

```
In[ ]:= PrintProfile[]
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Out[ ]:=
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ProfileRoot is root. Profiled time: 9823.62
(      1)      0.062/      2.219 above sol[1, 3]
(      1)      0.220/     81.218 above sol[1, 4]
(      1)      0.203/     29.406 above sol[2, 3]
(      1)      0.986/  9710.781 above sol[2, 4]
TTR: called 4 times, time in 8836.26/9443.
(      1)      1.062/      1.359 under sol[1, 3]
(      1)     54.440/     63.718 under sol[1, 4]
(      1)     19.684/     23.875 under sol[2, 3]
(      1)  8761.079/  9354.046 under sol[2, 4]
(   1120)      0.217/   398.976 above Ad
(    588)     11.738/     11.738 above CF
( 287643)     88.496/   196.019 above UCF
UB: called 1969 times, time in 352.397/511.085
(   1951)    352.318/   511.006 under Ad
(     18)      0.079/      0.079 under TT
( 455511)     97.043/   158.688 above UCF
CF: called 1014684 times, time in 253.783/253.783
(   6171)     64.105/     64.105 under DSolve
(    588)     11.738/     11.738 under TTR
(    296)      0.906/      0.906 under SG
( 1007629)    177.034/   177.034 under UCF
UCF: called 746824 times, time in 189.181/366.215
(      4)      0.297/      1.266 under diff
( 287643)     88.496/   196.019 under TTR
(      8)      0.359/      0.579 under SG
(   3602)      2.255/      5.930 under Ad
(     56)      0.731/      3.733 under TT
( 455511)     97.043/   158.688 under UB
( 1007629)    177.034/   177.034 above CF
SG: called 4 times, time in 175.683/206.36
(      1)      0.078/      0.125 under sol[1, 3]
(      1)      7.500/      8.094 under sol[1, 4]
(      1)      1.736/      2.188 under sol[2, 3]
(      1)   166.369/   195.953 under sol[2, 4]
(    495)      0.062/     29.192 above Ad
(    296)      0.906/      0.906 above CF
(      8)      0.359/      0.579 above UCF
DSolve: called 4 times, time in 14.098/78.203
(      1)      0.265/      0.313 under sol[1, 3]
(      1)      1.065/      2.312 under sol[1, 4]
(      1)      0.673/      1.203 under sol[2, 3]
(      1)     12.095/     74.375 under sol[2, 4]
(   6171)     64.105/     64.105 above CF
sol[2, 4]: called 1 times, time in 0.986/9710.78
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(      1)      0.986/ 9710.781 under ProfileRoot
(      1)      0.016/   1.141 above diff
(      1)     12.095/   74.375 above DSolve
(      1)      0.218/   0.218 above eqns
(      1)       0/    84.062 above TTL
(      1)    8761.079/ 9354.046 above TTR
(      1)    166.369/  195.953 above SG
eqns: called 4 times, time in 0.342/0.342
(      1)       0/       0 under sol[1, 3]
(      1)     0.062/   0.062 under sol[1, 4]
(      1)     0.062/   0.062 under sol[2, 3]
(      1)     0.218/   0.218 under sol[2, 4]
Ad: called 1651 times, time in 0.31/517.246
(   1120)     0.217/  398.976 under TTR
(    495)     0.062/   29.192 under SG
(     36)     0.031/   89.078 under TT
(   1951)    352.318/  511.006 above UB
(   3602)     2.255/    5.930 above UCF
sol[1, 4]: called 1 times, time in 0.22/81.218
(      1)     0.220/   81.218 under ProfileRoot
(      1)       0/    0.078 above diff
(      1)     1.065/   2.312 above DSolve
(      1)     0.062/   0.062 above eqns
(      1)     0.015/   6.734 above TTL
(      1)    54.440/   63.718 above TTR
(      1)     7.500/    8.094 above SG
sol[2, 3]: called 1 times, time in 0.203/29.406
(      1)     0.203/   29.406 under ProfileRoot
(      1)       0/    0.047 above diff
(      1)     0.673/   1.203 above DSolve
(      1)     0.062/   0.062 above eqns
(      1)       0/    1.828 above TTL
(      1)    19.684/   23.875 above TTR
(      1)     1.736/    2.188 above SG
TT: called 56 times, time in 0.063/509.338
(      4)       0/    92.953 under TTL
(     52)     0.063/  416.385 under TT
(     36)     0.031/   89.078 above Ad
(     52)     0.063/  416.385 above TT
(     18)     0.079/    0.079 above UB
(     56)     0.731/    3.733 above UCF
sol[1, 3]: called 1 times, time in 0.062/2.219
(      1)     0.062/    2.219 under ProfileRoot
(      1)       0/    0.016 above diff
(      1)     0.265/    0.313 above DSolve
(      1)       0/       0 above eqns
(      1)       0/    0.344 above TTL

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(      1)      1.062/      1.359 above TTR
(      1)      0.078/      0.125 above SG
diff: called 4 times, time in 0.016/1.282
(      1)      0/      0.016 under sol[1, 3]
(      1)      0/      0.078 under sol[1, 4]
(      1)      0/      0.047 under sol[2, 3]
(      1)      0.016/      1.141 under sol[2, 4]
(      4)      0.297/      1.266 above UCF
TTL: called 4 times, time in 0.015/92.968
(      1)      0/      0.344 under sol[1, 3]
(      1)      0.015/      6.734 under sol[1, 4]
(      1)      0/      1.828 under sol[2, 3]
(      1)      0/      84.062 under sol[2, 4]
(      4)      0/      92.953 above TT

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```

In[ ]:= Module[{Bas = $PBWGens, P},
  Bas = Echo@Join[Bas, Table[A[RandomInteger[{1, $n - 1}]] [RandomInteger[{-2, 2}]], {10}]];
  DeleteCases[_ -> 0] [Flatten@Table[As[X, Bas], As[Y, Bas],
    P = U1[X] U2[Y];
    {X, Y} -> UCF[(P // m1,2 ->1 // Δ1 ->1,2) - (P // Δ2 ->3,4 // Δ1 ->1,2 // m1,3 ->1 // m2,4 ->2)]
  ]
]
» {x12, a1, A1[-1], A1[2], A1[1], A1[0], A1[-2], A1[2], A1[-2], A1[0], A1[0], A1[2]}

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Out[ ]:=
{}

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```

In[ ]:= $k

```

```

Out[ ]:=
1

```

```

In[ ]:= $k < ∞

```

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Out[ ]:=
True

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In[ ]:= ∞ < ∞

```

```

Out[ ]:=
False

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```

In[ ]:= ∑k=0$k  $\frac{(\hbar \in \gamma n)^k}{k!}$ 

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Out[ ]:=
en γ ∈ ħ

```

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In[ ]:= Block[{$k = 2}, CF[-4 γ2 ε2 ħ2 U0[x12, a1] - 4 γ ε2 ħ2 U0[x12, a1, a1]]]

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Out[ ]:=
-4 γ2 ε2 ħ2 U0[x12, a1] - 4 γ ε2 ħ2 U0[x12, a1, a1]

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In[*]:= Block[{$k = ∞}, Module[{Bas = {x12, x13, x23, a1, a2, A2[0],
    A1[-1], A1[-1], A1[1], A1[0], A2[0], A1[1], A1[2], A2[-1], A1[2]}, P},
    DeleteCases[_ → 0] [Flatten@Table[As[X, Bas], As[Y, Bas], As[Z, Bas],
        P = U1[X] U2[Y] U3[Z];
        {X, Y, Z} → UCF[(P // m1,2→2 // m2,3→0) - (P // m2,3→2 // m1,2→0)]
    ]
]]
Out[*]=
{ }

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In[*]:= Block[{$k = 1}, Module[{Bas = {x12, x13, x23, a1, a2, A2[0],
    A1[-1], A1[-1], A1[1], A1[0], A2[0], A1[1], A1[2], A2[-1], A1[2]}, P},
    DeleteCases[_ → 0] [Flatten@Table[As[X, Bas], As[Y, Bas], As[Z, Bas],
        P = U1[X] U2[Y] U3[Z];
        {X, Y, Z} → UCF[(P // m1,2→2 // m2,3→0) - (P // m2,3→2 // m1,2→0)]
    ]
]]
Out[*]=

```

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{ {a1, A2[-1], A2[0]} → -2 ∈ ℏ U0[a1, a2] + 2 ∈ ℏ U0[a2, a1],
  {a1, A2[-1], A1[-1]} → -2 ∈ ℏ U0[a1, a2] + 2 ∈ ℏ U0[a2, a1],
  {a1, A2[-1], A1[1]} → -2 ∈ ℏ U0[a1, a2] + 2 ∈ ℏ U0[a2, a1],
  {a1, A2[-1], A1[0]} → -2 ∈ ℏ U0[a1, a2] + 2 ∈ ℏ U0[a2, a1],
  {a1, A2[-1], A2[0]} → -2 ∈ ℏ U0[a1, a2] + 2 ∈ ℏ U0[a2, a1],
  {a1, A2[-1], A1[1]} → -2 ∈ ℏ U0[a1, a2] + 2 ∈ ℏ U0[a2, a1],
  {a1, A2[-1], A1[2]} → -2 ∈ ℏ U0[a1, a2] + 2 ∈ ℏ U0[a2, a1],
  {a1, A2[-1], A2[-1]} → -2 ∈ ℏ U0[a1, a2] + 2 ∈ ℏ U0[a2, a1],
  {a1, A2[-1], A1[2]} → -2 ∈ ℏ U0[a1, a2] + 2 ∈ ℏ U0[a2, a1],
  {a2, a1, A2[-1]} → 2 ∈ ℏ U0[a1, a2, a2] - 2 ∈ ℏ U0[a2, a2, a1] }

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In[*]:= $EPBWRule

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Out[*]=
{x12 → 1, x12i$ → {i$, 1}, x13 → 2, x13i$ → {i$, 2},
 x23 → 3, x23i$ → {i$, 3}, a1 → 4, a1i$ → {i$, 4}, A1[_] → 5,
 A1[_]i$ → {i$, 5}, a2 → 6, a2i$ → {i$, 6}, A2[_] → 7, A2[_]i$ → {i$, 7}}

```

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In[*]:= $EPBWGens

```

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Out[*]=
{x12, x13, x23, a1, A1[_], a2, A2[_]}

```

$$\begin{aligned}
In[] := & -2 e^{-\frac{1}{2} \gamma (2 \alpha_{1i} - \alpha_{2i})} \gamma \lambda^2 \hbar \xi_{12j} \xi_{14i} + \\
& \frac{2}{3} e^{-\frac{1}{2} \gamma (2 \alpha_{1i} - \alpha_{2i})} \gamma \lambda^3 \hbar \xi_{12i} \xi_{12j} \xi_{24i} - \frac{2}{3} e^{\frac{1}{2} \gamma (-2 \alpha_{1i} + \alpha_{2i})} \gamma \lambda^3 \hbar \xi_{12i} \xi_{12j} \xi_{24i} + \\
& \frac{2}{3} e^{-\gamma (2 \alpha_{1i} - \alpha_{2i})} \gamma \lambda^3 \hbar \xi_{12j}^2 \xi_{24i} - \frac{2}{3} e^{\gamma (-2 \alpha_{1i} + \alpha_{2i})} \gamma \lambda^3 \hbar \xi_{12j}^2 \xi_{24i} - \\
& \frac{2}{3} e^{\frac{1}{2} \gamma (\alpha_{2i} - 2 \alpha_{3i})} \gamma \lambda^3 \hbar \xi_{12i} \xi_{13i} \xi_{34j} + \frac{2}{3} e^{-\frac{1}{2} \gamma (-\alpha_{2i} + 2 \alpha_{3i})} \gamma \lambda^3 \hbar \xi_{12i} \xi_{13i} \xi_{34j} + \\
& \frac{2}{3} e^{\gamma (-\alpha_{1i} + \alpha_{2i} - \alpha_{3i})} \gamma \lambda^3 \hbar \xi_{12j} \xi_{13i} \xi_{34j} - \frac{2}{3} e^{-\gamma (\alpha_{1i} - \alpha_{2i} + \alpha_{3i})} \gamma \lambda^3 \hbar \xi_{12j} \xi_{13i} \xi_{34j} - \\
& \frac{2}{3} e^{\frac{1}{2} \gamma (-3 \alpha_{1i} + \alpha_{2i} - \alpha_{3i})} \gamma \lambda^3 \hbar \xi_{12j} \xi_{13j} \xi_{34j} + \frac{2}{3} e^{-\frac{1}{2} \gamma (3 \alpha_{1i} - \alpha_{2i} + \alpha_{3i})} \gamma \lambda^3 \hbar \xi_{12j} \xi_{13j} \xi_{34j}
\end{aligned}$$

Out[] =

$$\begin{aligned}
& -2 e^{-\frac{1}{2} \gamma (2 \alpha_{1i} - \alpha_{2i})} \gamma \lambda^2 \hbar \xi_{12j} \xi_{14i} + \\
& \frac{2}{3} e^{-\frac{1}{2} \gamma (2 \alpha_{1i} - \alpha_{2i})} \gamma \lambda^3 \hbar \xi_{12i} \xi_{12j} \xi_{24i} - \frac{2}{3} e^{\frac{1}{2} \gamma (-2 \alpha_{1i} + \alpha_{2i})} \gamma \lambda^3 \hbar \xi_{12i} \xi_{12j} \xi_{24i} + \\
& \frac{2}{3} e^{-\gamma (2 \alpha_{1i} - \alpha_{2i})} \gamma \lambda^3 \hbar \xi_{12j}^2 \xi_{24i} - \frac{2}{3} e^{\gamma (-2 \alpha_{1i} + \alpha_{2i})} \gamma \lambda^3 \hbar \xi_{12j}^2 \xi_{24i} - \\
& \frac{2}{3} e^{\frac{1}{2} \gamma (\alpha_{2i} - 2 \alpha_{3i})} \gamma \lambda^3 \hbar \xi_{12i} \xi_{13i} \xi_{34j} + \frac{2}{3} e^{-\frac{1}{2} \gamma (-\alpha_{2i} + 2 \alpha_{3i})} \gamma \lambda^3 \hbar \xi_{12i} \xi_{13i} \xi_{34j} + \\
& \frac{2}{3} e^{\gamma (-\alpha_{1i} + \alpha_{2i} - \alpha_{3i})} \gamma \lambda^3 \hbar \xi_{12j} \xi_{13i} \xi_{34j} - \frac{2}{3} e^{-\gamma (\alpha_{1i} - \alpha_{2i} + \alpha_{3i})} \gamma \lambda^3 \hbar \xi_{12j} \xi_{13i} \xi_{34j} - \\
& \frac{2}{3} e^{\frac{1}{2} \gamma (-3 \alpha_{1i} + \alpha_{2i} - \alpha_{3i})} \gamma \lambda^3 \hbar \xi_{12j} \xi_{13j} \xi_{34j} + \frac{2}{3} e^{-\frac{1}{2} \gamma (3 \alpha_{1i} - \alpha_{2i} + \alpha_{3i})} \gamma \lambda^3 \hbar \xi_{12j} \xi_{13j} \xi_{34j}
\end{aligned}$$