Pensieve header: An isolated attempt to compute the multiplication tensor of QU.

The implementation of QU is modified from pensieve://Projects/SL2Portfolio/SL2PortfolioProgram.nb.

Initialization / Utilities

In[∘]:=
```
$k = 1; $U = QU; $E := {$k};
$trim := {ϵ^(k_.) /; k > $k → 0};
SetAttributes[{SS, SST}, HoldAll];
q_ℏ_ := 𝕖^(γ ϵ ℏ);
(* Upper to lower and lower to Upper: *)
U2l = {B_i_^(p_.) → 𝕖^(-p ℏ γ b_i), B^(p_.) → 𝕖^(-p ℏ γ b), T_i_^(p_.) → 𝕖^(p ℏ t_i), T^(p_.) → 𝕖^(p ℏ t), 𝒜_i_^(p_.) → 𝕖^(p γ α_i), 𝒜^(p_.) → 𝕖^(p γ α)};
l2U = {𝕖^(c_. b_i_ + d_.) :→ B_i^(-c/(ℏ γ)) 𝕖^d, 𝕖^(c_. b + d_.) :→ B^(-c/(ℏ γ)) 𝕖^d,
    𝕖^(c_. t_i_ + d_.) :→ T_i^(c/ℏ) 𝕖^d, 𝕖^(c_. t + d_.) :→ T^(c/ℏ) 𝕖^d,
    𝕖^(c_. α_i_ + d_.) :→ 𝒜_i^(c/γ) 𝕖^d, 𝕖^(c_. α + d_.) :→ 𝒜^(c/γ) 𝕖^d,
    𝕖^ℰ_ :→ 𝕖^(Expand@ℰ)};
SS[ℰ_, op_] := Collect[Normal@Series[ℰ, {ϵ, 0, $k}], ϵ, op];
SS[ℰ_] := SS[ℰ, Together];
SST[ℰ_, op___] := SS[ℰ /. U2l, op];
Simp[ℰ_, op_] := Collect[ℰ, _CU | _QU, op];
Simp[ℰ_] := Simp[ℰ, SS[#, Expand] &];
SimpT[ℰ_] := Collect[ℰ, _CU | _QU, SST[#, Expand] &];
Kδ /: Kδ_(i_,j_) := If[i === j, 1, 0];
c_Integer_(k_Integer) := c + O[ϵ]^(k+1);
```

In[∘]:=
```
CF[ℰ_] := ExpandDenominator@
   ExpandNumerator@Together[Expand[ℰ] //. 𝕖^x_ 𝕖^y_ :→ 𝕖^(x+y) /. 𝕖^x_ :→ 𝕖^(CF[x])];
```

In[∘]:=
```
Unprotect[SeriesData];
SeriesData /: CF[sd_SeriesData] := MapAt[CF, sd, 3];
SeriesData /: Expand[sd_SeriesData] := MapAt[Expand, sd, 3];
SeriesData /: Simplify[sd_SeriesData] := MapAt[Simplify, sd, 3];
SeriesData /: Together[sd_SeriesData] := MapAt[Together, sd, 3];
SeriesData /: Collect[sd_SeriesData, specs__] := MapAt[Collect[#, specs] &, sd, 3];
Protect[SeriesData];
```

Self-Pair (SP):

In[∘]:=
```
SP_{}[P_] := P; SP_{ℰ_→x_,ps___}[P_] := Expand[P // SP_{ps}] /. f_. ℰ^(d_.) :→ ∂_{x,d} f
```

DeclareAlgebra

In[•]:=
```
Unprotect[NonCommutativeMultiply]; Attributes[NonCommutativeMultiply] = {};
(NCM = NonCommutativeMultiply)[x_] := x;
NCM[x_, y_, z__] := (x ⊠ y) ⊠ z;
0 ⊠ _ = _ ⊠ 0 = 0;
(x_Plus) ⊠ y_ := (# ⊠ y) & /@ x; x_ ⊠ (y_Plus) := (x ⊠ #) & /@ y;
B[x_, x_] = 0; B[x_, y_] := x ⊠ y - y ⊠ x;
B[x_, y_, e_] := B[x, y, e] = B[x, y];
```

In[•]:=
```
DeclareAlgebra[U_Symbol, opts__Rule] := Module[
  {gp, sr, g, cp, M, pow, k = 0,
   gs = Generators /. {opts},
   cs = Centrals /. {opts} /. Centrals → {}
  },
  U[Generators] = gs;
  (#_U = U@#) & /@ gs;
  gp = Alternatives @@ gs; gp = gp | gp_; (* gens *)
  sr = Flatten@Table[{g → ++k, g_i_ → {i, k}}, {g, gs}]; (* sorting → *)
  cp = Alternatives @@ cs; (* cents *)
  SetAttributes[M, HoldRest]; M[0, _] = 0; M[a_, x_] := a x;
  CE[ℰ_] := Collect[ℰ, _U, Expand] /. ⅇ^x_ ⧴ ⅇ^Expand[x] /. $trim;
  U_i_[ℰ_] := ℰ /. {t : cp ⧴ t_i, u_U ⧴ (#_i &) /@ u};
  U_i_[NCM[]] = pow[ℰ_, 0] = U@{} = 1_U = U[];
  B[U@(x_)_i_, U@(y_)_i_] := U_i@B[U@x, U@y];
  B[U@(x_)_i_, U@(y_)_j_] /; i =!= j := 0;
  B[U@y_, U@x_] := CE[-B[U@x, U@y]];
  x_ ⊠ (c_. 1_U) := CE[c x]; (c_. 1_U) ⊠ x_ := CE[c x];
  (a_. U[xx___, x_]) ⊠ (b_. U[y_, yy___]) := If[OrderedQ[{x, y} /. sr],
    CE@M[a b /. $trim, U[xx, x, y, yy]],
    U@xx ⊠ CE@M[a b /. $trim, U@y ⊠ U@x + B[U@x, U@y, $E]] ⊠ U@yy ];
  U@{c_. * (l : gp)^n_, r___} /; FreeQ[c, gp] := CE[c U@Table[l, {n}] ⊠ U@{r}];
  U@{c_. * l : gp, r___} := CE[c U[l] ⊠ U@{r}];
  U@{c_, r___} /; FreeQ[c, gp] := CE[c U@{r}];
  U@{l_Plus, r___} := CE[U@{#, r} & /@ l];
  U@{l_, r___} := U@{Expand[l], r};
  U[ℰ_NonCommutativeMultiply] := U /@ ℰ;
  𝕆_U[specs___, poly_] := Module[{sp, null, vs, us},
    sp = Replace[{specs}, l_List ⧴ l_null, {1}];
    vs = Join @@ (First /@ sp);
    us = Join @@ (sp /. l_s_ ⧴ (l /. x_i_ ⧴ x_s));
    CE[Total[
       CoefficientRules[poly, vs] /. (p_ → c_) ⧴ c U@(us^p)
     ]] /. x_null ⧴ x];
```

```
𝕆_U[specs___, 𝔼[L_, Q_, P_]] := 𝕆_U[specs, SS@Normal[P e^(L+Q)]];
pow[ℰ_, n_] := pow[ℰ, n - 1] ⊠ ℰ;
𝕊_U[ℰ_, ss___Rule] := CE@Total[
   CoefficientRules[ℰ, First /@ {ss}] /.
    (p_ → c_) ⧴ c NCM @@ MapThread[pow, {Last /@ {ss}, p}]];
σ_rs___[c_. * u_U] := (c /. (t : cp)_j_ ⧴ t_{j/.{rs}}) U[List @@ (u /. v_j_ ⧴ v_{j/.{rs}})];
m_{j_→k_}[c_. * u_U] := CE[((c /. (t : cp)_j → t_k) DeleteCases[u, _j|k]) ⊠
   U @@ Cases[u, w_j ⧴ w_k] ⊠ U @@ Cases[u, _k]];
U /: c_. * u_U * v_U := CE[c u ⊠ v];
S_i_[c_. * u_U] := CE[((c /. S_i[U, Centrals]) DeleteCases[u, _i]) ⊠
   U_i[NCM @@ Reverse@Cases[u, x_i ⧴ S@U@x]]];
Δ_{i_→j_,k_}[c_. * u_U] := CE[((c /. Δ_{i→j,k}[U, Centrals]) DeleteCases[u, _i]) ⊠
   (NCM @@ Cases[u, x_i ⧴ σ_{1→j,2→k}@Δ@U@x] /. NCM[] → U[])]; ]
```

In[•]:=
```
𝕊_QU[(a^2 + a) b, a → QU[y, x], b → 7 QU[]]
```

Out[•]=

$(14 - 7\,T)\,\mathrm{QU}[y, x] + 14\,T\,\epsilon\,\mathrm{QU}[y, a, x] + (7 + 7\,\epsilon)\,\mathrm{QU}[y, y, x, x]$

Meta-Operations

In[•]:=
```
σ_rs___[ℰ_Plus] := σ_rs /@ ℰ;
m_{j_→j_} = Identity; m_{j_→k_}[0] = 0;
m_{j_→k_}[ℰ_Plus] := Simp[m_{j→k} /@ ℰ];
m_{is___,i_,j_→k_}[ℰ_] := m_{j→k}@m_{is,i→j}@ℰ;
S_i_[ℰ_Plus] := Simp[S_i /@ ℰ];
Δ_is___[ℰ_Plus] := Simp[Δ_is /@ ℰ];
```

Implementing CU = $\mathcal{U}(sl_2^{\gamma\epsilon})$

Verify σ and Δ! Also Generalize Δ to $\Delta_{i,j_1,j_2,\ldots}$.

In[•]:=
```
DeclareAlgebra[CU, Generators → {y, a, x}, Centrals → {t}];
B[a_CU, y_CU] = -γ y_CU; B[x_CU, a_CU] = -γ x_CU;
B[x_CU, y_CU] = 2 ϵ a_CU - t 1_CU;
(S@y_CU = -y_CU; S@a_CU = -a_CU; S@x_CU = -x_CU;)
S_i_[CU, Centrals] = {t_i → -t_i};
Δ@y_CU = CU@y_1 + CU@y_2; Δ@a_CU = CU@a_1 + CU@a_2; Δ@x_CU = CU@x_1 + CU@x_2;
Δ_{i_→j_,k_}[CU, Centrals] = {t_i → t_j + t_k};
```

Implementing QU = $\mathcal{U}_q(sl_2^{\gamma\epsilon})$

In[∘]:=
```
ℏ = γ = 1;
DeclareAlgebra[QU, Generators → {y, a, x}, Centrals → {t, T}];
B[a_QU, y_QU] = -γ y_QU; B[x_QU, a_QU] = -γ QU@x;
B[x_QU, y_QU] := SS[q_ℏ - 1] QU@{y, x} + 𝕆_QU[{a}, SS[(1 - T e^(-2 ∈ a ℏ)) / ℏ]];
(S@y_QU := 𝕆_QU[{a, y}, SS[-T^-1 e^(ℏ ∈ a) y]]; S@a_QU = -a_QU; S@x_QU := 𝕆_QU[{a, x}, SS[-e^(ℏ ∈ a) x]];)
S_i_ [QU, Centrals] = {t_i → -t_i, T_i → T_i^-1};
Δ@y_QU := 𝕆_QU[{y_1, a_1}_1, {y_2}_2, SS[y_1 + T_1 e^(-ℏ ∈ a_1) y_2]];
Δ@a_QU = QU@a_1 + QU@a_2; Δ@x_QU := 𝕆_QU[{a_1, x_1}_1, {x_2}_2, SS[x_1 + e^(-ℏ ∈ a_1) x_2]];
Δ_i_→j_,k_ [QU, Centrals] = {t_i → t_j + t_k, T_i → T_j T_k};
```

In[∘]:=
```
B[x_QU, y_QU]
```

Out[∘]=

$(1 - T)\ \mathrm{QU}[\,] + 2\, T \in \mathrm{QU}[a] + \in \mathrm{QU}[y, x]$

tSW

Logoi from Pensieve://Talks/Toulouse-1705/DogmaDemo.nb and from Pensieve://Talks/Sydney-1708/ExtraDetails@@.nb.

Goal. In either U, compute $F = e^{-\eta y}\, e^{\xi x}\, e^{\eta y}\, e^{-\xi x}$. First compute $G = e^{\xi x}\, y\, e^{-\xi x}$, a finite sum. Now F satisfies the ODE $\partial_\eta F = \partial_\eta\left(e^{-\eta y}\, e^{\eta G}\right) = -yF + FG$ with initial conditions $F(\eta = 0) = 1$. So we set it up and solve:

In[∘]:=
```
SW_xy[U_, kk_] :=
  SW_xy[U, kk] = Block[{$U = U, $k = kk, $p = kk}, Module[{G, F, fs, f, bs, e, b, es},
     G = Simp[Table[ξ^k / k!, {k, 0, $k + 1}].NestList[Simp[B[x_U, #]] &, y_U, $k + 1]];
     fs = Flatten@Table[f_1,i,j,k[η], {l, 0, $k}, {i, 0, 1}, {j, 0, 1}, {k, 0, 1}];
     F = fs.(bs = fs /. f_l_,i_,j_,k_[η] ⧴ ϵ^l U@{y^i, a^j, x^k});
     es = Flatten[
        Table[Coefficient[e, b] == 0, {e, {F - 1_U /. η → 0, F ⊠ G - y_U ⊠ F - ∂_η F}}, {b, bs}]];
     F = F /. DSolve[es, fs, η][[1]];
     𝔼[0,
        ξ x + η y + (U /. {CU → -t η ξ, QU → η ξ (1 - T) / ℏ}),
        F + 0_$k /. {e- → 1, U → Times}
      ] /. (v : η | ξ | t | T | y | a | x) → v_1
    ]];
tSW_xy_,i_,j_→k_ := SW_xy[$U, $k] /. {ξ_1 → ξ_i, η_1 → η_j, (v : t | T | y | a | x)_1 → v_k};
tSW_xa,i_,j_→k_ := 𝔼[α_j a_k, e^(-γ α_j) ξ_i x_k, 1];
tSW_ay,i_,j_→k_ := 𝔼[α_i a_k, e^(-γ α_i) η_j y_k, 1];
```

Exponentials as needed.

Task. Define $\mathrm{Exp}_{U_i,k}[\xi, P]$ which computes $e^{\xi\, \mathbb{O}(P)}$ to ϵ^k in the algebra U_i, where ξ is a scalar, X is x_i or y_i, and P is an ϵ-dependent near-docile element, giving the answer in $\mathbb{E}$-form. Should satisfy

$U@\text{Exp}_{U_i,k}[\xi, P] == \mathbb{S}_U\left[e^{\xi x},\ x \to \mathbb{O}(P)\right]$.

Methodology. If $P_0 := P_{\epsilon=0}$ and $e^{\xi\,\mathbb{O}(P)} = \mathbb{O}\left(e^{\xi P_0} F(\xi)\right)$, then $F(\xi = 0) = 1$ and we have:

$\mathbb{O}\left(e^{\xi P_0}(P_0 F(\xi) + \partial_\xi F)\right) = \mathbb{O}\left(\partial_\xi e^{\xi P_0} F(\xi)\right) = \partial_\xi \mathbb{O}\left(e^{\xi P_0} F(\xi)\right) = \partial_\xi e^{\xi\,\mathbb{O}(P)} = e^{\xi\,\mathbb{O}(P)}\,\mathbb{O}(P) = \mathbb{O}\left(e^{\xi P_0} F(\xi)\right)\mathbb{O}(P)$.

This is an ODE for F. Setting inductively $F_k = F_{k-1} + \epsilon^k \varphi$ we find that $F_0 = 1$ and solve for φ.

In[∘]:=
```
(* Bug: The first line is valid only if 𝕆(e^P0)==e^𝕆(P0). *)
(* Bug: ξ must be a symbol. *)
Exp_{U_i_,0}[ξ_, P_] := Module[{LQ = Normal@P /. ϵ → 0},
   𝔼[ξ LQ /. (x | y)_i → 0, ξ LQ /. (t | a)_i → 0, 1]];
Exp_{U_i_,k_}[ξ_, P_] := Block[{$U = U, $k = k},
  Module[{P0, φ, φs, F, j, rhs, at0, atξ},
   P0 = Normal@P /. ϵ → 0;
   φs =
    Flatten@Table[φ_{j1,j2,j3}[ξ], {j2, 0, k}, {j1, 0, 2 k + 1 - j2}, {j3, 0, 2 k + 1 - j2 - j1}];
   F = Normal@Last@Exp_{U_i,k-1}[ξ, P] + ϵ^k φs.(φs /. φ_{js__}[ξ] :→ Times @@ {y_i, a_i, x_i}^{js});
   rhs = Normal@
     Last@m_{i,j→i}[𝔼[ξ P0 /. (x | y)_i → 0, ξ P0 /. (t | a)_i → 0, F + O_k] m_{i→j}@𝔼[0, 0, P + O_k]];
   at0 = (# == 0) & /@ Flatten@CoefficientList[F - 1 /. ξ → 0, {y_i, a_i, x_i}];
   atξ = (# == 0) & /@ Flatten@CoefficientList[(∂_ξ F) + P0 F - rhs, {y_i, a_i, x_i}];
   𝔼[ξ P0 /. (x | y)_i → 0, ξ P0 /. (t | a)_i → 0, F + O_k] /.
    DSolve[And @@ (at0 ⋃ atξ), φs, ξ][[1]] ]]
```

In[∘]:=
```
$k = 1
```

Out[∘]=

1

In[∘]:=
```
$Basis = {a_QU, x_QU, y_QU};
Module[{a, b}, Table[{a, b} → CE[B[a, b]], {a, $Basis}, {b, $Basis}]] // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} \{QU[a], QU[a]\} \to 0 & \{QU[a], QU[x]\} \to \gamma\, QU[x] & \\ \{QU[x], QU[a]\} \to -\gamma\, QU[x] & \{QU[x], QU[x]\} \to 0 & \{QU[x], \\ \{QU[y], QU[a]\} \to \gamma\, QU[y] & \{QU[y], QU[x]\} \to \left(-\frac{1}{\hbar} + \frac{T}{\hbar}\right) QU[\,] - 2\, T\, \epsilon\, QU[a] - \gamma\, \epsilon\, \hbar\, QU[y, x] & \end{pmatrix}$$

In[∘]:=
```
$Basis = {a_QU, x_QU, y_QU};
Module[{a, b},
 DeleteCases[
  Flatten[Table[{a, b, c} → CE[B[a, b] + B[b, a]], {a, $Basis}, {b, $Basis}]], _ → 0]
]
Module[{a, b, c},
 DeleteCases[Flatten[Table[{a, b, c} → CE[B[a, B[b, c]] + B[b, B[c, a]] + B[c, B[a, b]]],
    {a, $Basis}, {b, $Basis}, {c, $Basis}]], _ → 0]
]
$ABasis = {a_QU, x_QU, y_QU, QU[y, y], QU[y, a], QU[y, x], QU[a, a], QU[a, x], QU[x, x]};
Module[{a, b, c},
 DeleteCases[Flatten[Table[{a, b, c} → CE[a ⊠ CE[b ⊠ c] - CE[a ⊠ b] ⊠ c],
    {a, $ABasis}, {b, $ABasis}, {c, $ABasis}]], _ → 0]
]
```

Out[∘]=

{}

Out[∘]=

{}

Out[∘]=

{}

In[∘]:=
```
Module[{a, b, c},
  Table[{a, b, c} → CE[a ⊠ CE[b ⊠ c]], {a, $ABasis}, {b, $ABasis}, {c, $ABasis}]
 ];
```

In[∘]:= `B[x_QU, y_QU] // CE`

Out[∘]=

$$\left(\frac{1}{\hbar} - \frac{T}{\hbar}\right) \mathrm{QU}[\,] + 2\,T\,\epsilon\,\mathrm{QU}[a] + \gamma\,\epsilon\,\hbar\,\mathrm{QU}[y, x]$$

In[∘]:= `B[x_QU, B[x_QU, y_QU]] // CE`

Out[∘]=

$$(\gamma\,\epsilon - 3\,T\,\gamma\,\epsilon)\,\mathrm{QU}[x]$$

In[∘]:= `B[x_QU, B[x_QU, B[x_QU, y_QU]]] // CE`

Out[∘]=

0

In[∘]:= `B[x_QU, B[x_QU, B[x_QU, B[x_QU, y_QU]]]] // CE`

Out[∘]=

0

In[∘]:= `B[x_QU, B[x_QU, B[x_QU, B[x_QU, B[x_QU, y_QU]]]]] // Expand`

Out[∘]=

0

In[∘]:= `B[x_QU, B[x_QU, B[x_QU, B[x_QU, B[x_QU, B[x_QU, y_QU]]]]]] // Expand`

Out[∘]=

0

In[ ]:=
```
adPower[x_, k_, Env_][c_ * y_QU] := CE[c adPower[x, k, Env][y]];
adPower[x_, k_, Env_][y_Plus] := adPower[x, k, Env] /@ y;
adPower[_, 0, Env_][y_] := y;
adPower[x_, k_, Env_][0] = 0;
adPower[x_, k_, Env_][y_QU] :=
  adPower[x, k, Env][y] = CE@B[adPower[x, k - 1, Env][y], x];
adPower[x_, k_][y_] := adPower[x, k, $E][y];
```

In[ ]:= adPower[x_{QU}, 1][y_{QU}]

Out[ ]=

$$\left(-\frac{1}{\hbar}+\frac{T}{\hbar}\right) QU[\,] - 2\,T\,\epsilon\,QU[a] - \gamma\,\epsilon\,\hbar\,QU[y, x]$$

In[ ]:= adPower[x_{QU}, 2][y_{QU}]

Out[ ]=

$$(\gamma\,\epsilon - 3\,T\,\gamma\,\epsilon)\,QU[x]$$

In[ ]:= adPower[x_{QU}, 3][y_{QU}]

Out[ ]=

0

In[ ]:= adPower[x_{QU}, 4][y_{QU}]

Out[ ]=

0

In[ ]:= adPower[x_{QU}, 5][y_{QU}]

Out[ ]=

0

In[ ]:=
```
NilQ[x_QU] = NilQ[y_QU] = True; NilQ[a_QU] = False;
PotQ[c_. * x_QU] := ¬ NilQ[x]; (* Potent Q *)
NilQ[c_ * x_QU] := NilQ[x];
PotQ[x_Plus] := And @@ (PotQ /@ (List @@ x));
NilQ[x_Plus] := And @@ (NilQ /@ (List @@ x));
```

In[ ]:=
```
Ad[0] = Identity;
Ad[c_. a_QU][z_] := z /. w_QU :→ e^(c γ (Count[w,y]-Count[w,x])) w;
Ad[c_. QU[]][z_] := z;
```

In[ ]:=
```
Ad[x_][y_] /; NilQ[x] := Expand@Module[{k = 0, s = 0},
   While[adPower[x, k][y] =!= 0,
    s += adPower[x, k][y] / k!;
    ++k
   ];
   s
  ]
```

In[ ]:=
```
Ad[x_QU][y_QU]
```
Out[ ]=

$$-\mathrm{QU}[\,] + T\,\mathrm{QU}[\,] - 2\,T\,\epsilon\,\mathrm{QU}[a] + \frac{1}{2}\,\epsilon\,\mathrm{QU}[x] - \frac{3}{2}\,T\,\epsilon\,\mathrm{QU}[x] + \mathrm{QU}[y] - \epsilon\,\mathrm{QU}[y, x]$$

In[ ]:=
```
$k
```
Out[ ]=

1

In[ ]:=
```
Ad[c a_QU] /@ {QU[], x_QU, y_QU, QU[x, y]}
```
Out[ ]=

$$\{\mathrm{QU}[\,],\ e^{-c\,\gamma}\,\mathrm{QU}[x],\ e^{c\,\gamma}\,\mathrm{QU}[y],\ \mathrm{QU}[x, y]\}$$

In[ ]:=
```
Ad[(α1 + α2 + ϵ fa[λ]) a_QU] /@ {QU[], x_QU, y_QU, QU[x, y]}
```
Out[ ]=

$$\left\{\mathrm{QU}[\,],\ e^{-\gamma\,(\alpha1+\alpha2+\epsilon\,\mathrm{fa}[\lambda])}\,\mathrm{QU}[x],\ e^{\gamma\,(\alpha1+\alpha2+\epsilon\,\mathrm{fa}[\lambda])}\,\mathrm{QU}[y],\ \mathrm{QU}[x, y]\right\}$$

In[ ]:=
```
Normal@Series[e^(-γ (α1+α2+ϵ fa[λ])), {ϵ, 0, $k}]
```
Out[ ]=

$$e^{-((\alpha1+\alpha2)\,\gamma)} - e^{-((\alpha1+\alpha2)\,\gamma)}\,\gamma\,\epsilon\,\mathrm{fa}[\lambda]$$

In[ ]:=
```
CE[
 Ad[(α1 + α2 + ϵ fa[λ]) a_QU] /@ {QU[], x_QU, y_QU, QU[x, y]} /. e^ℰ_ :→ Normal@Series[e^ℰ, {ϵ, 0, $k}]]
```
Out[ ]=

$$\left\{\mathrm{QU}[\,],\ \left(e^{-\alpha1\,\gamma-\alpha2\,\gamma} - e^{-\alpha1\,\gamma-\alpha2\,\gamma}\,\gamma\,\epsilon\,\mathrm{fa}[\lambda]\right)\mathrm{QU}[x],\ \left(e^{\alpha1\,\gamma+\alpha2\,\gamma} + e^{\alpha1\,\gamma+\alpha2\,\gamma}\,\gamma\,\epsilon\,\mathrm{fa}[\lambda]\right)\mathrm{QU}[y],\ \mathrm{QU}[x, y]\right\}$$

In[ ]:=
```
Ad[a_QU][Ad[x_QU][y_QU]]
```
Out[ ]=

$$-\frac{\mathrm{QU}[\,]}{\hbar} + \frac{T\,\mathrm{QU}[\,]}{\hbar} - 2\,T\,\epsilon\,\mathrm{QU}[a] + \frac{1}{2}\,e^{-\gamma}\,\gamma\,\epsilon\,\mathrm{QU}[x] - \frac{3}{2}\,e^{-\gamma}\,T\,\gamma\,\epsilon\,\mathrm{QU}[x] + e^{\gamma}\,\mathrm{QU}[y] - \gamma\,\epsilon\,\hbar\,\mathrm{QU}[y, x]$$

In[ ]:=
```
λTangent[] = 0;
λTangent[xs___, x_] := CE[Ad[x][λTangent[xs]] + ∂_λ x];
λTangent[xs_List] := λTangent @@ xs
```

In[ ]:=
```
λTangent[λ y_QU, λ a_QU, λ x_QU]
```
Out[ ]=

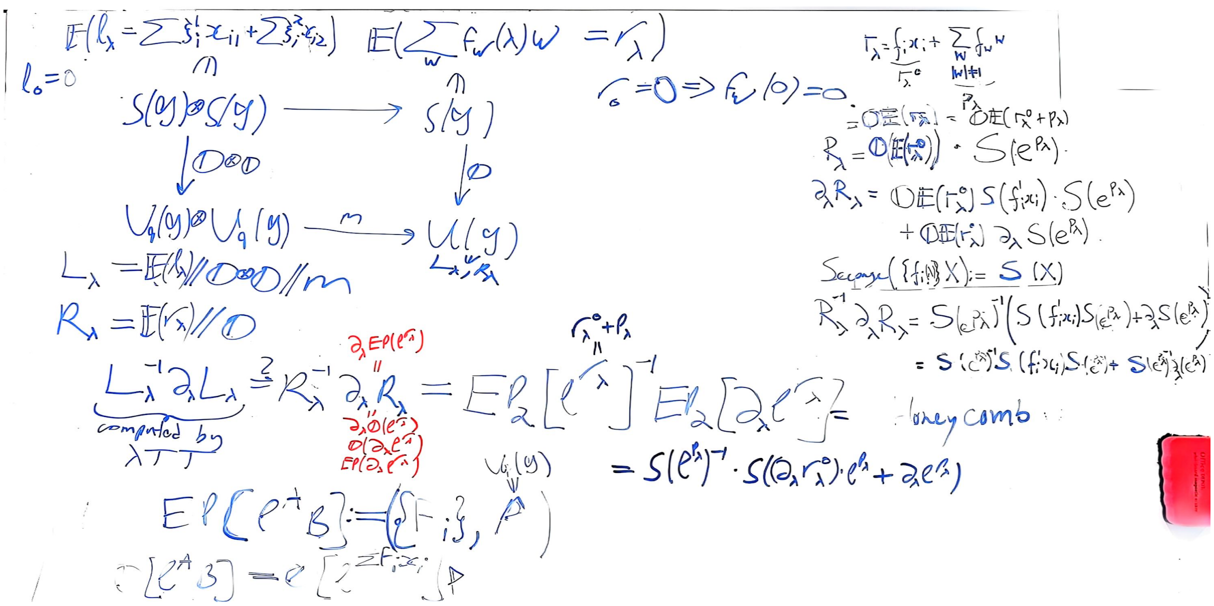
$$\left(-\frac{e^{\gamma\,\lambda}\,\lambda}{\hbar} + \frac{e^{\gamma\,\lambda}\,T\,\lambda}{\hbar}\right)\mathrm{QU}[\,] + \left(1 - 2\,e^{\gamma\,\lambda}\,T\,\epsilon\,\lambda\right)\mathrm{QU}[a] +$$
$$\left(1 + \gamma\,\lambda + \frac{1}{2}\,e^{\gamma\,\lambda}\,\gamma\,\epsilon\,\lambda^2 - \frac{3}{2}\,e^{\gamma\,\lambda}\,T\,\gamma\,\epsilon\,\lambda^2\right)\mathrm{QU}[x] + e^{\gamma\,\lambda}\,\mathrm{QU}[y] - e^{\gamma\,\lambda}\,\gamma\,\epsilon\,\lambda\,\hbar\,\mathrm{QU}[y, x]$$

In[ ]:=
```
Seepage_U_ [{}][c_] = c;
Seepage_U_ [ξs_List][P_] := Module[{gs = U[Generators], lx},
  lx = gs[[Length@ξs]];
  Collect[P, lx, (Ad[Last[ξs] lx_U][𝕆_U[gs, Seepage_U[Most@ξs]@#]] /. U → Times) &]
 ]
```

In[•]:= **$\text{Seepage}_{QU}[\{\eta, \alpha, \xi\}][y]$**

Out[•]=

$$e^{\alpha}\, y - e^{\alpha}\, \xi + e^{\alpha}\, T\, \xi - 2\, a\, e^{\alpha}\, T \in \xi - e^{\alpha}\, x\, y \in \xi + \frac{1}{2}\, e^{\alpha}\, x \in \xi^2 - \frac{3}{2}\, e^{\alpha}\, T\, x \in \xi^2$$



In[•]:= **$L\lambda = \lambda\text{Tangent}[\lambda\, \eta 1\, y_{QU}, \alpha 1\, a_{QU}, \lambda\, \xi 1\, x_{QU}, \lambda\, \eta 2\, y_{QU}, \alpha 2\, a_{QU}, \lambda\, \xi 2\, x_{QU}]$**

Out[•]=

$$\begin{aligned}
&\Big(-e^{\alpha 1}\, \eta 1\, \lambda\, \xi 1 + e^{\alpha 1}\, T\, \eta 1\, \lambda\, \xi 1 + \eta 2\, \lambda\, \xi 1 - T\, \eta 2\, \lambda\, \xi 1 + \frac{1}{2}\, e^{\alpha 1} \in \eta 1\, \eta 2\, \lambda^3\, \xi 1^2 - 2\, e^{\alpha 1}\, T \in \eta 1\, \eta 2\, \lambda^3\, \xi 1^2 + \\
&\quad \frac{3}{2}\, e^{\alpha 1}\, T^2 \in \eta 1\, \eta 2\, \lambda^3\, \xi 1^2 - e^{\alpha 1+\alpha 2}\, \eta 1\, \lambda\, \xi 2 + e^{\alpha 1+\alpha 2}\, T\, \eta 1\, \lambda\, \xi 2 - e^{\alpha 2}\, \eta 2\, \lambda\, \xi 2 + e^{\alpha 2}\, T\, \eta 2\, \lambda\, \xi 2 + \\
&\quad e^{\alpha 1+\alpha 2} \in \eta 1\, \eta 2\, \lambda^3\, \xi 1\, \xi 2 - 4\, e^{\alpha 1+\alpha 2}\, T \in \eta 1\, \eta 2\, \lambda^3\, \xi 1\, \xi 2 + 3\, e^{\alpha 1+\alpha 2}\, T^2 \in \eta 1\, \eta 2\, \lambda^3\, \xi 1\, \xi 2 - \\
&\quad \frac{1}{2}\, e^{\alpha 2} \in \eta 2^2\, \lambda^3\, \xi 1\, \xi 2 + 2\, e^{\alpha 2}\, T \in \eta 2^2\, \lambda^3\, \xi 1\, \xi 2 - \frac{3}{2}\, e^{\alpha 2}\, T^2 \in \eta 2^2\, \lambda^3\, \xi 1\, \xi 2\Big)\, QU[\,] + \\
&\big(-2\, e^{\alpha 1}\, T \in \eta 1\, \lambda\, \xi 1 + 2\, T \in \eta 2\, \lambda\, \xi 1 - 2\, e^{\alpha 1+\alpha 2}\, T \in \eta 1\, \lambda\, \xi 2 - 2\, e^{\alpha 2}\, T \in \eta 2\, \lambda\, \xi 2\big)\, QU[a] + \\
&\Big(e^{-\alpha 2}\, \xi 1 + \frac{1}{2}\, e^{\alpha 1-\alpha 2} \in \eta 1\, \lambda^2\, \xi 1^2 - \frac{3}{2}\, e^{\alpha 1-\alpha 2}\, T \in \eta 1\, \lambda^2\, \xi 1^2 + \xi 2 + e^{\alpha 1} \in \eta 1\, \lambda^2\, \xi 1\, \xi 2 - \\
&\quad 3\, e^{\alpha 1}\, T \in \eta 1\, \lambda^2\, \xi 1\, \xi 2 - \in \eta 2\, \lambda^2\, \xi 1\, \xi 2 + 3\, T \in \eta 2\, \lambda^2\, \xi 1\, \xi 2 + \frac{1}{2}\, e^{\alpha 1+\alpha 2} \in \eta 1\, \lambda^2\, \xi 2^2 - \\
&\quad \frac{3}{2}\, e^{\alpha 1+\alpha 2}\, T \in \eta 1\, \lambda^2\, \xi 2^2 + \frac{1}{2}\, e^{\alpha 2} \in \eta 2\, \lambda^2\, \xi 2^2 - \frac{3}{2}\, e^{\alpha 2}\, T \in \eta 2\, \lambda^2\, \xi 2^2\Big)\, QU[x] + \\
&\Big(e^{\alpha 1+\alpha 2}\, \eta 1 + e^{\alpha 2}\, \eta 2 - e^{\alpha 1+\alpha 2} \in \eta 1\, \eta 2\, \lambda^2\, \xi 1 + 3\, e^{\alpha 1+\alpha 2}\, T \in \eta 1\, \eta 2\, \lambda^2\, \xi 1 + \\
&\quad \frac{1}{2}\, e^{\alpha 2} \in \eta 2^2\, \lambda^2\, \xi 1 - \frac{3}{2}\, e^{\alpha 2}\, T \in \eta 2^2\, \lambda^2\, \xi 1\Big)\, QU[y] + \\
&\big(-e^{\alpha 1} \in \eta 1\, \lambda\, \xi 1 + \in \eta 2\, \lambda\, \xi 1 - e^{\alpha 1+\alpha 2} \in \eta 1\, \lambda\, \xi 2 - e^{\alpha 2} \in \eta 2\, \lambda\, \xi 2\big)\, QU[y, x]
\end{aligned}$$

In[∘]:=
```
r0λ = {fy0[λ] + ϵ fy1[λ], α1 + α2 + ϵ fa1[λ], fx0[λ] + ϵ fx1[λ]};
pλ = (ft0[λ] + ϵ ft1[λ]) t QU[] + (ϵ fyx1[λ]) QU[y, x];
rλ = pλ + r0λ.QU /@ (QU@Generators)
```

Out[∘]=

$$t\,(ft0[\lambda]+\epsilon\,ft1[\lambda])\,QU[\,]+(\alpha1+\alpha2+\epsilon\,fa1[\lambda])\,QU[a]+(fx0[\lambda]+\epsilon\,fx1[\lambda])\,QU[x]+(fy0[\lambda]+\epsilon\,fy1[\lambda])\,QU[y]+\epsilon\,fyx1[\lambda]\,QU[y,x]$$

In[∘]:=
```
SS[e^(pλ/.QU→Times)]
```

Out[∘]=

$$e^{t\,ft0[\lambda]}+e^{t\,ft0[\lambda]}\,\epsilon\,(t\,ft1[\lambda]+x\,y\,fyx1[\lambda])$$

In[∘]:=
```
Invert_QU[ℰ_] := Module[{A, B, β}, (* ℰ=A QU[]+ϵ B *)
  A = Coefficient[ℰ, 1_QU] /. ϵ → 0;
  B = CE[ϵ^-1 (ℰ - A 1_QU)];
  𝕊_QU[SS[(A + ϵ β)^-1], β → B]
 ]
```

In[∘]:=
```
ℰ = 𝕆_QU[{y, a, x}, CF@Seepage_QU[r0λ][SS[e^(pλ/.QU→Times)]] /. e^ℰ_ ⧴ Normal@Series[e^ℰ, {ϵ, 0, $k}]]
```

Out[∘]=

$$\left(e^{t\,ft0[\lambda]}+e^{t\,ft0[\lambda]}\,t\,\epsilon\,ft1[\lambda]\right)QU[\,]+\left(-e^{\alpha1+\alpha2+t\,ft0[\lambda]}\,\epsilon\,fx0[\lambda]\,fyx1[\lambda]+e^{\alpha1+\alpha2+t\,ft0[\lambda]}\,T\,\epsilon\,fx0[\lambda]\,fyx1[\lambda]\right)QU[x]+e^{\alpha1+\alpha2+t\,ft0[\lambda]}\,\epsilon\,fyx1[\lambda]\,QU[y,x]$$

In[∘]:=
```
Invert_QU[ℰ]
```

Out[∘]=

$$\left(e^{-t\,ft0[\lambda]}-e^{-t\,ft0[\lambda]}\,t\,\epsilon\,ft1[\lambda]\right)QU[\,]+\left(e^{\alpha1+\alpha2-t\,ft0[\lambda]}\,\epsilon\,fx0[\lambda]\,fyx1[\lambda]-e^{\alpha1+\alpha2-t\,ft0[\lambda]}\,T\,\epsilon\,fx0[\lambda]\,fyx1[\lambda]\right)QU[x]-e^{\alpha1+\alpha2-t\,ft0[\lambda]}\,\epsilon\,fyx1[\lambda]\,QU[y,x]$$

In[∘]:=
```
CE[Invert_QU[ℰ] ⊠ ℰ]
```

Out[∘]=

QU[ ]

In[∘]:=

$$= S(e^{p_\lambda})^{-1}\cdot S(\partial_\lambda r^\circ_\lambda)\cdot e^{p_\lambda}+\partial_\lambda e^{p_\lambda})$$

Out[∘]=

$$= S(e^{p_\lambda})^{-1}\cdot S(\partial_\lambda r^\circ_\lambda)\cdot e^{p_\lambda}+\partial_\lambda e^{p_\lambda})$$

In[∘]:=
```
Pλ =
  𝕆_QU[{y, a, x}, CF@Seepage_QU[r0λ][SS[e^(pλ/.QU→Times)]] /. e^ℰ_ ⧴ Normal@Series[e^ℰ, {ϵ, 0, $k}]];
Invert_QU[Pλ]
```

Out[∘]=

$$\left(e^{-t\,ft0[\lambda]}-e^{-t\,ft0[\lambda]}\,t\,\epsilon\,ft1[\lambda]\right)QU[\,]+\left(e^{\alpha1+\alpha2-t\,ft0[\lambda]}\,\epsilon\,fx0[\lambda]\,fyx1[\lambda]-e^{\alpha1+\alpha2-t\,ft0[\lambda]}\,T\,\epsilon\,fx0[\lambda]\,fyx1[\lambda]\right)QU[x]-e^{\alpha1+\alpha2-t\,ft0[\lambda]}\,\epsilon\,fyx1[\lambda]\,QU[y,x]$$

In[∘]:=
```
Rλ = CE@SS[Invert_QU[Pλ] ⊠ 𝕆_QU[{y, a, x},
    Seepage_QU[r0λ][(∂_λ (r0λ.QU@Generators)) (Pλ /. QU → Times) + ∂_λ (Pλ /. QU → Times)]]]
```

Out[∘]=

$$\begin{aligned}
&QU[y,x,x]\left(-e^{\alpha 1+\alpha 2}\in fyx1[\lambda]\,fx0'[\lambda]+e^{2\alpha 1+2\alpha 2}\in fyx1[\lambda]\,fx0'[\lambda]\right)+\\
&QU[x,x]\left(-e^{2\alpha 1+2\alpha 2}\in fx0[\lambda]\,fyx1[\lambda]\,fx0'[\lambda]+e^{2\alpha 1+2\alpha 2}\,T\in fx0[\lambda]\,fyx1[\lambda]\,fx0'[\lambda]\right)+\\
&QU[a]\left(\in fa1'[\lambda]-2\,e^{\alpha 1+\alpha 2}\,T\in fx0[\lambda]\,fy0'[\lambda]\right)+\\
&QU[y,y,x]\left(-e^{2\alpha 1+2\alpha 2}\in fyx1[\lambda]\,fy0'[\lambda]+e^{3\alpha 1+3\alpha 2}\in fyx1[\lambda]\,fy0'[\lambda]\right)+\\
&QU[y]\left(e^{\alpha 1+\alpha 2}\,fy0'[\lambda]+e^{\alpha 1+\alpha 2}\in fa1[\lambda]\,fy0'[\lambda]-\right.\\
&\quad\left.e^{2\alpha 1+2\alpha 2}\in fyx1[\lambda]\,fy0'[\lambda]+e^{2\alpha 1+2\alpha 2}\,T\in fyx1[\lambda]\,fy0'[\lambda]+e^{\alpha 1+\alpha 2}\in fy1'[\lambda]\right)+\\
&QU[]\left(t\,ft0'[\lambda]+t\in ft1'[\lambda]-e^{\alpha 1+\alpha 2}\,fx0[\lambda]\,fy0'[\lambda]+e^{\alpha 1+\alpha 2}\,T\,fx0[\lambda]\,fy0'[\lambda]-\right.\\
&\quad e^{\alpha 1+\alpha 2}\in fa1[\lambda]\,fx0[\lambda]\,fy0'[\lambda]+e^{\alpha 1+\alpha 2}\,T\in fa1[\lambda]\,fx0[\lambda]\,fy0'[\lambda]-\\
&\quad e^{\alpha 1+\alpha 2}\in fx1[\lambda]\,fy0'[\lambda]+e^{\alpha 1+\alpha 2}\,T\in fx1[\lambda]\,fy0'[\lambda]+e^{2\alpha 1+2\alpha 2}\in fx0[\lambda]\,fyx1[\lambda]\,fy0'[\lambda]-\\
&\quad 2\,e^{2\alpha 1+2\alpha 2}\,T\in fx0[\lambda]\,fyx1[\lambda]\,fy0'[\lambda]+e^{2\alpha 1+2\alpha 2}\,T^2\in fx0[\lambda]\,fyx1[\lambda]\,fy0'[\lambda]-\\
&\quad\left.e^{\alpha 1+\alpha 2}\in fx0[\lambda]\,fy1'[\lambda]+e^{\alpha 1+\alpha 2}\,T\in fx0[\lambda]\,fy1'[\lambda]\right)+\\
&QU[y,x]\left(-e^{\alpha 1+\alpha 2}\,t\in fyx1[\lambda]\,ft0'[\lambda]+e^{2\alpha 1+2\alpha 2}\,t\in fyx1[\lambda]\,ft0'[\lambda]-e^{\alpha 1+\alpha 2}\in fx0[\lambda]\,fy0'[\lambda]+\right.\\
&\quad e^{2\alpha 1+2\alpha 2}\in fx0[\lambda]\,fyx1[\lambda]\,fy0'[\lambda]-2\,e^{3\alpha 1+3\alpha 2}\in fx0[\lambda]\,fyx1[\lambda]\,fy0'[\lambda]-e^{2\alpha 1+2\alpha 2}\,T\in fx0[\lambda]\\
&\quad\ \left.fyx1[\lambda]\,fy0'[\lambda]+2\,e^{3\alpha 1+3\alpha 2}\,T\in fx0[\lambda]\,fyx1[\lambda]\,fy0'[\lambda]+e^{2\alpha 1+2\alpha 2}\in fyx1'[\lambda]\right)+QU[x]\\
&\left(\in fx0[\lambda]\,fa1'[\lambda]-e^{2\alpha 1+2\alpha 2}\,t\in fx0[\lambda]\,fyx1[\lambda]\,ft0'[\lambda]+e^{2\alpha 1+2\alpha 2}\,t\,T\in fx0[\lambda]\,fyx1[\lambda]\,ft0'[\lambda]+\right.\\
&\quad fx0'[\lambda]-e^{\alpha 1+\alpha 2}\in fyx1[\lambda]\,fx0'[\lambda]+e^{\alpha 1+\alpha 2}\,T\in fyx1[\lambda]\,fx0'[\lambda]+\\
&\quad \in fx1'[\lambda]+\frac{1}{2}\,e^{\alpha 1+\alpha 2}\in fx0[\lambda]^2\,fy0'[\lambda]-\frac{3}{2}\,e^{\alpha 1+\alpha 2}\,T\in fx0[\lambda]^2\,fy0'[\lambda]+\\
&\quad e^{3\alpha 1+3\alpha 2}\in fx0[\lambda]^2\,fyx1[\lambda]\,fy0'[\lambda]-2\,e^{3\alpha 1+3\alpha 2}\,T\in fx0[\lambda]^2\,fyx1[\lambda]\,fy0'[\lambda]+\\
&\quad e^{3\alpha 1+3\alpha 2}\,T^2\in fx0[\lambda]^2\,fyx1[\lambda]\,fy0'[\lambda]-e^{\alpha 1+\alpha 2}\in fx0[\lambda]\,fyx1'[\lambda]-\\
&\quad\left.e^{2\alpha 1+2\alpha 2}\in fx0[\lambda]\,fyx1'[\lambda]+e^{\alpha 1+\alpha 2}\,T\in fx0[\lambda]\,fyx1'[\lambda]+e^{2\alpha 1+2\alpha 2}\,T\in fx0[\lambda]\,fyx1'[\lambda]\right)
\end{aligned}$$

In[]:= **diff = CE[Lλ − Rλ]**

Out[]=

$$
\begin{aligned}
&\mathrm{QU}[y,x,x]\left(e^{\alpha 1+\alpha 2}\epsilon\,\mathrm{fyx1}[\lambda]\,\mathrm{fx0}'[\lambda]-e^{2\alpha 1+2\alpha 2}\epsilon\,\mathrm{fyx1}[\lambda]\,\mathrm{fx0}'[\lambda]\right)+\\
&\mathrm{QU}[x,x]\left(e^{2\alpha 1+2\alpha 2}\epsilon\,\mathrm{fx0}[\lambda]\,\mathrm{fyx1}[\lambda]\,\mathrm{fx0}'[\lambda]-e^{2\alpha 1+2\alpha 2}\,T\,\epsilon\,\mathrm{fx0}[\lambda]\,\mathrm{fyx1}[\lambda]\,\mathrm{fx0}'[\lambda]\right)+\\
&\mathrm{QU}[a]\left(-2e^{\alpha 1}T\epsilon\,\eta 1\,\lambda\,\xi 1+2T\epsilon\,\eta 2\,\lambda\,\xi 1-2e^{\alpha 1+\alpha 2}T\epsilon\,\eta 1\,\lambda\,\xi 2-\right.\\
&\quad\left.2e^{\alpha 2}T\epsilon\,\eta 2\,\lambda\,\xi 2-\epsilon\,\mathrm{fa1}'[\lambda]+2e^{\alpha 1+\alpha 2}T\epsilon\,\mathrm{fx0}[\lambda]\,\mathrm{fy0}'[\lambda]\right)+\\
&\mathrm{QU}[y,y,x]\left(e^{2\alpha 1+2\alpha 2}\epsilon\,\mathrm{fyx1}[\lambda]\,\mathrm{fy0}'[\lambda]-e^{3\alpha 1+3\alpha 2}\epsilon\,\mathrm{fyx1}[\lambda]\,\mathrm{fy0}'[\lambda]\right)+\\
&\mathrm{QU}[y]\left(e^{\alpha 1+\alpha 2}\eta 1+e^{\alpha 2}\eta 2-e^{\alpha 1+\alpha 2}\epsilon\,\eta 1\,\eta 2\,\lambda^2\,\xi 1+3e^{\alpha 1+\alpha 2}T\epsilon\,\eta 1\,\eta 2\,\lambda^2\,\xi 1+\right.\\
&\quad\frac{1}{2}e^{\alpha 2}\epsilon\,\eta 2^2\,\lambda^2\,\xi 1-\frac{3}{2}e^{\alpha 2}T\epsilon\,\eta 2^2\,\lambda^2\,\xi 1-e^{\alpha 1+\alpha 2}\mathrm{fy0}'[\lambda]-e^{\alpha 1+\alpha 2}\epsilon\,\mathrm{fa1}[\lambda]\,\mathrm{fy0}'[\lambda]+\\
&\quad\left.e^{2\alpha 1+2\alpha 2}\epsilon\,\mathrm{fyx1}[\lambda]\,\mathrm{fy0}'[\lambda]-e^{2\alpha 1+2\alpha 2}T\epsilon\,\mathrm{fyx1}[\lambda]\,\mathrm{fy0}'[\lambda]-e^{\alpha 1+\alpha 2}\epsilon\,\mathrm{fy1}'[\lambda]\right)+\\
&\mathrm{QU}[\,]\left(-e^{\alpha 1}\eta 1\,\lambda\,\xi 1+e^{\alpha 1}T\,\eta 1\,\lambda\,\xi 1+\eta 2\,\lambda\,\xi 1-T\,\eta 2\,\lambda\,\xi 1+\frac{1}{2}e^{\alpha 1}\epsilon\,\eta 1\,\eta 2\,\lambda^3\,\xi 1^2-2e^{\alpha 1}T\epsilon\,\eta 1\,\eta 2\,\lambda^3\,\xi 1^2+\right.\\
&\quad\frac{3}{2}e^{\alpha 1}T^2\epsilon\,\eta 1\,\eta 2\,\lambda^3\,\xi 1^2-e^{\alpha 1+\alpha 2}\eta 1\,\lambda\,\xi 2+e^{\alpha 1+\alpha 2}T\,\eta 1\,\lambda\,\xi 2-e^{\alpha 2}\eta 2\,\lambda\,\xi 2+e^{\alpha 2}T\,\eta 2\,\lambda\,\xi 2+\\
&\quad e^{\alpha 1+\alpha 2}\epsilon\,\eta 1\,\eta 2\,\lambda^3\,\xi 1\,\xi 2-4e^{\alpha 1+\alpha 2}T\epsilon\,\eta 1\,\eta 2\,\lambda^3\,\xi 1\,\xi 2+3e^{\alpha 1+\alpha 2}T^2\epsilon\,\eta 1\,\eta 2\,\lambda^3\,\xi 1\,\xi 2-\\
&\quad\frac{1}{2}e^{\alpha 2}\epsilon\,\eta 2^2\,\lambda^3\,\xi 1\,\xi 2+2e^{\alpha 2}T\epsilon\,\eta 2^2\,\lambda^3\,\xi 1\,\xi 2-\frac{3}{2}e^{\alpha 2}T^2\epsilon\,\eta 2^2\,\lambda^3\,\xi 1\,\xi 2-t\,\mathrm{ft0}'[\lambda]-t\,\epsilon\,\mathrm{ft1}'[\lambda]+\\
&\quad e^{\alpha 1+\alpha 2}\mathrm{fx0}[\lambda]\,\mathrm{fy0}'[\lambda]-e^{\alpha 1+\alpha 2}T\,\mathrm{fx0}[\lambda]\,\mathrm{fy0}'[\lambda]+e^{\alpha 1+\alpha 2}\epsilon\,\mathrm{fa1}[\lambda]\,\mathrm{fx0}[\lambda]\,\mathrm{fy0}'[\lambda]-\\
&\quad e^{\alpha 1+\alpha 2}T\epsilon\,\mathrm{fa1}[\lambda]\,\mathrm{fx0}[\lambda]\,\mathrm{fy0}'[\lambda]+e^{\alpha 1+\alpha 2}\epsilon\,\mathrm{fx1}[\lambda]\,\mathrm{fy0}'[\lambda]-e^{\alpha 1+\alpha 2}T\epsilon\,\mathrm{fx1}[\lambda]\,\mathrm{fy0}'[\lambda]-\\
&\quad e^{2\alpha 1+2\alpha 2}\epsilon\,\mathrm{fx0}[\lambda]\,\mathrm{fyx1}[\lambda]\,\mathrm{fy0}'[\lambda]+2e^{2\alpha 1+2\alpha 2}T\epsilon\,\mathrm{fx0}[\lambda]\,\mathrm{fyx1}[\lambda]\,\mathrm{fy0}'[\lambda]-\\
&\quad\left.e^{2\alpha 1+2\alpha 2}T^2\epsilon\,\mathrm{fx0}[\lambda]\,\mathrm{fyx1}[\lambda]\,\mathrm{fy0}'[\lambda]+e^{\alpha 1+\alpha 2}\epsilon\,\mathrm{fx0}[\lambda]\,\mathrm{fy1}'[\lambda]-e^{\alpha 1+\alpha 2}T\epsilon\,\mathrm{fx0}[\lambda]\,\mathrm{fy1}'[\lambda]\right)+\\
&\mathrm{QU}[y,x]\left(-e^{\alpha 1}\epsilon\,\eta 1\,\lambda\,\xi 1+\epsilon\,\eta 2\,\lambda\,\xi 1-e^{\alpha 1+\alpha 2}\epsilon\,\eta 1\,\lambda\,\xi 2-e^{\alpha 2}\epsilon\,\eta 2\,\lambda\,\xi 2+e^{\alpha 1+\alpha 2}t\,\epsilon\,\mathrm{fyx1}[\lambda]\,\mathrm{ft0}'[\lambda]-\right.\\
&\quad e^{2\alpha 1+2\alpha 2}t\,\epsilon\,\mathrm{fyx1}[\lambda]\,\mathrm{ft0}'[\lambda]+e^{\alpha 1+\alpha 2}\epsilon\,\mathrm{fx0}[\lambda]\,\mathrm{fy0}'[\lambda]-e^{2\alpha 1+2\alpha 2}\epsilon\,\mathrm{fx0}[\lambda]\,\mathrm{fyx1}[\lambda]\,\mathrm{fy0}'[\lambda]+\\
&\quad 2e^{3\alpha 1+3\alpha 2}\epsilon\,\mathrm{fx0}[\lambda]\,\mathrm{fyx1}[\lambda]\,\mathrm{fy0}'[\lambda]+e^{2\alpha 1+2\alpha 2}T\epsilon\,\mathrm{fx0}[\lambda]\,\mathrm{fyx1}[\lambda]\,\mathrm{fy0}'[\lambda]-\\
&\quad\left.2e^{3\alpha 1+3\alpha 2}T\epsilon\,\mathrm{fx0}[\lambda]\,\mathrm{fyx1}[\lambda]\,\mathrm{fy0}'[\lambda]-e^{2\alpha 1+2\alpha 2}\epsilon\,\mathrm{fyx1}'[\lambda]\right)+\\
&\mathrm{QU}[x]\left(e^{-\alpha 2}\xi 1+\frac{1}{2}e^{\alpha 1-\alpha 2}\epsilon\,\eta 1\,\lambda^2\,\xi 1^2-\frac{3}{2}e^{\alpha 1-\alpha 2}T\epsilon\,\eta 1\,\lambda^2\,\xi 1^2+\xi 2+e^{\alpha 1}\epsilon\,\eta 1\,\lambda^2\,\xi 1\,\xi 2-\right.\\
&\quad 3e^{\alpha 1}T\epsilon\,\eta 1\,\lambda^2\,\xi 1\,\xi 2-\epsilon\,\eta 2\,\lambda^2\,\xi 1\,\xi 2+3T\epsilon\,\eta 2\,\lambda^2\,\xi 1\,\xi 2+\frac{1}{2}e^{\alpha 1+\alpha 2}\epsilon\,\eta 1\,\lambda^2\,\xi 2^2-\\
&\quad\frac{3}{2}e^{\alpha 1+\alpha 2}T\epsilon\,\eta 1\,\lambda^2\,\xi 2^2+\frac{1}{2}e^{\alpha 2}\epsilon\,\eta 2\,\lambda^2\,\xi 2^2-\frac{3}{2}e^{\alpha 2}T\epsilon\,\eta 2\,\lambda^2\,\xi 2^2-\epsilon\,\mathrm{fx0}[\lambda]\,\mathrm{fa1}'[\lambda]+\\
&\quad e^{2\alpha 1+2\alpha 2}t\,\epsilon\,\mathrm{fx0}[\lambda]\,\mathrm{fyx1}[\lambda]\,\mathrm{ft0}'[\lambda]-e^{2\alpha 1+2\alpha 2}t\,T\epsilon\,\mathrm{fx0}[\lambda]\,\mathrm{fyx1}[\lambda]\,\mathrm{ft0}'[\lambda]-\mathrm{fx0}'[\lambda]+\\
&\quad e^{\alpha 1+\alpha 2}\epsilon\,\mathrm{fyx1}[\lambda]\,\mathrm{fx0}'[\lambda]-e^{\alpha 1+\alpha 2}T\epsilon\,\mathrm{fyx1}[\lambda]\,\mathrm{fx0}'[\lambda]-\epsilon\,\mathrm{fx1}'[\lambda]-\frac{1}{2}e^{\alpha 1+\alpha 2}\epsilon\,\mathrm{fx0}[\lambda]^2\,\mathrm{fy0}'[\lambda]+\\
&\quad\frac{3}{2}e^{\alpha 1+\alpha 2}T\epsilon\,\mathrm{fx0}[\lambda]^2\,\mathrm{fy0}'[\lambda]-e^{3\alpha 1+3\alpha 2}\epsilon\,\mathrm{fx0}[\lambda]^2\,\mathrm{fyx1}[\lambda]\,\mathrm{fy0}'[\lambda]+2e^{3\alpha 1+3\alpha 2}T\epsilon\,\mathrm{fx0}[\lambda]^2\\
&\quad\mathrm{fyx1}[\lambda]\,\mathrm{fy0}'[\lambda]-e^{3\alpha 1+3\alpha 2}T^2\epsilon\,\mathrm{fx0}[\lambda]^2\,\mathrm{fyx1}[\lambda]\,\mathrm{fy0}'[\lambda]+e^{\alpha 1+\alpha 2}\epsilon\,\mathrm{fx0}[\lambda]\,\mathrm{fyx1}'[\lambda]+\\
&\quad\left.e^{2\alpha 1+2\alpha 2}\epsilon\,\mathrm{fx0}[\lambda]\,\mathrm{fyx1}'[\lambda]-e^{\alpha 1+\alpha 2}T\epsilon\,\mathrm{fx0}[\lambda]\,\mathrm{fyx1}'[\lambda]-e^{2\alpha 1+2\alpha 2}T\epsilon\,\mathrm{fx0}[\lambda]\,\mathrm{fyx1}'[\lambda]\right)
\end{aligned}
$$

In[∘]:= `diff /. ϵ → 0`

Out[∘]=

$$\mathrm{QU}[x]\left(e^{-\alpha 2}\,\xi 1+\xi 2-\mathrm{fx0}'[\lambda]\right)+\mathrm{QU}[y]\left(e^{\alpha 1+\alpha 2}\,\eta 1+e^{\alpha 2}\,\eta 2-e^{\alpha 1+\alpha 2}\,\mathrm{fy0}'[\lambda]\right)+\mathrm{QU}[\,]\left(-e^{\alpha 1}\,\eta 1\,\lambda\,\xi 1+e^{\alpha 1}\,T\,\eta 1\,\lambda\,\xi 1+\eta 2\,\lambda\,\xi 1-T\,\eta 2\,\lambda\,\xi 1-e^{\alpha 1+\alpha 2}\,\eta 1\,\lambda\,\xi 2+e^{\alpha 1+\alpha 2}\,T\,\eta 1\,\lambda\,\xi 2-e^{\alpha 2}\,\eta 2\,\lambda\,\xi 2+e^{\alpha 2}\,T\,\eta 2\,\lambda\,\xi 2-t\,\mathrm{ft0}'[\lambda]+e^{\alpha 1+\alpha 2}\,\mathrm{fx0}[\lambda]\,\mathrm{fy0}'[\lambda]-e^{\alpha 1+\alpha 2}\,T\,\mathrm{fx0}[\lambda]\,\mathrm{fy0}'[\lambda]\right)$$

In[∘]:= `{sol1} = DSolve[Coefficient[diff /. ϵ → 0, x_QU] == 0 ∧ fx0[0] == 0, fx0, λ]`

Out[∘]=

$$\left\{\left\{\mathrm{fx0}\to\mathrm{Function}\left[\{\lambda\},\ e^{-\alpha 2}\,\lambda\left(\xi 1+e^{\alpha 2}\,\xi 2\right)\right]\right\}\right\}$$

In[∘]:= `CE[diff /. sol1 /. ϵ → 0]`

Out[∘]=

$$\mathrm{QU}[y]\left(e^{\alpha 1+\alpha 2}\,\eta 1+e^{\alpha 2}\,\eta 2-e^{\alpha 1+\alpha 2}\,\mathrm{fy0}'[\lambda]\right)+\mathrm{QU}[\,]\left(-e^{\alpha 1}\,\eta 1\,\lambda\,\xi 1+e^{\alpha 1}\,T\,\eta 1\,\lambda\,\xi 1+\eta 2\,\lambda\,\xi 1-T\,\eta 2\,\lambda\,\xi 1-e^{\alpha 1+\alpha 2}\,\eta 1\,\lambda\,\xi 2+e^{\alpha 1+\alpha 2}\,T\,\eta 1\,\lambda\,\xi 2-e^{\alpha 2}\,\eta 2\,\lambda\,\xi 2+e^{\alpha 2}\,T\,\eta 2\,\lambda\,\xi 2-t\,\mathrm{ft0}'[\lambda]+e^{\alpha 1}\,\lambda\,\xi 1\,\mathrm{fy0}'[\lambda]-e^{\alpha 1}\,T\,\lambda\,\xi 1\,\mathrm{fy0}'[\lambda]+e^{\alpha 1+\alpha 2}\,\lambda\,\xi 2\,\mathrm{fy0}'[\lambda]-e^{\alpha 1+\alpha 2}\,T\,\lambda\,\xi 2\,\mathrm{fy0}'[\lambda]\right)$$

In[∘]:= `{sol2} = DSolve[Coefficient[CE[diff /. sol1 /. ϵ → 0], y_QU] == 0 ∧ fy0[0] == 0, fy0, λ]`

Out[∘]=

$$\left\{\left\{\mathrm{fy0}\to\mathrm{Function}\left[\{\lambda\},\ e^{-\alpha 1}\left(e^{\alpha 1}\,\eta 1+\eta 2\right)\lambda\right]\right\}\right\}$$

In[∘]:= `CE[diff /. sol1 /. sol2 /. ϵ → 0]`

Out[∘]=

$$\mathrm{QU}[\,]\,(2\,\eta 2\,\lambda\,\xi 1-2\,T\,\eta 2\,\lambda\,\xi 1-t\,\mathrm{ft0}'[\lambda])$$

In[∘]:= `{sol3} = DSolve[Coefficient[CE[diff /. sol1 /. sol2 /. ϵ → 0], QU[]] == 0 ∧ ft0[0] == 0, ft0, λ]`

Out[∘]=

$$\left\{\left\{\mathrm{ft0}\to\mathrm{Function}\left[\{\lambda\},\ -\frac{(-1+T)\,\eta 2\,\lambda^2\,\xi 1}{t}\right]\right\}\right\}$$

In[∘]:= `CE[diff /. sol1 /. sol2 /. sol3] /. ϵ → 0`

Out[∘]=

0

In[•]:=
```
CE[diff /. sol1 /. sol2 /. sol3]
```

Out[•]=

$$\begin{aligned}
&\Big(\mathbb{e}^{2\alpha1}\epsilon\lambda\xi1^2\,\mathrm{fyx1}[\lambda]-\mathbb{e}^{2\alpha1}T\epsilon\lambda\xi1^2\,\mathrm{fyx1}[\lambda]+2\,\mathbb{e}^{2\alpha1+\alpha2}\epsilon\lambda\xi1\,\xi2\,\mathrm{fyx1}[\lambda]-\\
&\quad 2\,\mathbb{e}^{2\alpha1+\alpha2}T\epsilon\lambda\xi1\,\xi2\,\mathrm{fyx1}[\lambda]+\mathbb{e}^{2\alpha1+2\alpha2}\epsilon\lambda\xi2^2\,\mathrm{fyx1}[\lambda]-\mathbb{e}^{2\alpha1+2\alpha2}T\epsilon\lambda\xi2^2\,\mathrm{fyx1}[\lambda]\Big)\,\mathrm{QU}[x,x]+\\
&\Big(\mathbb{e}^{\alpha1}\epsilon\xi1\,\mathrm{fyx1}[\lambda]-\mathbb{e}^{2\alpha1+\alpha2}\epsilon\xi1\,\mathrm{fyx1}[\lambda]+\mathbb{e}^{\alpha1+\alpha2}\epsilon\xi2\,\mathrm{fyx1}[\lambda]-\mathbb{e}^{2\alpha1+2\alpha2}\epsilon\xi2\,\mathrm{fyx1}[\lambda]\Big)\,\mathrm{QU}[y,x,x]+\\
&\Big(\mathbb{e}^{2\alpha1+2\alpha2}\epsilon\eta1\,\mathrm{fyx1}[\lambda]-\mathbb{e}^{3\alpha1+3\alpha2}\epsilon\eta1\,\mathrm{fyx1}[\lambda]+\mathbb{e}^{\alpha1+2\alpha2}\epsilon\eta2\,\mathrm{fyx1}[\lambda]-\mathbb{e}^{2\alpha1+3\alpha2}\epsilon\eta2\,\mathrm{fyx1}[\lambda]\Big)\\
&\ \mathrm{QU}[y,y,x]+\mathrm{QU}[a]\,(4\,T\epsilon\eta2\,\lambda\xi1-\epsilon\,\mathrm{fa1}'[\lambda])+\\
&\mathrm{QU}[y]\,\Big(-\mathbb{e}^{\alpha1+\alpha2}\epsilon\eta1\,\eta2\,\lambda^2\xi1+3\,\mathbb{e}^{\alpha1+\alpha2}T\epsilon\eta1\,\eta2\,\lambda^2\xi1+\frac{1}{2}\mathbb{e}^{\alpha2}\epsilon\eta2^2\lambda^2\xi1-\\
&\quad\frac{3}{2}\mathbb{e}^{\alpha2}T\epsilon\eta2^2\lambda^2\xi1-\mathbb{e}^{\alpha1+\alpha2}\epsilon\eta1\,\mathrm{fa1}[\lambda]-\mathbb{e}^{\alpha2}\epsilon\eta2\,\mathrm{fa1}[\lambda]+\mathbb{e}^{2\alpha1+2\alpha2}\epsilon\eta1\,\mathrm{fyx1}[\lambda]-\\
&\quad\mathbb{e}^{2\alpha1+2\alpha2}T\epsilon\eta1\,\mathrm{fyx1}[\lambda]+\mathbb{e}^{\alpha1+2\alpha2}\epsilon\eta2\,\mathrm{fyx1}[\lambda]-\mathbb{e}^{\alpha1+2\alpha2}T\epsilon\eta2\,\mathrm{fyx1}[\lambda]-\mathbb{e}^{\alpha1+\alpha2}\epsilon\,\mathrm{fy1}'[\lambda]\Big)+\\
&\mathrm{QU}[\,]\,\Big(\frac{1}{2}\mathbb{e}^{\alpha1}\epsilon\eta1\,\eta2\,\lambda^3\xi1^2-2\,\mathbb{e}^{\alpha1}T\epsilon\eta1\,\eta2\,\lambda^3\xi1^2+\frac{3}{2}\mathbb{e}^{\alpha1}T^2\epsilon\eta1\,\eta2\,\lambda^3\xi1^2+\\
&\quad\mathbb{e}^{\alpha1+\alpha2}\epsilon\eta1\,\eta2\,\lambda^3\xi1\,\xi2-4\,\mathbb{e}^{\alpha1+\alpha2}T\epsilon\eta1\,\eta2\,\lambda^3\xi1\,\xi2+3\,\mathbb{e}^{\alpha1+\alpha2}T^2\epsilon\eta1\,\eta2\,\lambda^3\xi1\,\xi2-\\
&\quad\frac{1}{2}\mathbb{e}^{\alpha2}\epsilon\eta2^2\lambda^3\xi1\,\xi2+2\,\mathbb{e}^{\alpha2}T\epsilon\eta2^2\lambda^3\xi1\,\xi2-\frac{3}{2}\mathbb{e}^{\alpha2}T^2\epsilon\eta2^2\lambda^3\xi1\,\xi2+\mathbb{e}^{\alpha1}\epsilon\eta1\,\lambda\xi1\,\mathrm{fa1}[\lambda]-\\
&\quad\mathbb{e}^{\alpha1}T\epsilon\eta1\,\lambda\xi1\,\mathrm{fa1}[\lambda]+\epsilon\eta2\,\lambda\xi1\,\mathrm{fa1}[\lambda]-T\epsilon\eta2\,\lambda\xi1\,\mathrm{fa1}[\lambda]+\mathbb{e}^{\alpha1+\alpha2}\epsilon\eta1\,\lambda\xi2\,\mathrm{fa1}[\lambda]-\\
&\quad\mathbb{e}^{\alpha1+\alpha2}T\epsilon\eta1\,\lambda\xi2\,\mathrm{fa1}[\lambda]+\mathbb{e}^{\alpha2}\epsilon\eta2\,\lambda\xi2\,\mathrm{fa1}[\lambda]-\mathbb{e}^{\alpha2}T\epsilon\eta2\,\lambda\xi2\,\mathrm{fa1}[\lambda]+\mathbb{e}^{\alpha1+\alpha2}\epsilon\eta1\,\mathrm{fx1}[\lambda]-\\
&\quad\mathbb{e}^{\alpha1+\alpha2}T\epsilon\eta1\,\mathrm{fx1}[\lambda]+\mathbb{e}^{\alpha2}\epsilon\eta2\,\mathrm{fx1}[\lambda]-\mathbb{e}^{\alpha2}T\epsilon\eta2\,\mathrm{fx1}[\lambda]-\mathbb{e}^{2\alpha1+\alpha2}\epsilon\eta1\,\lambda\xi1\,\mathrm{fyx1}[\lambda]+\\
&\quad 2\,\mathbb{e}^{2\alpha1+\alpha2}T\epsilon\eta1\,\lambda\xi1\,\mathrm{fyx1}[\lambda]-\mathbb{e}^{2\alpha1+\alpha2}T^2\epsilon\eta1\,\lambda\xi1\,\mathrm{fyx1}[\lambda]-\mathbb{e}^{\alpha1+\alpha2}\epsilon\eta2\,\lambda\xi1\,\mathrm{fyx1}[\lambda]+\\
&\quad 2\,\mathbb{e}^{\alpha1+\alpha2}T\epsilon\eta2\,\lambda\xi1\,\mathrm{fyx1}[\lambda]-\mathbb{e}^{\alpha1+\alpha2}T^2\epsilon\eta2\,\lambda\xi1\,\mathrm{fyx1}[\lambda]-\mathbb{e}^{2\alpha1+2\alpha2}\epsilon\eta1\,\lambda\xi2\,\mathrm{fyx1}[\lambda]+\\
&\quad 2\,\mathbb{e}^{2\alpha1+2\alpha2}T\epsilon\eta1\,\lambda\xi2\,\mathrm{fyx1}[\lambda]-\mathbb{e}^{2\alpha1+2\alpha2}T^2\epsilon\eta1\,\lambda\xi2\,\mathrm{fyx1}[\lambda]-\mathbb{e}^{\alpha1+2\alpha2}\epsilon\eta2\,\lambda\xi2\,\mathrm{fyx1}[\lambda]+\\
&\quad 2\,\mathbb{e}^{\alpha1+2\alpha2}T\epsilon\eta2\,\lambda\xi2\,\mathrm{fyx1}[\lambda]-\mathbb{e}^{\alpha1+2\alpha2}T^2\epsilon\eta2\,\lambda\xi2\,\mathrm{fyx1}[\lambda]-t\epsilon\,\mathrm{ft1}'[\lambda]+\mathbb{e}^{\alpha1}\epsilon\lambda\xi1\,\mathrm{fy1}'[\lambda]-\\
&\quad\mathbb{e}^{\alpha1}T\epsilon\lambda\xi1\,\mathrm{fy1}'[\lambda]+\mathbb{e}^{\alpha1+\alpha2}\epsilon\lambda\xi2\,\mathrm{fy1}'[\lambda]-\mathbb{e}^{\alpha1+\alpha2}T\epsilon\lambda\xi2\,\mathrm{fy1}'[\lambda]\Big)+\mathrm{QU}[y,x]\\
&\ \Big(2\,\epsilon\eta2\,\lambda\xi1-\mathbb{e}^{2\alpha1+\alpha2}\epsilon\eta1\,\lambda\xi1\,\mathrm{fyx1}[\lambda]+2\,\mathbb{e}^{3\alpha1+2\alpha2}\epsilon\eta1\,\lambda\xi1\,\mathrm{fyx1}[\lambda]+\mathbb{e}^{2\alpha1+\alpha2}T\epsilon\eta1\,\lambda\xi1\,\mathrm{fyx1}[\lambda]-\\
&\quad 2\,\mathbb{e}^{3\alpha1+2\alpha2}T\epsilon\eta1\,\lambda\xi1\,\mathrm{fyx1}[\lambda]+\mathbb{e}^{\alpha1+\alpha2}\epsilon\eta2\,\lambda\xi1\,\mathrm{fyx1}[\lambda]-\mathbb{e}^{\alpha1+\alpha2}T\epsilon\eta2\,\lambda\xi1\,\mathrm{fyx1}[\lambda]-\\
&\quad\mathbb{e}^{2\alpha1+2\alpha2}\epsilon\eta1\,\lambda\xi2\,\mathrm{fyx1}[\lambda]+2\,\mathbb{e}^{3\alpha1+3\alpha2}\epsilon\eta1\,\lambda\xi2\,\mathrm{fyx1}[\lambda]+\mathbb{e}^{2\alpha1+2\alpha2}T\epsilon\eta1\,\lambda\xi2\,\mathrm{fyx1}[\lambda]-\\
&\quad 2\,\mathbb{e}^{3\alpha1+3\alpha2}T\epsilon\eta1\,\lambda\xi2\,\mathrm{fyx1}[\lambda]-\mathbb{e}^{\alpha1+2\alpha2}\epsilon\eta2\,\lambda\xi2\,\mathrm{fyx1}[\lambda]+2\,\mathbb{e}^{2\alpha1+3\alpha2}\epsilon\eta2\,\lambda\xi2\,\mathrm{fyx1}[\lambda]+\\
&\quad\mathbb{e}^{\alpha1+2\alpha2}T\epsilon\eta2\,\lambda\xi2\,\mathrm{fyx1}[\lambda]-2\,\mathbb{e}^{2\alpha1+3\alpha2}T\epsilon\eta2\,\lambda\xi2\,\mathrm{fyx1}[\lambda]-\mathbb{e}^{2\alpha1+2\alpha2}\epsilon\,\mathrm{fyx1}'[\lambda]\Big)+\\
&\mathrm{QU}[x]\,\Big(-\frac{1}{2}\mathbb{e}^{-\alpha2}\epsilon\eta2\,\lambda^2\xi1^2+\frac{3}{2}\mathbb{e}^{-\alpha2}T\epsilon\eta2\,\lambda^2\xi1^2-2\,\epsilon\eta2\,\lambda^2\xi1\,\xi2+6\,T\epsilon\eta2\,\lambda^2\xi1\,\xi2+\\
&\quad\mathbb{e}^{\alpha1}\epsilon\xi1\,\mathrm{fyx1}[\lambda]-\mathbb{e}^{\alpha1}T\epsilon\xi1\,\mathrm{fyx1}[\lambda]-\mathbb{e}^{3\alpha1+\alpha2}\epsilon\eta1\,\lambda^2\xi1^2\,\mathrm{fyx1}[\lambda]+\\
&\quad 2\,\mathbb{e}^{3\alpha1+\alpha2}T\epsilon\eta1\,\lambda^2\xi1^2\,\mathrm{fyx1}[\lambda]-\mathbb{e}^{3\alpha1+\alpha2}T^2\epsilon\eta1\,\lambda^2\xi1^2\,\mathrm{fyx1}[\lambda]+\mathbb{e}^{2\alpha1+\alpha2}\epsilon\eta2\,\lambda^2\xi1^2\,\mathrm{fyx1}[\lambda]-\\
&\quad 2\,\mathbb{e}^{2\alpha1+\alpha2}T\epsilon\eta2\,\lambda^2\xi1^2\,\mathrm{fyx1}[\lambda]+\mathbb{e}^{2\alpha1+\alpha2}T^2\epsilon\eta2\,\lambda^2\xi1^2\,\mathrm{fyx1}[\lambda]+\mathbb{e}^{\alpha1+\alpha2}\epsilon\xi2\,\mathrm{fyx1}[\lambda]-\\
&\quad\mathbb{e}^{\alpha1+\alpha2}T\epsilon\xi2\,\mathrm{fyx1}[\lambda]-2\,\mathbb{e}^{3\alpha1+2\alpha2}\epsilon\eta1\,\lambda^2\xi1\,\xi2\,\mathrm{fyx1}[\lambda]+4\,\mathbb{e}^{3\alpha1+2\alpha2}T\epsilon\eta1\,\lambda^2\xi1\,\xi2\,\mathrm{fyx1}[\lambda]-\\
&\quad 2\,\mathbb{e}^{3\alpha1+2\alpha2}T^2\epsilon\eta1\,\lambda^2\xi1\,\xi2\,\mathrm{fyx1}[\lambda]-\mathbb{e}^{3\alpha1+3\alpha2}\epsilon\eta1\,\lambda^2\xi2^2\,\mathrm{fyx1}[\lambda]+\\
&\quad 2\,\mathbb{e}^{3\alpha1+3\alpha2}T\epsilon\eta1\,\lambda^2\xi2^2\,\mathrm{fyx1}[\lambda]-\mathbb{e}^{3\alpha1+3\alpha2}T^2\epsilon\eta1\,\lambda^2\xi2^2\,\mathrm{fyx1}[\lambda]-\mathbb{e}^{2\alpha1+3\alpha2}\epsilon\eta2\,\lambda^2\xi2^2\,\mathrm{fyx1}[\lambda]+\\
&\quad 2\,\mathbb{e}^{2\alpha1+3\alpha2}T\epsilon\eta2\,\lambda^2\xi2^2\,\mathrm{fyx1}[\lambda]-\mathbb{e}^{2\alpha1+3\alpha2}T^2\epsilon\eta2\,\lambda^2\xi2^2\,\mathrm{fyx1}[\lambda]-\mathbb{e}^{-\alpha2}\epsilon\lambda\xi1\,\mathrm{fa1}'[\lambda]-\\
&\quad\epsilon\lambda\xi2\,\mathrm{fa1}'[\lambda]-\epsilon\,\mathrm{fx1}'[\lambda]+\mathbb{e}^{\alpha1}\epsilon\lambda\xi1\,\mathrm{fyx1}'[\lambda]+\mathbb{e}^{2\alpha1+\alpha2}\epsilon\lambda\xi1\,\mathrm{fyx1}'[\lambda]-\\
&\quad\mathbb{e}^{\alpha1}T\epsilon\lambda\xi1\,\mathrm{fyx1}'[\lambda]-\mathbb{e}^{2\alpha1+\alpha2}T\epsilon\lambda\xi1\,\mathrm{fyx1}'[\lambda]+\mathbb{e}^{\alpha1+\alpha2}\epsilon\lambda\xi2\,\mathrm{fyx1}'[\lambda]+\\
&\quad\mathbb{e}^{2\alpha1+2\alpha2}\epsilon\lambda\xi2\,\mathrm{fyx1}'[\lambda]-\mathbb{e}^{\alpha1+\alpha2}T\epsilon\lambda\xi2\,\mathrm{fyx1}'[\lambda]-\mathbb{e}^{2\alpha1+2\alpha2}T\epsilon\lambda\xi2\,\mathrm{fyx1}'[\lambda]\Big)
\end{aligned}$$

In[∘]:=
```
{sol4} =
  DSolve[Coefficient[CE[diff /. sol1 /. sol2 /. sol3], QU[y, x]] == 0 ∧ fyx1[0] == 0, fyx1, λ]
```

Out[∘]=

$$\Big\{\Big\{\text{fyx1}\to\text{Function}\Big[\{\lambda\},\ \Big(2\,e^{-\alpha1-\alpha2-\frac{1}{2}e^{-\alpha1-\alpha2}(-1+T)\lambda^2\left(-e^{\alpha1}\eta1\,\xi1+2e^{2\alpha1+\alpha2}\eta1\,\xi1+\eta2\,\xi1-e^{\alpha1+\alpha2}\eta1\,\xi2+2e^{2\alpha1+2\alpha2}\eta1\,\xi2-e^{\alpha2}\eta2\,\xi2+2e^{\alpha1+2\alpha2}\eta2\,\xi2\right)}\left(-1+e^{\frac{1}{2}e^{-\alpha1-\alpha2}(-1+T)\lambda^2\left(-e^{\alpha1}\eta1\,\xi1+2e^{2\alpha1+\alpha2}\eta1\,\xi1+\eta2\,\xi1-e^{\alpha1+\alpha2}\eta1\,\xi2+2e^{2\alpha1+2\alpha2}\eta1\,\xi2-e^{\alpha2}\eta2\,\xi2+2e^{\alpha1+2\alpha2}\eta2\,\xi2\right)}\right)\eta2\,\xi1\Big)\Big/\left((-1+T)\left(-e^{\alpha1}\eta1\,\xi1+2e^{2\alpha1+\alpha2}\eta1\,\xi1+\eta2\,\xi1-e^{\alpha1+\alpha2}\eta1\,\xi2+2e^{2\alpha1+2\alpha2}\eta1\,\xi2-e^{\alpha2}\eta2\,\xi2+2e^{\alpha1+2\alpha2}\eta2\,\xi2\right)\right)\Big]\Big\}\Big\}$$

In[∘]:=
```
CE[diff /. sol1 /. sol2 /. sol3 /. sol4]
```

Out[∘]=