Pensieve header: An isolated attempt to compute the multiplication tensor of QU.

The implementation of QU is modified from pensieve://Projects/SL2Portfolio/SL2PortfolioProgram.nb.

Initialization / Utilities

```
In[•]:= $k = 1; $U = QU; $E := {$k};
$trim := {ϵ^(k_·) /; k > $k → 0};
SetAttributes[{SS, SST}, HoldAll];
q_ħ_ := e^(γ ϵ ħ);
(* Upper to lower and lower to Upper: *)
U2l = {B_i^(p_·) → e^(-p ħ γ b_i), B^(p_·) → e^(-p ħ γ b), T_i^(p_·) → e^(p ħ t_i), T^(p_·) → e^(p ħ t), 𝒜_i^(p_·) → e^(p γ α_i), 𝒜^(p_·) → e^(p γ α)};
l2U = {e^(c_. b_i+d_.) :> B_i^(-c/(ħ γ)) e^d, e^(c_. b+d_.) :> B^(-c/(ħ γ)) e^d,
    e^(c_. t_i+d_.) :> T_i^(c/ħ) e^d, e^(c_. t+d_.) :> T^(c/ħ) e^d,
    e^(c_. α_i+d_.) :> 𝒜_i^(c/γ) e^d, e^(c_. α+d_.) :> 𝒜^(c/γ) e^d,
    e^ℰ_ :> e^(Expand@ℰ)};
SS[ℰ_, op_] := Collect[Normal@Series[ℰ, {ϵ, 0, $k}], ϵ, op];
SS[ℰ_] := SS[ℰ, Together];
SST[ℰ_, op___] := SS[ℰ /. U2l, op];
Simp[ℰ_, op_] := Collect[ℰ, _CU | _QU, op];
Simp[ℰ_] := Simp[ℰ, SS[#, Expand] &];
SimpT[ℰ_] := Collect[ℰ, _CU | _QU, SST[#, Expand] &];
Kδ /: Kδ_i_,j_ := If[i === j, 1, 0];
c_Integer_k_Integer := c + O[ϵ]^(k+1);
```

```
In[•]:= CF[ℰ_] := ExpandDenominator@
    ExpandNumerator@Together[Expand[ℰ] //. e^x_ e^y_ :> e^(x+y) /. e^x_ :> e^CF[x]];
```

```
In[•]:= Unprotect[SeriesData];
SeriesData /: CF[sd_SeriesData] := MapAt[CF, sd, 3];
SeriesData /: Expand[sd_SeriesData] := MapAt[Expand, sd, 3];
SeriesData /: Simplify[sd_SeriesData] := MapAt[Simplify, sd, 3];
SeriesData /: Together[sd_SeriesData] := MapAt[Together, sd, 3];
SeriesData /: Collect[sd_SeriesData, specs__] := MapAt[Collect[#, specs] &, sd, 3];
Protect[SeriesData];
```

Self-Pair (SP):

```
In[•]:= SP{}[P_] := P; SP{ξ_→x_,ps___}[P_] := Expand[P // SP{ps}] /. f_. ξ^d_. :> ∂{x,d} f
```

DeclareAlgebra

In[•]:=
```
Unprotect[NonCommutativeMultiply]; Attributes[NonCommutativeMultiply] = {};
(NCM = NonCommutativeMultiply)[x_] := x;
NCM[x_, y_, z__] := (x ⊠ y) ⊠ z;
0 ⊠ _ = _ ⊠ 0 = 0;
(x_Plus) ⊠ y_ := (# ⊠ y) & /@ x; x_ ⊠ (y_Plus) := (x ⊠ #) & /@ y;
B[x_, x_] = 0; B[x_, y_] := x ⊠ y - y ⊠ x;
B[x_, y_, e_] := B[x, y, e] = B[x, y];
```

In[•]:=
```
DeclareAlgebra[U_Symbol, opts__Rule] := Module[
  {gp, sr, g, cp, M, pow, k = 0,
   gs = Generators /. {opts},
   cs = Centrals /. {opts} /. Centrals → {}
  },
  U[Generators] = gs;
  (#_U = U@#) & /@ gs;
  gp = Alternatives @@ gs; gp = gp | gp_; (* gens *)
  sr = Flatten@Table[{g → ++k, g_i_ → {i, k}}, {g, gs}]; (* sorting → *)
  cp = Alternatives @@ cs; (* cents *)
  SetAttributes[M, HoldRest]; M[0, _] = 0; M[a_, x_] := a x;
  CE[ℰ_] := Collect[ℰ, _U, Expand] /. ℯ^x_ :↦ ℯ^Expand[x] /. $trim;
  U_i_[ℰ_] := ℰ /. {t : cp :↦ t_i, u_U :↦ (#_i &) /@ u};
  U_i_[NCM[]] = pow[ℰ_, 0] = U@{} = 1_U = U[];
  B[U@(x_)_i_, U@(y_)_i_] := U_i@B[U@x, U@y];
  B[U@(x_)_i_, U@(y_)_j_] /; i =!= j := 0;
  B[U@y_, U@x_] := CE[-B[U@x, U@y]];
  x_ ⊠ (c_. 1_U) := CE[c x]; (c_. 1_U) ⊠ x_ := CE[c x];
  (a_. U[xx___, x_]) ⊠ (b_. U[y_, yy___]) := If[OrderedQ[{x, y} /. sr],
    CE@M[a b /. $trim, U[xx, x, y, yy]],
    U@xx ⊠ CE@M[a b /. $trim, U@y ⊠ U@x + B[U@x, U@y, $E]] ⊠ U@yy ];
  U@{c_. * (l : gp)^n_, r___} /; FreeQ[c, gp] := CE[c U@Table[l, {n}] ⊠ U@{r}];
  U@{c_. * l : gp, r___} := CE[c U[l] ⊠ U@{r}];
  U@{c_, r___} /; FreeQ[c, gp] := CE[c U@{r}];
  U@{l_Plus, r___} := CE[U@{#, r} & /@ l];
  U@{l_, r___} := U@{Expand[l], r};
  U[ℰ_NonCommutativeMultiply] := U /@ ℰ;
  𝒪_U[specs___, poly_] := Module[{sp, null, vs, us},
    sp = Replace[{specs}, l_List :↦ l_null, {1}];
    vs = Join @@ (First /@ sp);
    us = Join @@ (sp /. l_s_ :↦ (l /. x_i_ :↦ x_s));
    CE[Total[
       CoefficientRules[poly, vs] /. (p_ → c_) :↦ c U@(us^p)
     ]] /. x_null :↦ x];
```

```
𝕆_U[specs___, 𝔼[L_, Q_, P_]] := 𝕆_U[specs, SS@Normal[P e^(L+Q)]];
pow[ℰ_, n_] := pow[ℰ, n-1] ⊠ ℰ;
𝕊_U[ℰ_, ss___Rule] := CE@Total[
    CoefficientRules[ℰ, First /@ {ss}] /.
     (p_ → c_) :↦ c NCM @@ MapThread[pow, {Last /@ {ss}, p}]];
σ_rs___[c_. * u_U] := (c /. (t : cp)_j :↦ t_(j/.{rs})) U[List @@ (u /. v_j_ :↦ v_(j/.{rs}))];
m_(j_→k_)[c_. * u_U] := CE[((c /. (t : cp)_j → t_k) DeleteCases[u, _(j|k)]) ⊠
    U @@ Cases[u, w_j :↦ w_k] ⊠ U @@ Cases[u, _k]];
U /: c_. * u_U * v_U := CE[c u ⊠ v];
S_i_[c_. * u_U] := CE[((c /. S_i[U, Centrals]) DeleteCases[u, _i]) ⊠
    U_i[NCM @@ Reverse@Cases[u, x_i :↦ S@U@x]]];
Δ_(i_→j_,k_)[c_. * u_U] := CE[((c /. Δ_(i→j,k)[U, Centrals]) DeleteCases[u, _i]) ⊠
    (NCM @@ Cases[u, x_i :↦ σ_(1→j,2→k)@Δ@U@x] /. NCM[] → U[])];
```

In[•]:=
```
𝕊_QU[(a^2 + a) b, a → QU[y, x], b → 7 QU[]]
```

Out[•]=
```
𝕊_QU[(a + a^2) b, a → QU[y, x], b → 7 QU[]]
```

Meta-Operations

In[•]:=
```
σ_rs___[ℰ_Plus] := σ_rs /@ ℰ;
m_(j_→j_) = Identity; m_(j_→k_)[0] = 0;
m_(j_→k_)[ℰ_Plus] := Simp[m_(j→k) /@ ℰ];
m_(is___,i_,j_→k_)[ℰ_] := m_(j→k)@m_(is,i→j)@ℰ;
S_i_[ℰ_Plus] := Simp[S_i /@ ℰ];
Δ_is___[ℰ_Plus] := Simp[Δ_is /@ ℰ];
```

Implementing CU = $\mathcal{U}(\mathrm{sl}_2^{\gamma\epsilon})$

Verify σ and Δ! Also Generalize Δ to $\Delta_{i,j_1,j_2,\dots}$.

In[•]:=
```
DeclareAlgebra[CU, Generators → {y, a, x}, Centrals → {t}];
B[a_CU, y_CU] = -γ y_CU; B[x_CU, a_CU] = -γ x_CU;
B[x_CU, y_CU] = 2 ϵ a_CU - t 1_CU;
(S@y_CU = -y_CU; S@a_CU = -a_CU; S@x_CU = -x_CU;)
S_i_[CU, Centrals] = {t_i → -t_i};
Δ@y_CU = CU@y_1 + CU@y_2; Δ@a_CU = CU@a_1 + CU@a_2; Δ@x_CU = CU@x_1 + CU@x_2;
Δ_(i_→j_,k_)[CU, Centrals] = {t_i → t_j + t_k};
```

Implementing QU = $\mathcal{U}_q(\mathrm{sl}_2^{y\epsilon})$

In[•]:=
```
ħ = γ = 1;
DeclareAlgebra[QU, Generators → {y, a, x}, Centrals → {t, T}];
B[a_QU, y_QU] = -γ y_QU; B[x_QU, a_QU] = -γ QU@x;
B[x_QU, y_QU] := SS[q_ħ - 1] QU@{y, x} + 𝕆_QU[{a}, SS[(1 - T ℯ^(-2 ϵ a ħ)) / ħ]];
(S@y_QU := 𝕆_QU[{a, y}, SS[-T^-1 ℯ^(ħ ϵ a) y]]; S@a_QU = -a_QU; S@x_QU := 𝕆_QU[{a, x}, SS[-ℯ^(ħ ϵ a) x]];)
S_i_[QU, Centrals] = {t_i → -t_i, T_i → T_i^-1};
Δ@y_QU := 𝕆_QU[{y_1, a_1}_1, {y_2}_2, SS[y_1 + T_1 ℯ^(-ħ ϵ a_1) y_2]];
Δ@a_QU = QU@a_1 + QU@a_2; Δ@x_QU := 𝕆_QU[{a_1, x_1}_1, {x_2}_2, SS[x_1 + ℯ^(-ħ ϵ a_1) x_2]];
Δ_i_→j_,k_[QU, Centrals] = {t_i → t_j + t_k, T_i → T_j T_k};
```

In[•]:=
```
B[x_QU, y_QU]
```

Out[•]=
```
(1 - T) QU[] + 2 T ϵ QU[a] + ϵ QU[y, x]
```

tSW

Logoi from Pensieve://Talks/Toulouse-1705/DogmaDemo.nb and from Pensieve://Talks/Sydney-1708/ExtraDetails@@.nb.

Goal. In either U, compute $F = e^{-\eta y} e^{\xi x} e^{\eta y} e^{-\xi x}$. First compute $G = e^{\xi x}\, y\, e^{-\xi x}$, a finite sum. Now F satisfies the ODE $\partial_\eta F = \partial_\eta\left(e^{-\eta y} e^{\eta G}\right) = -yF + FG$ with initial conditions $F(\eta = 0) = 1$. So we set it up and solve:

In[•]:=
```
SW_xy[U_, kk_] :=
  SW_xy[U, kk] = Block[{$U = U, $k = kk, $p = kk}, Module[{G, F, fs, f, bs, e, b, es},
    G = Simp[Table[ξ^k / k!, {k, 0, $k + 1}].NestList[Simp[B[x_U, #]] &, y_U, $k + 1]];
    fs = Flatten@Table[f_1,i,j,k[η], {l, 0, $k}, {i, 0, l}, {j, 0, l}, {k, 0, l}];
    F = fs.(bs = fs /. f_L_,i_,j_,k_[η] :> ϵ^L U@{y^i, a^j, x^k});
    es = Flatten[
      Table[Coefficient[e, b] == 0, {e, {F - 1_U /. η → 0, F ⊠ G - y_U ⊠ F - ∂_η F}}, {b, bs}]];
    F = F /. DSolve[es, fs, η][[1]];
    𝔼[0,
      ξ x + η y + (U /. {CU → -t η ξ, QU → η ξ (1 - T) / ħ}),
      F + 0_$k /. {ℯ → 1, U → Times}
     ] /. (v : η | ξ | t | T | y | a | x) → v_1
   ]];
tSW_xy_,i_,j_→k_ := SW_xy[$U, $k] /. {ξ_1 → ξ_i, η_1 → η_j, (v : t | T | y | a | x)_1 → v_k};
tSW_xa,i_,j_→k_ := 𝔼[α_j a_k, ℯ^(-γ α_j) ξ_i x_k, 1];
tSW_ay,i_,j_→k_ := 𝔼[α_i a_k, ℯ^(-γ α_i) η_j y_k, 1];
```

Exponentials as needed.

Task. Define $\mathrm{Exp}_{U_i,k}[\xi, P]$ which computes $e^{\xi\, \mathbb{D}(P)}$ to ϵ^k in the algebra U_i, where ξ is a scalar, X is x_i or y_i, and P is an ϵ-dependent near-docile element, giving the answer in $\mathbb{E}$-form. Should satisfy

$U@\text{Exp}_{U_i,k}[\xi, P] == \mathbb{S}_U\left[e^{\xi x},\ x \to \mathbb{O}(P)\right]$.

Methodology. If $P_0 := P_{\epsilon=0}$ and $e^{\xi\,\mathbb{O}(P)} = \mathbb{O}\left(e^{\xi P_0}\,F(\xi)\right)$, then $F(\xi=0)=1$ and we have:

$\mathbb{O}\left(e^{\xi P_0}(P_0\,F(\xi)+\partial_\xi F)\right) = \mathbb{O}\left(\partial_\xi e^{\xi P_0}\,F(\xi)\right) = \partial_\xi\,\mathbb{O}\left(e^{\xi P_0}\,F(\xi)\right) = \partial_\xi e^{\xi\,\mathbb{O}(P)} = e^{\xi\,\mathbb{O}(P)}\,\mathbb{O}(P) = \mathbb{O}\left(e^{\xi P_0}\,F(\xi)\right)\mathbb{O}(P)$.

This is an ODE for F. Setting inductively $F_k = F_{k-1} + \epsilon^k\,\varphi$ we find that $F_0 = 1$ and solve for φ.

In[•]:=
```
(* Bug: The first line is valid only if 𝕆(e^P0) == e^𝕆(P0). *)
(* Bug: ξ must be a symbol. *)
Exp_{U_i_,0}[ξ_, P_] := Module[{LQ = Normal@P /. ϵ → 0},
   𝔼[ξ LQ /. (x | y)_i → 0, ξ LQ /. (t | a)_i → 0, 1] ];
Exp_{U_i_,k_}[ξ_, P_] := Block[{$U = U, $k = k},
  Module[{P0, φ, φs, F, j, rhs, at0, atξ},
   P0 = Normal@P /. ϵ → 0;
   φs =
    Flatten@Table[φ_{j1,j2,j3}[ξ], {j2, 0, k}, {j1, 0, 2 k + 1 - j2}, {j3, 0, 2 k + 1 - j2 - j1}];
   F = Normal@Last@Exp_{U_i,k-1}[ξ, P] + ϵ^k φs. (φs /. φ_js__[ξ] :> Times @@ {y_i, a_i, x_i}^{js});
   rhs = Normal@
     Last@m_{i,j→i}[𝔼[ξ P0 /. (x | y)_i → 0, ξ P0 /. (t | a)_i → 0, F + 0_k] m_{i→j}@𝔼[0, 0, P + 0_k]];
   at0 = (# == 0) & /@ Flatten@CoefficientList[F - 1 /. ξ → 0, {y_i, a_i, x_i}];
   atξ = (# == 0) & /@ Flatten@CoefficientList[(∂_ξ F) + P0 F - rhs, {y_i, a_i, x_i}];
   𝔼[ξ P0 /. (x | y)_i → 0, ξ P0 /. (t | a)_i → 0, F + 0_k] /.
    DSolve[And @@ (at0 ⋃ atξ), φs, ξ][[1]] ]]
```

In[•]:=
```
$k = 1
```

Out[•]=
```
1
```

In[•]:=
```
$Basis = {a_QU, x_QU, y_QU};
Module[{a, b}, Table[{a, b} → CE[B[a, b]], {a, $Basis}, {b, $Basis}]] // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} \{QU[a], QU[a]\} \to 0 & \{QU[a], QU[x]\} \to QU[x] & \{QU[x], QU[y]\} \to QU[y] \\ \{QU[x], QU[a]\} \to -QU[x] & \{QU[x], QU[x]\} \to 0 & \{QU[y], QU[x]\} \to (-1+T)\,QU[\,] - 2\,T \in QU[a] - \in QU[y, x] \end{pmatrix}$$

```
In[•]:= $Basis = {a_QU, x_QU, y_QU};
        Module[{a, b},
         DeleteCases[
          Flatten[Table[{a, b, c} → CE[B[a, b] + B[b, a]], {a, $Basis}, {b, $Basis}]], _ → 0]
        ]
        Module[{a, b, c},
         DeleteCases[Flatten[Table[{a, b, c} → CE[B[a, B[b, c]] + B[b, B[c, a]] + B[c, B[a, b]]],
             {a, $Basis}, {b, $Basis}, {c, $Basis}]], _ → 0]
        ]
        $ABasis = {a_QU, x_QU, y_QU, QU[y, y], QU[y, a], QU[y, x], QU[a, a], QU[a, x], QU[x, x]};
        Module[{a, b, c},
         DeleteCases[Flatten[Table[{a, b, c} → CE[a ⊠ CE[b ⊠ c] - CE[a ⊠ b] ⊠ c],
             {a, $ABasis}, {b, $ABasis}, {c, $ABasis}]], _ → 0]
        ]
Out[•]=
        {}
Out[•]=
        {}
Out[•]=
        {}

In[•]:= Module[{a, b, c},
          Table[{a, b, c} → CE[a ⊠ CE[b ⊠ c]], {a, $ABasis}, {b, $ABasis}, {c, $ABasis}]
         ];

In[•]:= B[x_QU, y_QU] // CE
Out[•]=
        (1 - T) QU[] + 2 T ∈ QU[a] + ∈ QU[y, x]

In[•]:= B[x_QU, B[x_QU, y_QU]] // CE
Out[•]=
        (∈ - 3 T ∈) QU[x]

In[•]:= B[x_QU, B[x_QU, B[x_QU, y_QU]]] // CE
Out[•]=
        0

In[•]:= B[x_QU, B[x_QU, B[x_QU, B[x_QU, y_QU]]]] // CE
Out[•]=
        0

In[•]:= B[x_QU, B[x_QU, B[x_QU, B[x_QU, B[x_QU, y_QU]]]]] // Expand
Out[•]=
        0

In[•]:= B[x_QU, B[x_QU, B[x_QU, B[x_QU, B[x_QU, B[x_QU, y_QU]]]]]] // Expand
Out[•]=
        0
```

```
In[•]:= adPower[x_, k_, Env_][c_ * y_QU] := CE[c adPower[x, k, Env][y]];
        adPower[x_, k_, Env_][y_Plus] := adPower[x, k, Env] /@ y;
        adPower[_, 0, Env_][y_] := y;
        adPower[x_, k_, Env_][0] = 0;
        adPower[x_, k_, Env_][y_QU] :=
          adPower[x, k, Env][y] = CE@B[adPower[x, k - 1, Env][y], x];
        adPower[x_, k_][y_] := adPower[x, k, $E][y];
```

```
In[•]:= adPower[x_QU, 1][y_QU]
Out[•]=
        (-1 + T) QU[] - 2 T ∈ QU[a] - ∈ QU[y, x]
```

```
In[•]:= adPower[x_QU, 2][y_QU]
Out[•]=
        (∈ - 3 T ∈) QU[x]
```

```
In[•]:= adPower[x_QU, 3][y_QU]
Out[•]=
        0
```

```
In[•]:= adPower[x_QU, 4][y_QU]
Out[•]=
        0
```

```
In[•]:= adPower[x_QU, 5][y_QU]
Out[•]=
        0
```

```
In[•]:= NilQ[x_QU] = NilQ[y_QU] = True; NilQ[a_QU] = False;
        PotQ[c_. * x_QU] := ¬ NilQ[x]; (* Potent Q *)
        NilQ[c_ * x_QU] := NilQ[x];
        PotQ[x_Plus] := And @@ (PotQ /@ (List @@ x));
        NilQ[x_Plus] := And @@ (NilQ /@ (List @@ x));
```

```
In[•]:= Ad[0] = Identity;
        Ad[c_. a_QU][z_] := z /. w_QU :> ℯ^(c ϒ (Count[w,y]-Count[w,x])) w;
        Ad[c_. QU[]][z_] := z;
```

```
In[•]:= Ad[x_][y_] /; NilQ[x] := Expand@Module[{k = 0, s = 0},
           While[adPower[x, k][y] =!= 0,
            s += adPower[x, k][y] / k!;
            ++k
           ];
           s
          ]
```

In[•]:=
```
Ad[x_QU][y_QU]
```
Out[•]=
$$-QU[] + T\,QU[] - 2\,T\in QU[a] + \frac{1}{2}\in QU[x] - \frac{3}{2}\,T\in QU[x] + QU[y] - \in QU[y, x]$$

In[•]:=
```
$k
```
Out[•]=
$$1$$

In[•]:=
```
Ad[c a_QU] /@ {QU[], x_QU, y_QU, QU[x, y]}
```
Out[•]=
$$\{QU[],\ e^{-c}\,QU[x],\ e^{c}\,QU[y],\ QU[x, y]\}$$

In[•]:=
```
Ad[(α1 + α2 + ϵ fa[λ]) a_QU] /@ {QU[], x_QU, y_QU, QU[x, y]}
```
Out[•]=
$$\left\{QU[],\ e^{-\alpha 1-\alpha 2-\epsilon\,fa[\lambda]}\,QU[x],\ e^{\alpha 1+\alpha 2+\epsilon\,fa[\lambda]}\,QU[y],\ QU[x, y]\right\}$$

In[•]:=
```
Normal@Series[e^(-γ (α1+α2+ϵ fa[λ])), {ϵ, 0, $k}]
```
Out[•]=
$$e^{-\alpha 1-\alpha 2} - e^{-\alpha 1-\alpha 2}\in fa[\lambda]$$

In[•]:=
```
CE[
 Ad[(α1 + α2 + ϵ fa[λ]) a_QU] /@ {QU[], x_QU, y_QU, QU[x, y]} /. e^ℰ_ :> Normal@Series[e^ℰ, {ϵ, 0, $k}]]
```
Out[•]=
$$\left\{QU[],\ \left(e^{-\alpha 1-\alpha 2} - e^{-\alpha 1-\alpha 2}\in fa[\lambda]\right)\,QU[x],\ \left(e^{\alpha 1+\alpha 2} + e^{\alpha 1+\alpha 2}\in fa[\lambda]\right)\,QU[y],\ QU[x, y]\right\}$$

In[•]:=
```
Ad[a_QU][Ad[x_QU][y_QU]]
```
Out[•]=
$$-QU[] + T\,QU[] - 2\,T\in QU[a] + \frac{\in QU[x]}{2\,e} - \frac{3\,T\in QU[x]}{2\,e} + e\,QU[y] - \in QU[y, x]$$

In[•]:=
```
TT_λ_[] = 0; (* Translated Tangent *)
TT_λ_[xs___, x_] := CE[Ad[x][TT_λ[xs]] + ∂_λ x];
TT_λ_[xs_List] := TT_λ @@ xs
```

In[•]:=
```
TT_λ[λ y_QU, λ a_QU, λ x_QU]
```
Out[•]=
$$\left(-e^{\lambda}\,\lambda + e^{\lambda}\,T\,\lambda\right)\,QU[] + \left(1 - 2\,e^{\lambda}\,T\in\lambda\right)\,QU[a] + \left(1 + \lambda + \frac{1}{2}\,e^{\lambda}\in\lambda^2 - \frac{3}{2}\,e^{\lambda}\,T\in\lambda^2\right)\,QU[x] + e^{\lambda}\,QU[y] - e^{\lambda}\in\lambda\,QU[y, x]$$

In[•]:=
```
Seepage_U_[{}, c_] = c;
Seepage_U_[ℱs_List, P_] := Module[{gs = U[Generators], lx},
  lx = gs[[Length@ℱs]];
  Collect[P, lx, (Ad[Last[ℱs]] lx_U][𝔒_U[gs, Seepage_U[Most@ℱs, #]]] /. U → Times) &]
 ]
```

In[•]:= **B[y_{QU}, $\alpha\, a_{QU}$]**

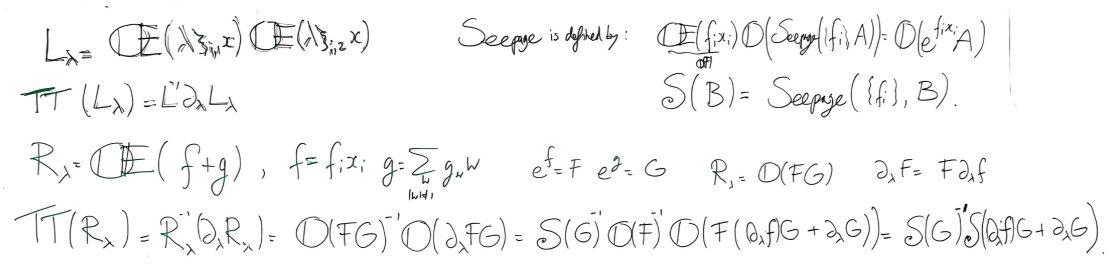
Out[•]=

$$\alpha\, \text{QU}[y]$$

In[•]:= **Seepage_{QU}[{η, α, ξ}, y]**

Out[•]=

$$e^{\alpha} y - e^{\alpha}\, \xi + e^{\alpha}\, T\, \xi - 2\, a\, e^{\alpha}\, T \in \xi - e^{\alpha}\, x\, y \in \xi + \frac{1}{2}\, e^{\alpha}\, x \in \xi^2 - \frac{3}{2}\, e^{\alpha}\, T\, x \in \xi^2$$



In[•]:= **TTL = TT_{λ}[$\lambda\, \eta 1\, y_{QU}$, $\alpha 1\, a_{QU}$, $\lambda\, \xi 1\, x_{QU}$, $\lambda\, \eta 2\, y_{QU}$, $\alpha 2\, a_{QU}$, $\lambda\, \xi 2\, x_{QU}$]**

Out[•]=

$$\begin{aligned}
&\Big(-e^{\alpha 1}\, \eta 1\, \lambda\, \xi 1 + e^{\alpha 1}\, T\, \eta 1\, \lambda\, \xi 1 + \eta 2\, \lambda\, \xi 1 - T\, \eta 2\, \lambda\, \xi 1 + \frac{1}{2}\, e^{\alpha 1} \in \eta 1\, \eta 2\, \lambda^3\, \xi 1^2 - 2\, e^{\alpha 1}\, T \in \eta 1\, \eta 2\, \lambda^3\, \xi 1^2 + \\
&\quad \frac{3}{2}\, e^{\alpha 1}\, T^2 \in \eta 1\, \eta 2\, \lambda^3\, \xi 1^2 - e^{\alpha 1+\alpha 2}\, \eta 1\, \lambda\, \xi 2 + e^{\alpha 1+\alpha 2}\, T\, \eta 1\, \lambda\, \xi 2 - e^{\alpha 2}\, \eta 2\, \lambda\, \xi 2 + e^{\alpha 2}\, T\, \eta 2\, \lambda\, \xi 2 + \\
&\quad e^{\alpha 1+\alpha 2} \in \eta 1\, \eta 2\, \lambda^3\, \xi 1\, \xi 2 - 4\, e^{\alpha 1+\alpha 2}\, T \in \eta 1\, \eta 2\, \lambda^3\, \xi 1\, \xi 2 + 3\, e^{\alpha 1+\alpha 2}\, T^2 \in \eta 1\, \eta 2\, \lambda^3\, \xi 1\, \xi 2 - \\
&\quad \frac{1}{2}\, e^{\alpha 2} \in \eta 2^2\, \lambda^3\, \xi 1\, \xi 2 + 2\, e^{\alpha 2}\, T \in \eta 2^2\, \lambda^3\, \xi 1\, \xi 2 - \frac{3}{2}\, e^{\alpha 2}\, T^2 \in \eta 2^2\, \lambda^3\, \xi 1\, \xi 2\Big)\, \text{QU}[\,] + \\
&\Big(-2\, e^{\alpha 1}\, T \in \eta 1\, \lambda\, \xi 1 + 2\, T \in \eta 2\, \lambda\, \xi 1 - 2\, e^{\alpha 1+\alpha 2}\, T \in \eta 1\, \lambda\, \xi 2 - 2\, e^{\alpha 2}\, T \in \eta 2\, \lambda\, \xi 2\Big)\, \text{QU}[a] + \\
&\Big(e^{-\alpha 2}\, \xi 1 + \frac{1}{2}\, e^{\alpha 1-\alpha 2} \in \eta 1\, \lambda^2\, \xi 1^2 - \frac{3}{2}\, e^{\alpha 1-\alpha 2}\, T \in \eta 1\, \lambda^2\, \xi 1^2 + \xi 2 + e^{\alpha 1} \in \eta 1\, \lambda^2\, \xi 1\, \xi 2 - \\
&\quad 3\, e^{\alpha 1}\, T \in \eta 1\, \lambda^2\, \xi 1\, \xi 2 - \epsilon \eta 2\, \lambda^2\, \xi 1\, \xi 2 + 3\, T \in \eta 2\, \lambda^2\, \xi 1\, \xi 2 + \frac{1}{2}\, e^{\alpha 1+\alpha 2} \in \eta 1\, \lambda^2\, \xi 2^2 - \\
&\quad \frac{3}{2}\, e^{\alpha 1+\alpha 2}\, T \in \eta 1\, \lambda^2\, \xi 2^2 + \frac{1}{2}\, e^{\alpha 2} \in \eta 2\, \lambda^2\, \xi 2^2 - \frac{3}{2}\, e^{\alpha 2}\, T \in \eta 2\, \lambda^2\, \xi 2^2\Big)\, \text{QU}[x] + \\
&\Big(e^{\alpha 1+\alpha 2}\, \eta 1 + e^{\alpha 2}\, \eta 2 - e^{\alpha 1+\alpha 2} \in \eta 1\, \eta 2\, \lambda^2\, \xi 1 + 3\, e^{\alpha 1+\alpha 2}\, T \in \eta 1\, \eta 2\, \lambda^2\, \xi 1 + \\
&\quad \frac{1}{2}\, e^{\alpha 2} \in \eta 2^2\, \lambda^2\, \xi 1 - \frac{3}{2}\, e^{\alpha 2}\, T \in \eta 2^2\, \lambda^2\, \xi 1\Big)\, \text{QU}[y] + \\
&\Big(-e^{\alpha 1} \in \eta 1\, \lambda\, \xi 1 + \epsilon \eta 2\, \lambda\, \xi 1 - e^{\alpha 1+\alpha 2} \in \eta 1\, \lambda\, \xi 2 - e^{\alpha 2} \in \eta 2\, \lambda\, \xi 2\Big)\, \text{QU}[y, x]
\end{aligned}$$

```
In[•]:= f = {fy0[λ] + ϵ fy1[λ], α1 + α2 + ϵ fa1[λ], fx0[λ] + ϵ fx1[λ]}
        g = (ft0[λ] + ϵ ft1[λ]) + (ϵ fyx1[λ]) y x
        G = SS[e^g]
```

Out[•]=

$$\{\text{fy0}[\lambda] + \epsilon\, \text{fy1}[\lambda],\ \alpha 1 + \alpha 2 + \epsilon\, \text{fa1}[\lambda],\ \text{fx0}[\lambda] + \epsilon\, \text{fx1}[\lambda]\}$$

Out[•]=

$$\text{ft0}[\lambda] + \epsilon\, \text{ft1}[\lambda] + x\, y\, \epsilon\, \text{fyx1}[\lambda]$$

Out[•]=

$$e^{\text{ft0}[\lambda]} + e^{\text{ft0}[\lambda]}\, \epsilon\, (\text{ft1}[\lambda] + x\, y\, \text{fyx1}[\lambda])$$

```
In[•]:= 𝒮[B_] := CE[SS@𝔒QU[{y, a, x}, SeepageQU[f, SS@B]]];
        𝒮[G]
```

Out[•]=

$$\left(e^{\text{ft0}[\lambda]} + e^{\text{ft0}[\lambda]}\, \epsilon\, \text{ft1}[\lambda]\right) \text{QU}[\,] + \left(-e^{\alpha 1+\alpha 2+\text{ft0}[\lambda]}\, \epsilon\, \text{fx0}[\lambda]\, \text{fyx1}[\lambda] + e^{\alpha 1+\alpha 2+\text{ft0}[\lambda]}\, T\, \epsilon\, \text{fx0}[\lambda]\, \text{fyx1}[\lambda]\right) \text{QU}[x] + e^{\alpha 1+\alpha 2+\text{ft0}[\lambda]}\, \epsilon\, \text{fyx1}[\lambda]\, \text{QU}[y, x]$$

```
In[•]:= InvertU_[ℰ_] := Module[{A, B, β}, (* ℰ=A U[]+ϵ B *)
          A = Coefficient[ℰ, 1U] /. ϵ → 0;
          B = CE[ϵ^-1 (ℰ - A 1U)];
          𝔖U[SS[(A + ϵ β)^-1], β → B]
         ]
```

```
In[•]:= InvertQU[𝒮[G]]
```

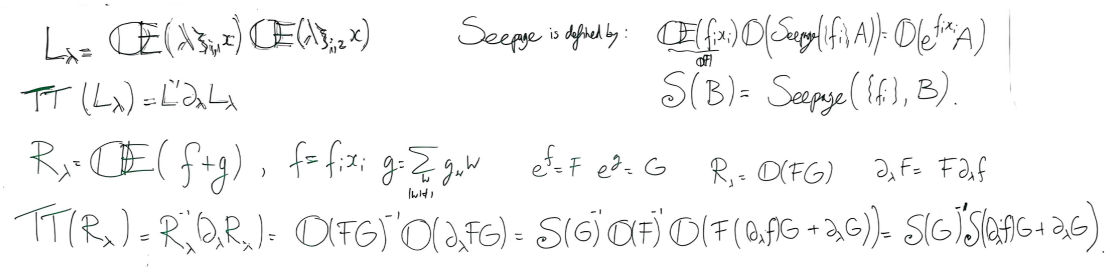
Out[•]=

$$\left(e^{-\text{ft0}[\lambda]} - e^{-\text{ft0}[\lambda]}\, \epsilon\, \text{ft1}[\lambda]\right) \text{QU}[\,] + \left(e^{\alpha 1+\alpha 2-\text{ft0}[\lambda]}\, \epsilon\, \text{fx0}[\lambda]\, \text{fyx1}[\lambda] - e^{\alpha 1+\alpha 2-\text{ft0}[\lambda]}\, T\, \epsilon\, \text{fx0}[\lambda]\, \text{fyx1}[\lambda]\right) \text{QU}[x] - e^{\alpha 1+\alpha 2-\text{ft0}[\lambda]}\, \epsilon\, \text{fyx1}[\lambda]\, \text{QU}[y, x]$$

```
In[•]:= CE[InvertQU[𝒮[G]] ⊠ 𝒮[G]]
```

Out[•]=

$$\text{QU}[\,]$$




```
In[•]:= G
```

Out[•]=

$$e^{\text{ft0}[\lambda]} + e^{\text{ft0}[\lambda]}\, \epsilon\, (\text{ft1}[\lambda] + x\, y\, \text{fyx1}[\lambda])$$

In[•]:=
```
(∂λ (f.QU@Generators) G + ∂λ G)
```

Out[•]=

$$\mathbb{e}^{\mathtt{ft0}[\lambda]}\,\mathtt{ft0}'[\lambda] + \mathbb{e}^{\mathtt{ft0}[\lambda]} \in (\mathtt{ft1}[\lambda] + \mathtt{x}\,\mathtt{y}\,\mathtt{fyx1}[\lambda])\,\mathtt{ft0}'[\lambda] + \left(\mathbb{e}^{\mathtt{ft0}[\lambda]} + \mathbb{e}^{\mathtt{ft0}[\lambda]} \in (\mathtt{ft1}[\lambda] + \mathtt{x}\,\mathtt{y}\,\mathtt{fyx1}[\lambda])\right)$$
$$(\mathtt{a} \in \mathtt{fa1}'[\lambda] + \mathtt{x}\,(\mathtt{fx0}'[\lambda] + \in \mathtt{fx1}'[\lambda]) + \mathtt{y}\,(\mathtt{fy0}'[\lambda] + \in \mathtt{fy1}'[\lambda])) +$$
$$\mathbb{e}^{\mathtt{ft0}[\lambda]} \in (\mathtt{ft1}'[\lambda] + \mathtt{x}\,\mathtt{y}\,\mathtt{fyx1}'[\lambda])$$

In[•]:=
```
TTR = CE[Invert_QU[S[G]] ⊠ S[(∂λ (f.QU@Generators) G + ∂λ G)]]
```

Out[•]=

$$\mathtt{QU}[\mathtt{a}]\left(\in \mathtt{fa1}'[\lambda] - 2\,\mathbb{e}^{\alpha 1+\alpha 2}\,\mathtt{T} \in \mathtt{fx0}[\lambda]\,\mathtt{fy0}'[\lambda]\right) +$$
$$\mathtt{QU}[\mathtt{y}]\left(\mathbb{e}^{\alpha 1+\alpha 2}\,\mathtt{fy0}'[\lambda] + \mathbb{e}^{\alpha 1+\alpha 2} \in \mathtt{fa1}[\lambda]\,\mathtt{fy0}'[\lambda] - \mathbb{e}^{2\,\alpha 1+2\,\alpha 2} \in \mathtt{fyx1}[\lambda]\,\mathtt{fy0}'[\lambda] + \mathbb{e}^{2\,\alpha 1+2\,\alpha 2}\,\mathtt{T} \in \mathtt{fyx1}[\lambda]\,\mathtt{fy0}'[\lambda] + \mathbb{e}^{\alpha 1+\alpha 2} \in \mathtt{fy1}'[\lambda]\right) +$$
$$\mathtt{QU}[]\left(\mathtt{ft0}'[\lambda] + \in \mathtt{ft1}'[\lambda] - \mathbb{e}^{\alpha 1+\alpha 2}\,\mathtt{fx0}[\lambda]\,\mathtt{fy0}'[\lambda] + \mathbb{e}^{\alpha 1+\alpha 2}\,\mathtt{T}\,\mathtt{fx0}[\lambda]\,\mathtt{fy0}'[\lambda] - \mathbb{e}^{\alpha 1+\alpha 2} \in \mathtt{fa1}[\lambda]\,\mathtt{fx0}[\lambda]\,\mathtt{fy0}'[\lambda] + \mathbb{e}^{\alpha 1+\alpha 2}\,\mathtt{T} \in \mathtt{fa1}[\lambda]\,\mathtt{fx0}[\lambda]\,\mathtt{fy0}'[\lambda] - \mathbb{e}^{\alpha 1+\alpha 2} \in \mathtt{fx1}[\lambda]\,\mathtt{fy0}'[\lambda] + \mathbb{e}^{\alpha 1+\alpha 2}\,\mathtt{T} \in \mathtt{fx1}[\lambda]\,\mathtt{fy0}'[\lambda] + \mathbb{e}^{2\,\alpha 1+2\,\alpha 2} \in \mathtt{fx0}[\lambda]\,\mathtt{fyx1}[\lambda]\,\mathtt{fy0}'[\lambda] - 2\,\mathbb{e}^{2\,\alpha 1+2\,\alpha 2}\,\mathtt{T} \in \mathtt{fx0}[\lambda]\,\mathtt{fyx1}[\lambda]\,\mathtt{fy0}'[\lambda] + \mathbb{e}^{2\,\alpha 1+2\,\alpha 2}\,\mathtt{T}^2 \in \mathtt{fx0}[\lambda]\,\mathtt{fyx1}[\lambda]\,\mathtt{fy0}'[\lambda] - \mathbb{e}^{\alpha 1+\alpha 2} \in \mathtt{fx0}[\lambda]\,\mathtt{fy1}'[\lambda] + \mathbb{e}^{\alpha 1+\alpha 2}\,\mathtt{T} \in \mathtt{fx0}[\lambda]\,\mathtt{fy1}'[\lambda]\right) +$$
$$\mathtt{QU}[\mathtt{y},\,\mathtt{x}]\left(-\mathbb{e}^{\alpha 1+\alpha 2} \in \mathtt{fx0}[\lambda]\,\mathtt{fy0}'[\lambda] + \mathbb{e}^{\alpha 1+\alpha 2} \in \mathtt{fyx1}'[\lambda]\right) +$$
$$\mathtt{QU}[\mathtt{x}]\left(\in \mathtt{fx0}[\lambda]\,\mathtt{fa1}'[\lambda] + \mathtt{fx0}'[\lambda] + \in \mathtt{fx1}'[\lambda] + \frac{1}{2}\,\mathbb{e}^{\alpha 1+\alpha 2} \in \mathtt{fx0}[\lambda]^2\,\mathtt{fy0}'[\lambda] - \frac{3}{2}\,\mathbb{e}^{\alpha 1+\alpha 2}\,\mathtt{T} \in \mathtt{fx0}[\lambda]^2\,\mathtt{fy0}'[\lambda] - \mathbb{e}^{\alpha 1+\alpha 2} \in \mathtt{fx0}[\lambda]\,\mathtt{fyx1}'[\lambda] + \mathbb{e}^{\alpha 1+\alpha 2}\,\mathtt{T} \in \mathtt{fx0}[\lambda]\,\mathtt{fyx1}'[\lambda]\right)$$

In[•]:=
```
diff = CE[TTL - TTR]
```

Out[•]=

$$\begin{aligned}
&\mathtt{QU}[\mathtt{a}]\ \left(-2\,e^{\alpha 1}\,\mathtt{T}\in\eta 1\,\lambda\,\xi 1+2\,\mathtt{T}\in\eta 2\,\lambda\,\xi 1-\right.\\
&\quad\left.2\,e^{\alpha 1+\alpha 2}\,\mathtt{T}\in\eta 1\,\lambda\,\xi 2-2\,e^{\alpha 2}\,\mathtt{T}\in\eta 2\,\lambda\,\xi 2-\epsilon\,\mathtt{fa1}'[\lambda]+2\,e^{\alpha 1+\alpha 2}\,\mathtt{T}\in\mathtt{fx0}[\lambda]\,\mathtt{fy0}'[\lambda]\right)+\\
&\mathtt{QU}[\mathtt{y}]\ \left(e^{\alpha 1+\alpha 2}\,\eta 1+e^{\alpha 2}\,\eta 2-e^{\alpha 1+\alpha 2}\in\eta 1\,\eta 2\,\lambda^2\,\xi 1+3\,e^{\alpha 1+\alpha 2}\,\mathtt{T}\in\eta 1\,\eta 2\,\lambda^2\,\xi 1+\right.\\
&\quad\frac{1}{2}\,e^{\alpha 2}\in\eta 2^2\,\lambda^2\,\xi 1-\frac{3}{2}\,e^{\alpha 2}\,\mathtt{T}\in\eta 2^2\,\lambda^2\,\xi 1-e^{\alpha 1+\alpha 2}\,\mathtt{fy0}'[\lambda]-e^{\alpha 1+\alpha 2}\in\mathtt{fa1}[\lambda]\,\mathtt{fy0}'[\lambda]+\\
&\quad\left.e^{2\,\alpha 1+2\,\alpha 2}\in\mathtt{fyx1}[\lambda]\,\mathtt{fy0}'[\lambda]-e^{2\,\alpha 1+2\,\alpha 2}\,\mathtt{T}\in\mathtt{fyx1}[\lambda]\,\mathtt{fy0}'[\lambda]-e^{\alpha 1+\alpha 2}\in\mathtt{fy1}'[\lambda]\right)+\\
&\mathtt{QU}[\,]\ \left(-e^{\alpha 1}\,\eta 1\,\lambda\,\xi 1+e^{\alpha 1}\,\mathtt{T}\,\eta 1\,\lambda\,\xi 1+\eta 2\,\lambda\,\xi 1-\mathtt{T}\,\eta 2\,\lambda\,\xi 1+\frac{1}{2}\,e^{\alpha 1}\in\eta 1\,\eta 2\,\lambda^3\,\xi 1^2-2\,e^{\alpha 1}\,\mathtt{T}\in\eta 1\,\eta 2\,\lambda^3\,\xi 1^2+\right.\\
&\quad\frac{3}{2}\,e^{\alpha 1}\,\mathtt{T}^2\in\eta 1\,\eta 2\,\lambda^3\,\xi 1^2-e^{\alpha 1+\alpha 2}\,\eta 1\,\lambda\,\xi 2+e^{\alpha 1+\alpha 2}\,\mathtt{T}\,\eta 1\,\lambda\,\xi 2-e^{\alpha 2}\,\eta 2\,\lambda\,\xi 2+e^{\alpha 2}\,\mathtt{T}\,\eta 2\,\lambda\,\xi 2+\\
&\quad e^{\alpha 1+\alpha 2}\in\eta 1\,\eta 2\,\lambda^3\,\xi 1\,\xi 2-4\,e^{\alpha 1+\alpha 2}\,\mathtt{T}\in\eta 1\,\eta 2\,\lambda^3\,\xi 1\,\xi 2+3\,e^{\alpha 1+\alpha 2}\,\mathtt{T}^2\in\eta 1\,\eta 2\,\lambda^3\,\xi 1\,\xi 2-\\
&\quad\frac{1}{2}\,e^{\alpha 2}\in\eta 2^2\,\lambda^3\,\xi 1\,\xi 2+2\,e^{\alpha 2}\,\mathtt{T}\in\eta 2^2\,\lambda^3\,\xi 1\,\xi 2-\frac{3}{2}\,e^{\alpha 2}\,\mathtt{T}^2\in\eta 2^2\,\lambda^3\,\xi 1\,\xi 2-\mathtt{ft0}'[\lambda]-\epsilon\,\mathtt{ft1}'[\lambda]+\\
&\quad e^{\alpha 1+\alpha 2}\,\mathtt{fx0}[\lambda]\,\mathtt{fy0}'[\lambda]-e^{\alpha 1+\alpha 2}\,\mathtt{T}\,\mathtt{fx0}[\lambda]\,\mathtt{fy0}'[\lambda]+e^{\alpha 1+\alpha 2}\in\mathtt{fa1}[\lambda]\,\mathtt{fx0}[\lambda]\,\mathtt{fy0}'[\lambda]-\\
&\quad e^{\alpha 1+\alpha 2}\,\mathtt{T}\in\mathtt{fa1}[\lambda]\,\mathtt{fx0}[\lambda]\,\mathtt{fy0}'[\lambda]+e^{\alpha 1+\alpha 2}\in\mathtt{fx1}[\lambda]\,\mathtt{fy0}'[\lambda]-e^{\alpha 1+\alpha 2}\,\mathtt{T}\in\mathtt{fx1}[\lambda]\,\mathtt{fy0}'[\lambda]-\\
&\quad e^{2\,\alpha 1+2\,\alpha 2}\in\mathtt{fx0}[\lambda]\,\mathtt{fyx1}[\lambda]\,\mathtt{fy0}'[\lambda]+2\,e^{2\,\alpha 1+2\,\alpha 2}\,\mathtt{T}\in\mathtt{fx0}[\lambda]\,\mathtt{fyx1}[\lambda]\,\mathtt{fy0}'[\lambda]-\\
&\quad\left.e^{2\,\alpha 1+2\,\alpha 2}\,\mathtt{T}^2\in\mathtt{fx0}[\lambda]\,\mathtt{fyx1}[\lambda]\,\mathtt{fy0}'[\lambda]+e^{\alpha 1+\alpha 2}\in\mathtt{fx0}[\lambda]\,\mathtt{fy1}'[\lambda]-e^{\alpha 1+\alpha 2}\,\mathtt{T}\in\mathtt{fx0}[\lambda]\,\mathtt{fy1}'[\lambda]\right)+\\
&\mathtt{QU}[\mathtt{y},\mathtt{x}]\ \left(-e^{\alpha 1}\in\eta 1\,\lambda\,\xi 1+\epsilon\,\eta 2\,\lambda\,\xi 1-e^{\alpha 1+\alpha 2}\in\eta 1\,\lambda\,\xi 2-e^{\alpha 2}\in\eta 2\,\lambda\,\xi 2+\right.\\
&\quad\left.e^{\alpha 1+\alpha 2}\in\mathtt{fx0}[\lambda]\,\mathtt{fy0}'[\lambda]-e^{\alpha 1+\alpha 2}\in\mathtt{fyx1}'[\lambda]\right)+\mathtt{QU}[\mathtt{x}]\\
&\quad\left(e^{-\alpha 2}\,\xi 1+\frac{1}{2}\,e^{\alpha 1-\alpha 2}\in\eta 1\,\lambda^2\,\xi 1^2-\frac{3}{2}\,e^{\alpha 1-\alpha 2}\,\mathtt{T}\in\eta 1\,\lambda^2\,\xi 1^2+\xi 2+e^{\alpha 1}\in\eta 1\,\lambda^2\,\xi 1\,\xi 2-3\,e^{\alpha 1}\,\mathtt{T}\in\eta 1\,\lambda^2\,\xi 1\,\xi 2-\right.\\
&\quad\epsilon\in\eta 2\,\lambda^2\,\xi 1\,\xi 2+3\,\mathtt{T}\in\eta 2\,\lambda^2\,\xi 1\,\xi 2+\frac{1}{2}\,e^{\alpha 1+\alpha 2}\in\eta 1\,\lambda^2\,\xi 2^2-\frac{3}{2}\,e^{\alpha 1+\alpha 2}\,\mathtt{T}\in\eta 1\,\lambda^2\,\xi 2^2+\frac{1}{2}\,e^{\alpha 2}\in\eta 2\,\lambda^2\,\xi 2^2-\\
&\quad\frac{3}{2}\,e^{\alpha 2}\,\mathtt{T}\in\eta 2\,\lambda^2\,\xi 2^2-\epsilon\,\mathtt{fx0}[\lambda]\,\mathtt{fa1}'[\lambda]-\mathtt{fx0}'[\lambda]-\epsilon\,\mathtt{fx1}'[\lambda]-\frac{1}{2}\,e^{\alpha 1+\alpha 2}\in\mathtt{fx0}[\lambda]^2\,\mathtt{fy0}'[\lambda]+\\
&\quad\left.\frac{3}{2}\,e^{\alpha 1+\alpha 2}\,\mathtt{T}\in\mathtt{fx0}[\lambda]^2\,\mathtt{fy0}'[\lambda]+e^{\alpha 1+\alpha 2}\in\mathtt{fx0}[\lambda]\,\mathtt{fyx1}'[\lambda]-e^{\alpha 1+\alpha 2}\,\mathtt{T}\in\mathtt{fx0}[\lambda]\,\mathtt{fyx1}'[\lambda]\right)
\end{aligned}$$

In[•]:=
```
diff /. ϵ → 0
```

Out[•]=

$$\begin{aligned}
&\mathtt{QU}[\mathtt{x}]\ \left(e^{-\alpha 2}\,\xi 1+\xi 2-\mathtt{fx0}'[\lambda]\right)+\mathtt{QU}[\mathtt{y}]\ \left(e^{\alpha 1+\alpha 2}\,\eta 1+e^{\alpha 2}\,\eta 2-e^{\alpha 1+\alpha 2}\,\mathtt{fy0}'[\lambda]\right)+\\
&\mathtt{QU}[\,]\ \left(-e^{\alpha 1}\,\eta 1\,\lambda\,\xi 1+e^{\alpha 1}\,\mathtt{T}\,\eta 1\,\lambda\,\xi 1+\eta 2\,\lambda\,\xi 1-\mathtt{T}\,\eta 2\,\lambda\,\xi 1-e^{\alpha 1+\alpha 2}\,\eta 1\,\lambda\,\xi 2+e^{\alpha 1+\alpha 2}\,\mathtt{T}\,\eta 1\,\lambda\,\xi 2-\right.\\
&\quad\left.e^{\alpha 2}\,\eta 2\,\lambda\,\xi 2+e^{\alpha 2}\,\mathtt{T}\,\eta 2\,\lambda\,\xi 2-\mathtt{ft0}'[\lambda]+e^{\alpha 1+\alpha 2}\,\mathtt{fx0}[\lambda]\,\mathtt{fy0}'[\lambda]-e^{\alpha 1+\alpha 2}\,\mathtt{T}\,\mathtt{fx0}[\lambda]\,\mathtt{fy0}'[\lambda]\right)
\end{aligned}$$

In[•]:=
```
{sol1} = DSolve[Coefficient[diff /. ϵ → 0, x_QU] == 0 ∧ fx0[0] == 0, fx0, λ]
```

Out[•]=

$$\left\{\left\{\mathtt{fx0}\to\mathtt{Function}\left[\{\lambda\},\ e^{-\alpha 2}\,\lambda\,\left(\xi 1+e^{\alpha 2}\,\xi 2\right)\right]\right\}\right\}$$

In[•]:=
```
CE[diff /. sol1 /. ϵ → 0]
```

Out[•]=
$$\begin{aligned}&\text{QU}[y]\ \left(e^{\alpha 1+\alpha 2}\,\eta 1+e^{\alpha 2}\,\eta 2-e^{\alpha 1+\alpha 2}\,\text{fy0}'[\lambda]\right)+\\&\ \text{QU}[]\ \left(-e^{\alpha 1}\,\eta 1\,\lambda\,\xi 1+e^{\alpha 1}\,T\,\eta 1\,\lambda\,\xi 1+\eta 2\,\lambda\,\xi 1-T\,\eta 2\,\lambda\,\xi 1-e^{\alpha 1+\alpha 2}\,\eta 1\,\lambda\,\xi 2+\right.\\&\quad e^{\alpha 1+\alpha 2}\,T\,\eta 1\,\lambda\,\xi 2-e^{\alpha 2}\,\eta 2\,\lambda\,\xi 2+e^{\alpha 2}\,T\,\eta 2\,\lambda\,\xi 2-\text{ft0}'[\lambda]+e^{\alpha 1}\,\lambda\,\xi 1\,\text{fy0}'[\lambda]-\\&\quad\left.e^{\alpha 1}\,T\,\lambda\,\xi 1\,\text{fy0}'[\lambda]+e^{\alpha 1+\alpha 2}\,\lambda\,\xi 2\,\text{fy0}'[\lambda]-e^{\alpha 1+\alpha 2}\,T\,\lambda\,\xi 2\,\text{fy0}'[\lambda]\right)\end{aligned}$$

In[•]:=
```
{sol2} = DSolve[Coefficient[CE[diff /. sol1 /. ϵ → 0], y_QU] == 0 ∧ fy0[0] == 0, fy0, λ]
```

Out[•]=
$$\left\{\left\{\text{fy0}\to\text{Function}\left[\{\lambda\},\ e^{-\alpha 1}\ \left(e^{\alpha 1}\,\eta 1+\eta 2\right)\ \lambda\right]\right\}\right\}$$

In[•]:=
```
CE[diff /. sol1 /. sol2 /. ϵ → 0]
```

Out[•]=
$$\text{QU}[]\ (2\,\eta 2\,\lambda\,\xi 1-2\,T\,\eta 2\,\lambda\,\xi 1-\text{ft0}'[\lambda])$$

In[•]:=
```
{sol3} = DSolve[Coefficient[CE[diff /. sol1 /. sol2 /. ϵ → 0], QU[]] == 0 ∧ ft0[0] == 0, ft0, λ]
```

Out[•]=
$$\left\{\left\{\text{ft0}\to\text{Function}\left[\{\lambda\},\ -\left((-1+T)\ \eta 2\,\lambda^2\,\xi 1\right)\right]\right\}\right\}$$

In[•]:=
```
CE[diff /. sol1 /. sol2 /. sol3] /. ϵ → 0
```

Out[•]=
$$0$$

In[•]:=
```
CE[diff /. sol1 /. sol2 /. sol3]
```

Out[•]=

$$\begin{aligned}
&\mathrm{QU}[\mathtt{a}]\ (4\,\mathsf{T}\in\eta 2\,\lambda\,\xi 1-\epsilon\,\mathtt{fa1}'[\lambda])+\\
&\mathrm{QU}[\mathtt{y}]\ \Big(-e^{\alpha 1+\alpha 2}\in\eta 1\,\eta 2\,\lambda^2\,\xi 1+3\,e^{\alpha 1+\alpha 2}\,\mathsf{T}\in\eta 1\,\eta 2\,\lambda^2\,\xi 1+\frac{1}{2}\,e^{\alpha 2}\in\eta 2^2\,\lambda^2\,\xi 1-\\
&\quad\frac{3}{2}\,e^{\alpha 2}\,\mathsf{T}\in\eta 2^2\,\lambda^2\,\xi 1-e^{\alpha 1+\alpha 2}\in\eta 1\,\mathtt{fa1}[\lambda]-e^{\alpha 2}\in\eta 2\,\mathtt{fa1}[\lambda]+e^{2\,\alpha 1+2\,\alpha 2}\in\eta 1\,\mathtt{fyx1}[\lambda]-\\
&\quad e^{2\,\alpha 1+2\,\alpha 2}\,\mathsf{T}\in\eta 1\,\mathtt{fyx1}[\lambda]+e^{\alpha 1+2\,\alpha 2}\in\eta 2\,\mathtt{fyx1}[\lambda]-e^{\alpha 1+2\,\alpha 2}\,\mathsf{T}\in\eta 2\,\mathtt{fyx1}[\lambda]-e^{\alpha 1+\alpha 2}\in\mathtt{fy1}'[\lambda]\Big)+\\
&\mathrm{QU}[\,]\ \Big(\frac{1}{2}\,e^{\alpha 1}\in\eta 1\,\eta 2\,\lambda^3\,\xi 1^2-2\,e^{\alpha 1}\,\mathsf{T}\in\eta 1\,\eta 2\,\lambda^3\,\xi 1^2+\frac{3}{2}\,e^{\alpha 1}\,\mathsf{T}^2\in\eta 1\,\eta 2\,\lambda^3\,\xi 1^2+\\
&\quad e^{\alpha 1+\alpha 2}\in\eta 1\,\eta 2\,\lambda^3\,\xi 1\,\xi 2-4\,e^{\alpha 1+\alpha 2}\,\mathsf{T}\in\eta 1\,\eta 2\,\lambda^3\,\xi 1\,\xi 2+3\,e^{\alpha 1+\alpha 2}\,\mathsf{T}^2\in\eta 1\,\eta 2\,\lambda^3\,\xi 1\,\xi 2-\\
&\quad\frac{1}{2}\,e^{\alpha 2}\in\eta 2^2\,\lambda^3\,\xi 1\,\xi 2+2\,e^{\alpha 2}\,\mathsf{T}\in\eta 2^2\,\lambda^3\,\xi 1\,\xi 2-\frac{3}{2}\,e^{\alpha 2}\,\mathsf{T}^2\in\eta 2^2\,\lambda^3\,\xi 1\,\xi 2+e^{\alpha 1}\in\eta 1\,\lambda\,\xi 1\,\mathtt{fa1}[\lambda]-\\
&\quad e^{\alpha 1}\,\mathsf{T}\in\eta 1\,\lambda\,\xi 1\,\mathtt{fa1}[\lambda]+\epsilon\,\eta 2\,\lambda\,\xi 1\,\mathtt{fa1}[\lambda]-\mathsf{T}\in\eta 2\,\lambda\,\xi 1\,\mathtt{fa1}[\lambda]+e^{\alpha 1+\alpha 2}\in\eta 1\,\lambda\,\xi 2\,\mathtt{fa1}[\lambda]-\\
&\quad e^{\alpha 1+\alpha 2}\,\mathsf{T}\in\eta 1\,\lambda\,\xi 2\,\mathtt{fa1}[\lambda]+e^{\alpha 2}\in\eta 2\,\lambda\,\xi 2\,\mathtt{fa1}[\lambda]-e^{\alpha 2}\,\mathsf{T}\in\eta 2\,\lambda\,\xi 2\,\mathtt{fa1}[\lambda]+e^{\alpha 1+\alpha 2}\in\eta 1\,\mathtt{fx1}[\lambda]-\\
&\quad e^{\alpha 1+\alpha 2}\,\mathsf{T}\in\eta 1\,\mathtt{fx1}[\lambda]+e^{\alpha 2}\in\eta 2\,\mathtt{fx1}[\lambda]-e^{\alpha 2}\,\mathsf{T}\in\eta 2\,\mathtt{fx1}[\lambda]-e^{2\,\alpha 1+\alpha 2}\in\eta 1\,\lambda\,\xi 1\,\mathtt{fyx1}[\lambda]+\\
&\quad 2\,e^{2\,\alpha 1+\alpha 2}\,\mathsf{T}\in\eta 1\,\lambda\,\xi 1\,\mathtt{fyx1}[\lambda]-e^{2\,\alpha 1+\alpha 2}\,\mathsf{T}^2\in\eta 1\,\lambda\,\xi 1\,\mathtt{fyx1}[\lambda]-e^{\alpha 1+\alpha 2}\in\eta 2\,\lambda\,\xi 1\,\mathtt{fyx1}[\lambda]+\\
&\quad 2\,e^{\alpha 1+\alpha 2}\,\mathsf{T}\in\eta 2\,\lambda\,\xi 1\,\mathtt{fyx1}[\lambda]-e^{\alpha 1+\alpha 2}\,\mathsf{T}^2\in\eta 2\,\lambda\,\xi 1\,\mathtt{fyx1}[\lambda]-e^{2\,\alpha 1+2\,\alpha 2}\in\eta 1\,\lambda\,\xi 2\,\mathtt{fyx1}[\lambda]+\\
&\quad 2\,e^{2\,\alpha 1+2\,\alpha 2}\,\mathsf{T}\in\eta 1\,\lambda\,\xi 2\,\mathtt{fyx1}[\lambda]-e^{2\,\alpha 1+2\,\alpha 2}\,\mathsf{T}^2\in\eta 1\,\lambda\,\xi 2\,\mathtt{fyx1}[\lambda]-e^{\alpha 1+2\,\alpha 2}\in\eta 2\,\lambda\,\xi 2\,\mathtt{fyx1}[\lambda]+\\
&\quad 2\,e^{\alpha 1+2\,\alpha 2}\,\mathsf{T}\in\eta 2\,\lambda\,\xi 2\,\mathtt{fyx1}[\lambda]-e^{\alpha 1+2\,\alpha 2}\,\mathsf{T}^2\in\eta 2\,\lambda\,\xi 2\,\mathtt{fyx1}[\lambda]-\epsilon\,\mathtt{ft1}'[\lambda]+\\
&\quad e^{\alpha 1}\in\lambda\,\xi 1\,\mathtt{fy1}'[\lambda]-e^{\alpha 1}\,\mathsf{T}\in\lambda\,\xi 1\,\mathtt{fy1}'[\lambda]+e^{\alpha 1+\alpha 2}\in\lambda\,\xi 2\,\mathtt{fy1}'[\lambda]-e^{\alpha 1+\alpha 2}\,\mathsf{T}\in\lambda\,\xi 2\,\mathtt{fy1}'[\lambda]\Big)+\\
&\mathrm{QU}[\mathtt{y},\mathtt{x}]\ \Big(2\in\eta 2\,\lambda\,\xi 1-e^{\alpha 1+\alpha 2}\in\mathtt{fyx1}'[\lambda]\Big)+\\
&\mathrm{QU}[\mathtt{x}]\ \Big(-\frac{1}{2}\,e^{-\alpha 2}\in\eta 2\,\lambda^2\,\xi 1^2+\frac{3}{2}\,e^{-\alpha 2}\,\mathsf{T}\in\eta 2\,\lambda^2\,\xi 1^2-2\in\eta 2\,\lambda^2\,\xi 1\,\xi 2+6\,\mathsf{T}\in\eta 2\,\lambda^2\,\xi 1\,\xi 2-\\
&\quad e^{-\alpha 2}\in\lambda\,\xi 1\,\mathtt{fa1}'[\lambda]-\epsilon\,\lambda\,\xi 2\,\mathtt{fa1}'[\lambda]-\epsilon\,\mathtt{fx1}'[\lambda]+e^{\alpha 1}\in\lambda\,\xi 1\,\mathtt{fyx1}'[\lambda]-\\
&\quad e^{\alpha 1}\,\mathsf{T}\in\lambda\,\xi 1\,\mathtt{fyx1}'[\lambda]+e^{\alpha 1+\alpha 2}\in\lambda\,\xi 2\,\mathtt{fyx1}'[\lambda]-e^{\alpha 1+\alpha 2}\,\mathsf{T}\in\lambda\,\xi 2\,\mathtt{fyx1}'[\lambda]\Big)
\end{aligned}$$

In[•]:=
```
{sol4} =
 DSolve[Coefficient[CE[diff /. sol1 /. sol2 /. sol3], QU[y, x]] == 0 ∧ fyx1[0] == 0, fyx1, λ]
```

Out[•]=

$$\left\{\left\{\mathtt{fyx1}\to\mathtt{Function}\left[\{\lambda\},\ e^{-\alpha 1-\alpha 2}\,\eta 2\,\lambda^2\,\xi 1\right]\right\}\right\}$$

In[•]:=
```
CE[diff /. sol1 /. sol2 /. sol3 /. sol4]
```

Out[•]=

$$\begin{aligned}
&\mathrm{QU}[a]\ (4\,T\in\eta 2\,\lambda\,\xi 1-\epsilon\,\mathrm{fa1}'[\lambda])+\\
&\mathrm{QU}[x]\left(\frac{3}{2}\,e^{-\alpha 2}\in\eta 2\,\lambda^2\,\xi 1^2-\frac{1}{2}\,e^{-\alpha 2}\,T\in\eta 2\,\lambda^2\,\xi 1^2+4\,T\in\eta 2\,\lambda^2\,\xi 1\,\xi 2-e^{-\alpha 2}\in\lambda\,\xi 1\,\mathrm{fa1}'[\lambda]-\right.\\
&\qquad\left.\in\lambda\,\xi 2\,\mathrm{fa1}'[\lambda]-\epsilon\,\mathrm{fx1}'[\lambda]\right)+\mathrm{QU}[y]\left(2\,e^{\alpha 1+\alpha 2}\,T\in\eta 1\,\eta 2\,\lambda^2\,\xi 1+\frac{3}{2}\,e^{\alpha 2}\in\eta 2^2\,\lambda^2\,\xi 1-\right.\\
&\qquad\left.\frac{5}{2}\,e^{\alpha 2}\,T\in\eta 2^2\,\lambda^2\,\xi 1-e^{\alpha 1+\alpha 2}\in\eta 1\,\mathrm{fa1}[\lambda]-e^{\alpha 2}\in\eta 2\,\mathrm{fa1}[\lambda]-e^{\alpha 1+\alpha 2}\in\mathrm{fy1}'[\lambda]\right)+\\
&\mathrm{QU}[]\left(-\frac{1}{2}\,e^{\alpha 1}\in\eta 1\,\eta 2\,\lambda^3\,\xi 1^2+\frac{1}{2}\,e^{\alpha 1}\,T^2\in\eta 1\,\eta 2\,\lambda^3\,\xi 1^2-\epsilon\,\eta 2^2\,\lambda^3\,\xi 1^2+2\,T\in\eta 2^2\,\lambda^3\,\xi 1^2-\right.\\
&\qquad T^2\in\eta 2^2\,\lambda^3\,\xi 1^2-2\,e^{\alpha 1+\alpha 2}\,T\in\eta 1\,\eta 2\,\lambda^3\,\xi 1\,\xi 2+2\,e^{\alpha 1+\alpha 2}\,T^2\in\eta 1\,\eta 2\,\lambda^3\,\xi 1\,\xi 2-\\
&\qquad\frac{3}{2}\,e^{\alpha 2}\in\eta 2^2\,\lambda^3\,\xi 1\,\xi 2+4\,e^{\alpha 2}\,T\in\eta 2^2\,\lambda^3\,\xi 1\,\xi 2-\frac{5}{2}\,e^{\alpha 2}\,T^2\in\eta 2^2\,\lambda^3\,\xi 1\,\xi 2+e^{\alpha 1}\in\eta 1\,\lambda\,\xi 1\,\mathrm{fa1}[\lambda]-\\
&\qquad e^{\alpha 1}\,T\in\eta 1\,\lambda\,\xi 1\,\mathrm{fa1}[\lambda]+\epsilon\,\eta 2\,\lambda\,\xi 1\,\mathrm{fa1}[\lambda]-T\in\eta 2\,\lambda\,\xi 1\,\mathrm{fa1}[\lambda]+e^{\alpha 1+\alpha 2}\in\eta 1\,\lambda\,\xi 2\,\mathrm{fa1}[\lambda]-\\
&\qquad e^{\alpha 1+\alpha 2}\,T\in\eta 1\,\lambda\,\xi 2\,\mathrm{fa1}[\lambda]+e^{\alpha 2}\in\eta 2\,\lambda\,\xi 2\,\mathrm{fa1}[\lambda]-e^{\alpha 2}\,T\in\eta 2\,\lambda\,\xi 2\,\mathrm{fa1}[\lambda]+\\
&\qquad e^{\alpha 1+\alpha 2}\in\eta 1\,\mathrm{fx1}[\lambda]-e^{\alpha 1+\alpha 2}\,T\in\eta 1\,\mathrm{fx1}[\lambda]+e^{\alpha 2}\in\eta 2\,\mathrm{fx1}[\lambda]-e^{\alpha 2}\,T\in\eta 2\,\mathrm{fx1}[\lambda]-\epsilon\,\mathrm{ft1}'[\lambda]+\\
&\qquad\left.e^{\alpha 1}\in\lambda\,\xi 1\,\mathrm{fy1}'[\lambda]-e^{\alpha 1}\,T\in\lambda\,\xi 1\,\mathrm{fy1}'[\lambda]+e^{\alpha 1+\alpha 2}\in\lambda\,\xi 2\,\mathrm{fy1}'[\lambda]-e^{\alpha 1+\alpha 2}\,T\in\lambda\,\xi 2\,\mathrm{fy1}'[\lambda]\right)
\end{aligned}$$

In[•]:=
```
{sol5} =
 DSolve[Coefficient[CE[diff /. sol1 /. sol2 /. sol3 /. sol4], QU[a]] == 0 ∧ fa1[0] == 0, fa1, λ]
```

Out[•]=

$$\left\{\left\{\mathrm{fa1}\to\mathrm{Function}\left[\{\lambda\},\,2\,T\,\eta 2\,\lambda^2\,\xi 1\right]\right\}\right\}$$

In[•]:=
```
CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5]
```

Out[•]=

$$\begin{aligned}
&\mathrm{QU}[x]\left(\frac{3}{2}\,e^{-\alpha 2}\in\eta 2\,\lambda^2\,\xi 1^2-\frac{9}{2}\,e^{-\alpha 2}\,T\in\eta 2\,\lambda^2\,\xi 1^2-\epsilon\,\mathrm{fx1}'[\lambda]\right)+\\
&\mathrm{QU}[y]\left(\frac{3}{2}\,e^{\alpha 2}\in\eta 2^2\,\lambda^2\,\xi 1-\frac{9}{2}\,e^{\alpha 2}\,T\in\eta 2^2\,\lambda^2\,\xi 1-e^{\alpha 1+\alpha 2}\in\mathrm{fy1}'[\lambda]\right)+\mathrm{QU}[]\\
&\left(-\frac{1}{2}\,e^{\alpha 1}\in\eta 1\,\eta 2\,\lambda^3\,\xi 1^2+2\,e^{\alpha 1}\,T\in\eta 1\,\eta 2\,\lambda^3\,\xi 1^2-\frac{3}{2}\,e^{\alpha 1}\,T^2\in\eta 1\,\eta 2\,\lambda^3\,\xi 1^2-\epsilon\,\eta 2^2\,\lambda^3\,\xi 1^2+4\,T\in\eta 2^2\,\lambda^3\,\xi 1^2-\right.\\
&\qquad 3\,T^2\in\eta 2^2\,\lambda^3\,\xi 1^2-\frac{3}{2}\,e^{\alpha 2}\in\eta 2^2\,\lambda^3\,\xi 1\,\xi 2+6\,e^{\alpha 2}\,T\in\eta 2^2\,\lambda^3\,\xi 1\,\xi 2-\frac{9}{2}\,e^{\alpha 2}\,T^2\in\eta 2^2\,\lambda^3\,\xi 1\,\xi 2+\\
&\qquad e^{\alpha 1+\alpha 2}\in\eta 1\,\mathrm{fx1}[\lambda]-e^{\alpha 1+\alpha 2}\,T\in\eta 1\,\mathrm{fx1}[\lambda]+e^{\alpha 2}\in\eta 2\,\mathrm{fx1}[\lambda]-e^{\alpha 2}\,T\in\eta 2\,\mathrm{fx1}[\lambda]-\epsilon\,\mathrm{ft1}'[\lambda]+\\
&\qquad\left.e^{\alpha 1}\in\lambda\,\xi 1\,\mathrm{fy1}'[\lambda]-e^{\alpha 1}\,T\in\lambda\,\xi 1\,\mathrm{fy1}'[\lambda]+e^{\alpha 1+\alpha 2}\in\lambda\,\xi 2\,\mathrm{fy1}'[\lambda]-e^{\alpha 1+\alpha 2}\,T\in\lambda\,\xi 2\,\mathrm{fy1}'[\lambda]\right)
\end{aligned}$$

In[•]:=
```
{sol6} = DSolve[
  Coefficient[CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5], QU[y]] == 0 ∧ fy1[0] == 0, fy1, λ]
```

Out[•]=

$$\left\{\left\{\mathrm{fy1}\to\mathrm{Function}\left[\{\lambda\},\,-\frac{1}{2}\,e^{-\alpha 1}\,(-1+3\,T)\,\eta 2^2\,\lambda^3\,\xi 1\right]\right\}\right\}$$

In[•]:=
```
CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6]
```

Out[•]=
$$\text{QU}[\,]\left(-\frac{1}{2}\,e^{\alpha 1}\in\eta 1\,\eta 2\,\lambda^3\,\xi 1^2+2\,e^{\alpha 1}\,T\in\eta 1\,\eta 2\,\lambda^3\,\xi 1^2-\frac{3}{2}\,e^{\alpha 1}\,T^2\in\eta 1\,\eta 2\,\lambda^3\,\xi 1^2+\frac{1}{2}\in\eta 2^2\,\lambda^3\,\xi 1^2-2\,T\in\eta 2^2\,\lambda^3\,\xi 1^2+\frac{3}{2}\,T^2\in\eta 2^2\,\lambda^3\,\xi 1^2+e^{\alpha 1+\alpha 2}\in\eta 1\,\text{fx1}[\lambda]-e^{\alpha 1+\alpha 2}\,T\in\eta 1\,\text{fx1}[\lambda]+e^{\alpha 2}\in\eta 2\,\text{fx1}[\lambda]-e^{\alpha 2}\,T\in\eta 2\,\text{fx1}[\lambda]-\in\text{ft1}'[\lambda]\right)+\text{QU}[x]\left(\frac{3}{2}\,e^{-\alpha 2}\in\eta 2\,\lambda^2\,\xi 1^2-\frac{9}{2}\,e^{-\alpha 2}\,T\in\eta 2\,\lambda^2\,\xi 1^2-\in\text{fx1}'[\lambda]\right)$$

In[•]:=
```
{sol7} =
 DSolve[Coefficient[CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6], QU[x]] == 0 ∧
   fx1[0] == 0, fx1, λ]
```

Out[•]=
$$\left\{\left\{\text{fx1}\to\text{Function}\left[\{\lambda\},-\frac{1}{2}\,e^{-\alpha 2}\,(-1+3\,T)\,\eta 2\,\lambda^3\,\xi 1^2\right]\right\}\right\}$$

In[•]:=
```
CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7]
```

Out[•]=
$$\text{QU}[\,]\left(\in\eta 2^2\,\lambda^3\,\xi 1^2-4\,T\in\eta 2^2\,\lambda^3\,\xi 1^2+3\,T^2\in\eta 2^2\,\lambda^3\,\xi 1^2-\in\text{ft1}'[\lambda]\right)$$

In[•]:=
```
{sol8} =
 DSolve[Coefficient[CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7], QU[]] == 0 ∧
   ft1[0] == 0, ft1, λ]
```

Out[•]=
$$\left\{\left\{\text{ft1}\to\text{Function}\left[\{\lambda\},\frac{1}{4}\left(\eta 2^2\,\lambda^4\,\xi 1^2-4\,T\,\eta 2^2\,\lambda^4\,\xi 1^2+3\,T^2\,\eta 2^2\,\lambda^4\,\xi 1^2\right)\right]\right\}\right\}$$

In[•]:=
```
CE[diff /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8]
```

Out[•]=
$$0$$

In[•]:=
```
r =
 Simplify[(f.QU@Generators + g) /. sol1 /. sol2 /. sol3 /. sol4 /. sol5 /. sol6 /. sol7 /. sol8]
```

Out[•]=
$$a\left(\alpha 1+\alpha 2+2\,T\in\eta 2\,\lambda^2\,\xi 1\right)+\frac{1}{4}\,e^{-\alpha 1-\alpha 2}\,\lambda\left(4\,x\,y\in\eta 2\,\lambda\,\xi 1-2\,e^{\alpha 2}\,y\,\eta 2\left(-2+(-1+3\,T)\in\eta 2\,\lambda^2\,\xi 1\right)-2\,e^{\alpha 1}\,x\,\xi 1\left(-2+(-1+3\,T)\in\eta 2\,\lambda^2\,\xi 1\right)+e^{\alpha 1+\alpha 2}\left(4\,y\,\eta 1-4\,(-1+T)\,\eta 2\,\lambda\,\xi 1+\left(1-4\,T+3\,T^2\right)\in\eta 2^2\,\lambda^3\,\xi 1^2+4\,x\,\xi 2\right)\right)$$

The following was imported from pensieve://Projects/SL2Portfolio2/PortfolioTesting.nb:

In[•]:=
```
km = Plus[a_k α_i + a_k α_j, y_k η_i + y_k η_j/𝒜_i + x_k ξ_i/𝒜_j + (1 - T) η_j ξ_i/ħ + x_k ξ_j,
  (2 T a_k η_j ξ_i + γ ħ x_k y_k η_j ξ_i/(𝒜_i 𝒜_j) + (γ - 3 T γ) y_k η_j^2 ξ_i/(2 𝒜_i) + (γ - 3 T γ) x_k η_j ξ_i^2/(2 𝒜_j) + (γ - 4 T γ + 3 T^2 γ) η_j^2 ξ_i^2/(4 ħ))
  ϵ] /. {ħ → 1, γ → 1, v_k :> v, η_i → η1,
  α_i → α1, 𝒜_i → e^α1, ξ_i → ξ1, η_j → η2, α_j → α2, 𝒜_j → e^α2, ξ_j → ξ2}
```

Out[•]=
$$a\,\alpha 1 + a\,\alpha 2 + y\,\eta 1 + e^{-\alpha 1}\,y\,\eta 2 + e^{-\alpha 2}\,x\,\xi 1 + (1-T)\,\eta 2\,\xi 1 + \epsilon\left(2\,a\,T\,\eta 2\,\xi 1 + e^{-\alpha 1-\alpha 2}\,x\,y\,\eta 2\,\xi 1 + \frac{1}{2}\,e^{-\alpha 1}\,(1-3\,T)\,y\,\eta 2^2\,\xi 1 + \frac{1}{2}\,e^{-\alpha 2}\,(1-3\,T)\,x\,\eta 2\,\xi 1^2 + \frac{1}{4}\left(1-4\,T+3\,T^2\right)\eta 2^2\,\xi 1^2\right) + x\,\xi 2$$

In[•]:=
```
Expand[(r /. λ → 1) - km]
```

Out[•]=
```
0
```