

```

In[*]:= $n = 9;
Do[As[i, 1, $n - 1],
  Δj, k[a[i]] = Uj[a[i]] Uk[] + Uj[] Uk[a[i]];
  Δj, k[A[i][n_]] = Uj[A[i][n]] Uk[A[i][n]];
  Δj, k[x[i, i + 1]] = Uj[x[i, i + 1]] Uk[A[i][1]] + Uj[A[i][-1]] Uk[x[i, i + 1]]
];
Do[As[i, 1, $n - 2], As[n, i + 2, $n],
  Δj, k[x[i, n]] = mk, k1→k[mj, j1→j[
    q Δj, k[x[i, n - 1]] Δj1, k1[x[n - 1, n]] - q-1 Δj, k[x[n - 1, n]] Δj1, k1[x[i, n - 1]]
  ]]
]

In[*]:= Column@Module[{Bas = $PBWGens, P},
  Table[As[P, Ui /@ Bas], P → Δi→j, k[P]]
]

Out[*]=
Ui[x12] → Uj[A1[-1]] Uk[x12] + Uj[x12] Uk[A1[1]]
Ui[x13] → Uj[A1[-1], A2[-1]] Uk[x13] +
  (-1/q2 + q2) Uj[x12, A2[-1]] Uk[x23, A1[1]] + Uj[x13] Uk[A1[1], A2[1]]
Ui[x14] →
  Uj[A1[-1], A2[-1], A3[-1]] Uk[x14] + (-1/q2 + q2) Uj[x12, A2[-1], A3[-1]] Uk[x24, A1[1]] +
  (-1/q2 + q2) Uj[x13, A3[-1]] Uk[x34, A1[1], A2[1]] + Uj[x14] Uk[A1[1], A2[1], A3[1]]
Ui[x15] → Uj[A1[-1], A2[-1], A3[-1], A4[-1]] Uk[x15] +
  (-1/q2 + q2) Uj[x12, A2[-1], A3[-1], A4[-1]] Uk[x25, A1[1]] +
  (-1/q2 + q2) Uj[x13, A3[-1], A4[-1]] Uk[x35, A1[1], A2[1]] +
  (-1/q2 + q2) Uj[x14, A4[-1]] Uk[x45, A1[1], A2[1], A3[1]] +
  Uj[x15] Uk[A1[1], A2[1], A3[1], A4[1]]
Ui[x16] → Uj[A1[-1], A2[-1], A3[-1], A4[-1], A5[-1]] Uk[x16] +
  (-1/q2 + q2) Uj[x12, A2[-1], A3[-1], A4[-1], A5[-1]] Uk[x26, A1[1]] +
  (-1/q2 + q2) Uj[x13, A3[-1], A4[-1], A5[-1]] Uk[x36, A1[1], A2[1]] +
  (-1/q2 + q2) Uj[x14, A4[-1], A5[-1]] Uk[x46, A1[1], A2[1], A3[1]] +
  (-1/q2 + q2) Uj[x15, A5[-1]] Uk[x56, A1[1], A2[1], A3[1], A4[1]] +
  Uj[x16] Uk[A1[1], A2[1], A3[1], A4[1], A5[1]]

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$$\begin{aligned}
U_i[x17] &\rightarrow U_j[A1[-1], A2[-1], A3[-1], A4[-1], A5[-1], A6[-1]] U_k[x17] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x12, A2[-1], A3[-1], A4[-1], A5[-1], A6[-1]] U_k[x27, A1[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x13, A3[-1], A4[-1], A5[-1], A6[-1]] U_k[x37, A1[1], A2[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x14, A4[-1], A5[-1], A6[-1]] U_k[x47, A1[1], A2[1], A3[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x15, A5[-1], A6[-1]] U_k[x57, A1[1], A2[1], A3[1], A4[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x16, A6[-1]] U_k[x67, A1[1], A2[1], A3[1], A4[1], A5[1]] + \\
&U_j[x17] U_k[A1[1], A2[1], A3[1], A4[1], A5[1], A6[1]] \\
U_i[x18] &\rightarrow U_j[A1[-1], A2[-1], A3[-1], A4[-1], A5[-1], A6[-1], A7[-1]] U_k[x18] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x12, A2[-1], A3[-1], A4[-1], A5[-1], A6[-1], A7[-1]] U_k[x28, A1[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x13, A3[-1], A4[-1], A5[-1], A6[-1], A7[-1]] U_k[x38, A1[1], A2[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x14, A4[-1], A5[-1], A6[-1], A7[-1]] U_k[x48, A1[1], A2[1], A3[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x15, A5[-1], A6[-1], A7[-1]] U_k[x58, A1[1], A2[1], A3[1], A4[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x16, A6[-1], A7[-1]] U_k[x68, A1[1], A2[1], A3[1], A4[1], A5[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x17, A7[-1]] U_k[x78, A1[1], A2[1], A3[1], A4[1], A5[1], A6[1]] + \\
&U_j[x18] U_k[A1[1], A2[1], A3[1], A4[1], A5[1], A6[1], A7[1]] \\
U_i[x19] &\rightarrow U_j[A1[-1], A2[-1], A3[-1], A4[-1], A5[-1], A6[-1], A7[-1], A8[-1]] U_k[x19] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x12, A2[-1], A3[-1], A4[-1], A5[-1], A6[-1], A7[-1], A8[-1]] U_k[x29, A1[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x13, A3[-1], A4[-1], A5[-1], A6[-1], A7[-1], A8[-1]] U_k[x39, A1[1], A2[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x14, A4[-1], A5[-1], A6[-1], A7[-1], A8[-1]] U_k[x49, A1[1], A2[1], A3[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x15, A5[-1], A6[-1], A7[-1], A8[-1]] U_k[x59, A1[1], A2[1], A3[1], A4[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x16, A6[-1], A7[-1], A8[-1]] U_k[x69, A1[1], A2[1], A3[1], A4[1], A5[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x17, A7[-1], A8[-1]] U_k[x79, A1[1], A2[1], A3[1], A4[1], A5[1], A6[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x18, A8[-1]] U_k[x89, A1[1], A2[1], A3[1], A4[1], A5[1], A6[1], A7[1]] + \\
&U_j[x19] U_k[A1[1], A2[1], A3[1], A4[1], A5[1], A6[1], A7[1], A8[1]] \\
U_i[x23] &\rightarrow U_j[A2[-1]] U_k[x23] + U_j[x23] U_k[A2[1]] \\
U_i[x24] &\rightarrow U_j[A2[-1], A3[-1]] U_k[x24] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x23, A3[-1]] U_k[x34, A2[1]] + U_j[x24] U_k[A2[1], A3[1]] \\
U_i[x25] &\rightarrow \\
&U_j[A2[-1], A3[-1], A4[-1]] U_k[x25] + \left(-\frac{1}{q^2} + q^2\right) U_j[x23, A3[-1], A4[-1]] U_k[x35, A2[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x24, A4[-1]] U_k[x45, A2[1], A3[1]] + U_j[x25] U_k[A2[1], A3[1], A4[1]]
\end{aligned}$$

$$\begin{aligned}
U_i[x_{26}] &\rightarrow U_j[A_2[-1], A_3[-1], A_4[-1], A_5[-1]] U_k[x_{26}] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x_{23}, A_3[-1], A_4[-1], A_5[-1]] U_k[x_{36}, A_2[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x_{24}, A_4[-1], A_5[-1]] U_k[x_{46}, A_2[1], A_3[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x_{25}, A_5[-1]] U_k[x_{56}, A_2[1], A_3[1], A_4[1]] + \\
&U_j[x_{26}] U_k[A_2[1], A_3[1], A_4[1], A_5[1]] \\
U_i[x_{27}] &\rightarrow U_j[A_2[-1], A_3[-1], A_4[-1], A_5[-1], A_6[-1]] U_k[x_{27}] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x_{23}, A_3[-1], A_4[-1], A_5[-1], A_6[-1]] U_k[x_{37}, A_2[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x_{24}, A_4[-1], A_5[-1], A_6[-1]] U_k[x_{47}, A_2[1], A_3[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x_{25}, A_5[-1], A_6[-1]] U_k[x_{57}, A_2[1], A_3[1], A_4[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x_{26}, A_6[-1]] U_k[x_{67}, A_2[1], A_3[1], A_4[1], A_5[1]] + \\
&U_j[x_{27}] U_k[A_2[1], A_3[1], A_4[1], A_5[1], A_6[1]] \\
U_i[x_{28}] &\rightarrow U_j[A_2[-1], A_3[-1], A_4[-1], A_5[-1], A_6[-1], A_7[-1]] U_k[x_{28}] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x_{23}, A_3[-1], A_4[-1], A_5[-1], A_6[-1], A_7[-1]] U_k[x_{38}, A_2[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x_{24}, A_4[-1], A_5[-1], A_6[-1], A_7[-1]] U_k[x_{48}, A_2[1], A_3[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x_{25}, A_5[-1], A_6[-1], A_7[-1]] U_k[x_{58}, A_2[1], A_3[1], A_4[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x_{26}, A_6[-1], A_7[-1]] U_k[x_{68}, A_2[1], A_3[1], A_4[1], A_5[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x_{27}, A_7[-1]] U_k[x_{78}, A_2[1], A_3[1], A_4[1], A_5[1], A_6[1]] + \\
&U_j[x_{28}] U_k[A_2[1], A_3[1], A_4[1], A_5[1], A_6[1], A_7[1]] \\
U_i[x_{29}] &\rightarrow U_j[A_2[-1], A_3[-1], A_4[-1], A_5[-1], A_6[-1], A_7[-1], A_8[-1]] U_k[x_{29}] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x_{23}, A_3[-1], A_4[-1], A_5[-1], A_6[-1], A_7[-1], A_8[-1]] U_k[x_{39}, A_2[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x_{24}, A_4[-1], A_5[-1], A_6[-1], A_7[-1], A_8[-1]] U_k[x_{49}, A_2[1], A_3[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x_{25}, A_5[-1], A_6[-1], A_7[-1], A_8[-1]] U_k[x_{59}, A_2[1], A_3[1], A_4[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x_{26}, A_6[-1], A_7[-1], A_8[-1]] U_k[x_{69}, A_2[1], A_3[1], A_4[1], A_5[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x_{27}, A_7[-1], A_8[-1]] U_k[x_{79}, A_2[1], A_3[1], A_4[1], A_5[1], A_6[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x_{28}, A_8[-1]] U_k[x_{89}, A_2[1], A_3[1], A_4[1], A_5[1], A_6[1], A_7[1]] + \\
&U_j[x_{29}] U_k[A_2[1], A_3[1], A_4[1], A_5[1], A_6[1], A_7[1], A_8[1]] \\
U_i[x_{34}] &\rightarrow U_j[A_3[-1]] U_k[x_{34}] + U_j[x_{34}] U_k[A_3[1]] \\
U_i[x_{35}] &\rightarrow U_j[A_3[-1], A_4[-1]] U_k[x_{35}] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x_{34}, A_4[-1]] U_k[x_{45}, A_3[1]] + U_j[x_{35}] U_k[A_3[1], A_4[1]] \\
U_i[x_{36}] &\rightarrow \\
&U_j[A_3[-1], A_4[-1], A_5[-1]] U_k[x_{36}] + \left(-\frac{1}{q^2} + q^2\right) U_j[x_{34}, A_4[-1], A_5[-1]] U_k[x_{46}, A_3[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x_{35}, A_5[-1]] U_k[x_{56}, A_3[1], A_4[1]] + U_j[x_{36}] U_k[A_3[1], A_4[1], A_5[1]]
\end{aligned}$$

$$\begin{aligned}
U_i[x37] &\rightarrow U_j[A3[-1], A4[-1], A5[-1], A6[-1]] U_k[x37] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x34, A4[-1], A5[-1], A6[-1]] U_k[x47, A3[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x35, A5[-1], A6[-1]] U_k[x57, A3[1], A4[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x36, A6[-1]] U_k[x67, A3[1], A4[1], A5[1]] + \\
&U_j[x37] U_k[A3[1], A4[1], A5[1], A6[1]] \\
U_i[x38] &\rightarrow U_j[A3[-1], A4[-1], A5[-1], A6[-1], A7[-1]] U_k[x38] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x34, A4[-1], A5[-1], A6[-1], A7[-1]] U_k[x48, A3[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x35, A5[-1], A6[-1], A7[-1]] U_k[x58, A3[1], A4[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x36, A6[-1], A7[-1]] U_k[x68, A3[1], A4[1], A5[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x37, A7[-1]] U_k[x78, A3[1], A4[1], A5[1], A6[1]] + \\
&U_j[x38] U_k[A3[1], A4[1], A5[1], A6[1], A7[1]] \\
U_i[x39] &\rightarrow U_j[A3[-1], A4[-1], A5[-1], A6[-1], A7[-1], A8[-1]] U_k[x39] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x34, A4[-1], A5[-1], A6[-1], A7[-1], A8[-1]] U_k[x49, A3[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x35, A5[-1], A6[-1], A7[-1], A8[-1]] U_k[x59, A3[1], A4[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x36, A6[-1], A7[-1], A8[-1]] U_k[x69, A3[1], A4[1], A5[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x37, A7[-1], A8[-1]] U_k[x79, A3[1], A4[1], A5[1], A6[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x38, A8[-1]] U_k[x89, A3[1], A4[1], A5[1], A6[1], A7[1]] + \\
&U_j[x39] U_k[A3[1], A4[1], A5[1], A6[1], A7[1], A8[1]] \\
U_i[x45] &\rightarrow U_j[A4[-1]] U_k[x45] + U_j[x45] U_k[A4[1]] \\
U_i[x46] &\rightarrow U_j[A4[-1], A5[-1]] U_k[x46] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x45, A5[-1]] U_k[x56, A4[1]] + U_j[x46] U_k[A4[1], A5[1]] \\
U_i[x47] &\rightarrow \\
&U_j[A4[-1], A5[-1], A6[-1]] U_k[x47] + \left(-\frac{1}{q^2} + q^2\right) U_j[x45, A5[-1], A6[-1]] U_k[x57, A4[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x46, A6[-1]] U_k[x67, A4[1], A5[1]] + U_j[x47] U_k[A4[1], A5[1], A6[1]] \\
U_i[x48] &\rightarrow U_j[A4[-1], A5[-1], A6[-1], A7[-1]] U_k[x48] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x45, A5[-1], A6[-1], A7[-1]] U_k[x58, A4[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x46, A6[-1], A7[-1]] U_k[x68, A4[1], A5[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x47, A7[-1]] U_k[x78, A4[1], A5[1], A6[1]] + \\
&U_j[x48] U_k[A4[1], A5[1], A6[1], A7[1]]
\end{aligned}$$

$$\begin{aligned}
U_i[x49] &\rightarrow U_j[A4[-1], A5[-1], A6[-1], A7[-1], A8[-1]] U_k[x49] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x45, A5[-1], A6[-1], A7[-1], A8[-1]] U_k[x59, A4[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x46, A6[-1], A7[-1], A8[-1]] U_k[x69, A4[1], A5[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x47, A7[-1], A8[-1]] U_k[x79, A4[1], A5[1], A6[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x48, A8[-1]] U_k[x89, A4[1], A5[1], A6[1], A7[1]] + \\
&U_j[x49] U_k[A4[1], A5[1], A6[1], A7[1], A8[1]] \\
U_i[x56] &\rightarrow U_j[A5[-1]] U_k[x56] + U_j[x56] U_k[A5[1]] \\
U_i[x57] &\rightarrow U_j[A5[-1], A6[-1]] U_k[x57] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x56, A6[-1]] U_k[x67, A5[1]] + U_j[x57] U_k[A5[1], A6[1]] \\
U_i[x58] &\rightarrow \\
&U_j[A5[-1], A6[-1], A7[-1]] U_k[x58] + \left(-\frac{1}{q^2} + q^2\right) U_j[x56, A6[-1], A7[-1]] U_k[x68, A5[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x57, A7[-1]] U_k[x78, A5[1], A6[1]] + U_j[x58] U_k[A5[1], A6[1], A7[1]] \\
U_i[x59] &\rightarrow U_j[A5[-1], A6[-1], A7[-1], A8[-1]] U_k[x59] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x56, A6[-1], A7[-1], A8[-1]] U_k[x69, A5[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x57, A7[-1], A8[-1]] U_k[x79, A5[1], A6[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x58, A8[-1]] U_k[x89, A5[1], A6[1], A7[1]] + \\
&U_j[x59] U_k[A5[1], A6[1], A7[1], A8[1]] \\
U_i[x67] &\rightarrow U_j[A6[-1]] U_k[x67] + U_j[x67] U_k[A6[1]] \\
U_i[x68] &\rightarrow U_j[A6[-1], A7[-1]] U_k[x68] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x67, A7[-1]] U_k[x78, A6[1]] + U_j[x68] U_k[A6[1], A7[1]] \\
U_i[x69] &\rightarrow \\
&U_j[A6[-1], A7[-1], A8[-1]] U_k[x69] + \left(-\frac{1}{q^2} + q^2\right) U_j[x67, A7[-1], A8[-1]] U_k[x79, A6[1]] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x68, A8[-1]] U_k[x89, A6[1], A7[1]] + U_j[x69] U_k[A6[1], A7[1], A8[1]] \\
U_i[x78] &\rightarrow U_j[A7[-1]] U_k[x78] + U_j[x78] U_k[A7[1]] \\
U_i[x79] &\rightarrow U_j[A7[-1], A8[-1]] U_k[x79] + \\
&\left(-\frac{1}{q^2} + q^2\right) U_j[x78, A8[-1]] U_k[x89, A7[1]] + U_j[x79] U_k[A7[1], A8[1]] \\
U_i[x89] &\rightarrow U_j[A8[-1]] U_k[x89] + U_j[x89] U_k[A8[1]] \\
U_i[a1] &\rightarrow U_j[a1] U_k[] + U_j[] U_k[a1] \\
U_i[a2] &\rightarrow U_j[a2] U_k[] + U_j[] U_k[a2] \\
U_i[a3] &\rightarrow U_j[a3] U_k[] + U_j[] U_k[a3] \\
U_i[a4] &\rightarrow U_j[a4] U_k[] + U_j[] U_k[a4] \\
U_i[a5] &\rightarrow U_j[a5] U_k[] + U_j[] U_k[a5] \\
U_i[a6] &\rightarrow U_j[a6] U_k[] + U_j[] U_k[a6] \\
U_i[a7] &\rightarrow U_j[a7] U_k[] + U_j[] U_k[a7] \\
U_i[a8] &\rightarrow U_j[a8] U_k[] + U_j[] U_k[a8]
\end{aligned}$$