in \lambda^2 \xi 1 \Omega 1 \Omega 2 + 3 e^{-3 \beta 2} s \in \lambda^2 \xi 1 \Omega 2^2 - e^{-3 \beta 1 - 3 \beta 2} \in f_{1_{YZ}}'[\lambda]$$


In[*]:= {sol21} = DSolve[
  Coefficient[CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /.
    sol10 /. sol11 /. sol12 /. sol13 /. sol14 /. sol15 /. sol16 /. sol17 /.
    sol18 /. sol19 /. sol20], QU[Y, Z]] == 0 & f_{1_{YZ}}[0] == 0, f_{1_{YZ}}, \lambda]
Out[*]=

$$\left\{ \left\{ f_{1_{YZ}} \rightarrow \text{Function} \left[ \{\lambda\}, e^{-\alpha 1 + 2 \beta 1} \lambda^2 \left( \Upsilon 2 \Omega 1 - s \lambda \xi 1 \Omega 1 \Omega 2 + e^{\alpha 1 + \beta 1} s \lambda \xi 1 \Omega 2^2 \right) \right] \right\} \right\}$$


In[*]:= Coefficient[
  CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /. sol10 /.
    sol11 /. sol12 /. sol13 /. sol14 /. sol15 /.
    sol16 /. sol17 /. sol18 /. sol19 /. sol20 /. sol21], QU[X, Z]]
Out[*]=

$$-2 e^{-\alpha 1 - 3 \alpha 2 - \beta 1} \in \lambda \Xi 2 \Omega 1 - 3 e^{-\alpha 1 - 3 \alpha 2 - \beta 1} \in \eta 1 \lambda^2 \Omega 1 \Omega 2 - 3 e^{-3 \alpha 2} \in \eta 1 \lambda^2 \Omega 2^2 - e^{-3 \alpha 1 - 3 \alpha 2} \in f_{1_{XZ}}'[\lambda]$$


In[*]:= {sol22} = DSolve[
  Coefficient[CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /.
    sol10 /. sol11 /. sol12 /. sol13 /. sol14 /. sol15 /. sol16 /. sol17 /.
    sol18 /. sol19 /. sol20 /. sol21], QU[X, Z]] == 0 & f_{1_{XZ}}[0] == 0, f_{1_{XZ}}, \lambda]
Out[*]=

$$\left\{ \left\{ f_{1_{XZ}} \rightarrow \text{Function} \left[ \{\lambda\}, -e^{2 \alpha 1 - \beta 1} \lambda^2 \left( \Xi 2 \Omega 1 + \eta 1 \lambda \Omega 1 \Omega 2 + e^{\alpha 1 + \beta 1} \eta 1 \lambda \Omega 2^2 \right) \right] \right\} \right\}$$


In[*]:= Coefficient[
  CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /. sol10 /.
    sol11 /. sol12 /. sol13 /. sol14 /. sol15 /. sol16 /.
    sol17 /. sol18 /. sol19 /. sol20 /. sol21 /. sol22], QU[a]]
Out[*]=

$$4 s \in \lambda \xi 1 \Xi 2 + 4 s t \in \zeta 1 \lambda \Omega 2 + 6 s \in \eta 1 \lambda^2 \xi 1 \Omega 2 - 6 s t \in \eta 1 \lambda^2 \xi 1 \Omega 2 - \in fa_1'[\lambda]$$


In[*]:= {sol23} = DSolve[
  Coefficient[CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /.
    sol10 /. sol11 /. sol12 /. sol13 /. sol14 /. sol15 /. sol16 /. sol17 /.
    sol18 /. sol19 /. sol20 /. sol21 /. sol22], QU[a]] == 0 & fa_1[0] == 0, fa_1, \lambda]
Out[*]=

$$\left\{ \left\{ fa_1 \rightarrow \text{Function} \left[ \{\lambda\}, -2 s \lambda^2 \left( -\xi 1 \Xi 2 - t \zeta 1 \Omega 2 - \eta 1 \lambda \xi 1 \Omega 2 + t \eta 1 \lambda \xi 1 \Omega 2 \right) \right] \right\} \right\}$$


In[*]:= {sol24} = DSolve[Coefficient[
  CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /. sol10 /.
    sol11 /. sol12 /. sol13 /. sol14 /. sol15 /. sol16 /. sol17 /. sol18 /.
    sol19 /. sol20 /. sol21 /. sol22 /. sol23], QU[b]] == 0 & fb_1[0] == 0, fb_1, \lambda]
Out[*]=

$$\left\{ \left\{ fb_1 \rightarrow \text{Function} \left[ \{\lambda\}, -2 t \lambda^2 \left( -\eta 1 \Upsilon 2 - s \zeta 1 \Omega 2 + s \eta 1 \lambda \xi 1 \Omega 2 \right) \right] \right\} \right\}$$


```

```
In[*]:= Coefficient[
  CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /. sol10 /.
    sol11 /. sol12 /. sol13 /. sol14 /. sol15 /. sol16 /. sol17 /. sol18 /.
    sol19 /. sol20 /. sol21 /. sol22 /. sol23 /. sol24], QU[Y, Y]]
```

```
Out[*]=

$$-3 e^{2\alpha 2-4\beta 2} s \in \lambda^2 \xi 1 \Upsilon 2 \Omega 2 - e^{2\alpha 1+2\alpha 2-4\beta 1-4\beta 2} \in f1_{Y^2}'[\lambda]$$

```

```
In[*]:= {sol25} =
  DSolve[Coefficient[CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /.
    sol9 /. sol10 /. sol11 /. sol12 /. sol13 /. sol14 /.
    sol15 /. sol16 /. sol17 /. sol18 /. sol19 /. sol20 /. sol21 /.
    sol22 /. sol23 /. sol24], QU[Y, Y]] == 0 & f1_{Y^2}[0] == 0, f1_{Y^2}, \lambda]
```

```
Out[*]=

$$\left\{ \left\{ f1_{Y^2} \rightarrow \text{Function}[\{\lambda\}, -e^{-2\alpha 1+4\beta 1} s \lambda^3 \xi 1 \Upsilon 2 \Omega 2] \right\} \right\}$$

```

```
In[*]:= Coefficient[
  CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /. sol10 /.
    sol11 /. sol12 /. sol13 /. sol14 /. sol15 /. sol16 /. sol17 /. sol18 /.
    sol19 /. sol20 /. sol21 /. sol22 /. sol23 /. sol24], QU[X, X]]
```

```
Out[*]=

$$3 e^{-4\alpha 2+2\beta 2} \in \eta 1 \lambda^2 \Xi 2 \Omega 2 - e^{-4\alpha 1-4\alpha 2+2\beta 1+2\beta 2} \in f1_{X^2}'[\lambda]$$

```

```
In[*]:= {sol26} =
  DSolve[Coefficient[CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /.
    sol9 /. sol10 /. sol11 /. sol12 /. sol13 /. sol14 /. sol15 /.
    sol16 /. sol17 /. sol18 /. sol19 /. sol20 /. sol21 /. sol22 /.
    sol23 /. sol24 /. sol25], QU[X, X]] == 0 & f1_{X^2}[0] == 0, f1_{X^2}, \lambda]
```

```
Out[*]=

$$\left\{ \left\{ f1_{X^2} \rightarrow \text{Function}[\{\lambda\}, e^{4\alpha 1-2\beta 1} \eta 1 \lambda^3 \Xi 2 \Omega 2] \right\} \right\}$$

```

```
In[*]:= Coefficient[
  CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /. sol10 /.
    sol11 /. sol12 /. sol13 /. sol14 /. sol15 /. sol16 /. sol17 /. sol18 /.
    sol19 /. sol20 /. sol21 /. sol22 /. sol23 /. sol24 /. sol25], QU[X, Y]]
```

```
Out[*]=

$$2 e^{\alpha 1-\alpha 2-2\beta 1-\beta 2} \in \lambda \Xi 2 \Upsilon 1 + 6 e^{-\alpha 2-\beta 2} s \in \lambda^2 \xi 1 \Xi 2 \Omega 2 + 3 e^{\alpha 1-\alpha 2-2\beta 1-\beta 2} \in \eta 1 \lambda^2 \Upsilon 1 \Omega 2 -$$


$$3 e^{-\alpha 2-\beta 2} \in \eta 1 \lambda^2 \Upsilon 2 \Omega 2 + 8 e^{-\alpha 2-\beta 2} s \in \eta 1 \lambda^3 \xi 1 \Omega 2^2 - e^{-\alpha 1-\alpha 2-\beta 1-\beta 2} \in f1_{XY}'[\lambda]$$

```

```
In[*]:= {sol27} =
  DSolve[Coefficient[CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /.
    sol9 /. sol10 /. sol11 /. sol12 /. sol13 /. sol14 /. sol15 /. sol16 /.
    sol17 /. sol18 /. sol19 /. sol20 /. sol21 /. sol22 /. sol23 /.
    sol24 /. sol25 /. sol26], QU[X, Y]] == 0 & f1_{XY}[0] == 0, f1_{XY}, \lambda]
```

```
Out[*]=

$$\left\{ \left\{ f1_{XY} \rightarrow \text{Function}[\{\lambda\}, \right.$$


$$\left. e^{\alpha 1-\beta 1} \lambda^2 \left( e^{\alpha 1} \Xi 2 \Upsilon 1 + 2 e^{2\beta 1} s \lambda \xi 1 \Xi 2 \Omega 2 + e^{\alpha 1} \eta 1 \lambda \Upsilon 1 \Omega 2 - e^{2\beta 1} \eta 1 \lambda \Upsilon 2 \Omega 2 + 2 e^{2\beta 1} s \eta 1 \lambda^2 \xi 1 \Omega 2^2 \right) \right\} \right\}$$

```

```
In[*]:= Coefficient[CE[
  diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /. sol10 /. sol11 /.
    sol12 /. sol13 /. sol14 /. sol15 /. sol16 /. sol17 /. sol18 /. sol19 /.
    sol20 /. sol21 /. sol22 /. sol23 /. sol24 /. sol25 /. sol26 /. sol27], QU[Z]]
```

```
Out[*]=
-4 eα1-α2-2β1-β2 ∈ λ Ξ2 Υ1 + 6 e-α2-β2 s ∈ λ2 ξ1 Ξ2 Ω2 - 6 eα1-α2-2β1-β2 ∈ η1 λ2 Υ1 Ω2 -
6 e-α2-β2 ∈ η1 λ2 Υ2 Ω2 + 6 e-α2-β2 t ∈ η1 λ2 Υ2 Ω2 - 3 e-α2-β2 ∈ ξ1 λ2 Ω22 + 9 e-α2-β2 s t ∈ ξ1 λ2 Ω22 +
12 e-α2-β2 s ∈ η1 λ3 ξ1 Ω22 - 12 e-α2-β2 s t ∈ η1 λ3 ξ1 Ω22 - e-α1-α2-β1-β2 ∈ fZ1' [λ]
```

```
In[*]:= {sol28} = DSolve[
  Coefficient[CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /.
    sol10 /. sol11 /. sol12 /. sol13 /. sol14 /. sol15 /. sol16 /.
    sol17 /. sol18 /. sol19 /. sol20 /. sol21 /. sol22 /. sol23 /.
    sol24 /. sol25 /. sol26 /. sol27], QU[Z]] == 0 ∧ fZ1[0] == 0, fZ1, λ]
```

```
Out[*]=
{ {fZ1 → Function[{λ}, -eα1-β1 λ2
(2 eα1 Ξ2 Υ1 - 2 e2β1 s λ ξ1 Ξ2 Ω2 + 2 eα1 η1 λ Υ1 Ω2 + 2 e2β1 η1 λ Υ2 Ω2 - 2 e2β1 t η1 λ Υ2 Ω2 +
e2β1 ξ1 λ Ω22 - 3 e2β1 s t ξ1 λ Ω22 - 3 e2β1 s η1 λ2 ξ1 Ω22 + 3 e2β1 s t η1 λ2 ξ1 Ω22) ] ] }
```

```
In[*]:= Coefficient[
  CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /. sol10 /.
    sol11 /. sol12 /. sol13 /. sol14 /. sol15 /.
    sol16 /. sol17 /. sol18 /. sol19 /. sol20 /. sol21 /. sol22 /.
    sol23 /. sol24 /. sol25 /. sol26 /. sol27 /. sol28], QU[y]]
```

```
Out[*]=
4 e-α2+2β2 ∈ ξ1 λ Ξ2 - 6 e-α2+2β2 s ∈ η1 λ2 ξ1 Ξ2 - 3 e-α2+2β2 ∈ η12 λ2 Υ2 +
9 e-α2+2β2 t ∈ η12 λ2 Υ2 + 3 e-α2+2β2 ∈ ξ1 η1 λ2 Ω2 + 3 e-α2+2β2 s t ∈ ξ1 η1 λ2 Ω2 -
4 e-α2+2β2 s ∈ η12 λ3 ξ1 Ω2 - 4 e-α2+2β2 s t ∈ η12 λ3 ξ1 Ω2 - ∈ fy1' [λ]
```

```
In[*]:= {sol29} = DSolve[
  Coefficient[CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /.
    sol10 /. sol11 /. sol12 /. sol13 /. sol14 /. sol15 /. sol16 /.
    sol17 /. sol18 /. sol19 /. sol20 /. sol21 /. sol22 /. sol23 /. sol24 /.
    sol25 /. sol26 /. sol27 /. sol28], QU[y]] == 0 ∧ fy1[0] == 0, fy1, λ]
```

```
Out[*]=
{ {fy1 → Function[{λ}, -e-α2+2β2 λ2 (-2 ξ1 Ξ2 + 2 s η1 λ ξ1 Ξ2 + η12 λ Υ2 -
3 t η12 λ Υ2 - ξ1 η1 λ Ω2 - s t ξ1 η1 λ Ω2 + s η12 λ2 ξ1 Ω2 + s t η12 λ2 ξ1 Ω2) ] ] }
```

```
In[*]:= {sol30} = DSolve[
  Coefficient[CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /.
    sol10 /. sol11 /. sol12 /. sol13 /. sol14 /. sol15 /. sol16 /.
    sol17 /. sol18 /. sol19 /. sol20 /. sol21 /. sol22 /. sol23 /. sol24 /.
    sol25 /. sol26 /. sol27 /. sol28 /. sol29], QU[x]] == 0 ∧ fx1[0] == 0, fx1, λ]
```

```
Out[*]=
{ {fx1 → Function[{λ}, -e2α2-β2 λ2 (λ ξ12 Ξ2 - 3 s λ ξ12 Ξ2 + 2 t ξ1 Υ2 - η1 λ ξ1 Υ2 +
t η1 λ ξ1 Υ2 + ξ1 λ ξ1 Ω2 - 3 s t ξ1 λ ξ1 Ω2 - 2 s η1 λ2 ξ12 Ω2 + 2 s t η1 λ2 ξ12 Ω2) ] ] }
```

```
In[*]:= Coefficient[
  CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /. sol10 /.
    sol11 /. sol12 /. sol13 /. sol14 /. sol15 /. sol16 /.
    sol17 /. sol18 /. sol19 /. sol20 /. sol21 /. sol22 /. sol23 /. sol24 /.
    sol25 /. sol26 /. sol27 /. sol28 /. sol29 /. sol30], QU[Y, z]]
```

```
Out[*]=
-2 e^{2\alpha-2\beta} \in \zeta^1 \lambda \Upsilon^2 + 3 e^{3\alpha-3\beta} \in \eta^2 \lambda^2 \xi^1 \Upsilon^2 +
9 e^{2\alpha-2\beta} s \in \zeta^1 \lambda^2 \xi^1 \Omega^2 + 8 e^{3\alpha-3\beta} s \in \eta^2 \lambda^3 \xi^1 \Omega^2 - e^{\alpha+2\beta-2\beta} \in f_{1\Upsilon z}'[\lambda]
```

```
In[*]:= {sol31} =
DSolve[Coefficient[CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /.
  sol9 /. sol10 /. sol11 /. sol12 /. sol13 /. sol14 /.
  sol15 /. sol16 /. sol17 /. sol18 /. sol19 /. sol20 /.
  sol21 /. sol22 /. sol23 /. sol24 /. sol25 /. sol26 /. sol27 /.
  sol28 /. sol29 /. sol30], QU[Y, z]] == 0 \wedge f_{1\Upsilon z}[0] == 0, f_{1\Upsilon z}, \lambda]
```

```
Out[*]=
{{f_{1\Upsilon z} \to Function[{ \lambda},
  e^{-\alpha+2\beta} \lambda^2 (-e^{2\beta} \zeta^1 \Upsilon^2 + e^{\alpha} \eta^2 \lambda \xi^1 \Upsilon^2 + 3 e^{2\beta} s \zeta^1 \lambda \xi^1 \Omega^2 + 2 e^{\alpha} s \eta^2 \lambda^2 \xi^1 \Omega^2)]}}
```

```
In[*]:= Coefficient[
  CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /. sol10 /.
    sol11 /. sol12 /. sol13 /. sol14 /. sol15 /. sol16 /. sol17 /.
    sol18 /. sol19 /. sol20 /. sol21 /. sol22 /. sol23 /. sol24 /.
    sol25 /. sol26 /. sol27 /. sol28 /. sol29 /. sol30 /. sol31], QU[Y]]
```

```
Out[*]=
-6 e^{\alpha-2\beta} s \in \lambda^2 \xi^1 \Xi^2 \Upsilon^2 - 3 e^{\alpha-2\beta} \in \eta^1 \lambda^2 \Upsilon^2^2 +
9 e^{\alpha-2\beta} t \in \eta^1 \lambda^2 \Upsilon^2^2 - 2 e^{\alpha-2\beta} s \in \lambda \xi^1 \Omega^2 + 4 e^{\alpha-2\beta} s \in \lambda^3 \xi^1 \Xi^2 \Omega^2 -
4 e^{\alpha-2\beta} s^2 \in \lambda^3 \xi^1 \Xi^2 \Omega^2 - 3 e^{\alpha-2\beta} \in \zeta^1 \lambda^2 \Upsilon^2 \Omega^2 + 9 e^{\alpha-2\beta} s t \in \zeta^1 \lambda^2 \Upsilon^2 \Omega^2 -
16 e^{\alpha-2\beta} s t \in \eta^1 \lambda^3 \xi^1 \Upsilon^2 \Omega^2 + 8 e^{\alpha-2\beta} s \in \zeta^1 \lambda^3 \xi^1 \Omega^2^2 - 16 e^{\alpha-2\beta} s^2 t \in \zeta^1 \lambda^3 \xi^1 \Omega^2^2 -
5 e^{\alpha-2\beta} s^2 \in \eta^1 \lambda^4 \xi^1 \Omega^2^2 + 15 e^{\alpha-2\beta} s^2 t \in \eta^1 \lambda^4 \xi^1 \Omega^2^2 - e^{\alpha+2\beta-2\beta} \in f_{Y1}'[\lambda]
```

```
In[*]:= {sol32} =
DSolve[Coefficient[CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /.
  sol9 /. sol10 /. sol11 /. sol12 /. sol13 /. sol14 /. sol15 /.
  sol16 /. sol17 /. sol18 /. sol19 /. sol20 /. sol21 /.
  sol22 /. sol23 /. sol24 /. sol25 /. sol26 /. sol27 /. sol28 /.
  sol29 /. sol30 /. sol31], QU[Y]] == 0 \wedge f_{Y1}[0] == 0, f_{Y1}, \lambda]
```

```
Out[*]=
{{f_{Y1} \to Function[{ \lambda}, e^{-\alpha+2\beta} \lambda^2 (-2 s \lambda \xi^1 \Xi^2 \Upsilon^2 - \eta^1 \lambda \Upsilon^2^2 + 3 t \eta^1 \lambda \Upsilon^2^2 - s \xi^1 \Omega^2 +
  s \lambda^2 \xi^1 \Xi^2 \Omega^2 - s^2 \lambda^2 \xi^1 \Xi^2 \Omega^2 - \zeta^1 \lambda \Upsilon^2 \Omega^2 + 3 s t \zeta^1 \lambda \Upsilon^2 \Omega^2 - 4 s t \eta^1 \lambda^2 \xi^1 \Upsilon^2 \Omega^2 +
  2 s \zeta^1 \lambda^2 \xi^1 \Omega^2^2 - 4 s^2 t \zeta^1 \lambda^2 \xi^1 \Omega^2^2 - s^2 \eta^1 \lambda^3 \xi^1 \Omega^2^2 + 3 s^2 t \eta^1 \lambda^3 \xi^1 \Omega^2^2)]}}
```

```
In[*]:= {sol33} =
  DSolve[Coefficient[CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /.
    sol9 /. sol10 /. sol11 /. sol12 /. sol13 /. sol14 /. sol15 /.
    sol16 /. sol17 /. sol18 /. sol19 /. sol20 /. sol21 /. sol22 /.
    sol23 /. sol24 /. sol25 /. sol26 /. sol27 /. sol28 /. sol29 /.
    sol30 /. sol31 /. sol32], QU[X]] == 0 & fX1[0] == 0, fX1, λ]
```

```
Out[*]=
  { {fX1 → Function[{λ}, -e2α1-β1 λ3 (ξ1 Ξ22 - 3 s ξ1 Ξ22 - η1 Ξ2 Υ2 + t η1 Ξ2 Υ2 -
    ξ1 Ξ2 Ω2 - s t ξ1 Ξ2 Ω2 - 3 s η1 λ ξ1 Ξ2 Ω2 + s t η1 λ ξ1 Ξ2 Ω2 + η12 λ Υ2 Ω2 -
    t η12 λ Υ2 Ω2 - 2 s t ξ1 η1 λ Ω22 - 2 s η12 λ2 ξ1 Ω22 + 2 s t η12 λ2 ξ1 Ω22) ] } }
```

```
In[*]:= Coefficient[CE[
  diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /. sol10 /. sol11 /.
  sol12 /. sol13 /. sol14 /. sol15 /. sol16 /. sol17 /. sol18 /.
  sol19 /. sol20 /. sol21 /. sol22 /. sol23 /. sol24 /. sol25 /. sol26 /.
  sol27 /. sol28 /. sol29 /. sol30 /. sol31 /. sol32 /. sol33], QU[z]]
```

```
Out[*]=
  6 eα2+β2 s ∈ ξ1 λ2 ξ1 Ξ2 - 4 e2α2-β2 ∈ η2 λ3 ξ12 Ξ2 + 12 e2α2-β2 s ∈ η2 λ3 ξ12 Ξ2 -
  6 e2α2-β2 t ∈ ξ1 η2 λ2 Υ2 + 4 e2α2-β2 ∈ η1 η2 λ3 ξ1 Υ2 - 4 e2α2-β2 t ∈ η1 η2 λ3 ξ1 Υ2 -
  3 eα2+β2 ∈ ξ12 λ2 Ω2 + 9 eα2+β2 s t ∈ ξ12 λ2 Ω2 + 8 eα2+β2 s ∈ ξ1 η1 λ3 ξ1 Ω2 -
  8 eα2+β2 s t ∈ ξ1 η1 λ3 ξ1 Ω2 - 4 e2α2-β2 ∈ ξ1 η2 λ3 ξ1 Ω2 + 12 e2α2-β2 s t ∈ ξ1 η2 λ3 ξ1 Ω2 +
  10 e2α2-β2 s ∈ η1 η2 λ4 ξ12 Ω2 - 10 e2α2-β2 s t ∈ η1 η2 λ4 ξ12 Ω2 - ∈ fz1'[λ]
```

```
In[*]:= {sol34} = DSolve[
  Coefficient[CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /.
    sol10 /. sol11 /. sol12 /. sol13 /. sol14 /. sol15 /. sol16 /.
    sol17 /. sol18 /. sol19 /. sol20 /. sol21 /. sol22 /. sol23 /.
    sol24 /. sol25 /. sol26 /. sol27 /. sol28 /. sol29 /. sol30 /.
    sol31 /. sol32 /. sol33], QU[z]] == 0 & fz1[0] == 0, fz1, λ]
```

```
Out[*]=
  { {fz1 → Function[{λ},
    -eα2-β2 λ3 (-2 e2β2 s ξ1 ξ1 Ξ2 + eα2 η2 λ ξ12 Ξ2 - 3 eα2 s η2 λ ξ12 Ξ2 + 2 eα2 t ξ1 η2 Υ2 -
    eα2 η1 η2 λ ξ1 Υ2 + eα2 t η1 η2 λ ξ1 Υ2 + e2β2 ξ12 Ω2 - 3 e2β2 s t ξ12 Ω2 - 2 e2β2 s ξ1 η1 λ ξ1 Ω2 +
    2 e2β2 s t ξ1 η1 λ ξ1 Ω2 + eα2 ξ1 η2 λ ξ1 Ω2 - 3 eα2 s t ξ1 η2 λ ξ1 Ω2 -
    2 eα2 s η1 η2 λ2 ξ12 Ω2 + 2 eα2 s t η1 η2 λ2 ξ12 Ω2) ] } }
```

```
In[*]:= {sol35} = DSolve[
  Coefficient[CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /.
    sol10 /. sol11 /. sol12 /. sol13 /. sol14 /. sol15 /. sol16 /.
    sol17 /. sol18 /. sol19 /. sol20 /. sol21 /. sol22 /. sol23 /.
    sol24 /. sol25 /. sol26 /. sol27 /. sol28 /. sol29 /. sol30 /.
    sol31 /. sol32 /. sol33 /. sol34], QU[]] == 0 & ft1[0] == 0, ft1, λ]
```

```
Out[*]=
```

$$\left\{ \left\{ \text{ft1} \rightarrow \text{Function}\left[\{\lambda\}, \frac{1}{2} \left( -\lambda^4 \xi^2 \Xi^2 + 4 s \lambda^4 \xi^2 \Xi^2 - 3 s^2 \lambda^4 \xi^2 \Xi^2 + 4 \xi^2 \lambda^3 \Xi^2 \Upsilon^2 - 4 t \xi^2 \lambda^3 \Xi^2 \Upsilon^2 - 4 s \eta^2 \lambda^4 \xi^2 \Xi^2 \Upsilon^2 + 4 s t \eta^2 \lambda^4 \xi^2 \Xi^2 \Upsilon^2 - \eta^2 \lambda^4 \Upsilon^2 + 4 t \eta^2 \lambda^4 \Upsilon^2 - 3 t^2 \eta^2 \lambda^4 \Upsilon^2 - 2 s \eta^2 \lambda^3 \xi^2 \Omega^2 + 2 s t \eta^2 \lambda^3 \xi^2 \Omega^2 + 4 s t \xi^2 \lambda^4 \xi^2 \Xi^2 \Omega^2 - 4 s^2 t \xi^2 \lambda^4 \xi^2 \Xi^2 \Omega^2 + 2 s \eta^2 \lambda^5 \xi^2 \Xi^2 \Omega^2 - 2 s^2 \eta^2 \lambda^5 \xi^2 \Xi^2 \Omega^2 - 2 s t \eta^2 \lambda^5 \xi^2 \Xi^2 \Omega^2 + 2 s^2 t \eta^2 \lambda^5 \xi^2 \Xi^2 \Omega^2 + 4 s t \xi^2 \eta^2 \lambda^4 \Upsilon^2 \Omega^2 - 4 s t^2 \xi^2 \eta^2 \lambda^4 \Upsilon^2 \Omega^2 - 4 s t \eta^2 \lambda^5 \xi^2 \Upsilon^2 \Omega^2 + 4 s t^2 \eta^2 \lambda^5 \xi^2 \Upsilon^2 \Omega^2 - \xi^2 \lambda^4 \Omega^2 + 4 s t \xi^2 \lambda^4 \Omega^2 - 3 s^2 t^2 \xi^2 \lambda^4 \Omega^2 + 2 s \xi^2 \eta^2 \lambda^5 \xi^2 \Omega^2 - 2 s t \xi^2 \eta^2 \lambda^5 \xi^2 \Omega^2 - 6 s^2 t \xi^2 \eta^2 \lambda^5 \xi^2 \Omega^2 + 6 s^2 t^2 \xi^2 \eta^2 \lambda^5 \xi^2 \Omega^2 - s^2 \eta^2 \lambda^6 \xi^2 \Omega^2 + 4 s^2 t \eta^2 \lambda^6 \xi^2 \Omega^2 - 3 s^2 t^2 \eta^2 \lambda^6 \xi^2 \Omega^2 \right) \right\} \right\}$$

```
In[*]:= CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /. sol10 /. sol11 /.
  sol12 /. sol13 /. sol14 /. sol15 /. sol16 /. sol17 /. sol18 /.
  sol19 /. sol20 /. sol21 /. sol22 /. sol23 /. sol24 /. sol25 /. sol26 /.
  sol27 /. sol28 /. sol29 /. sol30 /. sol31 /. sol32 /. sol33 /. sol34 /. sol35]
```

```
Out[*]=
```

0

```
In[*]:= g
```

```
Out[*]=
```

$$\text{ft0}[\lambda] + \epsilon \text{ft1}[\lambda] + \left( x X f_{1_{xx}}[\lambda] + X^2 f_{1_{x^2}}[\lambda] + x y f_{1_{xy}}[\lambda] + X y f_{1_{xy}}[\lambda] + a Y f_{1_{ay}}[\lambda] + x Y f_{1_{xy}}[\lambda] + X Y f_{1_{xy}}[\lambda] + y Y f_{1_{yy}}[\lambda] + Y^2 f_{1_{y^2}}[\lambda] + x z f_{1_{xz}}[\lambda] + X z f_{1_{xz}}[\lambda] + y z f_{1_{yz}}[\lambda] + Y z f_{1_{yz}}[\lambda] + z^2 f_{1_{z^2}}[\lambda] + x Z f_{1_{xz}}[\lambda] + X Z f_{1_{xz}}[\lambda] + y Z f_{1_{yz}}[\lambda] + Y Z f_{1_{yz}}[\lambda] + z Z f_{1_{zz}}[\lambda] \right)$$

In[\*]:= Simplify[

(f.QU@Generators + g) /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8 /. sol9 /.  
 sol10 /. sol11 /. sol12 /. sol13 /. sol14 /. sol15 /.  
 sol16 /. sol17 /. sol18 /. sol19 /. sol20 /. sol21 /.  
 sol22 /. sol23 /. sol24 /. sol25 /. sol26 /. sol27 /. sol28 /.  
 sol29 /. sol30 /. sol31 /. sol32 /. sol33 /. sol34 /. sol35]

Out[\*]=

$$\begin{aligned}
 & b \left( \beta_1 + \beta_2 + 2t \in \lambda^2 (\eta_1 \Upsilon_2 + s \zeta_1 \Omega_2 - s \eta_1 \lambda \xi_1 \Omega_2) \right) + \\
 & a \left( \alpha_1 + \alpha_2 + 2s \in \lambda^2 \left( t \zeta_1 \Omega_2 + \xi_1 \left( \Xi_2 + e^{-\alpha_1+2\beta_1} \Upsilon \Omega_2 - (-1+t) \eta_1 \lambda \Omega_2 \right) \right) \right) - \\
 & \frac{1}{2} e^{-2\alpha_1-2\alpha_2-2\beta_1-\beta_2} \lambda \left( -2 e^{4\alpha_1+\beta_1+3\beta_2} x y \in \eta_1 \lambda \Xi_2 + 4 e^{\alpha_1+4\beta_1+3\beta_2} y \Upsilon \in \eta_1 \lambda \Upsilon_2 + \right. \\
 & 4 e^{3\alpha_1+2\alpha_2+3\beta_1+2\beta_2} z z \in \zeta_1 \lambda \Omega_2 + 2 e^{3(\alpha_1+\beta_1+\beta_2)} y z \in \eta_1 \lambda \Omega_2 + 2 e^{3(\alpha_1+\alpha_2+\beta_1)} z \in \lambda (x + z \eta_2 \lambda) \xi_1 \Omega_2 - \\
 & 2 e^{6\alpha_1+\alpha_2+\beta_2} x^2 \in \eta_1 \lambda^2 \Xi_2 \Omega_2 + 2 e^{\alpha_2+6\beta_1+\beta_2} s \Upsilon^2 \in \lambda^2 \xi_1 \Upsilon_2 \Omega_2 + 2 e^{5\alpha_1+\alpha_2+2\beta_1+\beta_2} x z \in \eta_1 \lambda^2 \Omega_2^2 - \\
 & 2 e^{2\alpha_1+\alpha_2+5\beta_1+\beta_2} s \Upsilon z \in \lambda^2 \xi_1 \Omega_2^2 + 2 e^{4\alpha_1+2\alpha_2+\beta_1+2\beta_2} x z \in \zeta_1 \lambda (\Xi_2 + \eta_1 \lambda \Omega_2) + \\
 & 2 e^{4\alpha_1+3\alpha_2+\beta_1} x \in \lambda (x + z \eta_2 \lambda) \xi_1 (2 \Xi_2 + \eta_1 \lambda \Omega_2) - 2 e^{\alpha_1+3\alpha_2+4\beta_1} \Upsilon \in \lambda (x + z \eta_2 \lambda) \\
 & \xi_1 (\Upsilon_2 + 2 s \lambda \xi_1 \Omega_2) - 2 e^{\alpha_1+2(\alpha_2+2\beta_1+\beta_2)} \Upsilon z \in \zeta_1 \lambda (-\Upsilon_2 + 3 s \lambda \xi_1 \Omega_2) + \\
 & e^{2\alpha_1+\alpha_2+2\beta_1+\beta_2} (-2 z \zeta_2 - 2 y \eta_2 - 2 x \Xi_1 - 2 x \xi_2 - 2 \lambda \xi_1 \Xi_2 + 2 s \lambda \xi_1 \Xi_2 + e \lambda^3 \xi_1^2 \Xi_2^2 - \\
 & 4 s \in \lambda^3 \xi_1^2 \Xi_2^2 + 3 s^2 \in \lambda^3 \xi_1^2 \Xi_2^2 - 2 \Upsilon \Upsilon_1 - 2 \eta_1 \lambda \Upsilon_2 + 2 t \eta_1 \lambda \Upsilon_2 - 4 \in \zeta_1 \lambda^2 \Xi_2 \Upsilon_2 + \\
 & 4 t \in \zeta_1 \lambda^2 \Xi_2 \Upsilon_2 + 4 s \in \eta_1 \lambda^3 \xi_1 \Xi_2 \Upsilon_2 - 4 s t \in \eta_1 \lambda^3 \xi_1 \Xi_2 \Upsilon_2 + e \eta_1^2 \lambda^3 \Upsilon_2^2 - \\
 & 4 t \in \eta_1^2 \lambda^3 \Upsilon_2^2 + 3 t^2 \in \eta_1^2 \lambda^3 \Upsilon_2^2 - 2 z \Omega_1 - 2 \zeta_1 \lambda \Omega_2 + 2 s t \zeta_1 \lambda \Omega_2 + 2 s \eta_1 \lambda^2 \xi_1 \Omega_2 - \\
 & 2 s t \eta_1 \lambda^2 \xi_1 \Omega_2 + 2 s \in \eta_1 \lambda^2 \xi_1 \Omega_2 - 2 s t \in \eta_1 \lambda^2 \xi_1 \Omega_2 - 4 s t \in \zeta_1 \lambda^3 \xi_1 \Xi_2 \Omega_2 + \\
 & 4 s^2 t \in \zeta_1 \lambda^3 \xi_1 \Xi_2 \Omega_2 - 2 s \in \eta_1 \lambda^4 \xi_1^2 \Xi_2 \Omega_2 + 2 s^2 \in \eta_1 \lambda^4 \xi_1^2 \Xi_2 \Omega_2 + 2 s t \in \eta_1 \lambda^4 \xi_1^2 \Xi_2 \Omega_2 - \\
 & 2 s^2 t \in \eta_1 \lambda^4 \xi_1^2 \Xi_2 \Omega_2 - 4 s t \in \zeta_1 \eta_1 \lambda^3 \Upsilon_2 \Omega_2 + 4 s t^2 \in \zeta_1 \eta_1 \lambda^3 \Upsilon_2 \Omega_2 + 4 s t \in \eta_1^2 \lambda^4 \xi_1 \Upsilon_2 \Omega_2 - \\
 & 4 s t^2 \in \eta_1^2 \lambda^4 \xi_1 \Upsilon_2 \Omega_2 + e \zeta_1^2 \lambda^3 \Omega_2^2 - 4 s t \in \zeta_1^2 \lambda^3 \Omega_2^2 + 3 s^2 t^2 \in \zeta_1^2 \lambda^3 \Omega_2^2 - \\
 & 2 s \in \zeta_1 \eta_1 \lambda^4 \xi_1 \Omega_2^2 + 2 s t \in \zeta_1 \eta_1 \lambda^4 \xi_1 \Omega_2^2 + 6 s^2 t \in \zeta_1 \eta_1 \lambda^4 \xi_1 \Omega_2^2 - 6 s^2 t^2 \in \zeta_1 \eta_1 \lambda^4 \xi_1 \Omega_2^2 + \\
 & s^2 \in \eta_1^2 \lambda^5 \xi_1^2 \Omega_2^2 - 4 s^2 t \in \eta_1^2 \lambda^5 \xi_1^2 \Omega_2^2 + 3 s^2 t^2 \in \eta_1^2 \lambda^5 \xi_1^2 \Omega_2^2) - 2 e^{2(\alpha_1+\alpha_2+\beta_1+\beta_2)} z \\
 & (\zeta_1 - y \in \eta_1 \eta_2 \lambda^2 \xi_1 + (-1+3 s t) \in \zeta_1^2 \lambda^2 \Omega_2 + 2 s \in \zeta_1 \lambda^2 \xi_1 (\Xi_2 - (-1+t) \eta_1 \lambda \Omega_2)) + \\
 & 2 e^{2\alpha_1+3\alpha_2+2\beta_1} (x + z \eta_2 \lambda) (2 t \in \zeta_1 \lambda \Upsilon_2 + e \lambda^2 \xi_1^2 (\Xi_2 - 3 s \Xi_2 + 2 s (-1+t) \eta_1 \lambda \Omega_2) + \\
 & \xi_1 (-1 - z \in \zeta_2 \lambda + y \in \eta_2 \lambda - e \eta_1 \lambda^2 \Upsilon_2 + t \in \eta_1 \lambda^2 \Upsilon_2 + e \zeta_1 \lambda^2 \Omega_2 - 3 s t \in \zeta_1 \lambda^2 \Omega_2)) + \\
 & 2 e^{2\alpha_1+2\beta_1+3\beta_2} y (-2 \in \zeta_1 \lambda \Xi_2 + e \eta_1^2 \lambda^2 (\Upsilon_2 - 3 t \Upsilon_2 + s (1+t) \lambda \xi_1 \Omega_2) + \\
 & \eta_1 (-1 + z \in \zeta_2 \lambda - e \zeta_1 \lambda^2 \Omega_2 + s \in \lambda^2 (2 \xi_1 \Xi_2 - t \zeta_1 \Omega_2))) + 2 e^{3\alpha_1+\alpha_2+3\beta_1+\beta_2} \Omega_2 \\
 & (x y \in \lambda^2 (\eta_1 \Upsilon_2 - 2 s \xi_1 (\Xi_2 + \eta_1 \lambda \Omega_2)) + z (-1 + e \lambda^2 (-2 (-1+t) \eta_1 \Upsilon_2 + \zeta_1 \Omega_2) + s \in \lambda^2 \\
 & (-2 \zeta_1 \Xi_2 - 3 t \zeta_1 \Omega_2 + 3 (-1+t) \eta_1 \lambda \xi_1 \Omega_2))) + 2 e^{\alpha_1+\alpha_2+4\beta_1+\beta_2} \Upsilon ((1-3 t) \in \eta_1 \lambda^2 \Upsilon_2^2 + \\
 & \Upsilon_2 (-1 - z \in \lambda \Omega_1 + e \zeta_1 \lambda^2 \Omega_2 + s \in \lambda^2 (2 \xi_1 \Xi_2 - 3 t \zeta_1 \Omega_2 + 4 t \eta_1 \lambda \xi_1 \Omega_2)) + s \lambda \xi_1 \\
 & \Omega_2 (1 + e (1 + z \lambda \Omega_1 + s (1-3 t) \eta_1 \lambda^3 \xi_1 \Omega_2 + \lambda^2 ((-1+s) \xi_1 \Xi_2 + 2 (-1+2 s t) \zeta_1 \Omega_2)))) - \\
 & 2 e^{4\alpha_1+\alpha_2+\beta_1+\beta_2} (-2 z \in \lambda \Upsilon_1 (\Xi_2 + \eta_1 \lambda \Omega_2) + x ((-1+3 s) \in \lambda^2 \xi_1 \Xi_2^2 + \\
 & \eta_1 \lambda \Omega_2 (1 + y \in \lambda \Upsilon_1 + e \lambda (-z \Omega_1 + 2 s t \zeta_1 \lambda \Omega_2 + (-1+t) \eta_1 \lambda (\Upsilon_2 - 2 s \lambda \xi_1 \Omega_2))) + \\
 & \Xi_2 (1 + y \in \lambda \Upsilon_1 + e \lambda (-z \Omega_1 + (1+s t) \zeta_1 \lambda \Omega_2 + \eta_1 \lambda (\Upsilon_2 - t \Upsilon_2 - s (-3+t) \lambda \xi_1 \Omega_2))))))
 \end{aligned}$$